Non-local quantum entanglement in macroscopic matter

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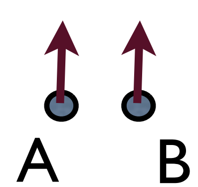
ICTS Chandrasekhar lectures, Jan 2015.

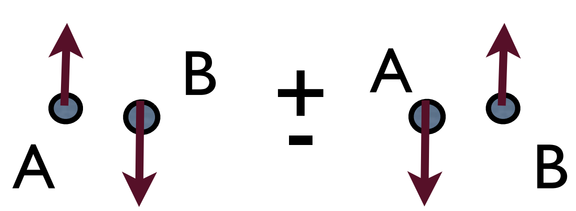
Entanglement in quantum mechanics

Two parts A and B of a quantum mechanical system may be ``entangled'' with each other.

Example: Spin orientations of two electrons in a simple molecule



unentangled; each spin by itself in a definite quantum state

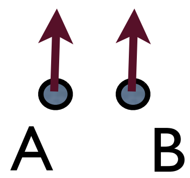


entangled: each spin by itself not in a definite quantum state though full system is.

I would not call entanglement ***one*** but rather ***the*** characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

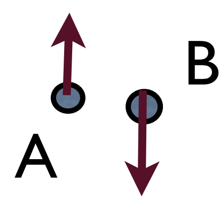
E. Schrodinger, 1935

The relation of a part to the whole

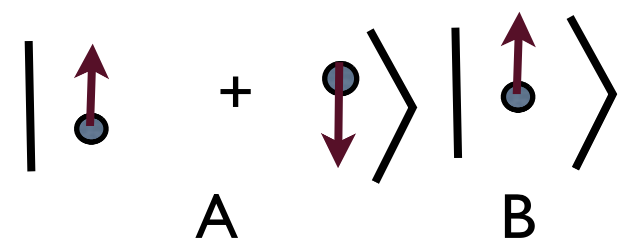


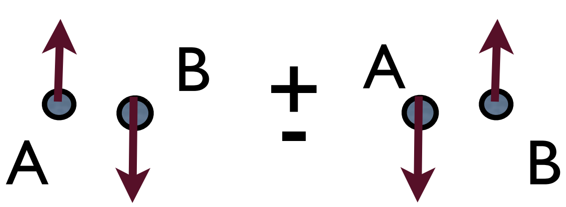
Unentangled parts: wavefunction of whole system factorizes into a product of wavefunctions of parts.

Other examples:



or even





Entangled parts: Wavefunction of full system does not factorize as products of wavefunction of parts.

Cannot describe one part fully without the other.

Macroscopic quantum matter

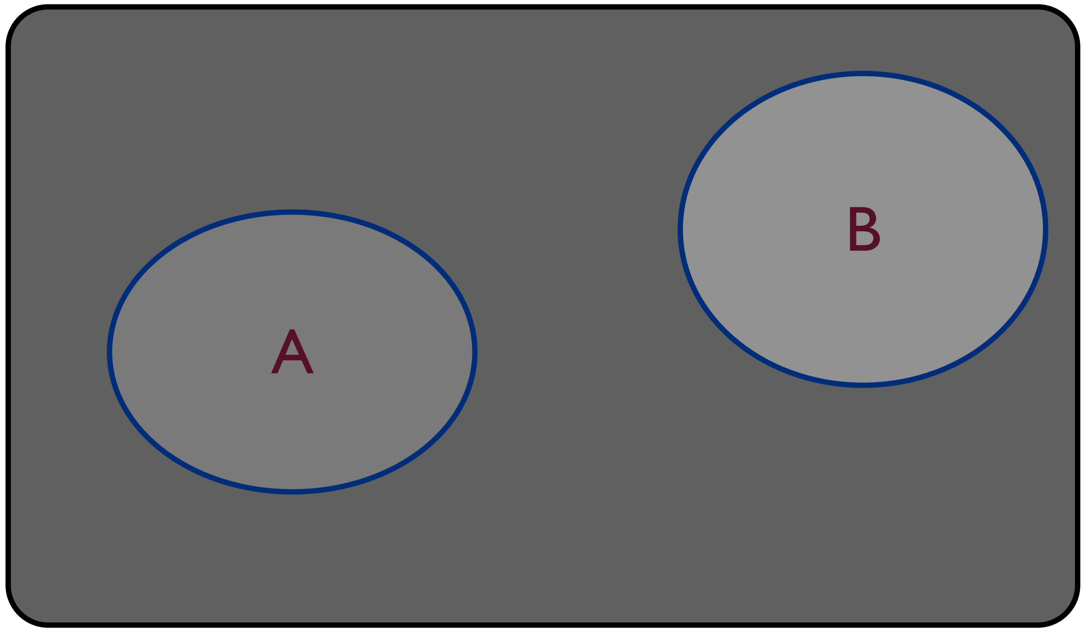
Ordinary macroscopic matter: Large number (10^{23}) of interacting degrees of freedom which must be treated quantum mechanically.

Example: electrons inside a macroscopic piece of solid.

Room temperature is cold: electron motion is quantum rather than thermal.

Entanglement and macroscopic matter

How are the different parts of a piece of macroscopic matter entangled quantum mechanically with each other?



