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The H_{xxz} Line Ensemble and KPZ* Universality

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* Kardar, Parisi, Zhang 1986

1D quantum many-body $\Longleftrightarrow$ 2D statistical mechanics

specific model

→ long term goal: KPZ universality

1. H_{XXZ} line ensemble

$$H_{XXZ} = -\sum_j \left\{ \sigma_j^- \sigma_{j+1}^+ + \sigma_j^+ \sigma_{j+1}^- + \Delta\, \sigma_j^z \sigma_{j+1}^z \right\} \quad \text{on } \mathbb{Z}$$

0	1		
-1	1	↓	↑

σ^z - representation background ↓ ↑ spin at $\vec{x} = (x_1, \dots, x_n)$

$$\langle \vec{x} \,|\, e^{-TH_{XXZ}} \,|\, \vec{y} \rangle = \mathbb{E}_{(\vec{x},0) \to (\vec{y},T)} \left(\chi(\text{no intersection})\, e^{\Delta \int_0^T dt \sum_j \eta_j(t)\, \eta_{j+1}(t)} \right)$$

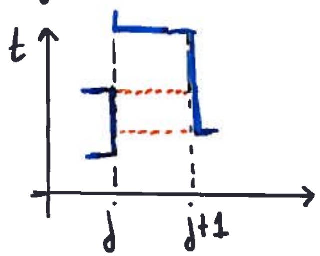
n independent symmetric, nearest neighbor, time continuous, random walks on $\mathbb{Z}$ jump rate 1

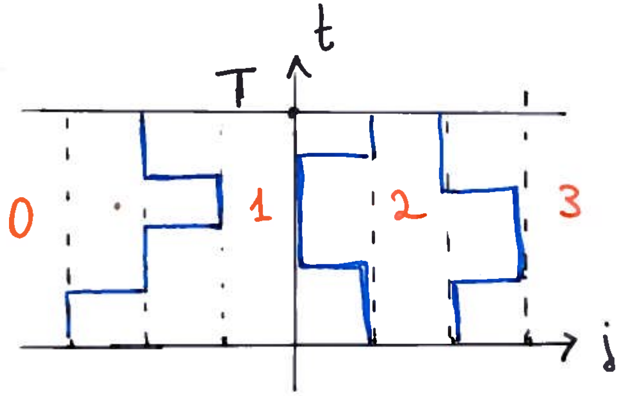
start $\vec{x}$, $t=0$. end $\vec{y}$, $t=T$

CONSTRAINT

// NO Ising //

$$\eta_j(t) = \begin{cases} 1 & \text{line} \\ 0 & \text{no line} \end{cases}$$





$T \to \infty$ ground state

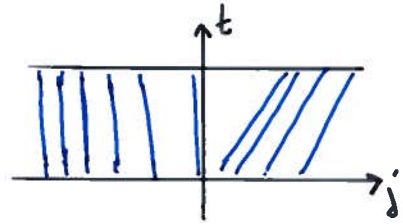
random walk lines $\Longleftrightarrow$ level lines of height-function

|| random surface over $\mathbb{R}^2$ ||

|| no Ising ||

free energy depends on boundary conditions ||

2. Asymmetric H_{xxz} - surface j-slope, t-slope



- j-slope $\quad - h \int_0^T dt \sum_j \eta_j(t)$

- t-slope

$$H_{xxz} = -\sum_j \{ e^{\theta} \sigma_j^- \sigma_{j+1}^+ + e^{-\theta} \sigma_j^+ \sigma_{j+1}^- + \Delta \sum_j \sigma_j^z \sigma_{j+1}^z \} + h \sum_j \sigma_j^z$$

