Time reversal symmetric U(1) quantum spin liquids

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Collaborator: Chong Wang (MIT grad student)

Wang, TS, forthcoming.

A solid as a universe

Different phases of quantum matter define different kinds of `universes'
as seen by a microbe living inside.

Example:

Conventional band insulator

Universe with elementary particles - phonon (gapless) and electron (gapped).

Designer universes

Phases of quantum matter with low energy effective field theory corresponding to any universe one can imagine?

Are there phases where the `standard model' of particle physics emerges?

Research program: Volovik, Xiao-gang Wen, Laughlin,............

Let there be (artificial) light.....

Are there quantum phases with an emergent excitation that behaves like a photon?

Long distance physics: emergence of Maxwell equations in a quantum spin/boson system with short range interactions?

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They are particular `quantum spin liquid’ phases of spin/boson systems in three space dimensions.

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They are `quantum spin liquid' phases of spin/boson systems in three space dimensions.

Terminology: U(1) quantum spin liquid (as there is an emergent U(1) gauge field associated with the photon).

Microscopic models with emergent photons

1. O. Motrunich and T. Senthil, Phys. Rev. Lett. (2002)
(Boson models)

2. Michael Hermele, Matthew P.A. Fisher, and Leon Balents, Phys. Rev. B (2004).
(Quantum spin models)

3. R. Moessner and S. L. Sondhi, Phys. Rev. B (2003)
(Quantum dimer models)

Numerical simulations:

1. Argha Banerjee, Sergei V. Isakov, Kedar Damle, and Yong Baek Kim
Phys. Rev. Lett. (2008).

2. Nic Shannon, Olga Sikora, Frank Pollmann, Karlo Penc, and Peter Fulde
Phys. Rev. Lett. (2012).

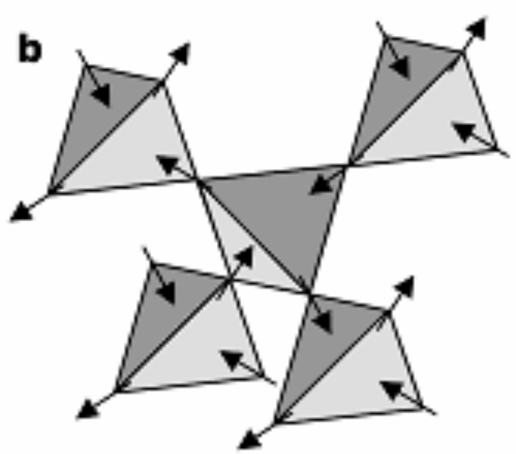
Recent revival

Possibility of U(1) quantum spin liquids in ``quantum spin ice" materials

Next few slides: borrowed and adapted from L. Balents

Classical spin ice

Spin ice: Ising spins on 3d pyrochlore lattice with interactions enforcing 2 in - 2 out \`ice rule'