A deep and fundamental question.........

Importance only became clear in last few years.

Very fruitful in our ongoing attempt to characterize distinct phases of quantum matter.

Entanglement between A and B?

Phases of matter

Macroscopic matter in equilibrium organizes itself into phases.

Solids, liquids, gases.......

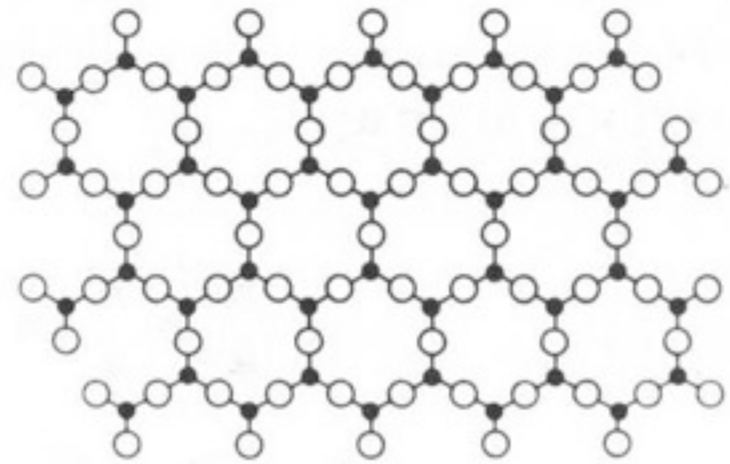
Magnets........

Superconductors.............

Organizing principles: Long Range Order(LRO) and broken symmetry

Example: crystalline solid.

Atoms arrange themselves into an ordered array.



Pattern of atomic positions in one region determines atomic positions far away.

Broken symmetry: Microscopic interactions invariant under translating all atoms but equilibrium state is not.

General consequences of broken symmetry

Pattern of broken symmetry determines many macroscopic properties of ordered matter.

Examples: rigidity of solids, persistence of currents in a superconductor, etc.

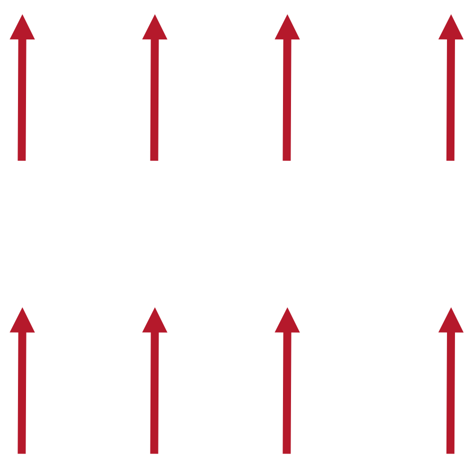
Broken symmetry point of view: unifying theoretical framework for many seemingly distinct properties of matter.

Magnetism: an illustrative example

Most familiar form of magnetism:
ferromagnetism.

Discovered may be around 600 BC.

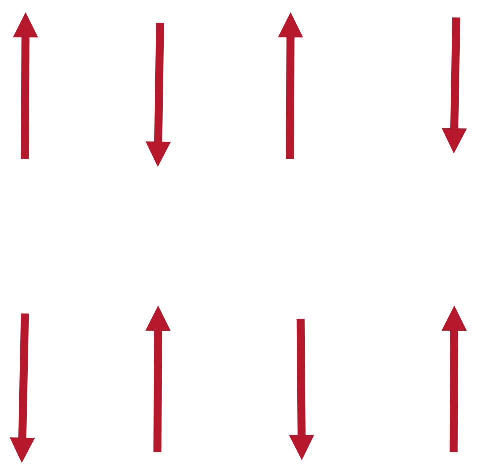
Microscopic picture: Electron spins inside
magnet are all pointed in same direction.



Example of broken symmetry: Microscopic interactions do not pick direction for spin but macroscopic magnetized state has specific spin orientation.

Antiferromagnetism: The more common magnetism

Actually the more common form of magnetism is not the familiar ferromagnetism but is ``antiferromagnetism”.



Also a broken symmetry state -
spin orientation frozen in time but oscillates in space
Microscopic interactions allow any orientation.

Despite being more common antiferromagnetism was discovered only in the 1930s!

Ferromagnetism: easily detected.

Antiferromagnetism: need microscopic probes that sense spin orientation with atomic spatial resolution.

Quantum description of magnetism

The essential properties of these magnetic states of matter is contained in their ground state wavefunction.

Example: Prototypical wavefunctions

Ferromagnet $|\uparrow\uparrow\uparrow\uparrow \cdots\cdots\cdots\rangle$

Antiferromagnet $|\uparrow\downarrow\uparrow\downarrow \cdots\cdots\cdots\rangle$

Prototypical wavefunctions capture the pattern of broken symmetry which holds the key to many macroscopic properties of these phases.

A crucial observation

To determine pattern of symmetry breaking need to only look at a small part of the full system.

$$| \uparrow\uparrow\uparrow\uparrow \ldots\ldots\ldots \rangle$$

$$| \uparrow\downarrow\uparrow\downarrow \ldots\ldots\ldots \rangle$$

Information on broken symmetry is stored locally.

