Time Scales and Self-Averaging in Many-Body Quantum Dynamics

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Thouless and relaxation time scales in many-body quantum systems, PRB **99**, 174313 (2019)

Self-averaging in many-body quantum systems out of equilibrium, ArXiv: 1906.11856

Self-averaging behavior at the metal-insulator transition…, ArXiv: 1910.11332

Time Scales and Self-Averaging

Many-body quantum systems out of equilibrium.
Time-independent Hamiltonians.

Questions:

- What are the **time scales** involved in relaxation process?
  (**long time scales**)
  *) time for the properties of the spectrum to get manifested
  *) time to reach equilibrium

PRB **99**, 174313 (2019)

- How does the self-averaging behavior depend on?
  *) quantity
  *) model (**chaos** does **not** imply self-averaging)
  *) **time scale**

ArXiv: 1906.11856

ArXiv: 1910.11332

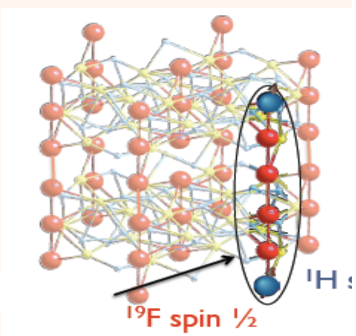
Coherent Evolution in Experiments

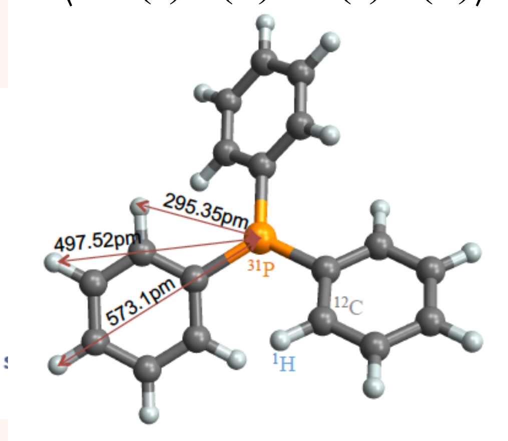
NMR

They are collectively addressed with magnetic pulses;
Slow relaxation

Li, Fan, Wang, Zeng et al (Beijing, Anhui, Hubei)
Levstein, Pastawski (Cordoba)
Cory (Waterloo)
Cappellaro (MIT)

$$\langle W^{+}(t)V(0)^{+}W(t)V(0)\rangle$$



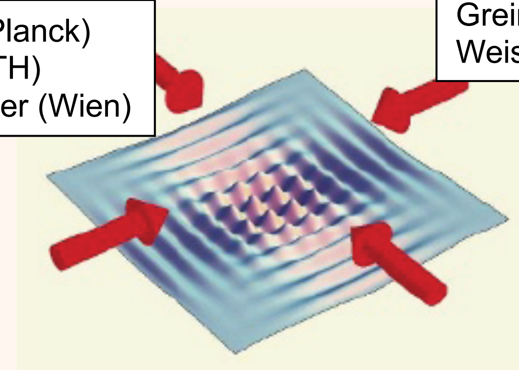


Ultracold Gases

Dynamics under designed potentials.

Bloch (Max Planck)
Esslinger (ETH)
Schmiedmayer (Wien)

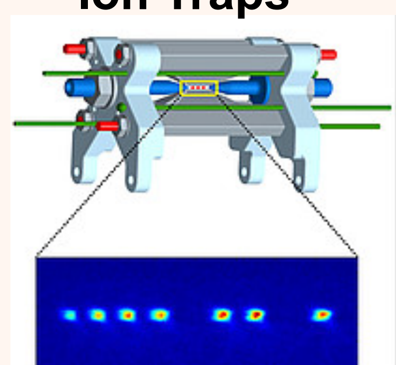
Greiner (Harvard)
Weiss (Penn Sate)



- highly controllable systems – interactions, level of disorder, 1,2,3D (simple models)
- quasi-isolated -- study evolution for very long time

Ion Traps

Ions trapped via electric and magnetic fields.
Laser used to induce couplings.
Isolated from an external environment.



Blatt (Innsbrück)

Monroe (Maryland)

Equilibration

Time-independent Hamiltonian

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

$$\Big\downarrow \quad C_\alpha^{ini} = \langle\alpha|\Psi(0)\rangle$$

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha^{ini} e^{-iE_\alpha t}|\alpha\rangle$$

Time-independent Hamiltonian

$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

$$C_\alpha^{ini} = \langle \alpha | \Psi(0)\rangle$$

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha^{ini} e^{-iE_\alpha t} |\alpha\rangle$$

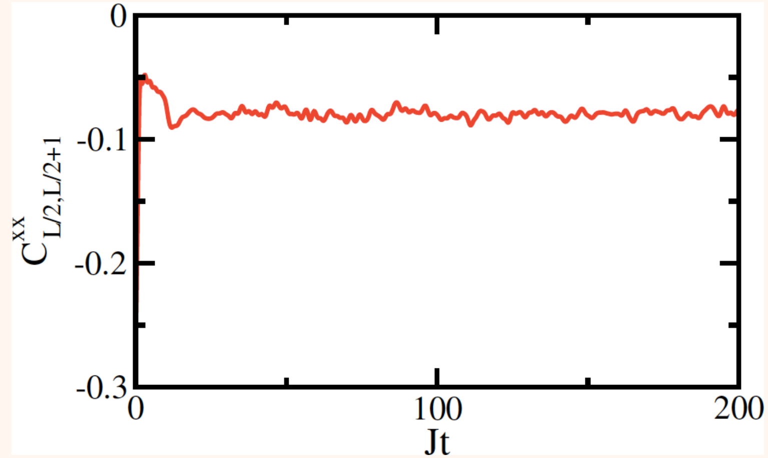
$$\boxed{\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}}$$

$$\langle \Psi(t)| = \sum_\beta C_\beta^{ini*} e^{iE_\beta t} \langle \beta|$$

$$\langle \beta|O|\alpha\rangle \qquad \langle \alpha|O|\alpha\rangle$$

Equilibration

Equilibration in
an isolated many-body quantum system



$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

$$C_\alpha^{ini} = \langle \alpha | \Psi(0)\rangle$$

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} \underbrace{O_{\beta\alpha}}_{\langle\beta|O|\alpha\rangle} + \boxed{\sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}}$$

Proximity to equilibrium for the majority of time

Fluctuations decay exponentially with system size.

Zangara et al,
PRE **88**, 032913 (2013)

Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \boxed{\sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}}$$

Infinite time average   Thermodynamic average

$$O_{\alpha\alpha} = \langle\alpha|O|\alpha\rangle$$

$$\boxed{\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{\mathcal{N}_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha| < \Delta E}} O_{\alpha\alpha}}$$

depends on the initial conditions   depends only on the energy

Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics,
L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol,
Adv. Phys. **65**, 239 (2016)

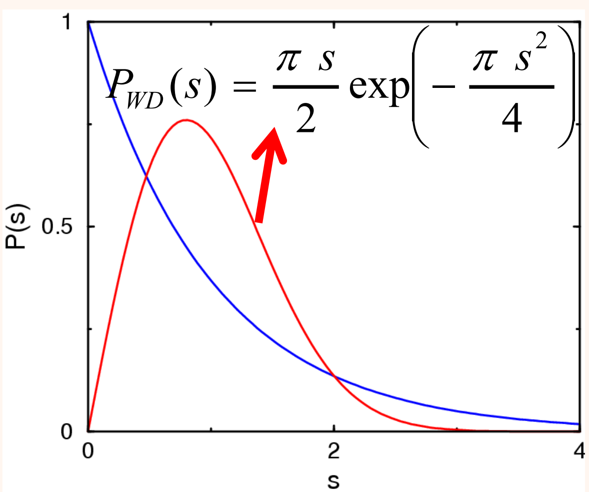
Chaos in Realistic Many-Body Quantum Systems

Full Random Matrices vs Realistic Models

Full random matrices

Correlated eigenvalues
(level repulsion)

Wigner-Dyson distribution
(time reversal symmetry)

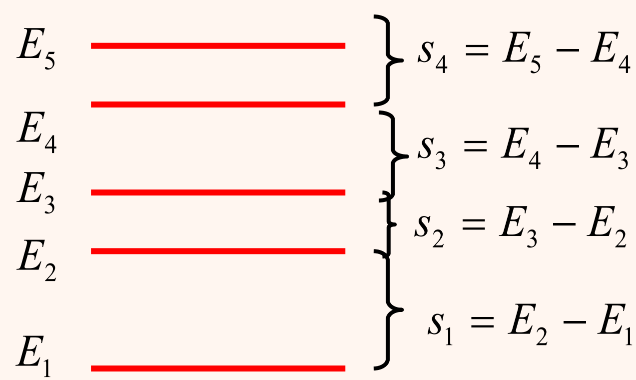


Realistic chaotic models:

*) Spin models
*) Bose Hubbard model

*) Embedded Random Ensembles
(= SYK model)
*) Wigner Banded Random Matrices

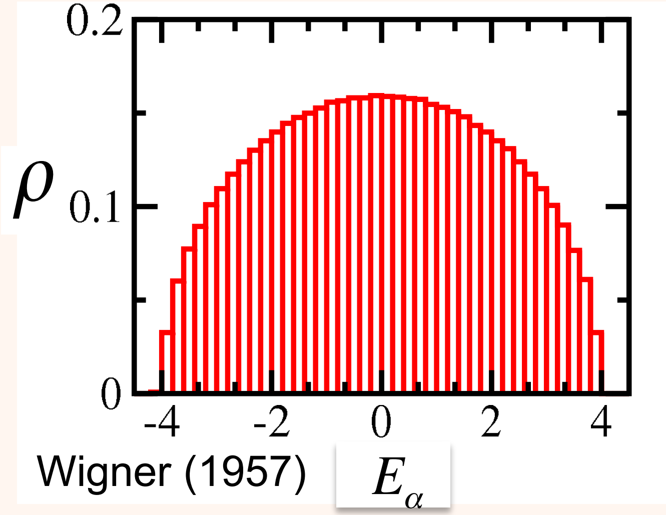
Level spacing distribution



Full Random Matrices vs Two-Body Interaction

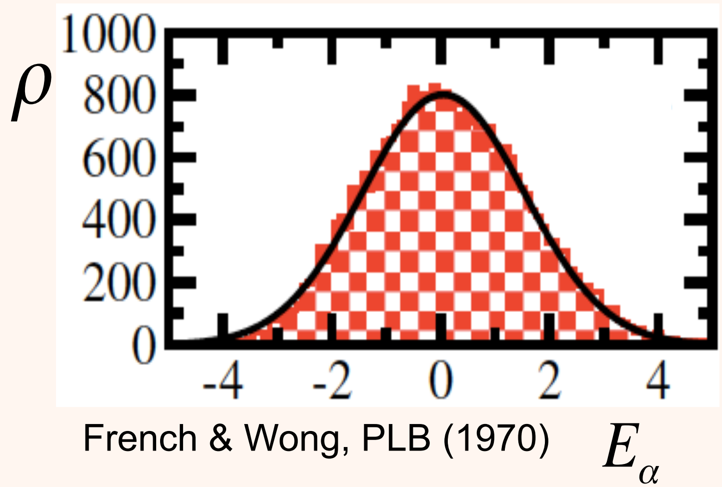
Density of States (Energy Distribution)

Full random matrices: semicircular



Wigner (1957)

Two-body interactions: Gaussian



French & Wong, PLB (1970)

