UNIVERSALITY IN RANDOM STRUCTURES: INTERFACES, MATRICES SANDPILES

Titles and Abstracts for Week 1, 14 January -18 January, 2019
Focus Area: Random Matrices and Point Processes

Some Open Questions About Scaling Limits in Probability Theory

Ramanujan Lectures by **Sourav Chatterjee** (Stanford University).

Lecture 1: Yang-Mills for mathematicians. Making sense of quantum field theories is one of the most important open problems of modern mathematics. It is not very well known in the mathematics community that many small parts of this big problem are in fact well-posed questions in probability theory. In this talk I will describe a number of probabilistic open questions, which, if solved, would contribute greatly towards the goal of rigorous construction of quantum field theories. Specifically, I will discuss Yang-Mills theories, lattice gauge theories, quark confinement, mass gap and gauge-string duality, all as problems in probability.

Lecture 2: Gauge-string duality in lattice gauge theories. Quantum gauge theories are the mathematically ill-defined building blocks of the Standard Model of quantum mechanics. String theories, on the other hand, were built to serve as models of quantum gravity. Physicists have long been aware of the existence of a duality between quantum Yang-Mills theories and string theories. This is sometimes called gauge-string duality or gauge-gravity duality. Making sense of the duality formulas is still well beyond the reach of rigorous mathematics, partly because the models in question have not yet been rigorously defined. In this talk I will present a rigorously proved version of gauge-string duality in a discrete setting. Specifically, I will take a lattice gauge theory, which is a discrete approximation of a quantum gauge theory, and prove an explicit duality with a kind of string theory on the lattice. The duality has the appearance of a natural discrete analog of the formulas conjectured for the continuum models. The question of proving a continuum version of this duality remains open. These and other open questions will be discussed. (Partly based on joint work with Jafar Jafarov.)

Lecture 3: Constructing a solution of the 2D Kardar-Parisi-Zhang equation. The Kardar-Parisi-Zhang (KPZ) equation has become accepted as the canonical model for the growth of random surfaces. While the 1D KPZ equation has now a vast amount of rigorous mathematical work behind it, the physically important case of 2D surfaces has remained mathematically intractable. I will describe a first step towards constructing a solution of the 2D KPZ equation, by showing the existence of certain subsequential scaling limits if the parameters of the equation are renormalized in a suitable way. Many open questions remain, and these will be discussed. (Based on recent joint work with Alex Dunlap.)

Mini-Course by **Charles Bordenave**, University of Toulouse

Title: High trace methods in random matrix theory
Abstract: In 1955, Eugene Wigner has established the semi-circular law by computing expected traces of random matrices. In 1981, Fredi and Komlos have refined the computation of Wigner and studied the spectral radius of random matrices. Since then, there have been numerous successful extensions of their approach notably in connexion with the non-backtracking matrices. In this mini-course, we will introduce the high trace method of Furedi-Komlos and present some its

latest developments: the use of non-backtracking matrices, the tangle-free random graphs and the comparison argument of Bandeira and Van Handel.

Titles and Abstracts for Other Speakers

(1) **Fanny Augeri**, Weizmann Institute.
Title: Nonlinear large deviations bounds with applications to sparse Erdős-Rényi graphs.
Abstract: In this talk, I will present the framework of the so-called nonlinear large deviations introduced by Chatterjee and Dembo. In a seminal paper, they provided a sufficient criterion in order that the large deviations of a function on the discrete hypercube to be due by only changing the mean of the background measure. This sufficient condition was formulated in terms of the complexity of the gradient of the function of interest. I will present general nonlinear large deviation estimates similar to Chatterjee-Dembo's original bounds except that we do not require any second order smoothness. The approach relies on convex analysis arguments and is valid for a broad class of distributions. Then, I will detail an application of this nonlinear large deviations bounds to the problem of estimating the upper tail of cycles counts in sparse Erdős-Rényi graphs down to the connectivity parameter $n^{-1/2}$.

(2) **Anirban Basak**, ICTS.
Title: Sharp transition of invertibility of sparse random matrices.
Abstract: Consider an $n \times n$ matrix with i.i.d. Bernoulli(p) entries. It is well known that for $p = \Omega(1)$, i.e. p is bounded below by some positive constant, the matrix is invertible with high probability. If $p \ll \frac{\log n}{n}$ then the matrix contains zero rows and columns with high probability and hence it is singular with high probability. In this talk, we will discuss the sharp transition of the invertibility of this matrix at $p = \frac{logn}{n}$. This phenomenon extends to the adjacency matrices of directed and undirected Erdős-Rényi graphs, and random bipartite graph. This is joint work with Mark Rudelson.

