Questions: GRBs

August 14, 2018

Physics constants and astrophysical units.

$$
\begin{aligned}
&G && 6.67384 \times 10^{-8}\ \mathrm{cm}^3\ \mathrm{g}^{-1}\mathrm{s}^{-2} \\
&c && 2.99792458 \times 10^{10}\ \mathrm{cm\ s}^{-1} \\
&h && 6.626070040 \times 10^{-27}\ \mathrm{erg\ s} \\
&\hbar && 1.054571628 \times 10^{-27}\ \mathrm{erg\ s} \\
&m_p && 1.6726217 \times 10^{-24}\ \mathrm{g} \\
&m_u && 1.6605389 \times 10^{-24}\ \mathrm{g} \\
&m_e && 9.10938291 \times 10^{-28}\ \mathrm{g} \\
&e && 4.80320425 \times 10^{-10}\ \mathrm{erg}^{1/2}\mathrm{cm}^{1/2} \\
&\alpha = \frac{e^2}{\hbar c} && \frac{1}{137.035999139} \\
&\sigma_{\mathrm{T}} = \frac{8\pi e^4}{3m_e^2c^4} && 6.6524574 \times 10^{-25}\ \mathrm{cm}^2 \\
&a_{\mathrm{B}} = \frac{\hbar}{m_e c\alpha} && 5.2917721067 \times 10^{-9}\ \mathrm{cm} \\
&k_{\mathrm{B}} && 1.3806488 \times 10^{-16}\ \mathrm{erg\ K}^{-1} \\
&\sigma_{\mathrm{SB}} && 5.6704 \times 10^{-5}\ \mathrm{erg\ cm}^{-2}\mathrm{s}^{-1}\mathrm{K}^{-4} \\
&a && 7.5657 \times 10^{-15}\ \mathrm{erg\ cm}^{-3}\mathrm{K}^{-4} \\
&G_F/(\hbar c)^3 && 1.1663787 \times 10^{-5}\ \mathrm{GeV}^{-2} \\
&M_\odot && 1.9884 \times 10^{33}\ \mathrm{g} \\
&GM_\odot && 1.32712440018 \times 10^{26}\ \mathrm{cm}^3\mathrm{s}^{-2} \\
&R_\odot && 6.955 \times 10^{10}\ \mathrm{cm} \\
&L_\odot && 3.828 \times 10^{33}\ \mathrm{erg/s} \\
&\mathrm{Jy} && 10^{-23}\ \mathrm{erg\ s}^{-1}\mathrm{cm}^{-2}\mathrm{Hz}^{-1} \\
&\mathrm{AU} && 1.495978707 \times 10^{13}\ \mathrm{cm} \\
&\mathrm{pc} && 3.08568 \times 10^{18}\ \mathrm{cm}
\end{aligned} \tag{1}
$$

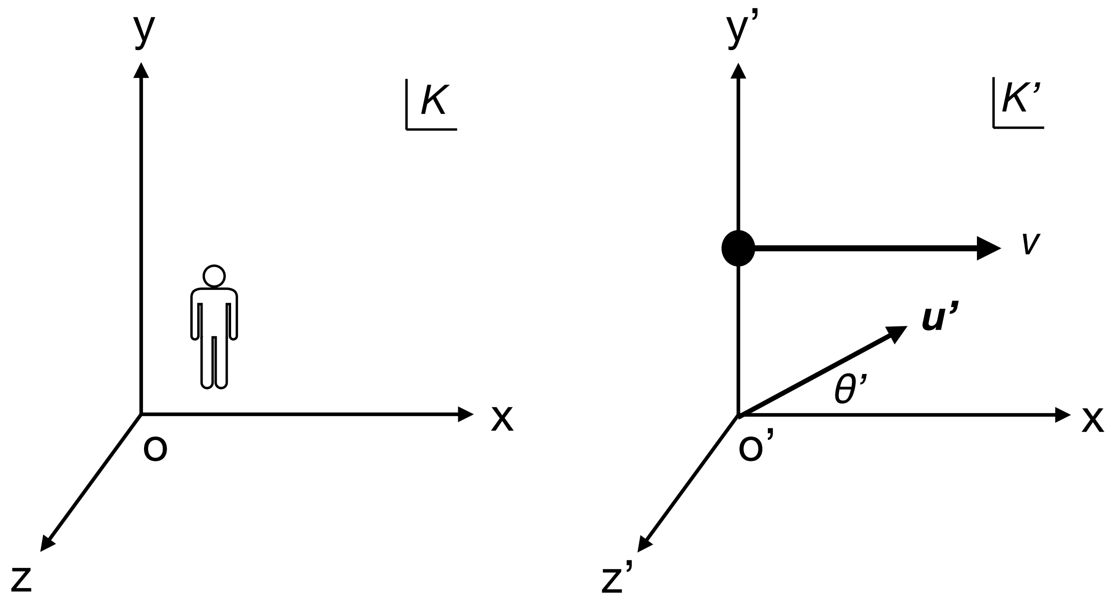

Figure 1: Lorentz transformation.