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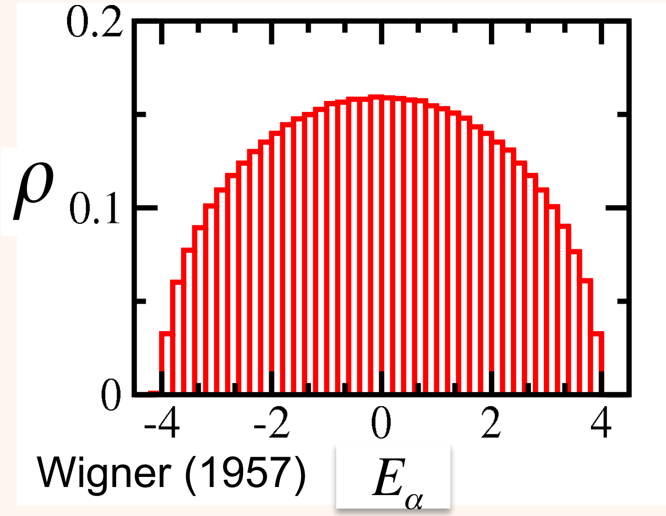
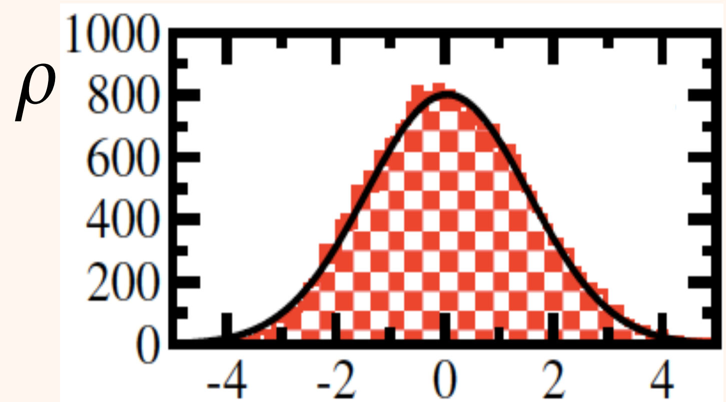
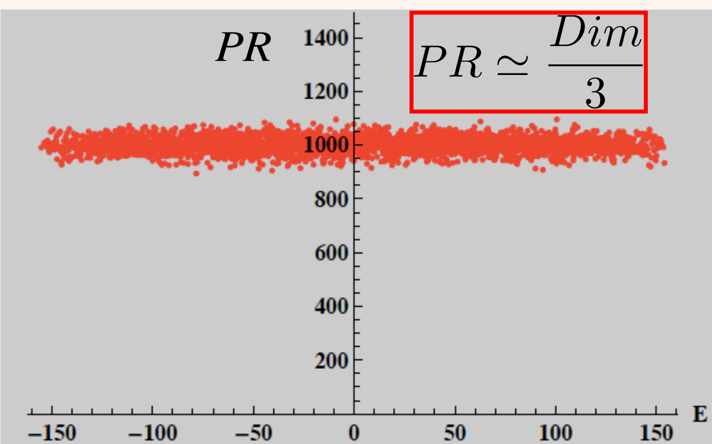


Wigner (1957)



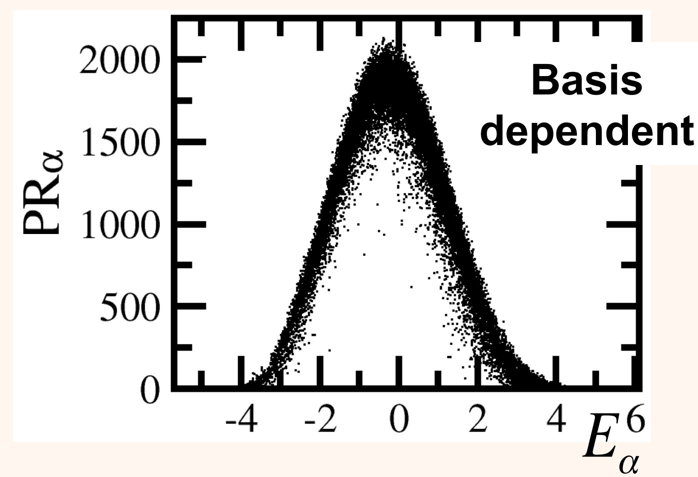
French & Wong, PLB (1970) E_α

Participation Ratio



$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} | c_i^{(\alpha)} |^4}$$

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$



DYNAMICS
and
TIME SCALES

- Manifestations of spectral correlations in the time domain
- Dynamics in the many-body Hilbert space

Dynamics

$$H_0|\Psi(0)\rangle = \mathcal{E}|\Psi(0)\rangle$$

$$H = H_0 + V$$ (very strong perturbation)

Dynamics

$$H_0|\Psi(0)\rangle = \mathcal{E}|\Psi(0)\rangle$$

$$\uparrow$$

$$H = \underbrace{H_0 + V}$$ (very strong perturbation)

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

Energy of the initial state:

$$E_0 = \langle\Psi(0)|H|\Psi(0)\rangle \sim 0$$

Models

$$\boxed{H = H_0 + V}$$

GOE Full Random Matrices

H_0 Diagonal matrix,
Gaussian real random numbers

$$\langle H_0^2 \rangle = 2$$

V Off-diagonal matrix,
Gaussian real random numbers

$$\langle V^2 \rangle = 1$$

Models

$$\boxed{H = H_0 + V}$$

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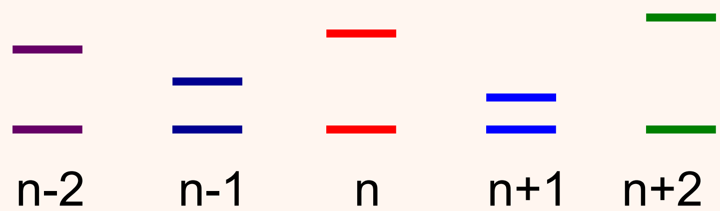
<u>Disordered Spin Model</u>

$$H_0 = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} \sigma_n^z \sigma_{n+1}^z$$



Random numbers
$h_n \in [-h, h]$

Models

$$\boxed{H = H_0 + V}$$

$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

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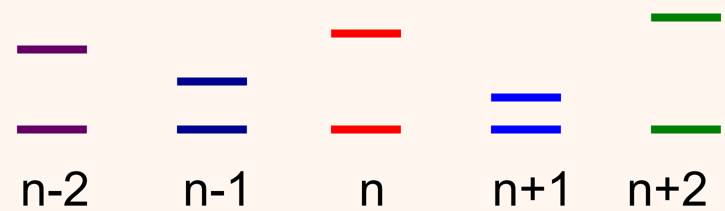

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Random numbers
$h_n \in [-h, h]$

$$V = \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

LFS, Rigolin, Escobar
PRA **69**, 042304 (2004)

Models

$$\boxed{H = H_0 + V}$$

$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

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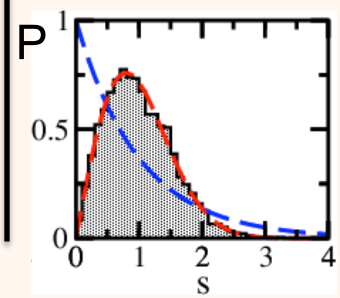

Chaotic h=0.5J

Models

$$\boxed{H = H_0 + V}$$

$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$

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Random numbers $h_n \in [-h, h]$

$$V = \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

AVERAGES 10^4-10^5 data

Chaotic h=0.5J

Survival Probability

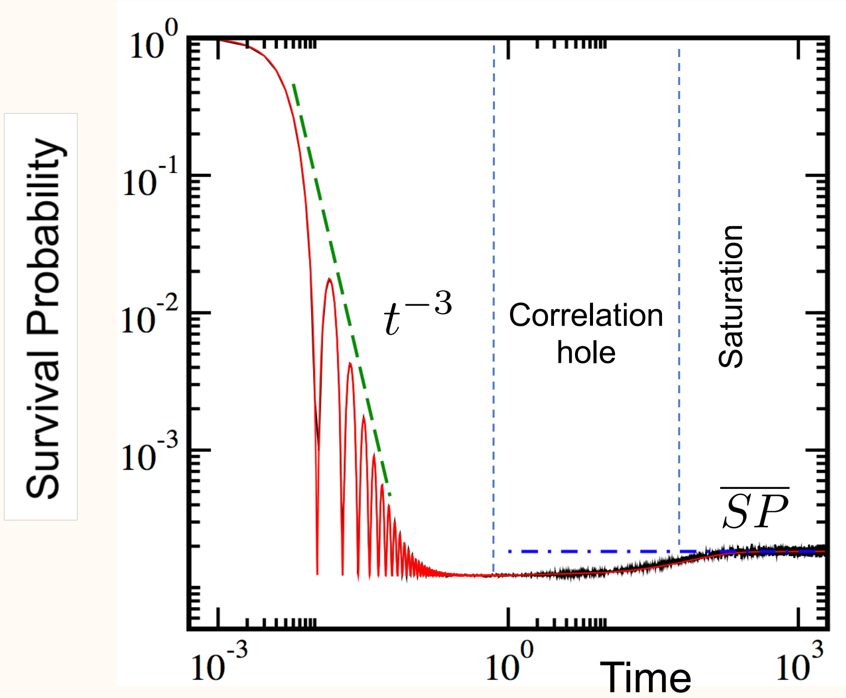
$$\left|\left\langle \Psi(0) \mid \Psi(t) \right\rangle\right|^2$$

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

Survival Probability

GOE Full Random Matrices $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2}-b_2\left(\frac{\Gamma t}{2D}\right)\right]+\overline{SP}$$



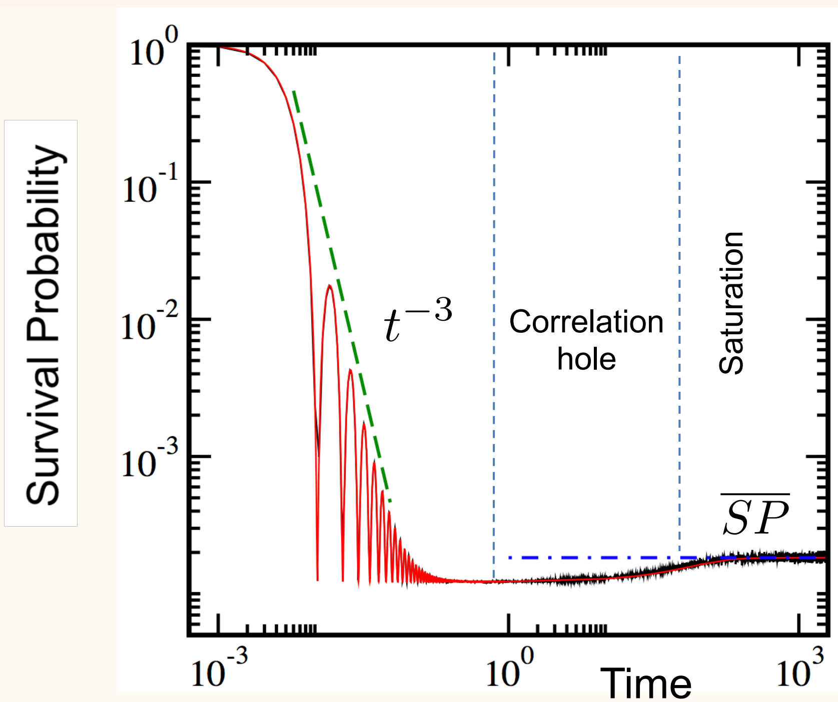
Torres, Garcia-Garcia, LFS
PRB **97**, 060303 (R) (2018)

Survival Probability

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

GOE Full Random Matrices

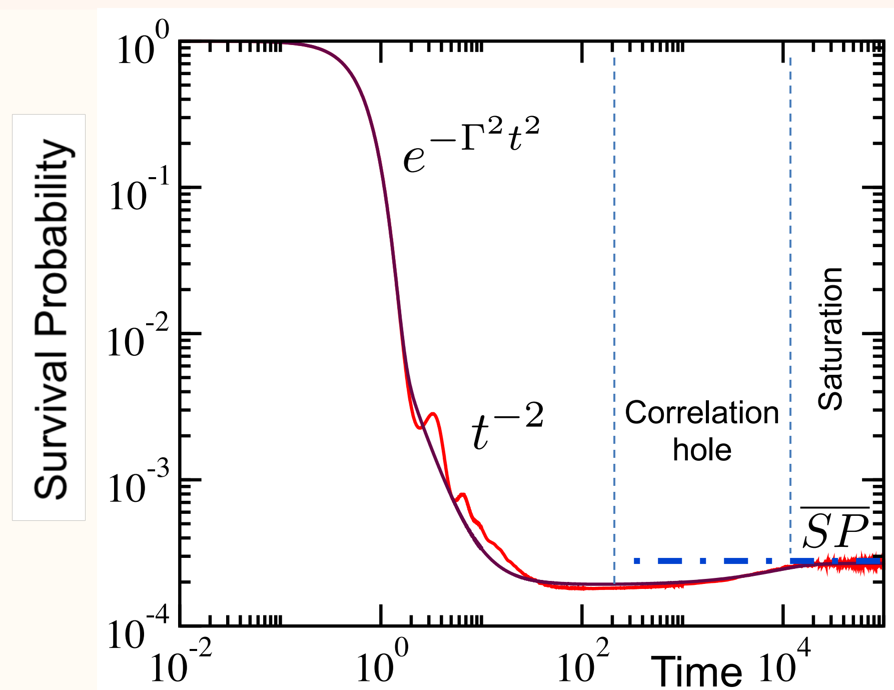
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$



Disordered Spin Model

$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

$$\mathcal{F}(t) = \left|\mathrm{erf}\left(\frac{E_{\max}+it\Gamma^2}{\sqrt{2}\Gamma}\right) - \mathrm{erf}\left(\frac{E_{\min}+it\Gamma^2}{\sqrt{2}\Gamma}\right)\right|^2$$