More precise: short range entanglement

For familiar magnetic states,
prototypical ground state wavefunction factorizes as
direct product of local degrees of freedom

$$|\uparrow\uparrow\uparrow\uparrow\ \cdots\cdots\cdots\rangle$$

$$|\uparrow\downarrow\uparrow\downarrow\ \cdots\cdots\cdots\rangle$$

Quantum entanglement short ranged in space.

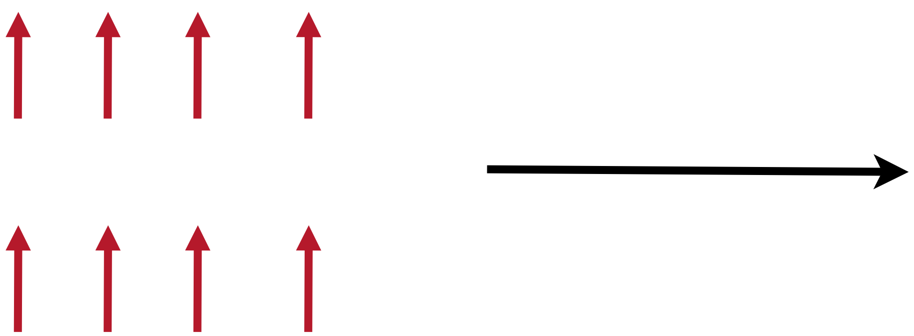
1930s- present: elaboration of broken symmetry and other states with Short Range Entanglement (SRE).

Emergence of classical physics

Broken symmetry states of magnetism:

Macroscopic description in terms of classical physics of the ``thing'' that orders.

Example: spontaneous magnetization of a ferromagnet.



Microscopic quantum spins

Macroscopic classical magnet

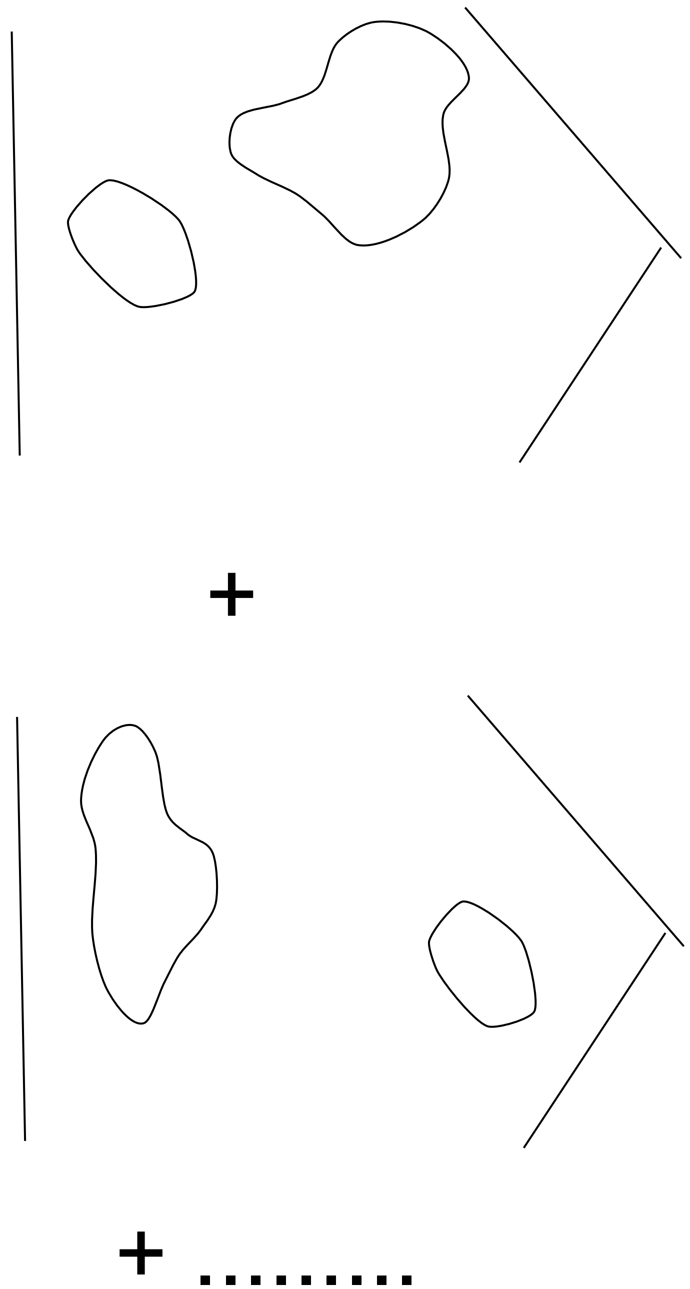
Modern times

Experimental discovery of a *qualitatively* new kind of magnetic matter.

Popular name: ``quantum spin liquid"

Prototypical ground state wavefunction

Not a direct product of local degrees of freedom.

