UNIVERSALITY IN RANDOM STRUCTURES: INTERFACES, MATRICES SANDPILES

Titles and Abstracts for Week 4, 04-08 February, 2019
Focus Area: Sandpiles and Laplacian Growth Models

Mini-Course by **Lionel Levine**, Cornell University

Title: Abelian networks and sandpile models
Abstract: In this mini-course, we explore particle systems with an abelian property: the order of certain interactions does not matter. Deepak Dhar [3] observed that many dynamical questions (what does this particle system do?) have a computational side (what can this network of automata compute?). The computational counterpart of the abelian property is a least action principle: the particles conspire to solve a certain optimization problem, by reaching stability in the most efficient possible way.

We are interested in the phase transition between activity and fixation, and in universal properties of the threshold state that separates the two phases. The dynamical question will this system fixate? corresponds to the computational question will this program halt? Alan Turing proved in 1936 that the latter question is undecidable in general. Hence, we should expect these systems to be hard, and there will be no completely satisfactory general theory; but questions in the neighborhood of an undecidable question are where the most fruitful mathematics lies!

References

[1] Bond, Levine, https://arxiv.org/abs/1309.3445
[2] Cairns, https://arxiv.org/abs/1508.00161
[3] Dhar, https://doi.org/10.1016/j.physa.2006.04.004
[4] Holroyd et al., https://arxiv.org/abs/0801.3306
[5] Jarai, https://arxiv.org/abs/1401.0354
[6] Levine, https://arxiv.org/abs/1402.3283
[7] Levine, Peres, https://arxiv.org/abs/1611.00411
[8] Rolla,Sidoravicius,Zindy https://arxiv.org/abs/1707.06081

Titles and Abstracts for Other Speakers

(1) **Alessandra Cipriani**, TU Delft.
Title: The discrete Gaussian free field on a compact manifold.
Abstract: In this talk we aim at defining the discrete Gaussian free field (DGFF) on a compact Riemannian manifold. Since there is no canonical grid approximation of a manifold, we construct a suitable random graph that replaces the square lattice $\mathbb{Z}^d$ in Euclidean space, and prove that the scaling limit of the DGFF is given by the manifold continuum Gaussian free field. Joint work with Bart van Ginkel (TU Delft).

(2) **Rahul Dandekar**, IMSc.
Title: Recurrence-Transience transition and Tracy-Widom growth in the Rotor-router model.
Abstract: We describe the growing patterns formed in the rotor-router model, starting from noisy initial conditions. By the detailed study of two cases, we show that: (a) the boundary of the pattern for a certain class of initial conditions displays KPZ fluctuations with a Tracy-Widom distribution, (b) by changing the amount of randomness, one can

induce a transition in which the rotor-router path changes from recurrent to transient. We show that this transition falls in the 3D Anisotropic Directed Percolation universality class.

(3) **Deepak Dhar**, IISER Pune.
Title: The active-absorbing state transition in the fixed energy sand-pile models
Abstract: The active absorbing state transition in the fixed energy sand-pile models has been a topic of much interest. In these model, the steady state shows a transition from the inactive to active phase, as the density of particles is increased. The relationship of this transition to the directed percolation universality class, or the conserved directed percolation class has been a topic of some controversy. I will discuss our recent work on the directed Oslo model, which shows different critical behaviors for different parameter ranges. I will also discuss the equivalence of this model to a generalization of the Edwards -Wilkinson model of interface growth with additive but non-linear noise.

(4) **Rajat Hazra**,ISI Kolkata.
Title: A PDE approach to scaling limit of random interface models on $\mathbb{Z}^d$.
Abstract: In this talk we shall discuss the scaling limit of Gaussian interface models, where the covariance structure comes from a discrete partial differential equation. In some models of random interfaces, the explicit description of the covariance is either lacking or sometimes difficult to derive. We suggest an approach through the approximation of solutions of continuum PDEs through discrete solutions by finite difference methods. We discuss the implications of such approximation results in the cases of the discrete Gaussian free field, the Membrane model, and the mixed model containing both gradient and Laplacian interaction. We derive the weak convergence in appropriate spaces, depending on the dimension of the lattice. This talk is based on joint and on-going works with Biltu Dan (ISI, Kolkata) and Alessandra Cipriani (TU, Delft).

(5) **Christopher Hoffman**, University of Washington.
Title: Frogs on Trees.
Abstract: The frog model is a branching random walk on a graph in which particles branch only at unvisited sites. I will discuss a collection of results that are joint with Tobias Johnson and Matthew Junge. These papers all discuss the frog model on regular trees. On infinite trees we consider the question of transience and recurrence. On finite depth trees we examine the time it takes before every vertex on the graph is visited.

(6) **Pradeep Kumar Mohanty**, SINP, Kolkata.
Title: Universality in sandpile models
Abstract: The self-organized critical (SOC) state of sndpile models can be understood as the critical state of an absorbing state phase transition (APT) occurring in fixed energy sandpiles. It is still debated, whether the most generic sandpile models belongs to the universality class of Directed percolation -the most robust universality class of APT. I will discuss this issue focusing on recent simulation results of Manna and Oslo models.