$e^{\theta} \rightsquigarrow e^{-i\theta}$ magnetic flux $\quad (H_{xxz}^* = H_{xxz})$

$$Z(\theta) = \mathrm{tr}\, e^{-H(\theta)T}$$

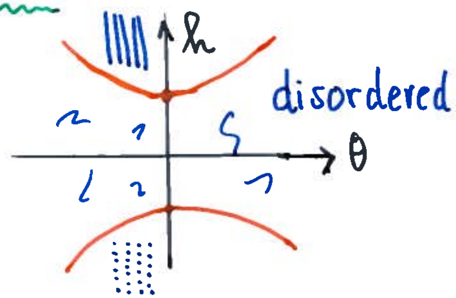
$$\frac{d}{d\theta} \log Z(\theta) = \frac{1}{Z} \mathrm{tr}\, e^{-H(\theta)T} \left(\sum_j \left(e^{\theta} \sigma_j^- \sigma_{j+1}^+ - e^{-\theta} \sigma_j^+ \sigma_{j+1}^- \right) \right)$$

average flux = t-slope

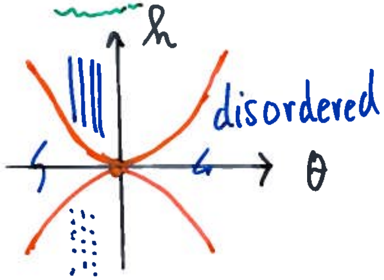
3. Phase diagram

ground state energy $E_\Delta(\theta, h)$

$\Delta = 0$ (free fermions)



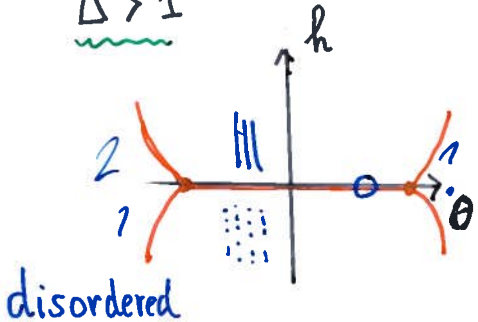
$\Delta = 1$



• stochastic point SSEP

ferro Heisenberg H_{xxx}

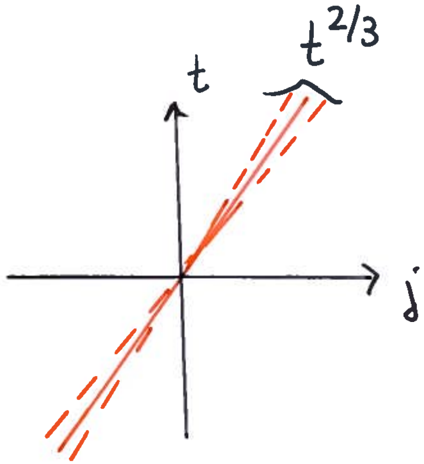
$\Delta > 1$



o domain wall Koma, Nachtergaele 1997

• conical point Gwa, HS. 1992 ASEP spectral gap $N^{-3/2}$

KPZ I



scaling $t^{-2/3} f_{KPZ}(t^{-2/3} j)$

Aggarwal 2016

stationary

fixed density

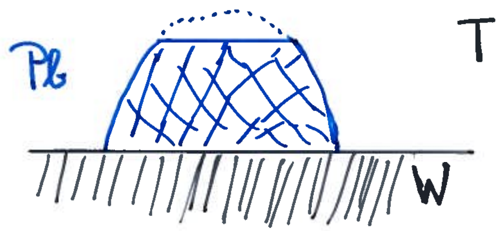
→ two-point function of stationary Burgers

two-point $\langle \eta_j(t)\, \eta_0(0) \rangle^c$

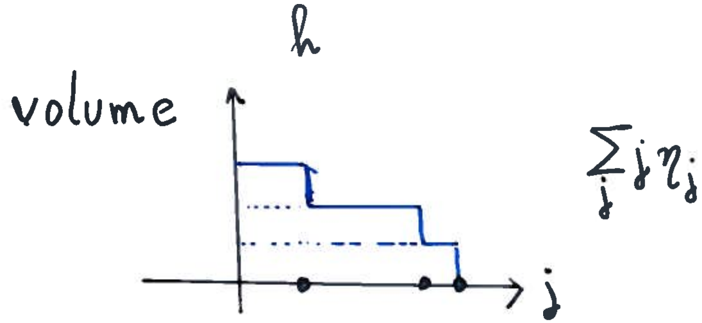
$$\partial_t u + \partial_x (u^2 - \partial_x u + \xi) = 0$$