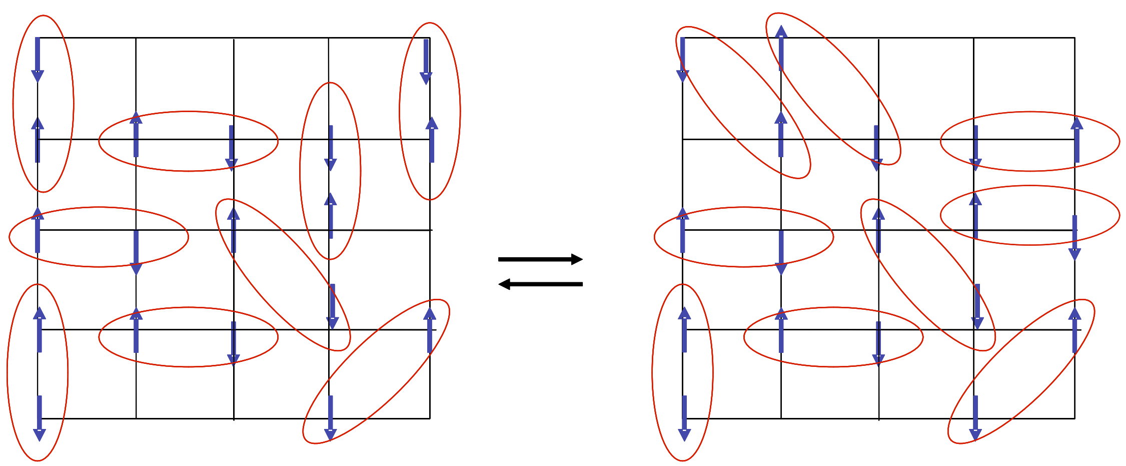


Quantum entanglement is long ranged in space.

* In d > 1

What is a quantum spin liquid?

Rough description: Spins do not freeze but fluctuate in time and space due to quantum zero point motion.



Resonance between many different configurations (like in benzene)
In each configuration each spin forms an ***entangled pair*** with one other partner spin.

Envisaged by P.W. Anderson (1973, 1987); older suggestion of Pauling (1950s)

Long Range Entangled (LRE) phases

Universal information about state not visible by looking only at small part of system.

Passage from microscopic to macroscopic scales - classical physics does not emerge.
(contrast with broken symmetry phases, eg, ferromagnet)

Older very famous example: ``Fractional quantum Hall states”.

Other fascinating examples: metallic ground state of electrons.

Conventional metals: ``Fermi Liquid” state (oldest familiar Long Range Entangled state)

Many new metals: ``Non-fermi liquids”

Can quantum spin liquid phases exist?
Question for theory

Yes!!! (work of many people over last 20 years)

Many dramatic phenomena seen to be theoretically possible.

Examples:

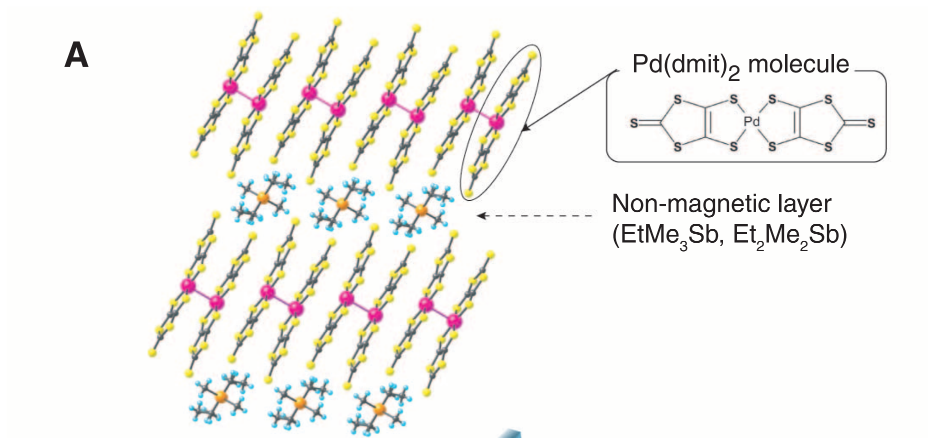
1. Electron can break apart into fractions

2. Emergence of long range quantum mechanical interactions between fractional pieces of electron.

Similar phenomena established in FQHE in two dimensions but now are known to be possible in much less restrictive situations.

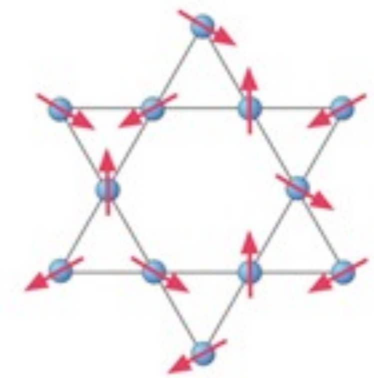
Do quantum spin liquid phases exist?
Question for experiment

Yes - many interesting candidate materials in last few years!!