(7) **Kirone Mallick**, IPHT.
Title: Continuous-time Quantum Walks
Abstract: Quantum analogs of classical random walks have been defined in quantum information theory as a useful concept to implement original algorithms. Due to interference effects, statistical properties of quantum walks can drastically differ from their classical counterparts, leading to much faster computations. In this talk, we shall discuss various statistical properties of continuous time quantum walks on a lattice, such as: survival properties of quantum particles in the presence of traps (i.e. a quantum generalization of the Donsker-Varadhan stretched exponential law), the growth of a quantum population in the presence of a source, quantum return probabilities and Loschmidt echoes.

(8) **Punyabrata Pradahn**, S. N. Bose Institute.
Title: Hydrodynamics of conserved stochastic sandpiles.
Abstract: We shall discuss conserved stochastic sandpiles (CSSs), which exhibit an active-absorbing phase transition upon tuning density. We demonstrate that a broad class of CSSs possesses a remarkable hydrodynamic structure: There is an Einstein relation, which connects bulk-diffusion coefficient, conductivity, and mass fluctuation. Consequently, density large-deviations are governed by an equilibrium-like chemical potential. We also derive two scaling relations, which could help us to settle the long standing issue of universality in such systems.

(9) **Leonardo Rolla**, NYU Shanghai.
Title: Absorbing-State Phase Transitions.
Abstract: Modern statistical mechanics offers a large class of driven-dissipative stochastic systems that naturally evolve to a critical state, of which Activated Random Walks are perhaps the best example. The main pursuit in this field is to show universality of critical parameters, describe the critical behavior, the scaling relations and critical exponents of such systems, and the connection between driven-dissipative dynamics and conservative dynamics in infinite space. The study of this model was an open challenge for a long time, then it had signifi cant partial progress a decade ago, and got stuck again. Through the last 5 years it has seen exciting progress thanks to contributions by Asselah, Basu, Cabezas, Ganguly, Hoffman, Richey, Schapira, Sidoravicius, Sousi, Stauffer, Taggi, Teixeira, Tournier, Zindy, and myself. These covered most of the questions regarding existence of an absorbing and an active phase for different ranges of parameters, and current efforts are drifting towards the description of critical states, scaling limits, etc. We will summarize the current state of art and discuss some of the many open problems.

(10) **Vladas Sidoravicius**, NYU Shanghai.
Title: Mathematics of Multi-particle diffusion limited aggregation.
Abstract: In 1980 H. Rosenstock and C. Marquardt introduced the following stochastic aggregation model on $\mathbb{Z}^d$: Start with particles distributed according to the product Bernoulli measure with parameter $0 < \mu < 1$, conditioned not to have particle at the origin. At the origin we place a special particle, which is called the "seed of aggregate". Non-aggregated particles move as continuous-time simple random walks obeying the exclusion rule. The aggregate grows by attaching particles to its surface whenever a particle attempts to jump onto it. Aggregated particles do not move. This evolution is called Multi-particle Diffusion Limited Aggregation (MDLA). MDLA model even in dimension 1 has highly nontrivial behavior: If exclusion rule is removed and non-aggregated particles move as simple symmetric random walks (in this case the initial density is product of Poisson with parameter $0 < \rho$, H. Kesten and V. Sidoravicius (2008) proved that if $\rho < 1$, then the aggregate is growing sub-linearly, with exponent $1/2$, and A. Sly (2016) showed that if $\rho > 1$ it advances linearly, establishing a phase transition.

In my talk I will briefly review known results and will focus on the progress in dimensions $d \geq 2$. Our main result (joint with A. Stauffer) states that for the exclusion version of the process if $d > 1$ and the initial density of particles is close enough to 1, then with positive probability the aggregate has linearly growing arms; that is, there exists a constant $c > 0$ so that at time t the aggregate contains a point at distance at least ct from the origin, for all t, and also it obeys certain type of the shape theorem. The key conceptual element of our analysis is the introduction and study of a new FPP type growth process. Study of this process indicates that high density MDLA may belong to KPZ universality class.

(11) **Debleena Thacker**, Uppsala University.
Title: Border Aggregation Model.

Abstract: Start with a graph with a subset of vertices called the border. A particle released from the origin performs a random walk on the graph until it comes to the immediate neighbourhood of the border, at which point it joins this subset thus increasing the border by one point. Then a new particle is released from the origin and the process repeats until the origin becomes a part of the border itself. We are interested in the total number ξ of particles to be released by this final moment. We show that this model covers OK Corral model as well as the erosion model, and obtain distributions and bounds for ξ in cases where the graph is star graph, regular tree, and a $d$dimensional lattice.

Levine and Peres (2007) observed that the border aggregation model on $d$dimensional lattice can be considered as an inversion of the classical diffusion-limited-aggregation model (DLA). We strengthen the bounds obtained in Kesten (1987) for DLA model to obtain a lower bound for ξ.

(12) **Wioletta Ruszel**, TU Delft.
Title: Scaling limits of odometers in sandpile models.
Abstract: The divisible sandpile model is a special case of the class of continuous sandpile models on a graph V where the initial configuration is random and the evolution deterministic. Under certain conditions on the initial configuration the model will stabilize to the all 1 configuration. The amount of mass $(u(x))_{x\in V}$ that is emitted from $x \in V$ during stabilization is called the odometer. Depending on the initial configuration and the way how mass is distributed one can show that the scaling limit of u can be either a fractional Gaussian field or alpha-stable. The results presented in this talk are joint work with L.Chiarini (IMPA/TU Delft), A. Cipriani (TU Delft), J. de Graaff (TU Delft), M. Jara (IMPA) and R. Hazra (ISI Kolkatta).