white noise

4. Facet, volume constraint



$T < T_r$ (roughening)



$$H_\lambda = H_{XXZ} - \lambda \sum_j j\, \sigma_j^z$$ unique ground state ψ_λ

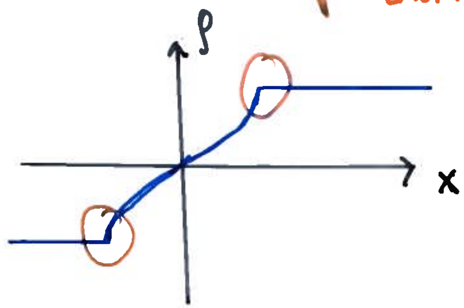
domain wall

$\Delta > 1$, H_{XXZ} has spectral gap $\qquad \lim_{\lambda \to 0} \langle \psi_\lambda, \sigma_j^z \psi_\lambda \rangle = \langle \psi_0, \sigma_j^z \psi_0 \rangle$

$\surd$ universal

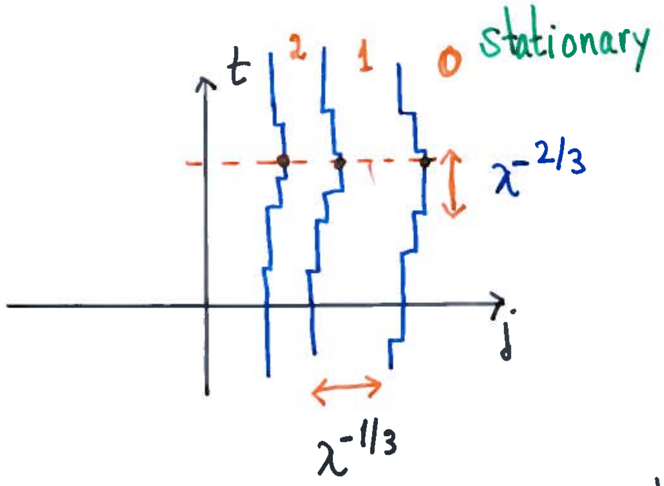
$|\Delta| < 1$ conjecture

$$\lim_{\lambda \to 0} \langle \psi_\lambda, \sigma^z_{\lfloor \lambda^{-1} x \rfloor} \psi_\lambda \rangle = \rho(x)$$



fine structure at facet edge

Ferrari, Prähofer, HS 2004



limit $\lambda \to 0$ stationary Airy process $\mathcal{A}i(t)$

NOT Airy point process

construction $N \times N$ complex hermitian

$$dA = -A\,dt + dB_t$$

matrix valued Brownian motion $U^* B_t U \cong B_t$ stationary

eigenvalues $\lambda_1(t) < \dots < \lambda_N(t)$

$$\lim_{N\to\infty} N^{-1/3} \lambda_N(N^{2/3} t) = \mathcal{A}i(t)$$

continuous sample paths

fixed t : $\mathcal{A}i(t)$ is Tracy-Widom distributed

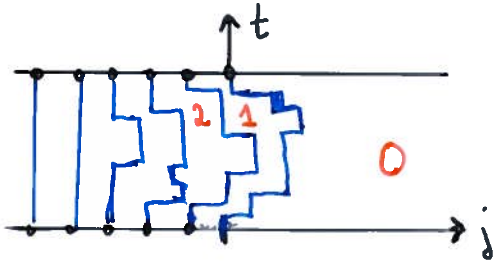
$$\mathbb{E}\left(\mathcal{A}i(t) - \mathcal{A}i(0)\right)^2 \cong \frac{2}{t^2}$$