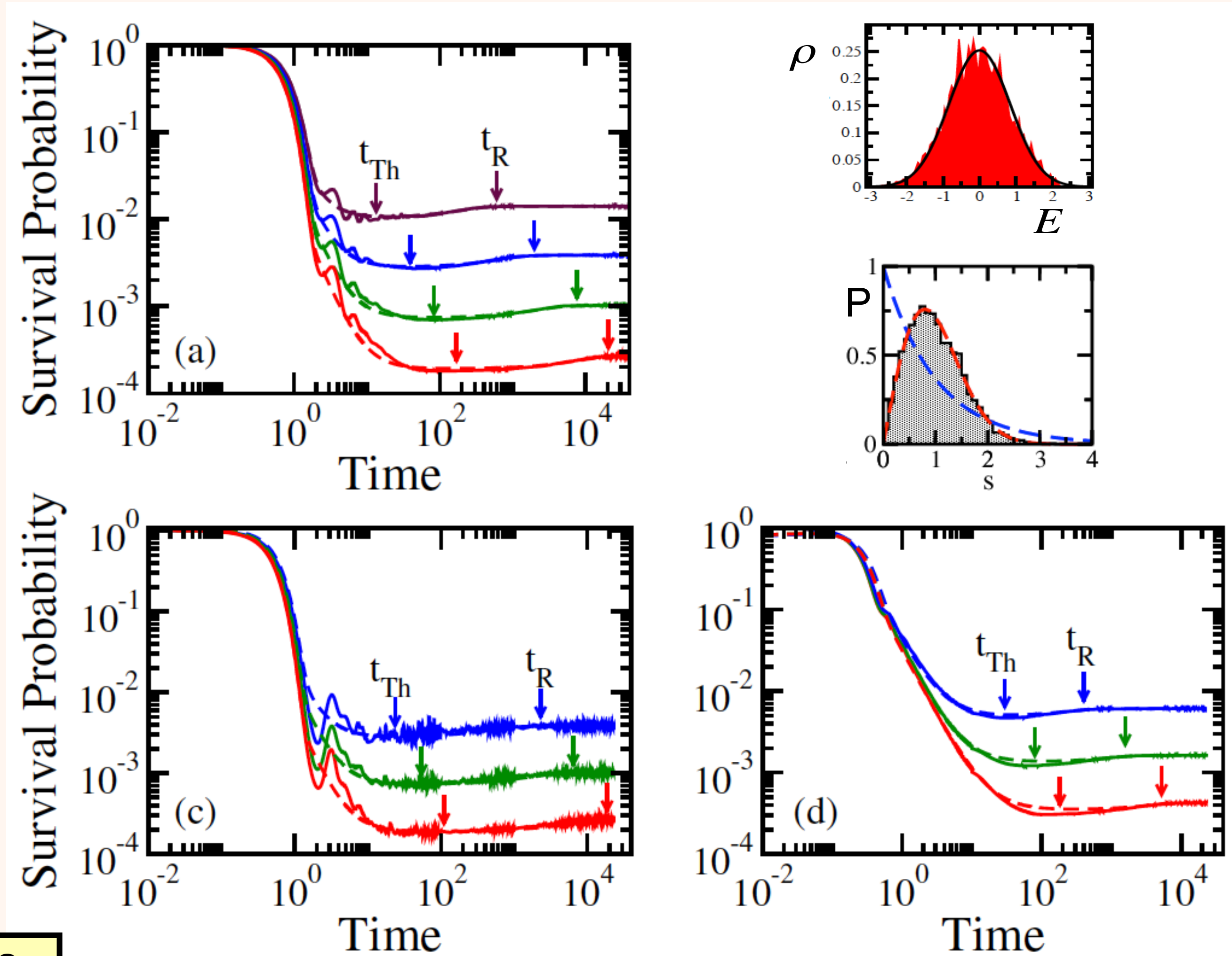


Survival Probability

Chaotic h=0.5J

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

Coefficients
Eigenvalues
Coupling parameters
No assumptions
No approximations
Exact



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Survival Probability

Chaotic h=0.5J

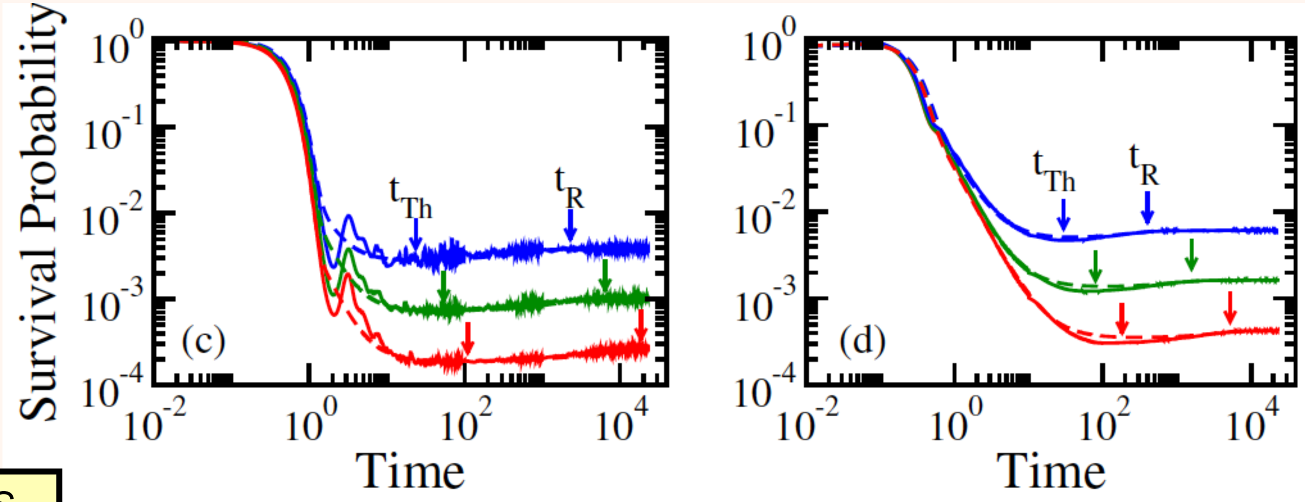
$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

$$H^{\text{cl}} = H_0^{\text{cl}} + V^{\text{cl}},$$

$$H_0^{\text{cl}} = J\Delta \sum_{k=1}^{L} \left(S_k^z S_{k+1}^z + \lambda S_k^z S_{k+2}^z \right),$$

$$V^{\text{cl}} = J \sum_{k=1}^{L} \left[S_k^x S_{k+1}^x + S_k^y S_{k+1}^y + \lambda \left(S_k^x S_{k+2}^x + S_k^y S_{k+2}^y \right) \right].$$

Coefficients
Eigenvalues
Coupling parameters
No assumptions
No approximations
Exact



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Survival Probability

$$SP(t) = \left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

$$\downarrow$$

$$\rho_{ini}(E) = \sum_\alpha \left|C_\alpha^{ini}\right|^2 \delta(E - E_\alpha)$$

Fourier transform $\left\{\begin{array}{l}\text{of the weighted energy distribution of the initial state} \\ \text{of the } \underline{\textbf{LDOS}} \text{ (local density of states)}\end{array}\right.$

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha^{ini} e^{-iE_\alpha t} \,|\alpha\rangle$$

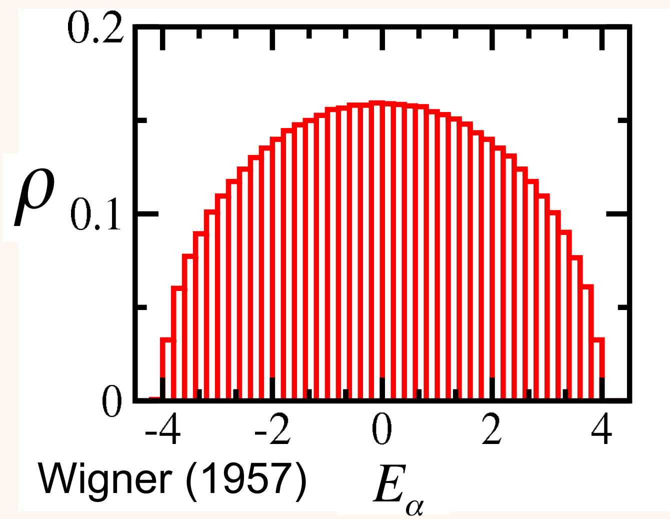
$$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$$

Full Random Matrices vs Two-Body Interaction

Density of States (Energy Distribution)

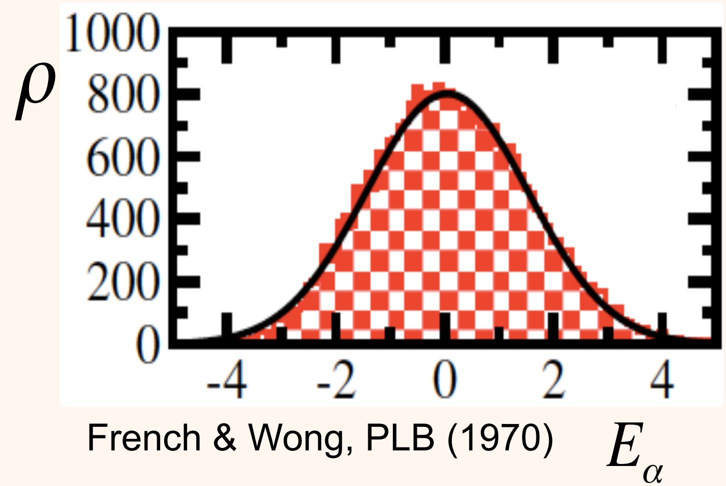
$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

Full random matrices: semicircular



Wigner (1957)

Two-body interactions: Gaussian



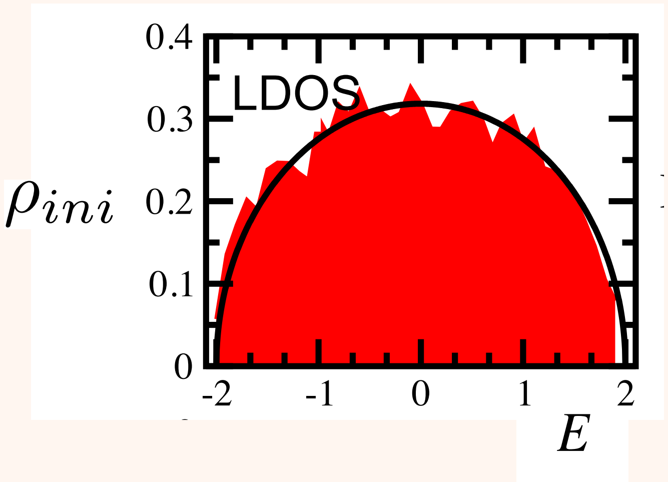
French & Wong, PLB (1970)

Full Random Matrices vs Two-Body Interaction

Local Density of States

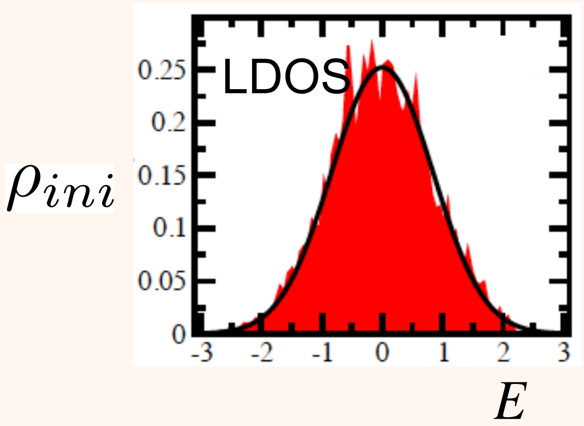
$$\rho_{ini}(E) = \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 \delta(E - E_{\alpha})$$

Full random matrices: semicircular



NOT universal

Two-body interactions: Gaussian



General:

- Chaotic many-body systems with 2-body interactions
- Perturbed far from equilibrium
- Initial state close to middle

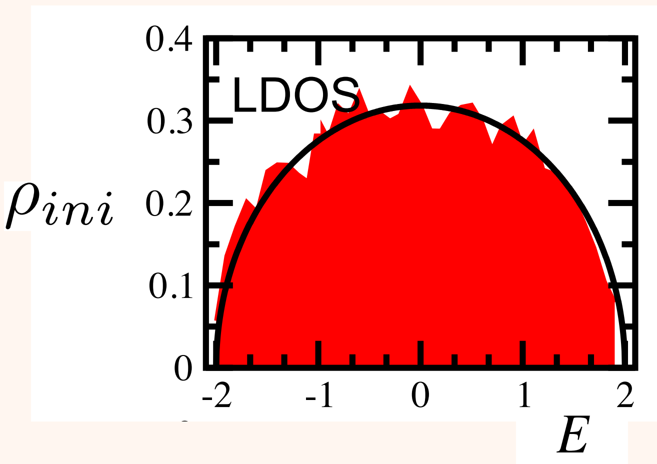
Initial Fast Decay

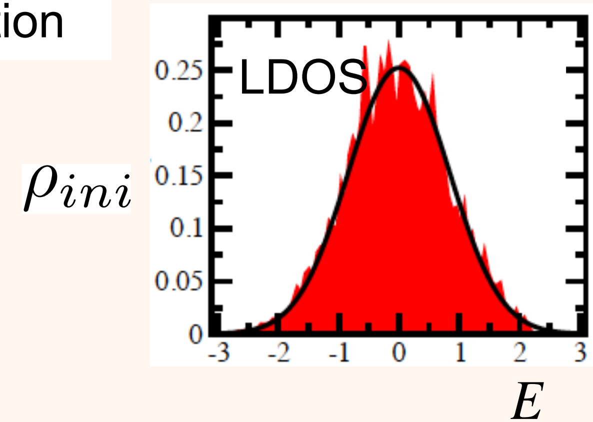
$$\left|\int \rho_{ini}(E)e^{-iEt}\,dE\right|^2$$