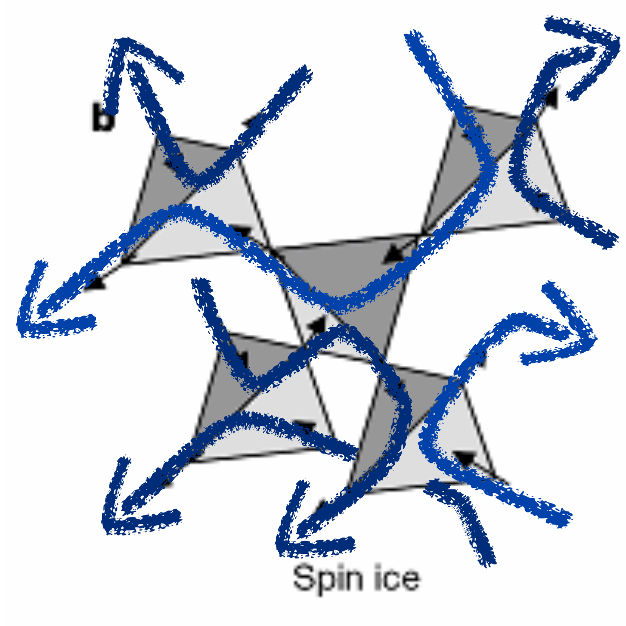


Spin ice

$$H \approx J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z$$

Materials: $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$

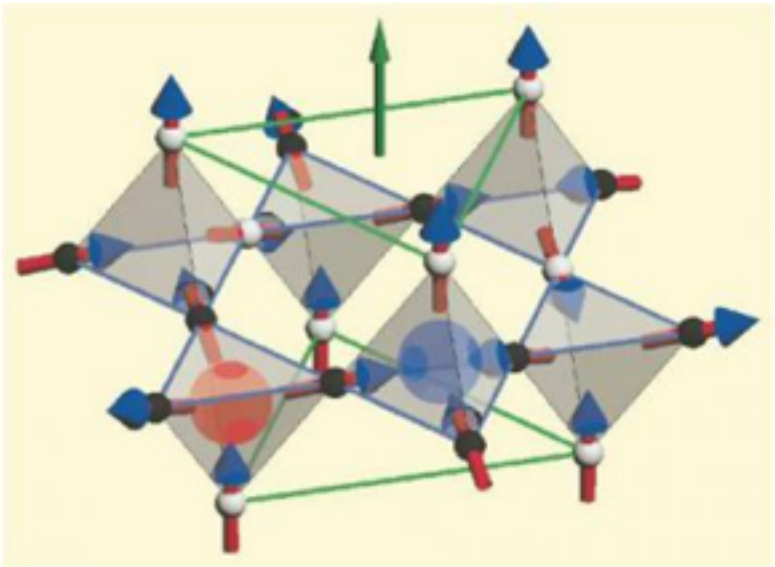
Classical spin ice: `artificial magnetostatics'



$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

Castelnovo *et al*, 2008



Defect tetrahedra (3 in - 1 out or 3 out- 1 in) in spin ice manifold: `magnetic monopoles'

Quantum spin ice

New spin ice materials where quantum effects on the Ising spins are clearly important.

Eg: $Yb_2Ti_2O_7$, $Pr_2Zr_2O_7$, $Pr_2Sn_2O_7$...?

Experiment: ($Yb_2Ti_2O_7$ Gaulin et al , $Pr_2Zr_2O_7$ Nakatsuji, Broholm et al) : Many deviations from classical spin ice behavior at low-T (eg continuum excitations in neutron scattering).

Eg: Large weight at $\omega >> T$ in inelastic neutron scattering in $Pr_2Zr_2O_7$ (Nakatsuji, Broholm et al, Nat. Comm. 2013).

S. Curnoe, 2008
S. Onoda, 2010

Hamiltonian

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z$$

classical NN spin ice

$$-J_{\pm} \sum_{\langle i,j \rangle} \left(S_i^+ S_j^- + S_i^- S_j^+ \right)$$

$$+ J_{z\pm} \sum_{\langle i,j \rangle} \left[S_i^z \left(\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^- \right) + i \leftrightarrow j \right]$$

$$+ J_{\pm\pm} \sum_{\langle i,j \rangle} \left(\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^- \right)$$

+ quantum fluctuations

= “quantum spin ice”

+ dipolar

Quantum spin ice Hamiltonian for $Yb_2Ti_2O_7$

Ross, Savary, Gaulin, Balents, 2011

$J_{zz} = 0.17 \pm 0.04$ meV

$J_{\pm} = 0.05 \pm 0.01$ meV $J_{z\pm} = 0.14 \pm 0.01$ meV $J_{\pm\pm} = 0.05 \pm 0.01$ meV

Reliably extracted from fitting spin wave dispersion in high field state.

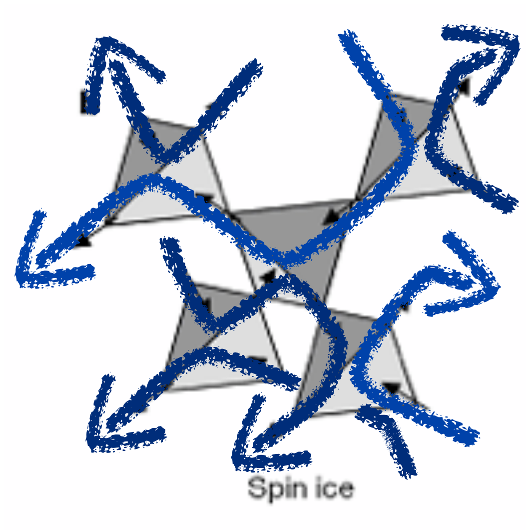
Parameters => appropriate to call this quantum spin ice.

Quantum spin ice and quantum spin liquids

Quantum fluctuations in spin ice manifold:

Magnetic field lines quantum fluctuate.

If field lines have zero tension => quantum spin liquid with an emergent U(1) gauge field.



$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

3d ``U(1) quantum spin liquid”

Excitations: 1. Gapless artificial photon

2. Gapped `magnetic monopole’ (3 in - 1 out defect tetrahedra) - the `m’ particle

3. Other gapped point particles carrying internal `electric’ charge - the `e’ particle

Many ongoing experiments to look for this!

Quantum spin ice, quantum spin liquids, and symmetry

Some crucial questions for theory:

1. What distinct kinds of quantum spin liquids with symmetry are possible for this kind of Hamiltonian?

Note: Only physical symmetry - Time reversal x space group.

2. How to theoretically access these distinct quantum spin liquids?

3. How to distinguish in experiments?

Quantum spin ice, quantum spin liquids, and symmetry

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2. How to theoretically access these distinct quantum spin liquids?

3. How to distinguish in experiments?

Focus on time reversal alone (leave space group for future).

Time reversal symmetric U(1) quantum spin liquids

Time reversal symmetric U(1) quantum spin liquids

Restrict to phases where only the photon is gapless.

e and m excitations are gapped.

