Tutorial: Kilonova heating

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Physical constants and astrophysical units.

$$
\begin{aligned}
&G && 6.67384\times10^{-8}\ \mathrm{cm}^3\ \mathrm{g}^{-1}\mathrm{s}^{-2} \\
&c && 2.99792458\times10^{10}\ \mathrm{cm\ s}^{-1} \\
&h && 6.626070040\times10^{-27}\ \mathrm{erg\ s} \\
&\hbar && 1.054571628\times10^{-27}\ \mathrm{erg\ s} \\
&m_p && 1.6726217\times10^{-24}\ \mathrm{g} \\
&m_u && 1.6605389\times10^{-24}\ \mathrm{g} \\
&m_e && 9.10938291\times10^{-28}\ \mathrm{g} \\
&e && 4.80320425\times10^{-10}\ \mathrm{erg}^{1/2}\mathrm{cm}^{1/2} \\
&\alpha=\frac{e^2}{\hbar c} && \frac{1}{137.035999139} \\
&\sigma_\mathrm{T}=\frac{8\pi e^4}{3m_e^2c^4} && 6.6524574\times10^{-25}\ \mathrm{cm}^2 \\
&a_\mathrm{B}=\frac{\hbar}{m_e c\alpha} && 5.2917721067\times10^{-9}\ \mathrm{cm} \\
&k_\mathrm{B} && 1.3806488\times10^{-16}\ \mathrm{erg\ K}^{-1} \\
&\sigma_\mathrm{SB} && 5.6704\times10^{-5}\ \mathrm{erg\ cm}^{-2}\mathrm{s}^{-1}\mathrm{K}^{-4} \\
&a && 7.5657\times10^{-15}\ \mathrm{erg\ cm}^{-3}\mathrm{K}^{-4} \\
&G_F/(\hbar c)^3 && 1.1663787\times10^{-5}\ \mathrm{GeV}^{-2} \\
&M_\odot && 1.9884\times10^{33}\ \mathrm{g} \\
&GM_\odot && 1.32712440018\times10^{26}\ \mathrm{cm}^3\mathrm{s}^{-2} \\
&R_\odot && 6.955\times10^{10}\ \mathrm{cm} \\
&L_\odot && 3.828\times10^{33}\ \mathrm{erg/s} \\
&\mathrm{Jy} && 10^{-23}\ \mathrm{erg\ s}^{-1}\mathrm{cm}^{-2}\mathrm{Hz}^{-1} \\
&\mathrm{AU} && 1.495978707\times10^{13}\ \mathrm{cm} \\
&\mathrm{pc} && 3.08568\times10^{18}\ \mathrm{cm}
\end{aligned}
\tag{1}
$$

Beta decay heating rate

(i) Compute the total energy generation rate for $0.03M_\odot$ of r-process elements by using the energy generation rate per nuclide:

$$\dot{q}(t) \sim \frac{m_e c^2}{t_F}\left(\frac{t}{t_F}\right)^{-1.2}, \tag{2}$$

where $t_F \approx 8600\,\mathrm{s}$. Here you can use the mean mass number $A \approx 200$.

(2) Perform the following integration in the three regimes (i) $E_0 \gg 1$ and $\eta \ll 1$, (ii) $E_0 \ll 1$ and $\eta \ll 1$, and (iii) $E_0 \ll 1$ and $\eta \gg 1$:

$$f(E_0, Z) = \int_0^{p(E_0)} dp F(Z,E) p^2 (E - E_0)^2, \tag{3}$$

$$F(Z,E) = \frac{2\pi\eta}{1 - \exp(-2\pi\eta)} \tag{4}$$

where p is the electron's momentum, $E_0 = \bar{E}_0 - 1$, $\bar{E}_0$ is the total disintegration energy, E is the electron's kinetic energy, and $\eta = Z\alpha/\beta$, and β is the electron's velocity. Here all the quantities are in units of m_e and c.

(3) Calculate the fraction of the electron kinetic energy to the total disintegration kinetic energy:

$$\epsilon_e \equiv \frac{\langle E\rangle}{E_0}, \tag{5}$$

$$= \frac{1}{\epsilon_0}\frac{\int_0^{p_0} E F(Z,E) p^2 (E_0 - E)^2 dp}{\int_0^{p_0} F(Z,E) p^2 (E_0 - E)^2 dp}, \tag{6}$$

for the three above regimes.