(3) **Arup Bose**, ISI Kolkata.
Title: Smallest singular value and limit eigenvalue distribution of a class of non-Hermitian random matrices with statistical application.
Abstract: Suppose X is an $N \times n$ complex matrix whose entries are centered, independent, and identically distributed random variables with variance $1/n$ and whose fourth moment is of order $\mathcal{O}(n^{-2})$. We first consider the non-Hermitian matrix $XAX^* - z$, where A is a deterministic matrix whose smallest and largest singular values are bounded below and above respectively, and $z \neq 0$ is a complex number. Asymptotic probability bounds for the smallest singular value of this model are obtained in the large dimensional regime where N and n diverge to infinity at the same rate.

We then consider the special case where $A = J = [\mathbf{1}_{i-j=1 \mod n}]$ is a circulant matrix. Using the result of the first part, it is shown that the limit eigenvalue distribution of XJX^* exists in the large dimensional regime, and we determine this limit explicitly. A statistical application of this result devoted towards testing the presence of correlations within a multivariate time series is considered. Assuming that X represents a $\mathbb{C}^N$-valued time series which is observed over a time window of length n, the matrix XJX^* represents the one-step sample autocovariance matrix of this time series. Guided by the result on the limit spectral measure of this matrix, a whiteness test against an MA correlation model on the time series is introduced. Numerical simulations show the excellent performance of this test.

This is joint work with Walid Hachem.

(4) **Arijit Chakrabarty**, ISI Kolkata.
Title: Spectra of Adjacency and Laplacian Matrices of inhomogeneous Erdős-Rényi Graphs.

Abstract: Inhomogeneous Erdős-Rényi random graphs $\mathbb{G}_N$ on N vertices in the non-dense regime are studied. The edge between the pair of vertices $\{i,j\}$ is retained with probability $\varepsilon_N f(\frac{i}{N},\frac{j}{N})$, $1 \le i,j \le N$, independently of other edges, where $f\colon [0,1]\times[0,1] \to [0,\infty)$ is a continuous function such that $f(x,y)=f(y,x)$ for all $x,y \in [0,1]$. We study the empirical distribution of both the adjacency matrix A_N and the Laplacian matrix Δ_N associated with $\mathbb{G}_N$ in the limit as $N \to \infty$ when $\lim_{N\to\infty} \varepsilon_N = 0$ and $\lim_{N\to\infty} N\varepsilon_N = \infty$. In particular, we show that the empirical distributions of $(N\varepsilon_N)^{-1/2}A_N$ and $(N\varepsilon_N)^{-1/2}\Delta_N$ converge to deterministic limits weakly in probability. For the special case where $f(x,y)=r(x)r(y)$ with $r\colon [0,1] \to [0,\infty)$ a continuous function, we give an explicit characterisation of the limiting distributions.

Authors: Arijit Chakrabarty, Rajat Subhra Hazra, Frank den Hollander, and Matteo Sfragara

(5) **Zhou Fan**, Yale University.
Title: Tracy-Widom at each edge of real covariance and MANOVA estimators.
Abstract: We study the sample covariance matrix for real-valued data with general population covariance, as well as MANOVA-type covariance estimators in statistical variance components models under null hypotheses of global sphericity. In the limit as matrix dimensions increase proportionally, the asymptotic spectra of such estimators may have multiple disjoint intervals of support, possibly intersecting the negative half line. We show that the distribution of the extremal eigenvalue at each regular edge of the support has a GOE Tracy-Widom limit. Our proof extends a universality argument of Lee and Schnelli, replacing a continuous Green function flow by a discrete Lindeberg swapping scheme. This is joint work with Iain M. Johnstone.

(6) **Shirshendu Ganguly**, UC Berkeley.
Title: Polymer geometry in the large deviation regime via eigenvalue rigidity.
Abstract: Polymer weights in certain two dimensional exactly solvable models of last passage percolation in the KPZ universality class are known to exhibit remarkable distributional equalities with eigenvalues of well known random matrix ensembles and other determinantal processes. A general goal of the talk will be to explore consequences of recent advances in the study of rigidity properties such point processes in the context of polymer geometry.

We will discuss results about precise transversal fluctuation behavior of the polymer in upper and lower tail large deviation regimes using various random matrix theory inputs as well as geometric arguments, sharpening a result obtained by Deuschel and Zeitouni (1999) and addressing an open question left by them. Time permitting, we shall also discuss how some of these results extend beyond the exactly solvable settings.