To distinguish different phases enough to focus on these e and m particles.

Claim: There are precisely 8 distinct such phases which however become equivalent if time reversal is broken.

Very strong connections to recent progress in understanding interacting topological insulators.

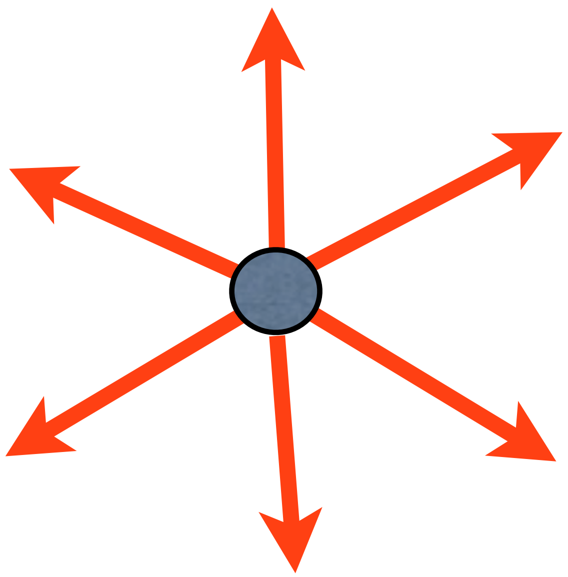
Some trivial observations

Microscopic Hilbert space:
All excitations created by local (i.e physical) operators must be bosons.

Emergent excitations - the e and m particles - are not created by local operators.

To create, eg, an electric charge, must also create associated electric field lines.

e (or m) particle may be either boson or fermion.



Time reversal (T)

For spin/boson systems, time reversal acts on physical states in a simple way.

In particular $T^2 = +1$ on all physical states.

Contrast with electronic systems where $T^2 = -1$ and there is a Kramers degeneracy.

Time reversal (T) on emergent particles

Electric charge is even under time reversal.

Magnetic charge is odd under time reversal.

=> e and Te (its time reversed partner) differ only by a local operator.

But as e itself is not local it could have $T^2 = -1$ and the associated Kramers degeneracy.

m and Tm are not related by a local operator and there is no meaning to whether it is Kramers or not (T^2 acting on m can be shifted by a gauge transformation).

More trivial observations

e or m may be fermion, and e may be Kramers.

But composite particles with zero electric and zero magnetic charge are `local'.
=> Must be bosons, and must transform trivially under time reversal.

Time reversal symmetric U(1) quantum spin liquids

Two broad classes:

m is either a boson or a fermion.

Discuss these separately.

Bosonic monopole

Four distinct phases depending on statistics of e particle (boson or fermion), and on realization of time reversal (trivial or Kramers).

Notation:

$e_b m_b$, $e_f m_b$, $e_b T m_b$, $e_f T m_b$

Name	e particle	m particle
$e_b m_b$	boson, $T^2 = 1$	boson
$e_f m_b$	fermion, $T^2 = 1$	boson
$e_b T m_b$	boson, $T^2 = -1$	boson
$e_f T m_b$	fermion, $T^2 = -1$	boson

Some of these are phases obtained through familiar constructions.

Eg: $e_b m_b$ - constructed in all existing microscopic models for the spin liquid

(realized for instance by XXZ spin-1/2 model on pyrochlore lattice).

$e_b T m_b$, $e_f T m_b$: constructed by Schwinger boson/Schwinger fermion representation of physical spins.