Layered organic crystals $\kappa-(ET)_2Cu_2(CN)_3$ Kanoda et al, 2003-now

$EtMe_3Sb[Pd(dmit)_2]_2$ Kato et al, 2008



Some layered inorganic minerals

$ZnCu_3(OH)_6Cl_2$ (Y. Lee, Nocera et al, 2007)

Herbertsmithite

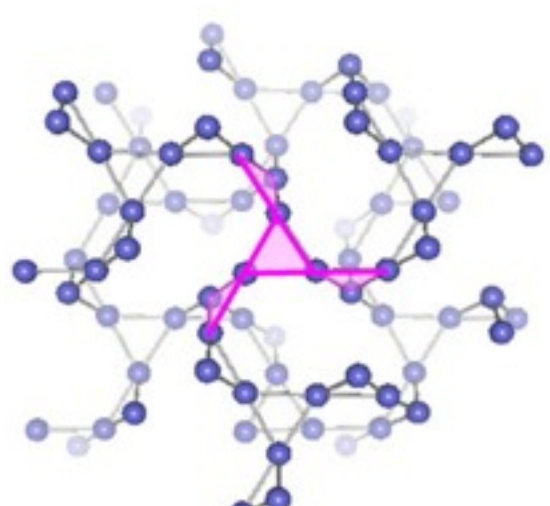
$Cu_3V_2O_7(OH)_2\cdot 2H_2O$ (Z. Hiroi et al, 2010)

Volborthite

Three dimensional transition metal oxide

$Na_4Ir_3O_8$

(H. Takagi et al, 2008)



Some phenomena in experiments

Quantum spin liquid materials are all electrical insulators.

Despite this many properties other than electrical conduction are very similar to that of a metal.

Two examples at low temperature:

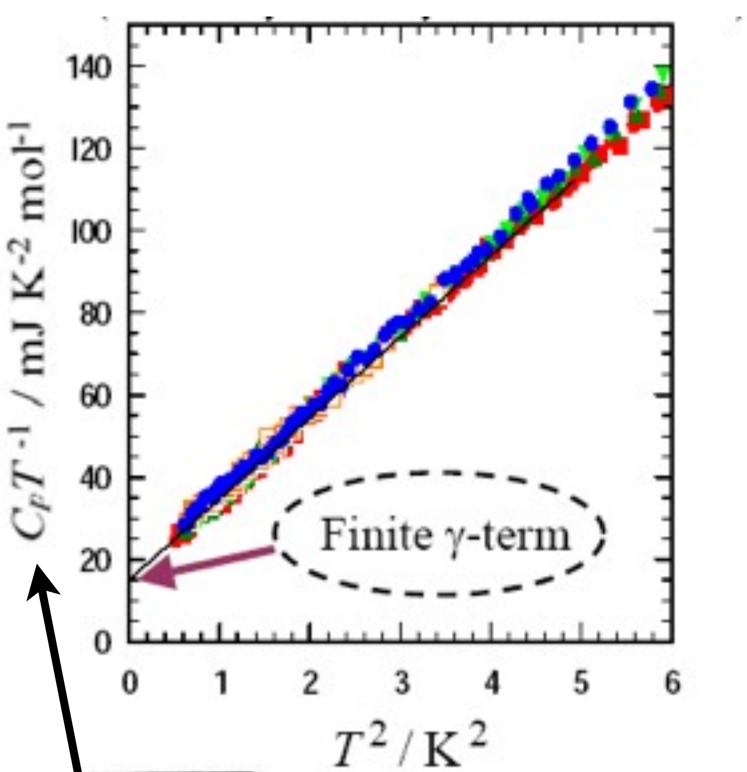
1. Entropy very similar to that of a metal at low temperature

2. Conduct heat just like a metal even though they are electrical insulators.

Very strange......not known to happen in any ordinary insulator.

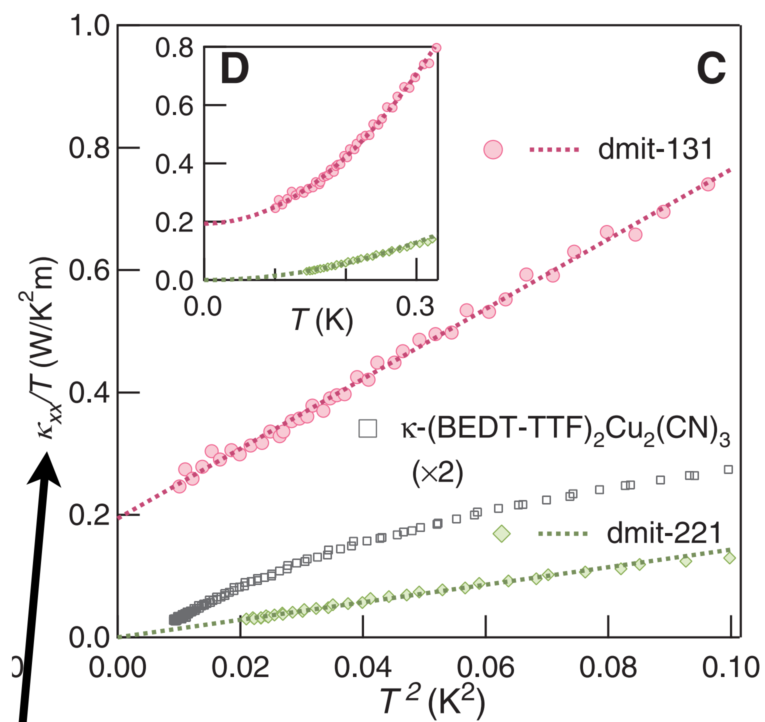
Some phenomena in experiments

S. Yamashita et al, Nat Phys, 2008



Heat capacity

M.Yamashita et al, Science 2010



Thermal conductivity

These are both exactly like in a metal but were measured in an insulator.

Towards understanding experiments

Low-T properties of metals are determined by mobile electrons obeying Pauli exclusion principle.