Tracy, Widom 2004

KPZ II

also from KPZ equation

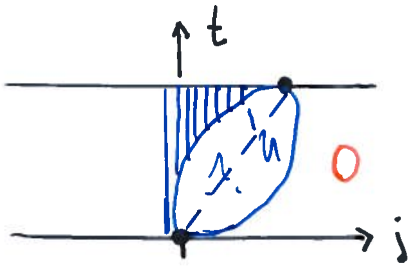
5. Domain wall b.c.



facet shape Colomo Pronko 2011
Stephan 2017 $\Delta < 1$ outer facet

fluctuations $\Delta = 0$ Prähofer HS 2001
$\Delta = 1$ HS. 2018
$\Delta > 1$ exponential

open problem also for $\Delta = 0$

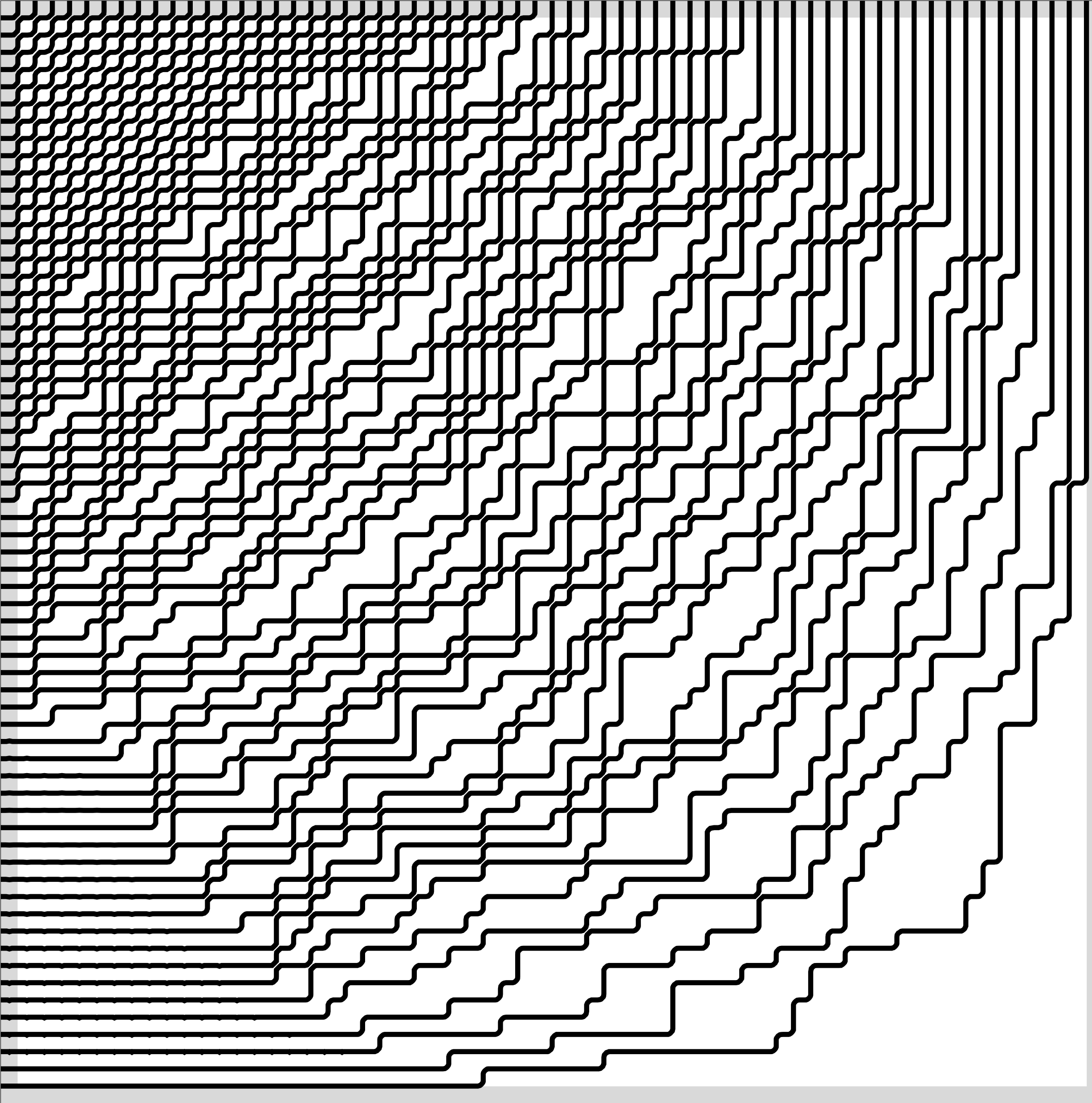


6. Monte Carlo

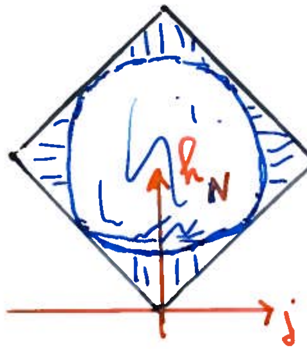