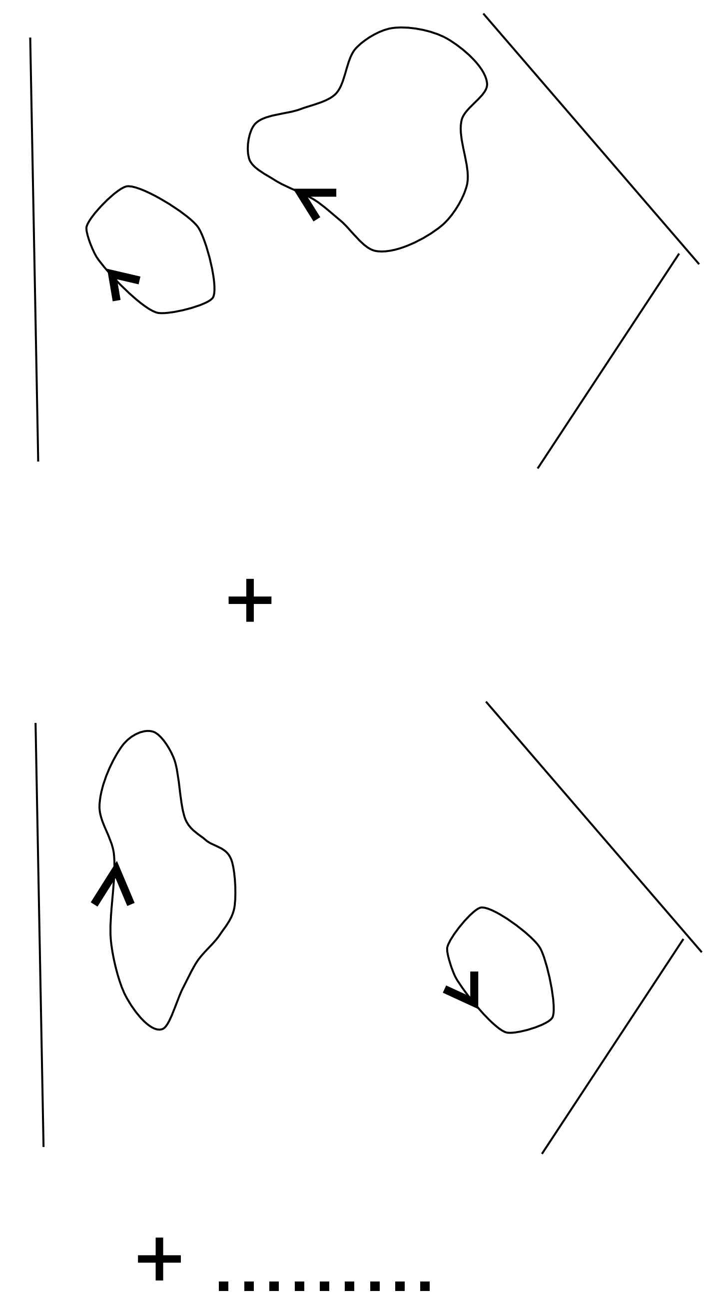
Understanding the 4 phases: wavefunctions

Useful to think in terms of fluctuating electric/ magnetic field lines.

$e_b m_b$: Electric picture.

E-field lines form oriented loops at low energies.

Ground state - superposition of oriented electric loops with positive weights.