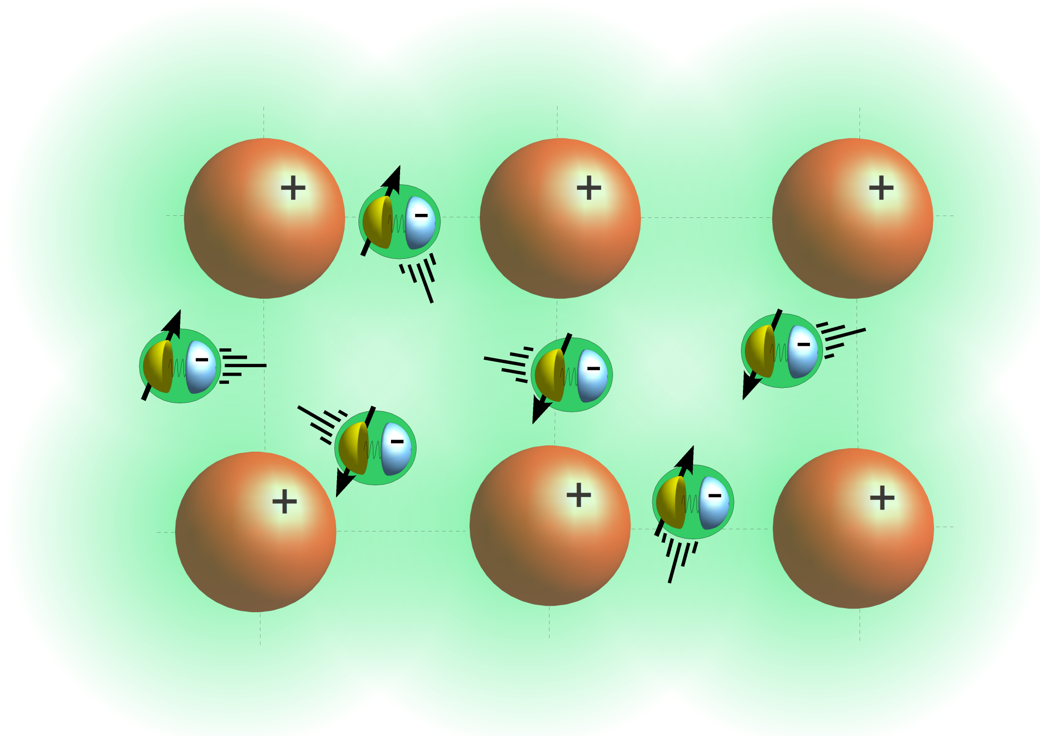
In an insulator there cannot be mobile electrons.

A promising idea: perhaps there are emergent particles obeying Pauli exclusion that carry the electron spin but not its charge inside these materials.

Such phenomena are known to be theoretically possible in Long Range Entangled phases (but are prohibited if there is only Short Range Entanglement)

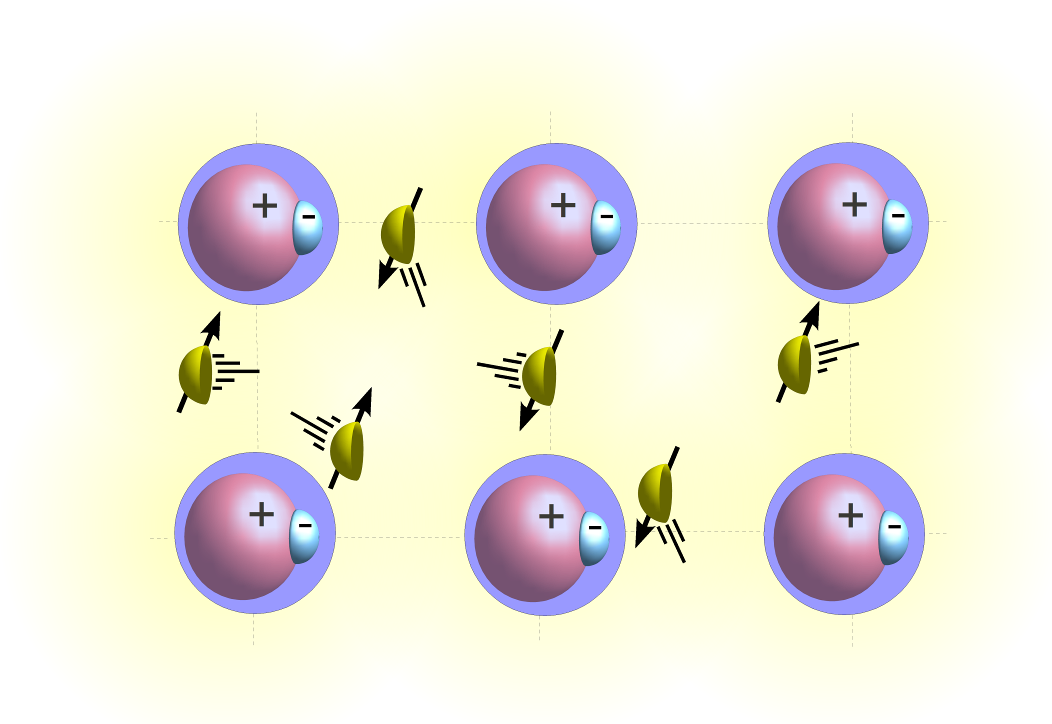
Picture of a quantum spin liquid

Metal



Electrons swimming in sea of +vely charged ions

A quantum spin liquid



Electron charge gets pinned to ionic lattice while spins continue to swim freely.