(7) **Subhroshekhar Ghosh**, National University of Singapore.
Title: Two manifestations of rigidity in point sets : forbidden regions and maximal degeneracy.
Abstract: A point process is said to be "rigid" if its local observables are completely determined (as deterministic functions of) the point configuration outside a local neighbourhood. For example, it has been shown in earlier work that, in the Ginibre ensemble (a.k.a. the 2D Coulomb gas at inverse temperature $\beta = 2$), the point configuration outside any bounded domain determines, almost surely, the number of points in the domain. In this talk, we will explore two recent manifestations of such rigidity phenomena. For the zeros of the planar Gaussian analytic function, we show that outside every large "hole", there is a "forbidden region" which contains a vanishing density of points. This should be seen in contrast with the corresponding situation for classically understood models (e.g. random matrix ensembles), where no such effects are known to occur. In the second part

of the talk, we will consider "stealthy" hyperuniform systems, which are systems whose structure function (i.e., the Fourier transform of the two-point correlation) vanishes near the origin. We show that such systems exhibit "maximal degeneracy", namely the points outside a bounded domain determine, almost surely, the entire point configuration inside the domain. En route, we establish a conjecture of Zhang, Stillinger and Torquato that such systems have (deterministically) bounded holes.

Based on joint works with Joel Lebowitz and Alon Nishry.

(8) **Satya Majumdar**, University Paris-Sud.
Title: Rotating trapped fermions in 2d and the complex Ginibre ensemble.
Abstract: We establish an exact mapping between the positions of N noninteracting fermions in a $2d$ rotating harmonic trap in its ground-state and the eigenvalues of the $N \times N$ complex Ginibre ensemble of Random Matrix Theory (RMT). Using RMT techniques, we make precise predictions for the statistics of the positions of the fermions, both in the bulk as well as at the edge of the trapped Fermi gas. In addition, we compute exactly, for any finite N, the Rényi entanglement entropy and the number variance of a disk of radius r in the ground-state. We show that while these two quantities are proportional to each other in the (extended) bulk, this is no longer the case very close to the trap center nor at the edge. Near the edge, and for large N, we provide exact expressions for the scaling functions associated with these two observables.

(9) **Ansh Mallick**, ICTS.
Title: Regularity properties of LSD for certain families of random patterned matrices.
Abstract: In random matrix theory, after defining a family of random matrix one of the first question one asks is about the existence and regularity of limiting empirical spectral distribution (LSD). Here, I will talk about the absolute continuity and bound on the density of LSD for random Hankel and Toeplitz matrices.

(10) **Nanda Kishore Reddy**, ISI Bangalore.
Title: Eigenvalues of product random matrices.
Abstract: Products of random matrices have always been a topic of interest in Mathematics, Physics and Statistics for various reasons. In this talk, we shall discuss, along with their relevance, the exact eigenvalue distributions of certain product random matrix models and also the asymptotic behaviour of the eigenvalues of products of random matrices with the matrix sizes fixed and the number of matrices in the product increasing.

(11) **David Renfrew**, Binghamton University.
Title: Eigenvalues of random non-Hermitian matrices and randomly coupled differential equations.
Abstract: We consider large random matrices with centered, independent entries but possibly different variances and compute the limiting distribution of eigenvalues. We then consider applications to long time asymptotics for systems of critically coupled differential equations with random coefficients.

(12) **Philip Wood**, University of Wisconsin-Madison.
Title: Outliers in the spectrum for products of independent random matrices.
Abstract: For fixed positive integers m, we consider the product of m independent n by n random matrices with i.i.d. entries as in the limit as n tends to infinity. Under suitable assumptions on the entries of each matrix, it is known that the limiting empirical distribution of the eigenvalues is described by the m-th power of the circular law. Moreover, this same limiting distribution continues to hold if each i.i.d. random matrix is additively perturbed by a bounded rank deterministic error. However, the bounded rank perturbations may create one or more outlier eigenvalues. We describe the asymptotic location of the

outlier eigenvalues, which extends a result of Terence Tao for the case of a single i.i.d. matrix. Our methods also allow us to consider several other types of perturbations, including multiplicative perturbations.

(13) **Ofer Zeitouni**, Weizmann Institute.
Title: On eigenvectors of non-normal random matrices.
Abstract: What is the typical inner product between eigenvectors of non-normal matrices from the invariant ensembles with density proportional to $e^{-\mathrm{Tr} V(XX^*)}dX$? In the Ginibre case (i.e. $V(x) = x$), when the eigenvectors are chosen to correspond to specific eigenvalues, a CLT can be proved (after proper re-scaling). In the general case, the scale is known, but no limit law is known. I will describe the known (to me) results and their proofs. I will also describe a particular large deviations problem that seems essential for the general case. (Joint work with Florent Benaych-Georges).