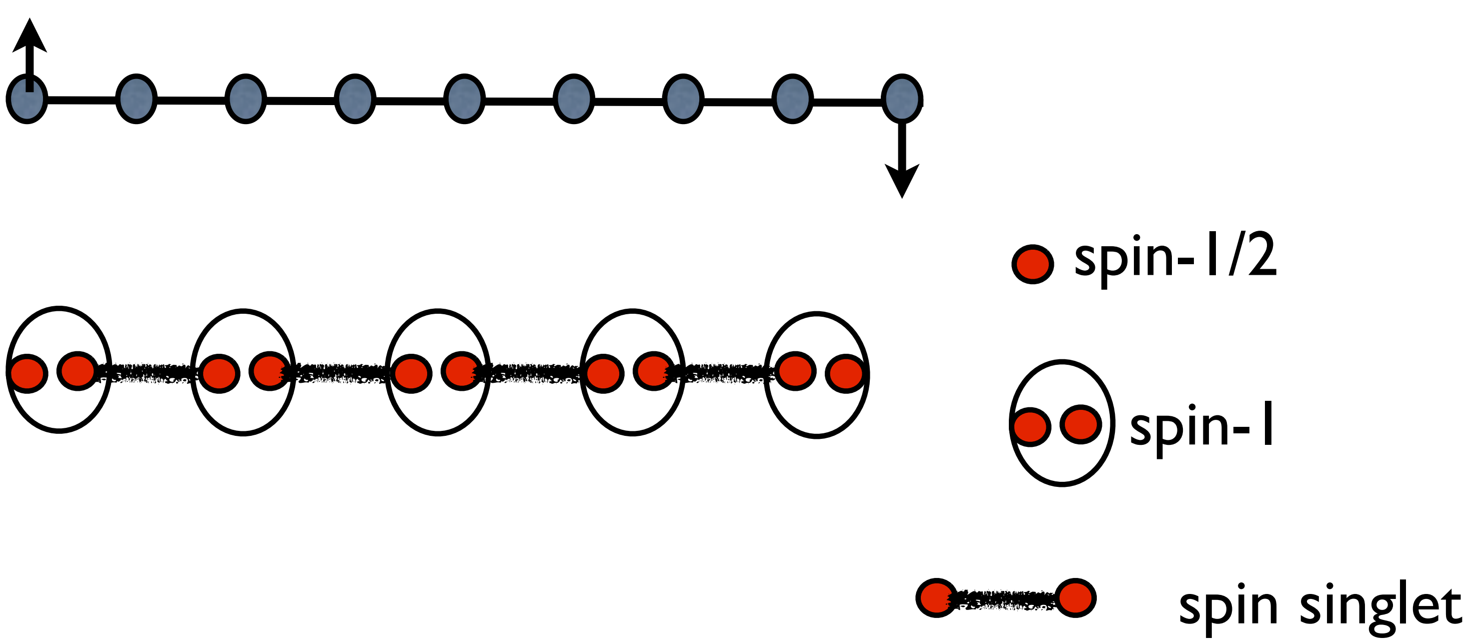
Kramers e-particle: Haldane loops

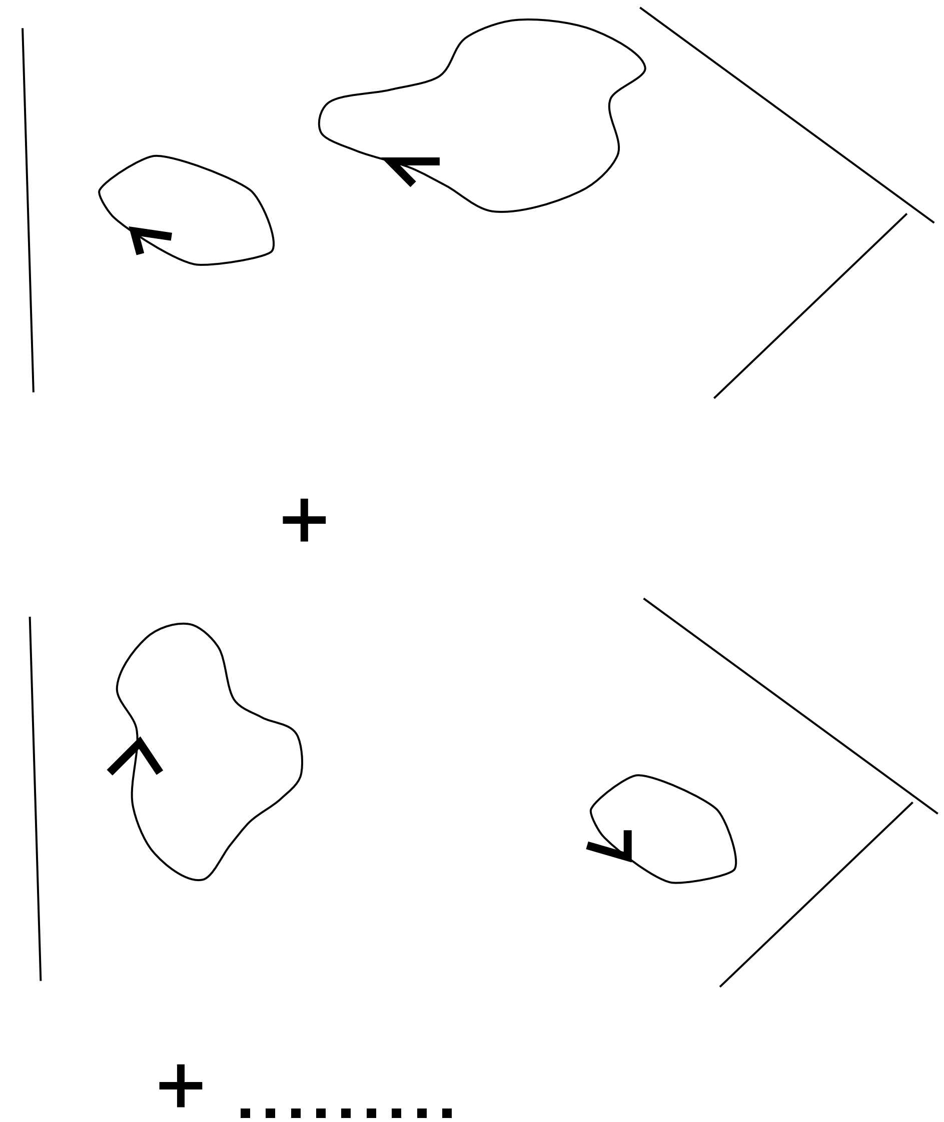
If in addition e is a Kramers doublet ($e_b T m_b$), then E-field lines are stuffed with Haldane spin-1 chains.

e-particle: Open end of E-field line

Haldane chain: gapped in bulk but at open end there is a Kramers doublet

=> achieve Kramers e-particle.