→ 6 vertex model at ASM (ice) point all tiles have weight 1 / $\Delta = \frac{1}{2}$ /

discrete version of H_{xxz}

DWBC



KPZ scaling theory



size N, facet h_N

conjecture: $$h_N(\lfloor Nx \rfloor) = v(x)N + (T(x)N)^{1/3}\,\xi_{TW}$$

GUE Tracy-Widom

macroscopic shape $v(x)$ $\Leftarrow$ Colomo + Pronko 2011

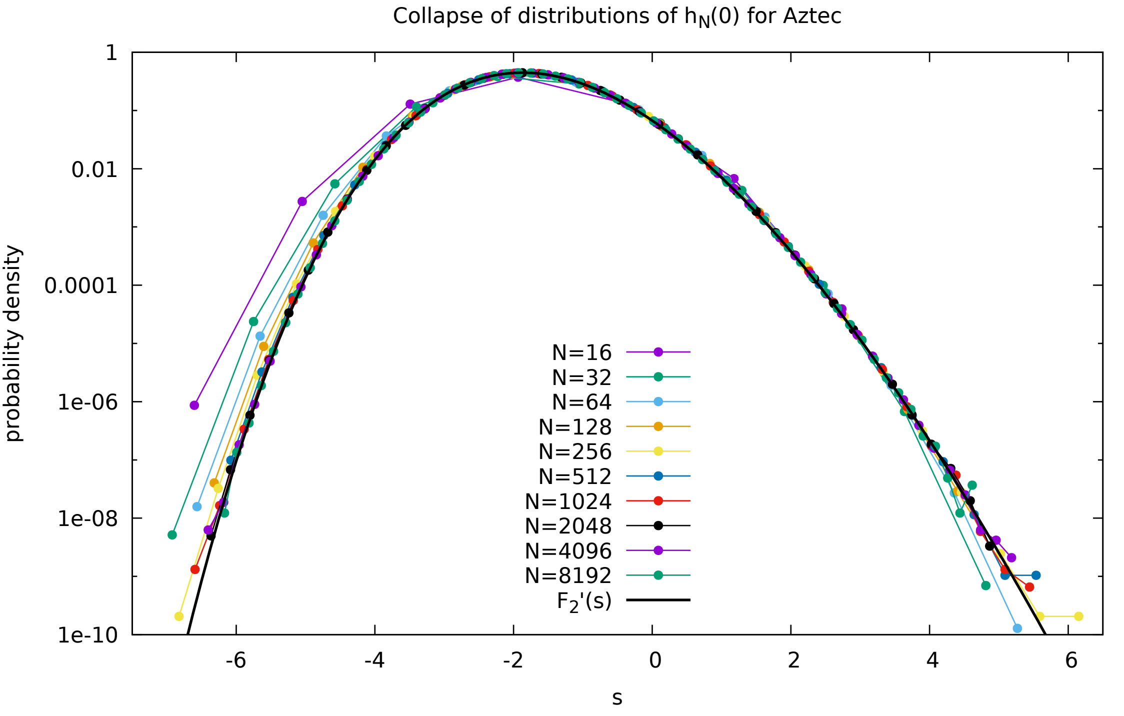
$$T = \left(\frac{1}{2}\frac{1}{v''}A^2\right)^{\frac{1}{3}} \qquad A = (1-u^2)\sqrt{u^2(1-b^2)+b^2} \qquad u = v'$$