Full Random Matrices

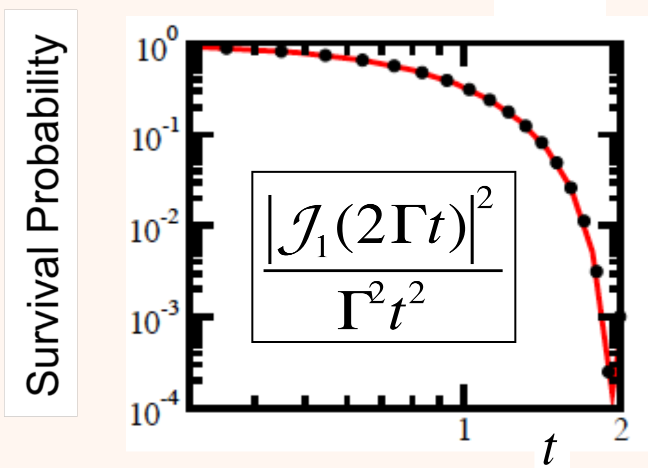
Spin Model

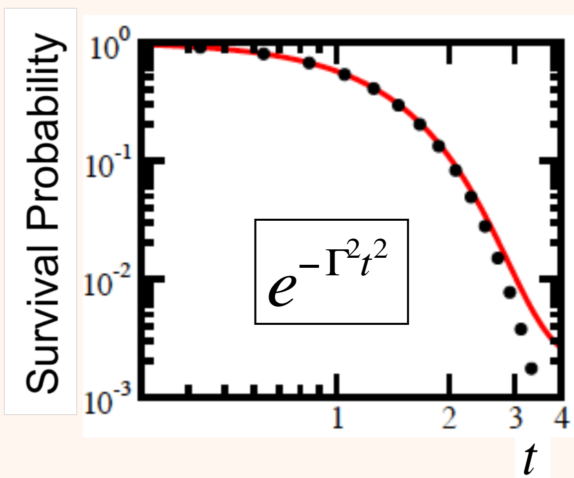
Γ Width of the distribution





$1/\Gamma$





Torres-Herrera & LFS
PRA, NJP (2014)

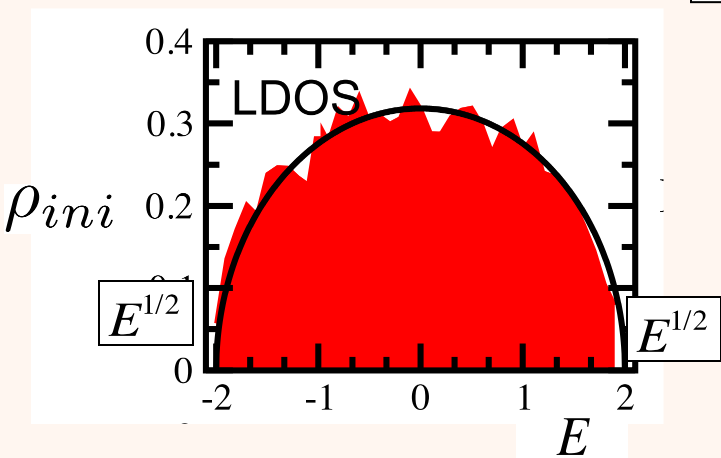
Power-law Decay

$$\left|\int_{E_{low}}^{E_{up}} \rho_{ini}(E)\, e^{-iEt}\, dE\right|^2$$

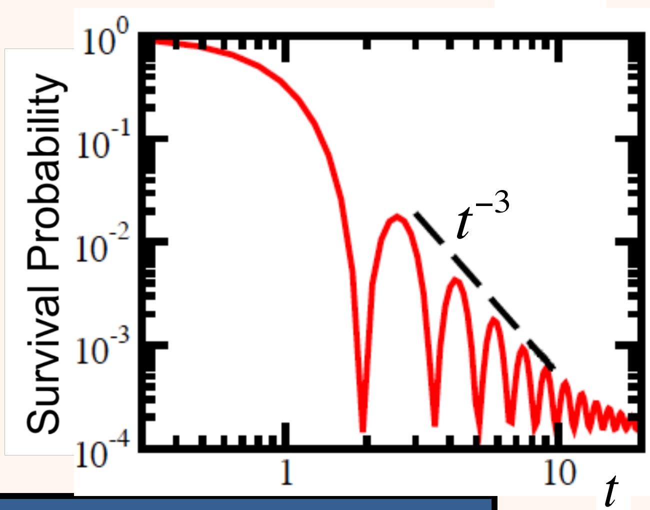
$$\rho_{ini}(E) = (E-E_0)^{\xi}\, \eta(E)$$

$$F(t) \propto t^{-2(\xi+1)}$$

Full Random Matrices



$\xi = 1/2$



Spin Model

Távora, Torres, LFS
PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

Power-law Decay

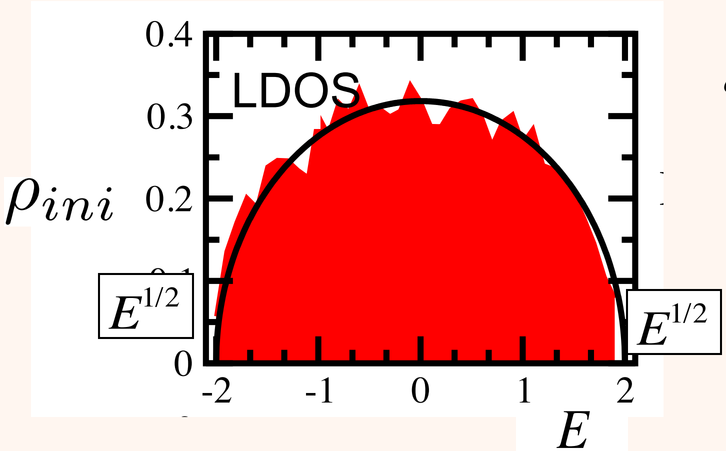
$$\left| \int_{E_{low}}^{E_{up}} \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$

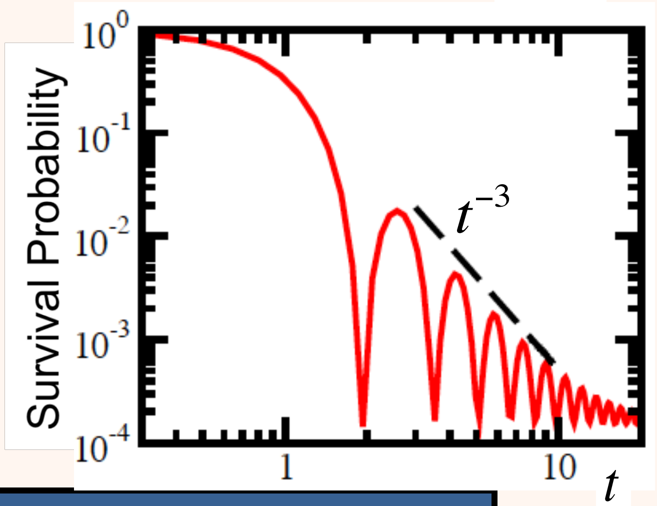
$$\rho_{ini}(E) = (E - E_0)^{\xi} \eta(E)$$