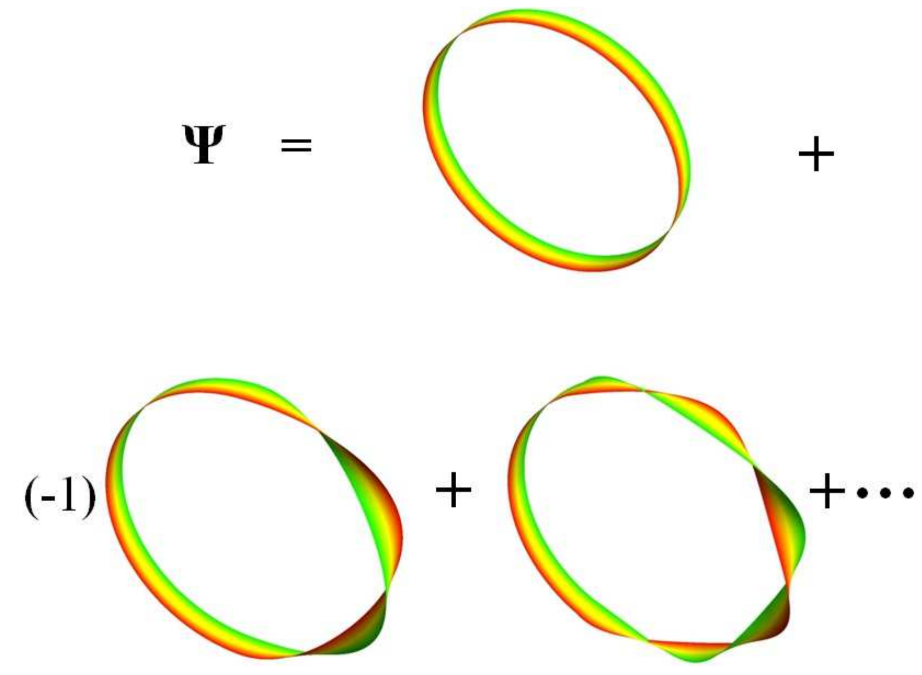


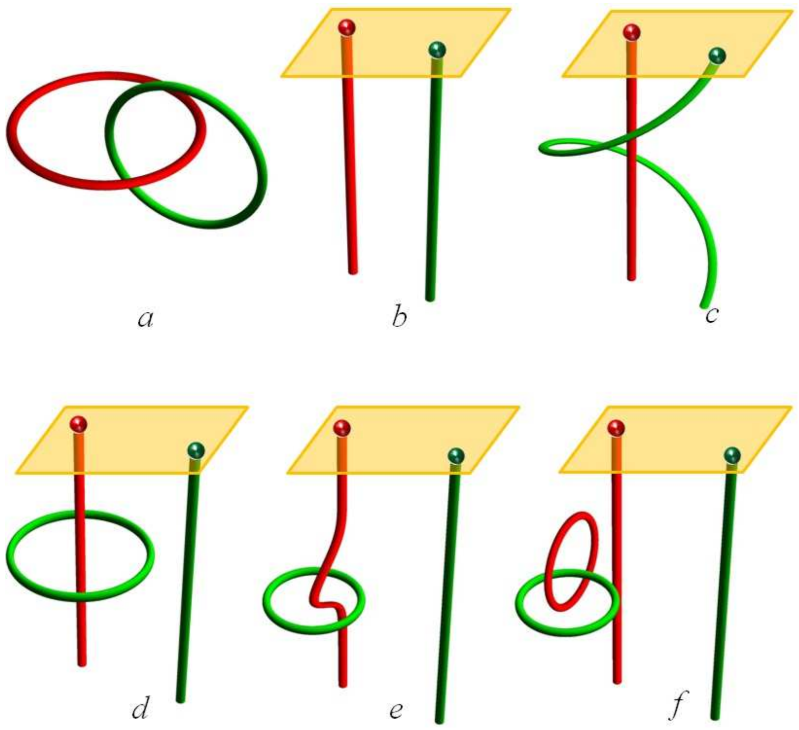
Fermionic e-particle ($e_f m_b$ or $e_f T m_b$)

Electric field loops are ribbons.

Phase (-1) for self-linking of any ribbon.

Open end point of such a ribbon is a fermion.





Aside: relationship between these phases

Interesting point of view:

Monopoles are gapped - regard as time reversal symmetric monopole insulators.

These distinct T-reversal symmetric U(1) spin liquids correspond to distinct ``interacting bosonic topological'' insulators formed by the monopoles (Wang, TS, 13)

A fifth phase: the ``topological Mott insulator”

Start with $e_f T m_b$

Put the Kramers fermion e-particle in a topological band insulator (Pesin, Balents, 2010).

=> new T-reversal symmetric U(1) quantum spin liquid.

(Proposed originally for the pyrochlore iridate $Y_2Ir_2O_7$.)

Notation: $(e_f T m_b)_\theta$

Key consequences: 1. Surface Dirac cone of the e-particle.

2. m-particle has internal electric charge-1/2 (see next slide)

Witten effect in topological band insulators

Response of a topological band insulator to a $U(1)$ gauge field includes a θ term:

$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Qi, Highes, Zhang 09
Essin, Moore, Vanderbilt, 10

with $\theta = \pi$.