Lorentz and Doppler transformation

(1) Consider frames K (lab/stellar frame) and K' (the emitter rest frame) with a relative motion of v along the x axis and show

$$u_x = \frac{u'_x + v}{1 + \beta u'_x/c}, \tag{2}$$

$$u_y = \frac{u'_y}{\gamma(1 + \beta u'_x/c)}, \tag{3}$$

$$u_y = \frac{u'_z}{\gamma(1 + \beta u'_x/c)}. \tag{4}$$

The relation between (x, y, z, t) and (x', y', z', t') (Lorentz transformation) is given by

$$dx' = \gamma(dx - vdt), \tag{5}$$

$$dy' = dy, \tag{6}$$

$$dz' = dz, \tag{7}$$

$$dt' = \gamma(dt - \frac{\beta}{c}dx), \tag{8}$$

where $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$.

(2) Consider a photon emitted to θ' in the K' frame and derive the relation between $\cos\theta$ and $\cos\theta'$, where θ and θ' are the angle measured from the x axis and x' axis.

(3) Consider the photons' energy dW' is emitted (0-th component of the four momentum vector) in a direction of θ' and get dW via Lorentz transformation. You can use $P^{\mu\prime} = (dW', dW' \cos\theta'/c, dW' \sin\theta'/c, 0)$. Then write down $dW/d\Omega$ in terms of $dW'/d\Omega'$, $\cos\theta$.

(4) Consider emission from a relativistic jet with a half-opening angle of θ_j. The spectral peak, E_p, and isotropic-equivalent energy $E_{\rm iso}$ for an off-axis observer at θ are related to those for an on-axis observer as

$$\frac{E_p(\theta)}{E_p(0)} \approx q^{-2}, \tag{9}$$

and

$$\frac{E_{\rm iso}(\theta)}{E_{\rm iso}(0)} \approx q^{-6}, \qquad (\theta_j < 1/\Gamma, \theta_j < \theta), \tag{10}$$

$$\frac{E_{\rm iso}(\theta)}{E_{\rm iso}(0)} \approx q^{-6}(\Gamma\theta_j)^2, \qquad (\theta_j > 1/\Gamma, \theta_j < \theta), \tag{11}$$

(12)

where Γ is the Lorentz factor of the jet and $q^{-2} = (\theta - \theta_j)\Gamma$.

Given the typical values of short GRB (assuming on-axis), $E_p \sim 700$ keV, $E_{\rm iso} \sim 10^{51}$ erg, and $\theta_j = 0.07$ rad, calculate E_p and $E_{\rm iso}$ for $\theta = 0.2,\ 0.4,\ 0.8$ for $\Gamma = 10,\ 30,\ 100$.

Compactness problem

(5) When high energy photons are trapped in a compact region, these photons may create electron-positron pairs. In order that the high energy photons are radiated from the region, the optical depth to the pair creation process should be ~ 1.

Pair creation occurs when the total energy of two photons in the center of mass frame exceeds $2m_ec^2$. Show the following general condition of pair production, a photon of energy e' in the fluid-comoving frame,

$$e_1' e_2' = (m_e c^2)^2. \tag{13}$$

Rewrite this condition in terms of the observed photons' energy when the fluid is moving to us with a Lorentz factor of Γ and calculate e_2 if the maximum observed photon energy, e_1, is 1 GeV assuming a Lorentz factor of 100.

Suppose a emission time variability of δT, Lorentz factor of Γ, the observed photon spectrum (erg/cm^2) is flat from $e_{\rm min} = 0.1$ MeV to $e_{\rm max} = 1$ GeV, and $E_{\rm iso} = 10^{51}$ erg. Obtain the number of photons with energy greater than the annihilation energy $e_{\rm an}$ for $e_{\rm max}$.

Derive the optical depth to the pair production using a cross section of $11\sigma_T/180$ and calculate the Lorentz factor in order that the fluid is optically thin, $\tau \sim 1$.