model parameter

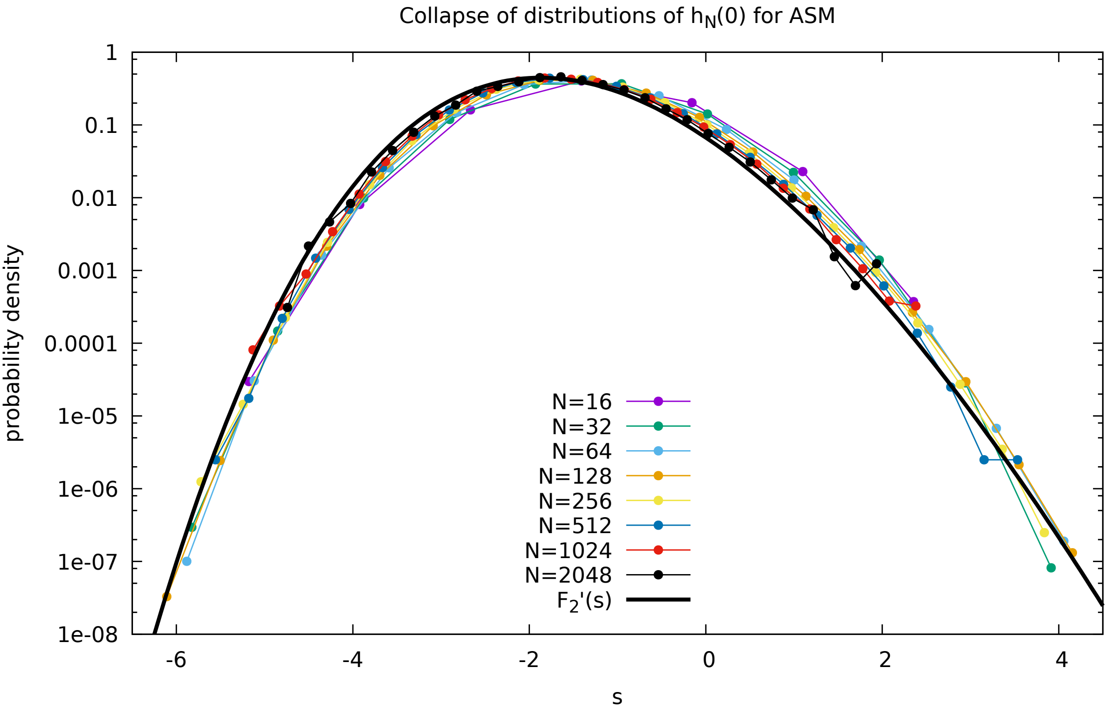
$\rightsquigarrow$ parameter free numerical fit

$\Delta = \frac{1}{2}$

compare with $\Delta = 0$ $\quad h_N \to h_{N+1}$ Markov



Collapse of distributions of h_N(0) for Aztec
probability density
s
1
0.01
0.0001
1e-06
1e-08
1e-10
-6
-4
-2
0
2
4
6
N=16
N=32
N=64
N=128
N=256
N=512
N=1024
N=2048
N=4096
N=8192
F_2'(s)


Collapse of distributions of $h_N(0)$ for ASM
probability density
s
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
-6
-4
-2
0
2
4
N=16
N=32
N=64
N=128
N=256
N=512
N=1024
N=2048
$F_2'(s)$

7. Conclusion

any progress towards universality welcome!

Example: $\{ b_j(t), j = 1, \dots, N \}$ independent OU processes stationary

condition on nonintersection free fermions $\rightsquigarrow$ $\lambda_N(t) \simeq \mathcal{A}i(t)$

$\to$ condition on $b_{j+1}(t) - b_j(t) > 1$ all t ??