θ term => monopole has electric charge 1/2 (the "Witten effect")

$$\begin{aligned}
\mathcal{L}_\theta &= \frac{\theta}{4\pi^2}\vec{E}.\vec{B} \\
&= -\frac{\theta}{4\pi^2}\vec{\nabla}A_0.\vec{B} + \ldots . \\
&= \frac{\theta}{4\pi^2}A_0\vec{\nabla}.\vec{B}
\end{aligned}$$

A remarkably simple wavefunction

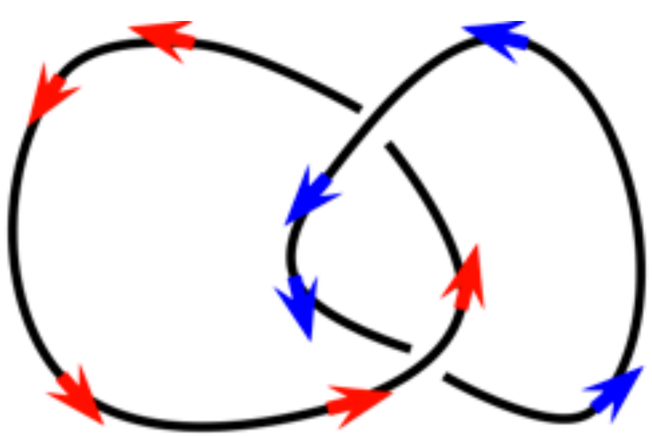
Wang, TS, 15

Simplest in terms of magnetic field lines.

Below monopole gap, magnetic field lines form closed loops.

Electric charge-1/2 of monopole => when 2 magnetic loops link there is a (-1) phase.

Ground state = superposition of (oriented) magnetic loops with this linking phase:

$\Psi\Big[$  $\Big] \sim -1$

Contrast with $e_b m_{b:}$

In magnetic field representation, $e_b m_{b:}$ is superposition of (oriented) magnetic loops with positive weights.

The different (-) sign linking phase changes the state dramatically:
e-particle becomes a Kramers fermion with topological band structure.

A better understanding of the linking phase

Why exactly does (-) sign linking phase of magnetic loops have these effects?

(-) linking phase => monopoles are ``dyons'' with charge ±1/2.

Consider bound state of charge-1/2 ``dyon'' with charge-1/2 ``antidyon''.

Bound state carries electric charge-1 and magnetic charge-0 => identify with e-particle.

What are its time reversal properties?

A better understanding of the linking phase (cont'd)

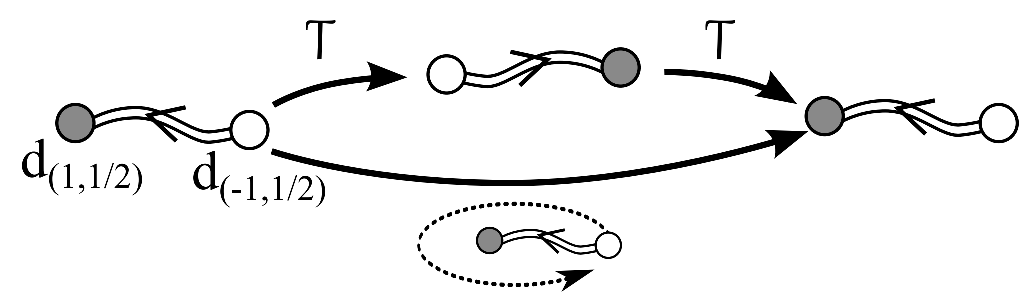
Wang, Potter, TS, 13
Metlitski, Kane, Fisher, 13

``Dyon" and ``Anti-dyon" with same electric charge transform into each other.

$$\mathcal{T}^{-1} d_{(1,1/2)} \mathcal{T} = d_{(-1,1/2)} \quad (1)$$
$$\mathcal{T}^{-1} d_{(-1,1/2)} \mathcal{T} = d_{(1,1/2)}$$

$d_{(q_m,q_e)}$ is dyon operator with magnetic charge q_m and electric charge q_e.

These two see each other as relative monopoles.



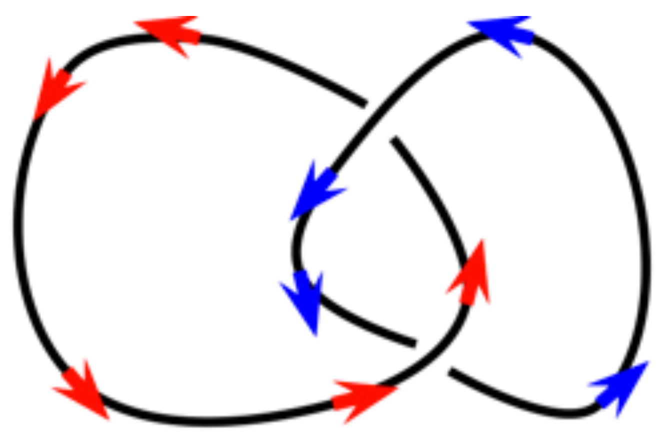
=> In bound state, EM field angular momentum is half-integer.

Under T^2 action, this `orbital' part leads to T^2 = -1.

=> Bound state (= e-particle) must be Kramers doublet, and must be a fermion.

End of proof.

A prediction for numerics

$\Psi\Big[$  $\Big] \sim -1$

At interface with vacuum (or with a state with no linking phase) this state will have powerlaw correlations characteristic of a fermion surface Dirac cone.