(4) Derive the electron energy generation rate of statistical assembly of radioactive nuclides in the above three regimes by using

$$\dot{Q}(t) \approx \frac{e^{-1}}{\langle A\rangle m_u}\frac{\langle E_e(t)\rangle}{t}, \tag{7}$$

where e is the Euler number and you can use the relation between E_0 and t:

$$f(E_0, Z)t \equiv \frac{1}{\langle|\mathcal{M}_N|^2\rangle} t_F. \tag{8}$$

The fraction of the electrons' energy is given by the question (3). Here one may take $\langle|\mathcal{M}_N|^2\rangle \approx 0.05$, $Z \approx 70$, and $A \approx 200$.

Thermalization efficiency

(5) Estimate the time scale on which the ejecta becomes transparent to γ-rays of energy 10 keV, 0.1 MeV, 1 MeV, and 10 MeV by using the Compton scattering cross section per electron

$$\sigma_{\mathrm{Com}} = \frac{3}{4}\sigma_T\left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right) + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2}\right], \tag{9}$$

where σ_T is the Thomson scattering cross section and $x = h\nu/m_e c^2$. Consider two ejecta parameters (i) supernova-like ejecta $(M_{\mathrm{ej}}, v) = (3M_\odot, 0.01c)$ and (ii) kilonova-like ejecta $(M_{\mathrm{ej}}, v) = (0.03M_\odot, 0.2c)$. The density is roughly $\rho 3M_{\mathrm{ej}}/4\pi v_{\mathrm{ej}}^3 t^3$. One may use the ejecta is composed of atoms of $Z/A \approx 0.5$ for supernovae and 0.3 for kilonovae. After this time, γ-rays no longer interact with the ejecta material so that the efficiency of γ-ray heating drops exponentially.

(6) Do the same excise as question (5) but for photoelectric absorption

$$\sigma_{\rm ph,K} = 4\pi\sqrt{2}Z^5\alpha^4\sigma_T\left(\frac{m_ec^2}{h\nu}\right)^{7/2}\Theta(h\nu - E_K), \tag{10}$$

where $\Theta(x)$ is the Heaviside function.

And pair production

$$\sigma_{\rm pair} = Z^2\alpha^2\left(\frac{e^2}{m_ec^2}\right)^2\left(\frac{28}{9}\ln\frac{2h\nu}{m_ec^2} - \frac{218}{27}\right)\Theta(h\nu - 2m_ec^2). \tag{11}$$

(7) Consider the energy loss of an electron with velocity v by colliding electrons bound by ions of Z. Derive the stopping power [erg $\cdot$ cm^2]:

$$\sigma_e(Z) = Z\int_{b_1}^{b_2} T_e 2\pi b db = \frac{4\pi Ze^4}{m_ev^2}\ln\frac{m_ev^2}{\langle I\rangle}, \tag{12}$$

where $b_1 = h/m_ev$, $b_2 = v/\nu$, and $\langle I\rangle = h\nu$.

(8) Consider the electron thermalization in r-process ejecta composed of ions with A and Z and calculate the time when the thermalization efficiency of electrons with $E \sim 0.5$ MeV and $v \sim c$ becomes in efficient, $t_{\rm ineff,e}$, corresponding to the dynamical time equals the energy loss time of electrons:

$$\begin{aligned} t_{\rm dy} &= E\left(\frac{dE}{dt}\right)^{-1}, & (13)\\ \frac{dE}{dt} &= \sigma_e(Z)n_Av, & (14)\\ n_A(t) &= \frac{3M_{\rm ej}}{4\pi v_{\rm ej}^3t^3Am_u}, & (15) \end{aligned}$$

e.g., $Z/A \approx 0.3$, $\langle I\rangle \approx 10$ eV, $M_{\rm ej} \approx 0.03M_\odot$, and $v_{\rm ej} \approx 0.2c$.

(9) Derive the γ-ray and electron heating rate of beta decay including the thermalization efficiency by using the energy generation rate (problem 4, the regime: $E_0 \ll 1$ and $\eta \gg 1$). Here assume the γ-ray energy generation rate is two times the electron's and the mean energy of γ-ray energy is 1 MeV. One can use the thermalization efficiencies:

$$\begin{aligned} f_{\rm th,\gamma} &\approx 1 - \exp\left(-\tau_\gamma(t)\right), & (16)\\ f_{\rm th,e} &\approx \left(\frac{t_{\rm ineff,e}}{t}\right)^2. \quad (\text{for}\, t > t_{\rm ineff,e}), & (17) \end{aligned}$$

where τ_γ is the Compton optical depth of the ejecta. You can use the same ejecta parameters of problem (8).

(10) Calculate the kilonova light curves by using the heating rate derived in problem (9) and compare them with the observed bolometric data.