$$F(t) \propto t^{-2(\xi+1)}$$

<u>Full Random Matrices</u>

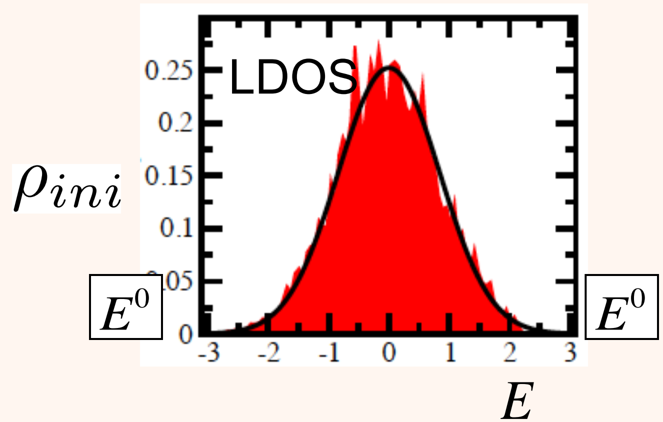
$\xi = 1/2$

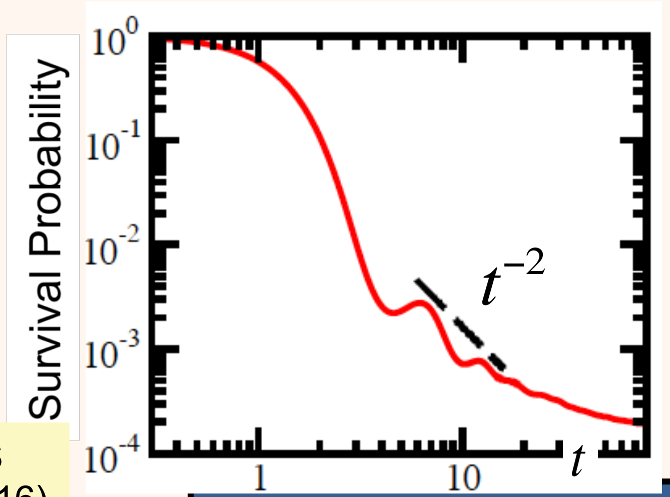




<u>Spin Model</u>

$\xi = 0$

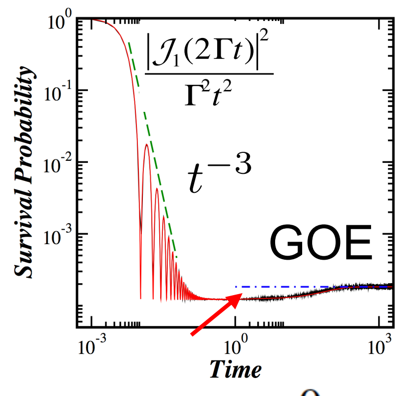


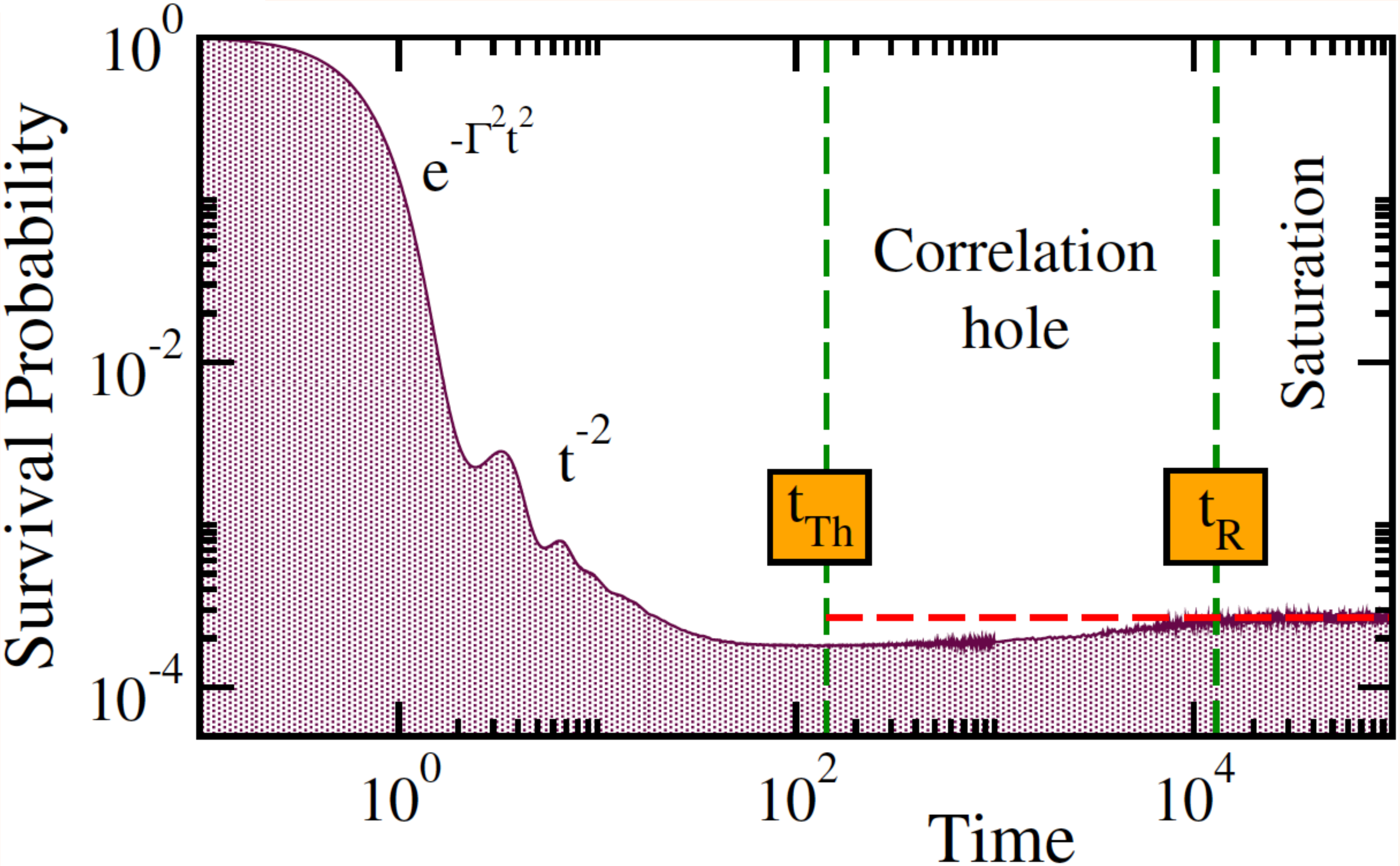


Távora, Torres, LFS
PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

Survival Probability

Chaotic h=0.5J



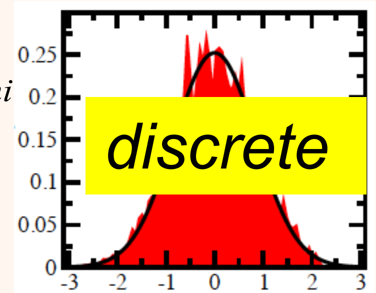


$$\overline{SP} = \sum_\alpha |C_\alpha^{ini}|^4 \propto \frac{1}{D}$$

Survival Probability

$$SP(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \left|\sum_\alpha |C_\alpha^{ini}|^2 e^{-iE_\alpha t}\right|^2$$

ρ_{ini}



E

$$SP(t) = \sum_{\alpha,\beta} \left|C_\alpha^{ini}\right|^2 \left|C_\beta^{ini}\right|^2 e^{-i(E_\alpha - E_\beta)t}$$

$$= \int \underbrace{\boxed{\sum_{\alpha,\beta}\left[\left|C_\alpha^{ini}\right|^2 \left|C_\beta^{ini}\right|^2 \delta(E - E_\alpha + E_\beta)\right]}} e^{-iEt}\, dE$$

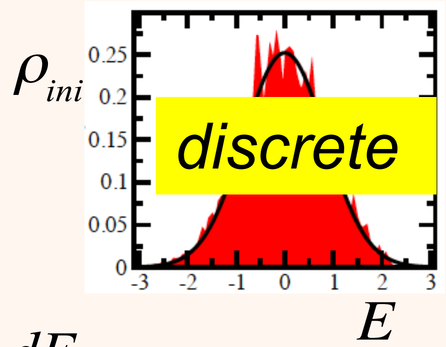
Spectral autocorrelation function

shape correlations

$$R_2(E_\alpha, E_\beta) = DOS(E_\alpha) DOS(E_\beta) - T_2(E_\alpha, E_\beta)$$

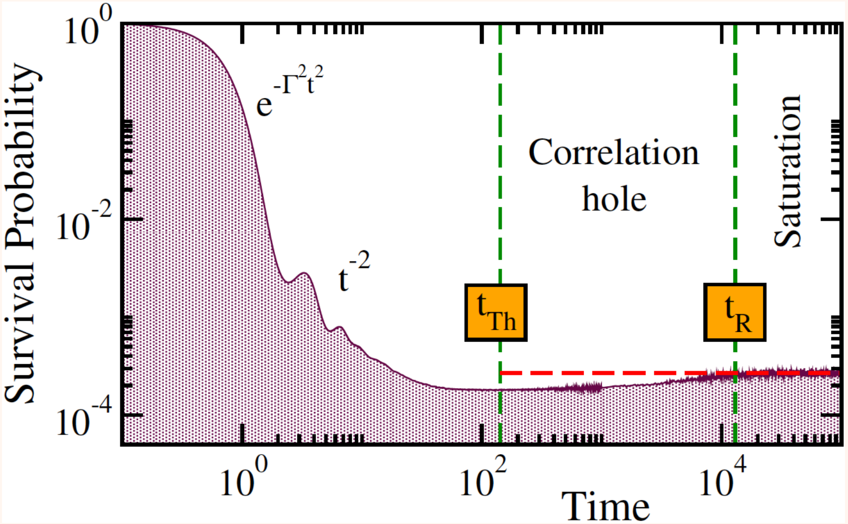
Survival Probability

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$



$$SP(t) = \sum_{\alpha,\beta} \left|C_\alpha^{ini}\right|^2 \left|C_\beta^{ini}\right|^2 e^{-i(E_\alpha - E_\beta)t}$$

$$= \int \boxed{\sum_{\alpha,\beta} \left[\left|C_\alpha^{ini}\right|^2 \left|C_\beta^{ini}\right|^2 \delta(E - E_\alpha + E_\beta) \right]} e^{-iEt}\, dE$$



Correlation hole:

- Long times
- Direct manifestation of level repulsion in the time domain

Correlation Hole

$$b_2(t) = \begin{cases} 1 - 2t + t\ln(1+2t), & t \le 1 \\ t\ln\left(\dfrac{2t+1}{2t-1}\right) - 1, & t > 1 \end{cases}$$

GOE FRM
(semicircle)

$$b_2\left(\frac{\Gamma t}{2D}\right)$$

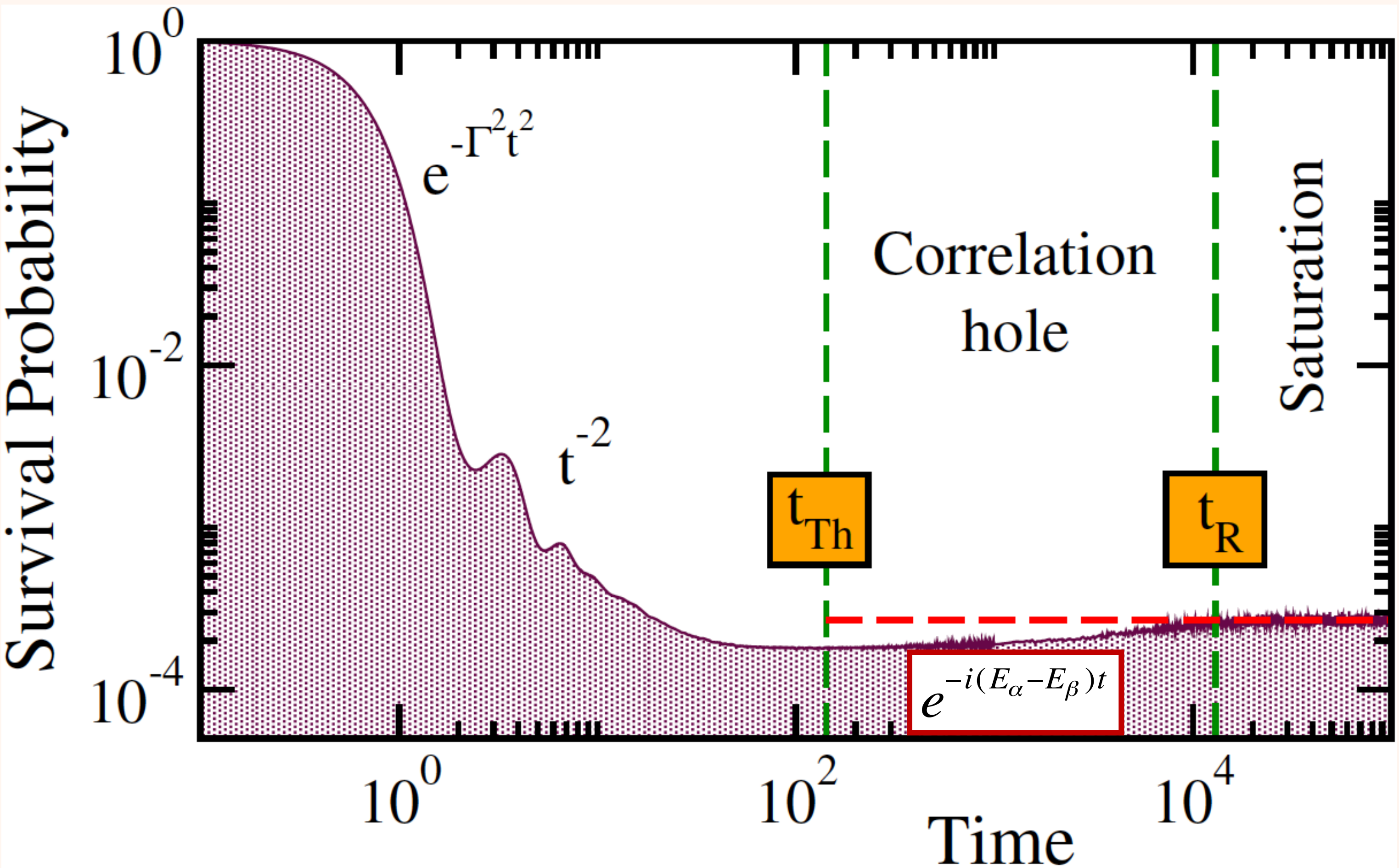
Chaotic spin model
(Gaussian)

$$b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)$$

$$\int\left(\int \delta\left(E-(E_1-E_2)\right)T_2(E_1,E_2)\,dE_1 dE_2\right)e^{-iEt}\,dE \Rightarrow b_2\left(\frac{\Gamma t}{2D}\right)$$

Two-level form factor

Survival Probability
Time Scales



$$\overline{SP} = \boxed{\sum_\alpha \left| C_\alpha^{ini} \right|^4} \propto \frac{1}{D}$$

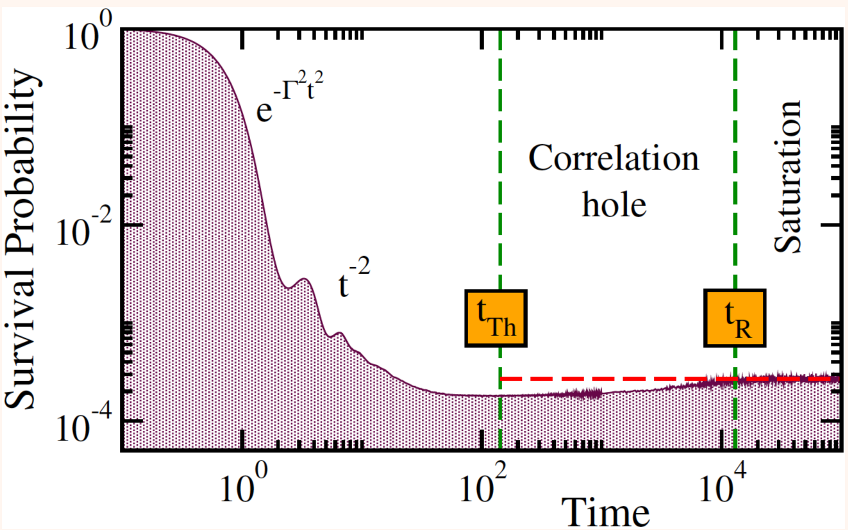
Time to Reach the Minimum of the Hole

Chaotic h=0.5J

$$SP(t) = \frac{1-\overline{SP}}{D-1}\left[D\frac{e^{-\Gamma^2 t^2}}{(\Gamma t)^2}g(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

$t \to \infty$

$t \to 0$



$$\underbrace{\frac{D}{(\Gamma t)^2} \qquad 1-\frac{2\Gamma t}{\sqrt{2\pi}D}}$$

$$D = \frac{L!}{(L/2)!^2}$$

$$\boxed{t_{Th} \propto \frac{D^{2/3}}{\Gamma}}$$

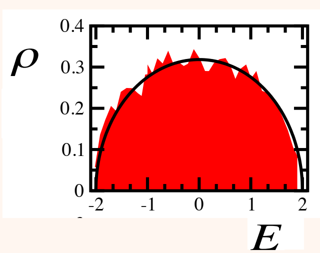
Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