Remarks

A wide variety of distinct kinds of quantum spin liquid phases can exist
- distinct physical properties and low energy effective field theories.

A gross distinction: Gapped versus gapless

Gapped spin liquids - many interesting properties (``topological order”).

Gapless spin liquids: ``Beyond topological order”
Relevant to current experiments, and as a platform for understanding other phenomena (eg, some metal-insulator phase transitions).

Challenge for theory

Construct effective field theories of distinct quantum spin liquid phases, other similar long range entangled phases /associated quantum phase transitions.

Effective field theory in condensed matter physics

Microscopic models (e.g, Hubbard/t-J, lattice spin Hamiltonians, etc)

`Low energy' effective field theory

`Low energy' experiments/ phenomenology

Effective field theory: *minimal* requirements/ challenges

1. **`Tractable':** Must be simpler to understand than original microscopic models and relate to experiments

- continuum field theory often useful but not necessarily of the kind familiar from high energy physics.

Effective field theory:*minimal* requirements/ challenges

1. **`Tractable':** Must be simpler to understand than original microscopic models and to relate to experiments

- continuum field theory often useful but not necessarily of the kind familiar from high energy physics.

2. **`Emergeable':** A proposed low energy field theory must (at the very least) be *capable of emerging* from microscopic lattice models in the *`right' physical Hilbert space* with the *right symmetries*.

- demonstrate by calculations on `designer' lattice Hamiltonians (see Kaul, Sandvik talks)

Designer Hamiltonians do not need to be realistic to serve their purpose.

Conventional condensed matter physics

Hartee-Fock + fluctuations

Structure of effective field theory:

Landau quasiparticles + broken symmetry order parameters (if any).

Phase	Field Theory	Sample designer Hamiltonian
Metals	Fermi Liquid Theory	Eg,Jellium
Superconductors	Landau-Ginzburg + Bogoliubov quasiparticles	Reduced BCS Hamiltonian
Antiferromagnets	Nonlinear sigma model	Eg, Nearest neighbour Heisenberg

Quantum spin liquids/other exotic phases

What are the useful degrees of freedom for formulating an effective field theory?

Field theory not necessarily in terms of electrons + Landau order parameters.

A powerful approach:

Slave particles (partons): Fractionalize spin/electron into partons which are then coupled to fluctuating emergent gauge fields.

Slave particle framework

Slave particle construction

$$\vec{S}_r = f^\dagger_{r\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} f_{r\beta}$$

$f_{r\alpha}$: fermionic 'spinon' with spin α.

Constraint $f^\dagger_r f_r = 1$ ensures physical Hilbert space.

Redundant description, e.g., can let $f_{r\alpha} \to e^{i\theta_r} f_{r\alpha}$.

Can alternately represent using bosons

$$\vec{S}_r = b^\dagger_{r\alpha} \frac{\sigma_{\alpha\beta}}{2} b_{r\beta}$$

with constraint $b^\dagger_r b_r = 1$.

Slave particle framework (cont'd)

Strategy: Put f in some mean field state with a quadratic Hamiltonian.

Examples:

1. 'spinon metal'

$$H_{MF} = -t_f \sum_{rr'} f_r^\dagger f_{r'} + h.c$$

Spin physics similar to metal

2. 'Paired'

$$H_{MF} = -t_f \sum_{rr'} f_r^\dagger f_{r'} + \Delta_{rr'}(f_{r\uparrow} f_{r'\downarrow} - f_{r\downarrow} f_{r'\uparrow}) + h.c$$

Spin physics similar to superconductor

Mean field theory for highly non-trivial quantum spin liquid insulators

Fluctuations

Mean field Hamiltonian breaks gauge redundancy down to a subgroup.

Fluctuations beyond mean field: must couple f to gauge fields in that subgroup.

Effective field theory: spinon + fluctuating gauge fields.

Use to address stability (`lower critical dimension', etc) and predict testable physical properties.

Example:

1. `Spinon metal':

Spinon Fermi surface + fluctuating U(1) gauge field.

2. Paired spin liquid

Spinons + fluctuating Z_2 gauge field.

Comments on slave particle framework

1. **Conceptually important** construction of effective field theories of several exotic quantum phases/phase transitions that are beyond standard quasiparticles

2. Slave particle effective field theories are ***emergeable***, and often tractable.

Theoretical demonstration of many unusual phenomena; some successful contact with experiments (eg, FQHE, 1/2-filled Landau level, some quantum spin liquids)

Open question: Are there quantum spin liquids that are `beyond slave particles'?

Some modern conceptual questions

1. Symmetry and long range entanglement (Wednesday talk)

2. Novel continuous quantum phase transitions (Thursday talk)

Some modern conceptual questions

1. Symmetry and long range entanglement (Wednesday talk)

2. Novel continuous quantum phase transitions (Thursday talk)

Symmetry and Long Range Entanglement (LRE)

Physical global symmetry may be realized by excitations in a non-trivial way in a Long Range Entangled phase

Eg: Fractional charge in the FQHE

Many open questions and surprises (see tomorrow talk).