T-reversal symmetric U(1) spin liquids with a bosonic monopole

Total of 5 distinct phases

Name	e particle	m particle
$e_b m_b$	boson, $T^2 = 1$	boson
$e_f m_b$	fermion, $T^2 = 1$	boson
$e_b T m_b$	boson, $T^2 = -1$	boson
$e_f T m_b$	fermion, $T^2 = -1$	boson
$(e_f T m_b)_\theta$	fermion, $T^2 = -1$	boson, `electric' charge 1/2

Fermionic monopole

Can e also be a fermion?

No!!

\`All-fermion' U(1) gauge theory with fermion statistics for both e and m forbidden in strict 3+1-d (*): a non-emergeable field theory.

Proof (biproduct of recent classification of interacting electronic topological insulators): Wang, Potter, TS, Science 2014 (Appendix).

Key idea: Can think of such a phase as a (gauged) putative topological insulator of fermionic e particles.

Show such a putative topological insulator does not have a consistent surface in the right Hilbert space.

(*): Can arise at boundary of a 4+1- d theory (Kravec, McGreevy, Swingle, 14).

Fermionic monopole

e must be a boson.

Distinct possibilities generated by endowing the fermionic monopole with topological band structure.

Topological insulators of the Fermionic monopole

Symmetries: Monopole transforms under $U_g(1)$ x T.

($U_g(1)$: magnetic gauge transformation).

Free fermions with *global* symmetry U(1) xT can form topological band structure classified by Z, i.e, indexed by an integer n. (Schnyder et al 2009, Kitaev 2009).

With interactions this collapses to a Z_8 classification (Wang, TS, 14).

Of these n = 4 is protected by T alone.

=> if the U(1) is gauged, only n = 0, 1, 2 give distinct U(1) spin liquids.

T-reversal symmetric U(1) spin liquids with a fermionic monopole

Distinguish different n by effect on e particle. (Wang, TS, 14)

n = 0: e is Kramers singlet boson => $e_b m_f$.

n = 1: $\theta = \pi$ response => `dual Witten effect'

e is boson with magnetic charge 1/2. (denote $(e_b m_f)_\theta$).

n = 2: e is Kramers doublet boson => $e_b T m_f$.

T-reversal symmetric U(1) spin liquids with a fermionic monopole

Total of 3 distinct phases.

Name	e particle	m particle
$e_b m_f$.	boson, $T^2 = 1$	fermion
$(e_b m_f)_\theta$	boson, `magnetic charge' 1/2	fermion
$e_b T m_f$.	boson, $T^2 = -1$	fermion

T-reversal symmetric U(1) spin liquids: Full classification

Wang, TS, 15

8 distinct phases.

Name	e particle	m particle
$e_b m_b$	boson, $T^2 = 1$	boson
$e_f m_b$	fermion, $T^2 = 1$	boson
$e_b T m_b$	boson, $T^2 = -1$	boson
$e_f T m_b$	fermion, $T^2 = -1$	boson
$(e_f T m_b)_\theta$	fermion, $T^2 = -1$	boson, \`electric' charge 1/2
$e_b m_f$.	boson, $T^2 = 1$	fermion
$(e_b m_f)_\theta$	boson, \`magnetic charge' 1/2	fermion
$e_b T m_f$.	boson, $T^2 = -1$	fermion

Quantum spin ice

Presumably monopole will be a boson.

If the microscopic spin is a Kramers doublet (as in $Yb_2Ti_2O_7$), then $e_b m_b$, $e_b T m_b$, $e_f T m_b$, $(e_f T m_b)_\theta$ are possible candidates.

Neutrons will see 2 thresholds for continuum scattering - one associated with the m gap, and the other associated with the e gap (if e is Kramers).

If the microscopic spin is a non-Kramers doublet (as in $Pr_2Zr_2O_7$), then $e_b m_b$, $e_f m_b$ are candidates (harder to distinguish).

Remarks

1. The different T-reversal symmetric U(1) spin liquids are obtained from one another by putting either e or m in an interacting topological insulator phase.

Various U(1) spin liquids = gauged versions of interacting topological insulators of bosons/fermions.

2. Statistics changing phase transition, eg

$$e_b m_b \longleftrightarrow e_f m_b$$

View alternately as phase transition between (gauged versions) of trivial insulator and a topological insulator of the bosonic monopoles.

=> Possible route to progress on a highly non-trivial phase transition.

Remarks (cont'd)
Symmetry and gapped quantum spin liquids

Non-trivial symmetry implementations in gapped quantum spin liquids:

- fractionalization of symmetry quantum numbers (``projective realizations'' of symmetry)

- symmetry operations may even exchange different topological sectors in a topological phase.

Much recent understanding for d = 2 gapped phases (eg, Essin, Hermele 13, Vishwanath, TS 13, Geraedts, Motrunich 13, Lu, Vishwanath, 13, Wang, TS 13, Fidkowski et al, 14, Cho et al 14,
Barkeshli et al, 14);

Some (incomplete) progress in d = 3 gapped Z_2 quantum spin liquids.