D: dimension of Hilbert space
L: number of sites

Time to Reach the Minimum of the Hole

GOE full random matrix

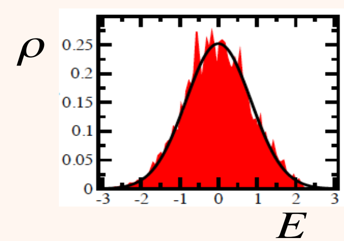
$$t^{-3} \Rightarrow \boxed{t_{Th} = \left(\frac{3}{\pi}\right)^{1/4} \frac{\sqrt{D}}{\Gamma}}$$



$$\Gamma \propto \sqrt{D}$$

Realistic CHAOTIC spin-1/2 model

$$t^{-2} \Rightarrow \boxed{t_{Th} \propto \frac{D^{2/3}}{\Gamma}}$$



$$\Gamma \propto \sqrt{L}$$

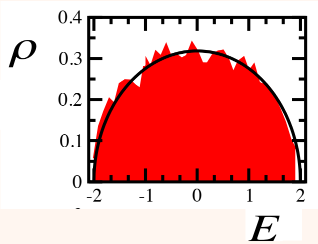
$$\Gamma = \sqrt{\sum_{\alpha} |C_{\alpha}^{ini}|^2 (E_{\alpha} - E_{ini})^2} = \sqrt{\sum_{n \neq ini} |\langle n|H|ini\rangle|^2}$$

Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Time to Reach the Minimum of the Hole

GOE full random matrix

$$t^{-3} \Rightarrow \boxed{t_{Th} = \left(\frac{3}{\pi}\right)^{1/4} \frac{\sqrt{D}}{\Gamma}}$$



$$\Gamma \propto \sqrt{D}$$

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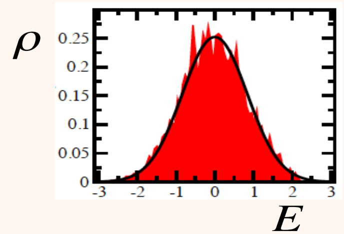


$$\Gamma \propto \sqrt{L}$$

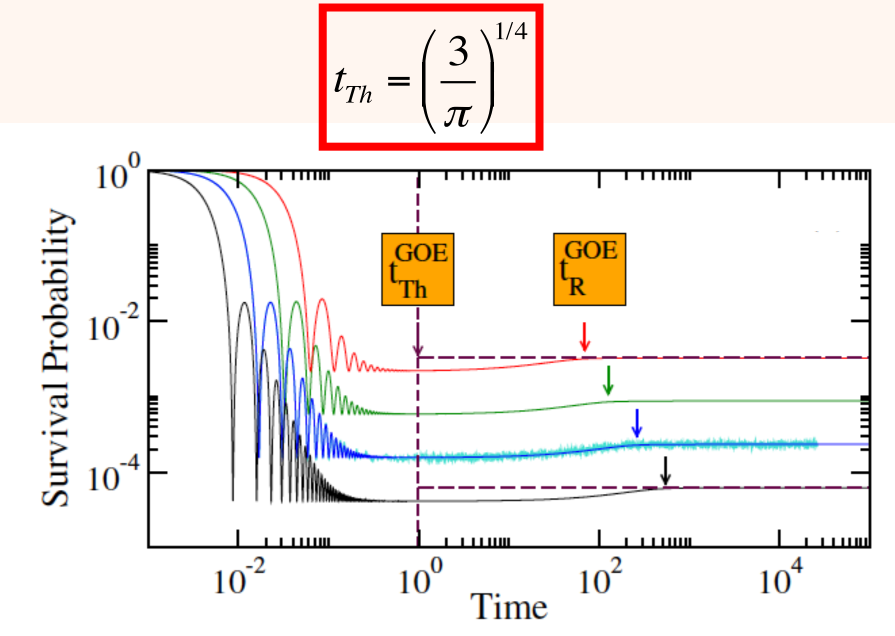
$$D = \frac{L!}{(L/2)!^2} \qquad \boxed{t_{Th} \propto \frac{e^{(2L\ln 2)/3}}{\sqrt{L}}}$$

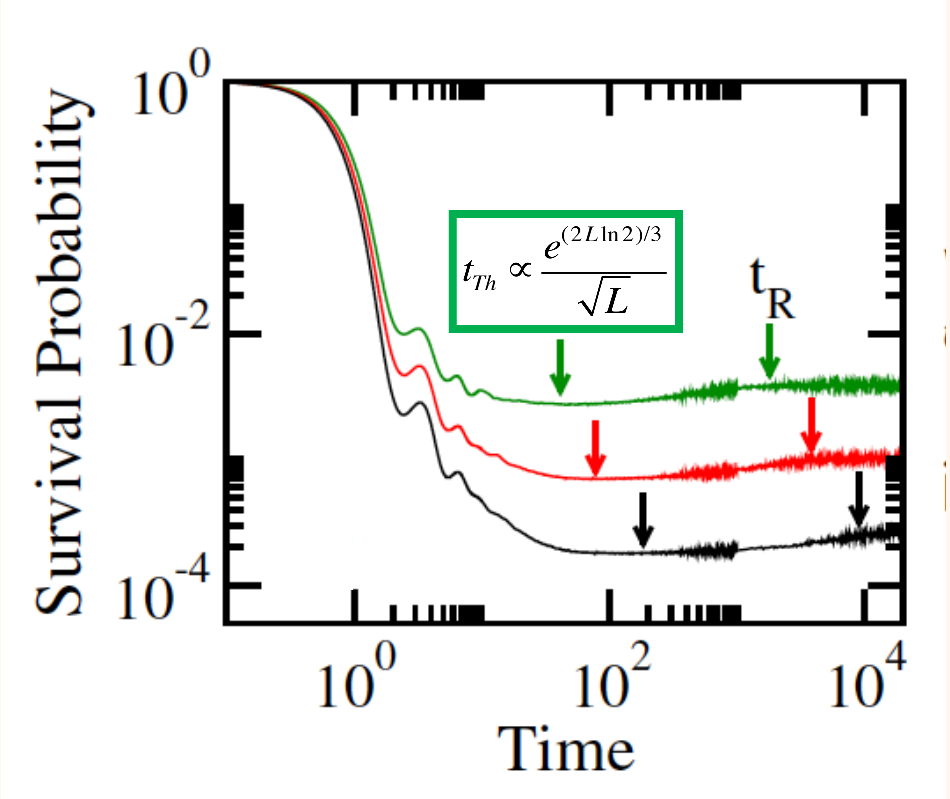
Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Time to Reach the Minimum of the Hole

GOE full random matrix

Realistic CHAOTIC spin-1/2 model

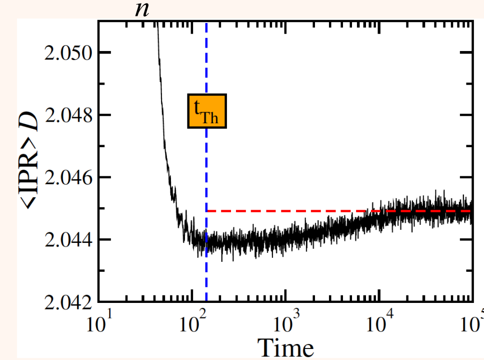




Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Spread in the many-body space

$$IPR(t) = \sum_n \left| \langle n | e^{-iHt} | \Psi(0) \rangle \right|^4$$



$$|\Psi(0)\rangle = |\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle \qquad |\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle$$

$$|\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\rangle \qquad \ldots \qquad \ldots$$

dimension of Hilbert space

$$D = \frac{L!}{(L/2)!^2}$$

L: number of sites

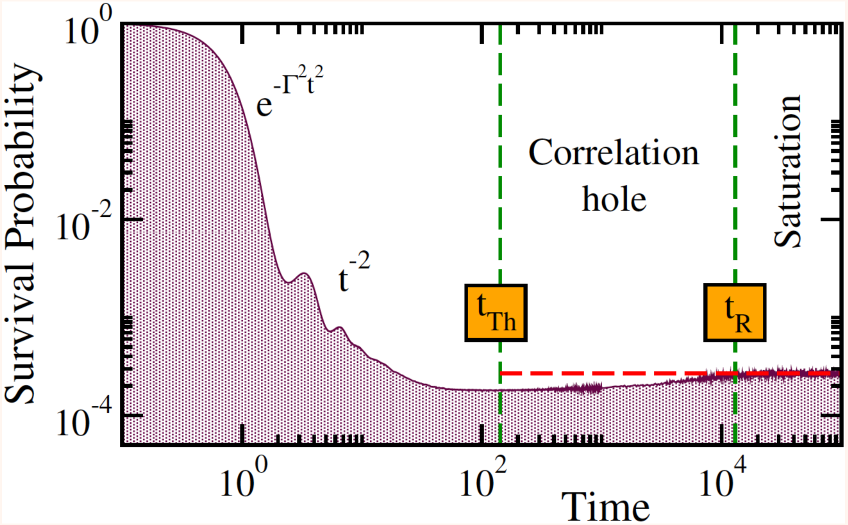
$$V = \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z)$$

Relaxation Time

Chaotic h=0.5J

$$SP(t) = \frac{1-\overline{SP}}{D-1}\left[D\frac{e^{-\Gamma^2 t^2}}{(\Gamma t)^2} g(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

$$\xrightarrow{t\to\infty} \frac{D^2}{\Gamma^2 t^2}$$



$$\frac{SP(t)-\overline{SP}}{\overline{SP}} \sim \delta$$

Inverse of mean level spacing

$$\boxed{t_R \propto \frac{D}{\Gamma\sqrt{\delta}}}$$

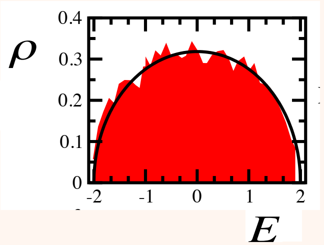
Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Relaxation Time: Heisenberg Time

GOE full random matrix

$$\boxed{t_R \propto \frac{D}{\Gamma\sqrt{\delta}}}$$

$$\Gamma \propto \sqrt{D}$$



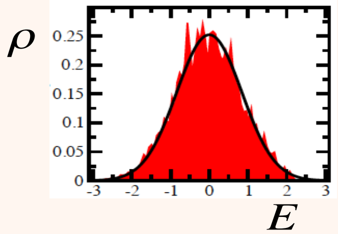
Inverse of mean level spacing

Heisenberg Time

Realistic CHAOTIC spin-1/2 model

$$\boxed{t_R \propto \frac{D}{\Gamma\sqrt{\delta}}}$$

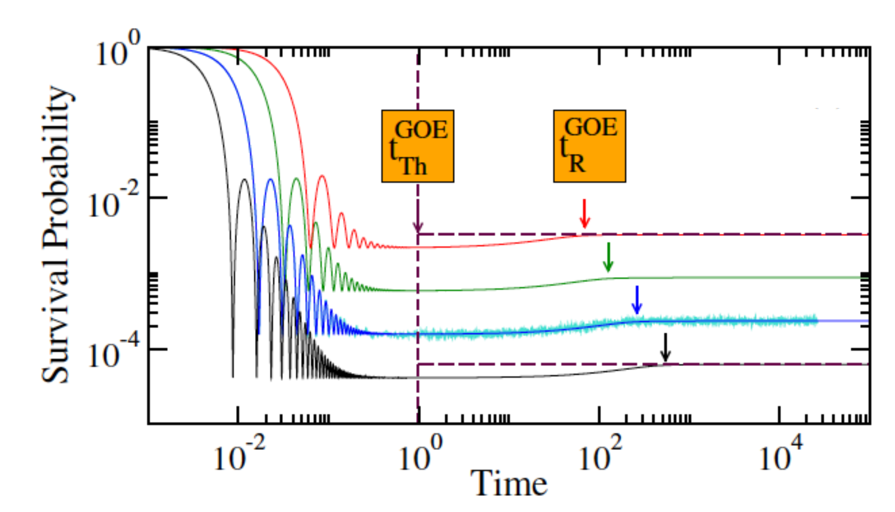
$$\Gamma \propto \sqrt{L}$$



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Relaxation Time: Heisenberg Time