A simpler question: Symmetry and Short Range Entanglement

Physical global symmetry may protect distinctions even between short range entangled states.

Eg: Topological band insulators, Haldane spin chains......

``Symmetry Protected Topological'' Phases.

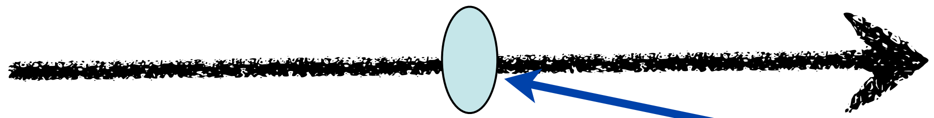
Surprisingly studying these has generated lots of new insights into more complex Long Range Entangled phases.

Some modern conceptual questions

1. Symmetry and long range entanglement (Wednesday talk)

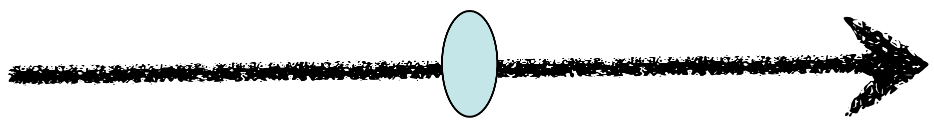
2. Novel continuous quantum phase transitions (Thursday talk)

Landscape of quantum phase transitions



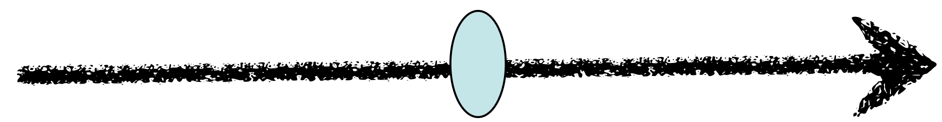
LRO, no LRE — no LRO, no LRE

Landau-Ginzburg-Wilson paradigm only works here!



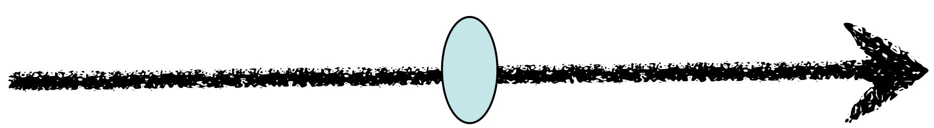
LRE, no LRO — no LRO, no LRE

Eg: Quantum spin liquid - trivial paramagnet



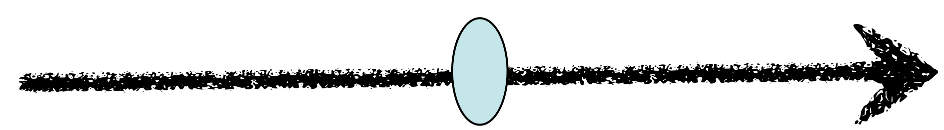
LRO, no LRE — LRE, no LRO

Eg: AF - quantum spin liquid



LRO, no LRE — LRO', no LRE

Eg: AF - Valence Bond Solid in spin-1/2 square lattice



LRE — LRE'

Eg: FQHE plateau transitions

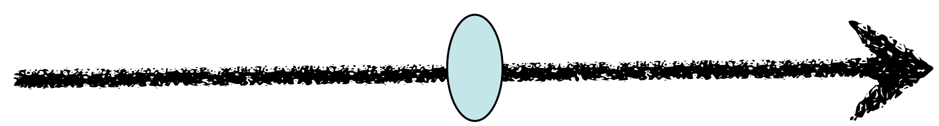
Note: quantum critical points have Long Range Entanglement

Landau-forbidden phase transitions between Landau-allowed phases

TS, Vishwanath, Balents, Fisher, Sachdev, 2004

Naive expectation: Breakdown of LGW paradigm at QCP when one of the two proximate phases has non-Landau order.

Very interesting that LGW can also break down at critical point between two Landau-allowed phases.



LRO, no LRE   LRO', no LRE

Eg: AF - Valence Bond Solid in spin-1/2 square lattice

Landau-forbidden critical point: Critical theory described in terms of emergent gapless spinons coupled to emergent gauge fields.

``Deconfined Quantum Critical Points"

Many other proposed examples by now.

Future prospects: short term

1. Combined theory/experiment effort to characterize currently existing quantum spin liquids/other LRE phases.

??Directly demonstrate non-local entanglement in experiment??

2. Theory predicts possibility of wide variety of such exotic phases of magnetic matter.

Future prospects: long term

In the last 3 decades, growing number of experimental discoveries* have dethroned all the ``textbook'' paradigms of the old field of solid state physics.

Some of these we understand; most of these we do not.

Our eyes have been opened to a new ***truly quantum*** world of 10^{23} electrons.

Characterizing ``***pattern of entanglement***'' in macroscopic quantum matter promises to be as rich and profound as the previous century's efforts at characterizing broken symmetry.

*FQHE (1982), high temperature superconductivity (1987), strange metals where electron-like charge carriers do not exist, quantum spin liquid magnets