GOE full random matrix



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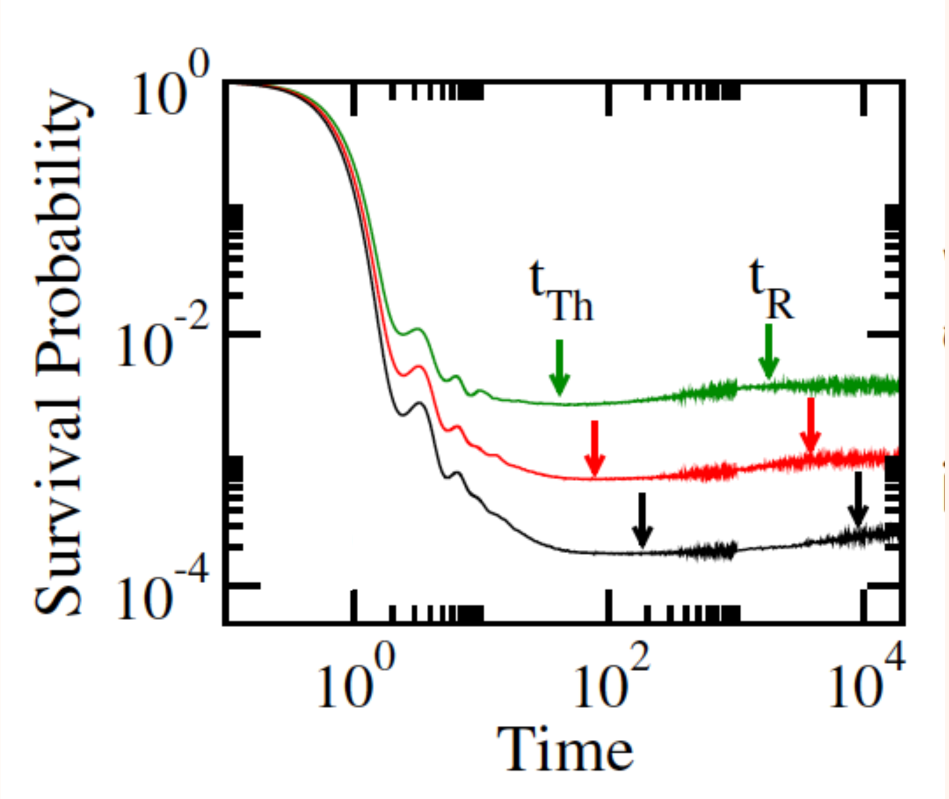


Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

Realistic spin-1/2 model

Chaotic h=0.5J

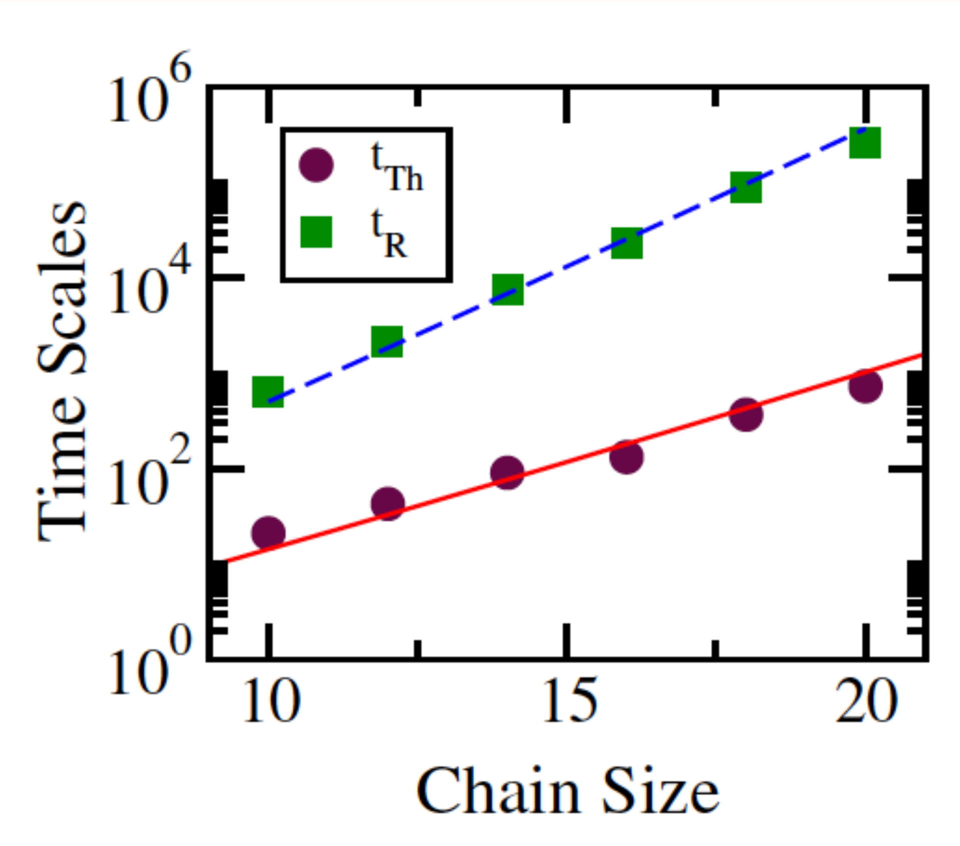
Thouless time

$$\boxed{t_{Th} \propto \frac{D^{2/3}}{\Gamma}}$$

$$\Gamma \propto \sqrt{L} \qquad D = \frac{L!}{(L/2)!^2}$$

Relaxation time

$$\boxed{t_R \propto \frac{D}{\Gamma\sqrt{\delta}}}$$



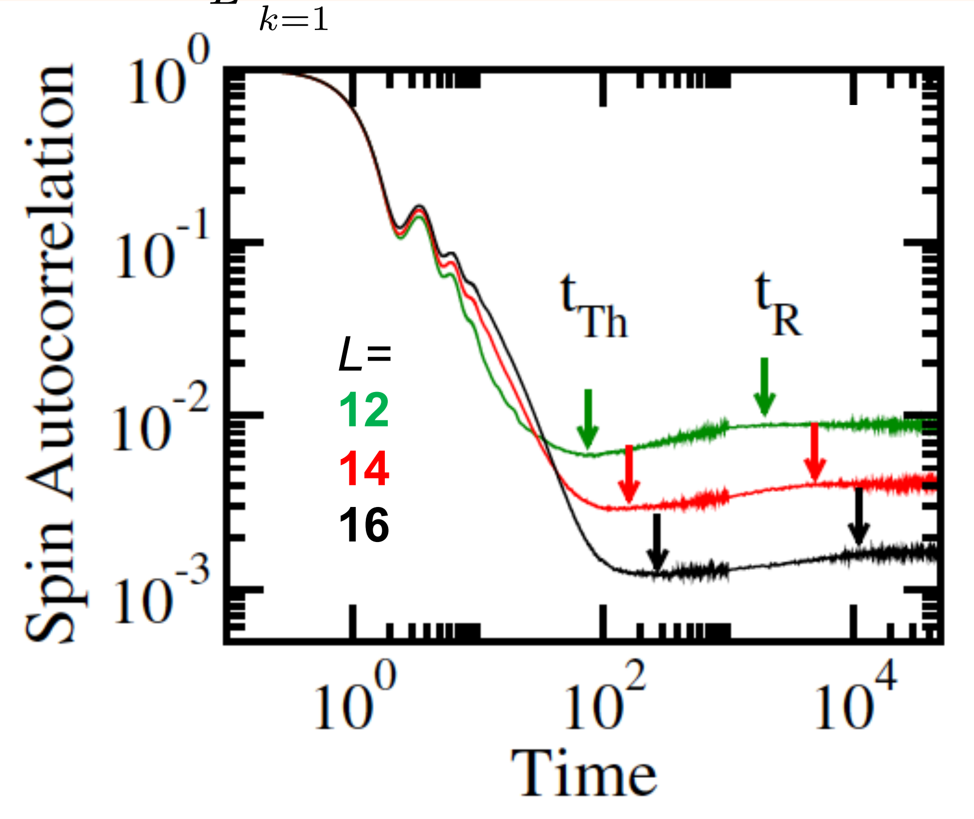
$$\boxed{t_R \propto D^{1/3} t_{Th}}$$

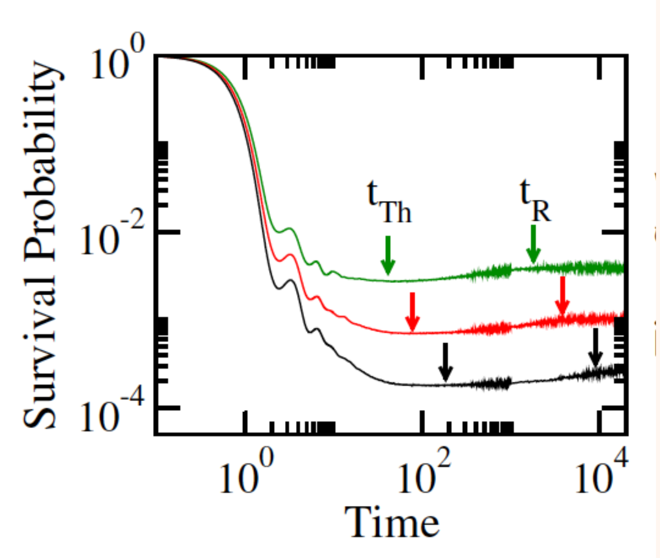
hole stretches with L

Spin Autocorrelation Function

Chaotic h=0.5J

$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$





Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Spin Autocorrelation Function

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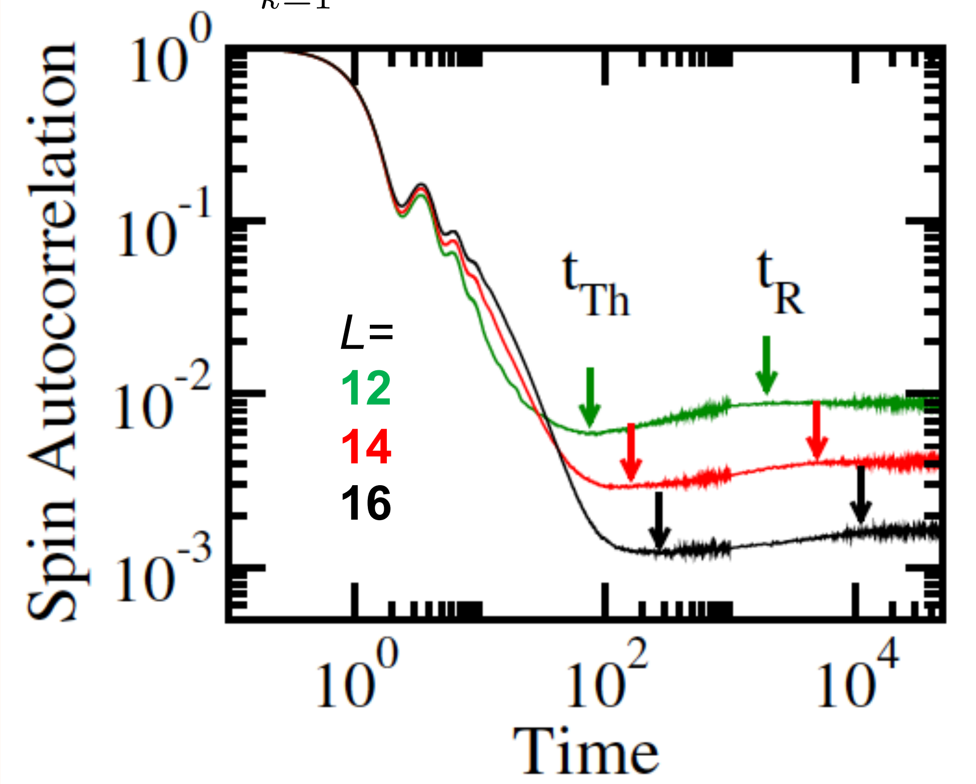
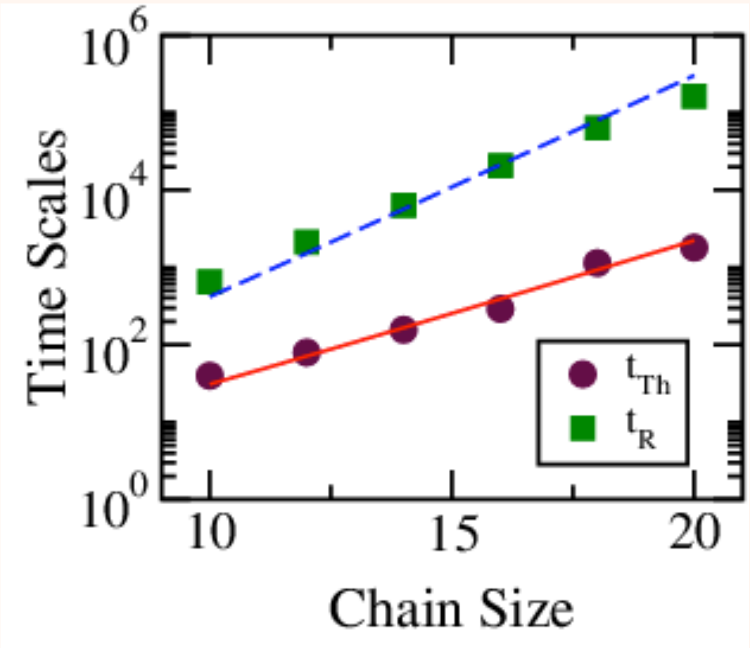




Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

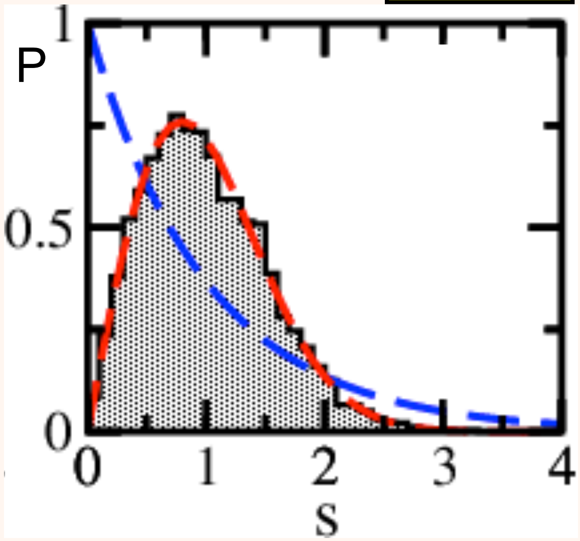
As the disorder increases

1D Disordered Spin-1/2 Model

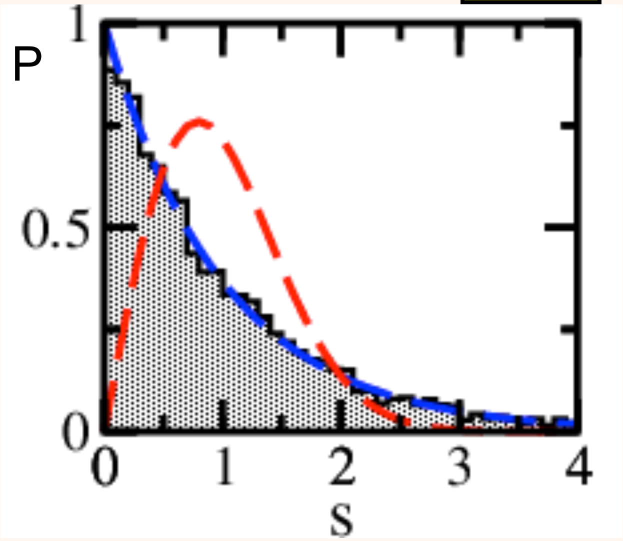
$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z)$$

$$h_n \in [-h, h]$$

Chaotic h=0.5J



Localized $h > h_c$

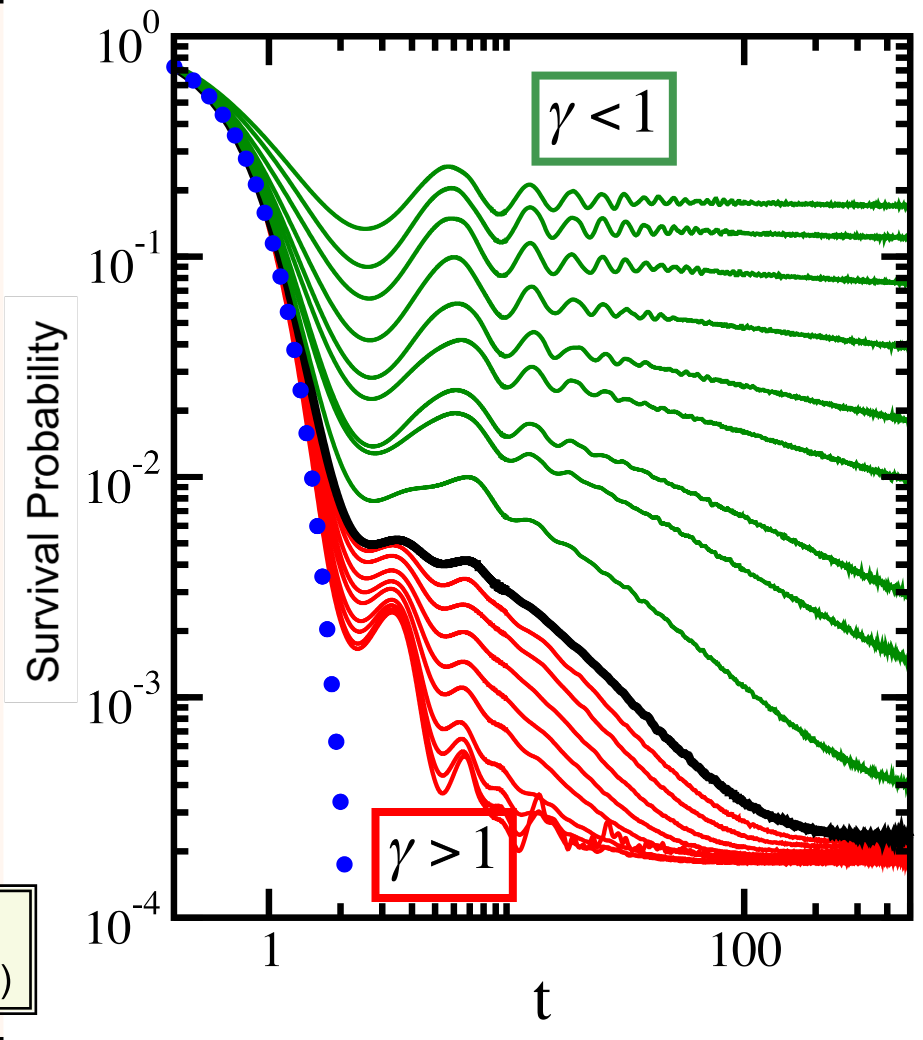


LFS, Rigolin, Escobar
Entanglement versus chaos in disordered spin chains
PRA **69**, 042304 (2004)

Power-law exponent: energy bounds

$$\boxed{t^{-\gamma}}$$

$$\boxed{PR^{(\alpha)} \propto Dim^{D_2}}$$



$$\boxed{PR^{(\alpha)} \propto Dim^{D_2}}$$

h>J

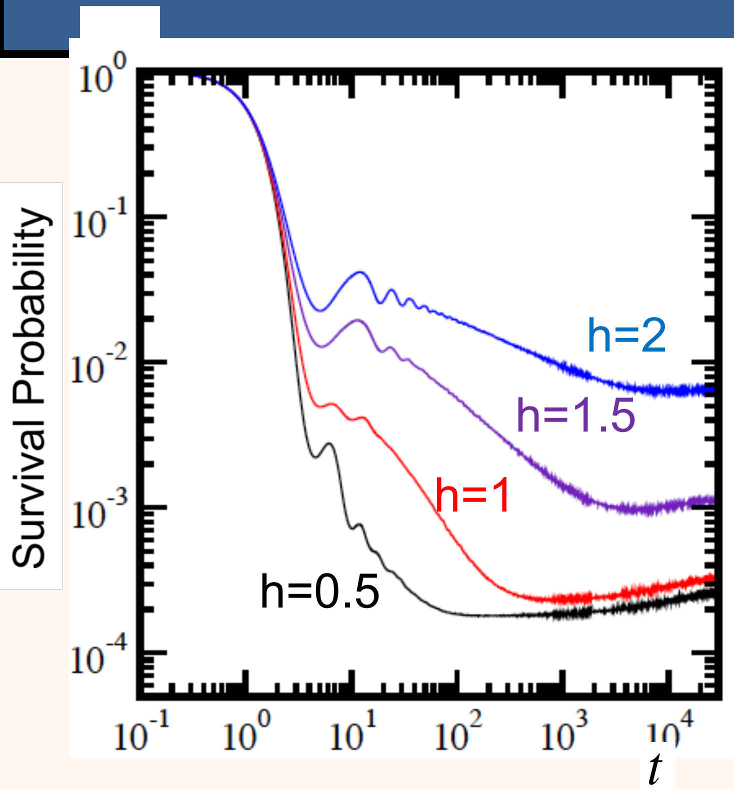
$$\boxed{t^{-\gamma}}$$

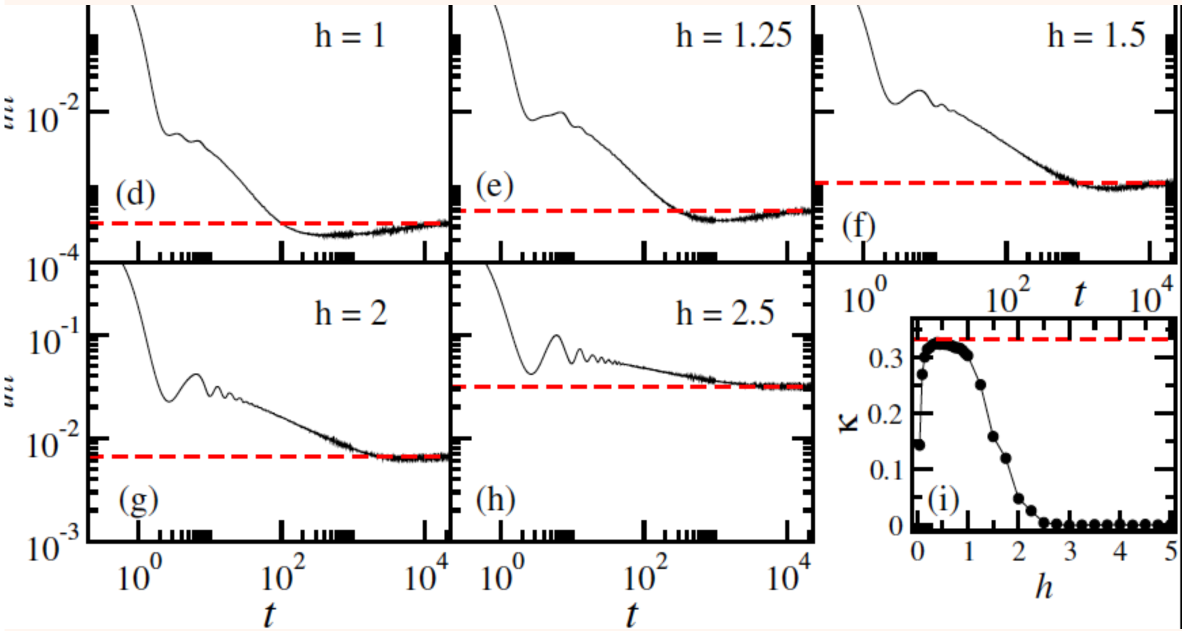
h<J

$$\boxed{PR^{(\alpha)} \propto Dim}$$

Torres & LFS
PRB **92**, 01420 (2015)

Correlation Hole and Disorder Strength





The hole becomes narrower and shallower.

Time for the minimum of the hole (T_{Th}) gets postponed as the disorder (h) increases.

Fixed system size L=16

Torres & LFS
Philos. Trans. A **65** (2017)

Thouless Dimensionless Conductance

Thouless dimensionless conductance $\frac{t_R}{t_{Th}} \propto \frac{E_{th}}{MLS}$

Delocalized (chaotic)

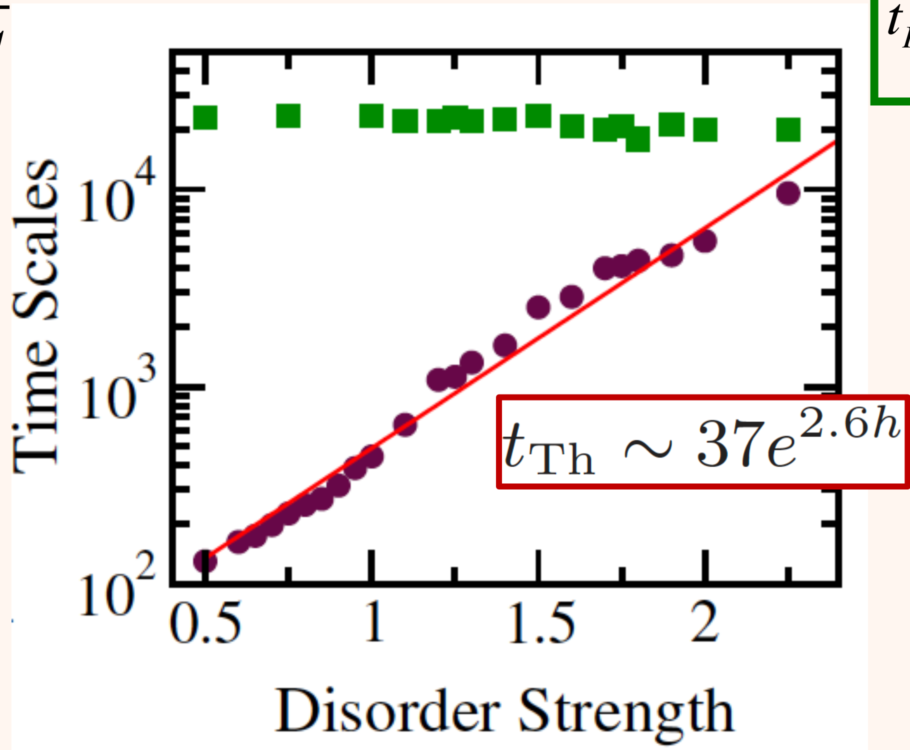
$$\frac{t_R}{t_{Th}} \propto e^{(L\ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \rightarrow 1$$

SURVIVAL PROBABILITY

$$\boxed{t_R \propto \frac{D}{\Gamma\sqrt{\delta}}}$$



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Thouless
Dimensionless Conductance

Thouless dimensionless conductance $\dfrac{t_R}{t_{Th}} \propto \dfrac{E_{th}}{MLS}$

Delocalized (chaotic)

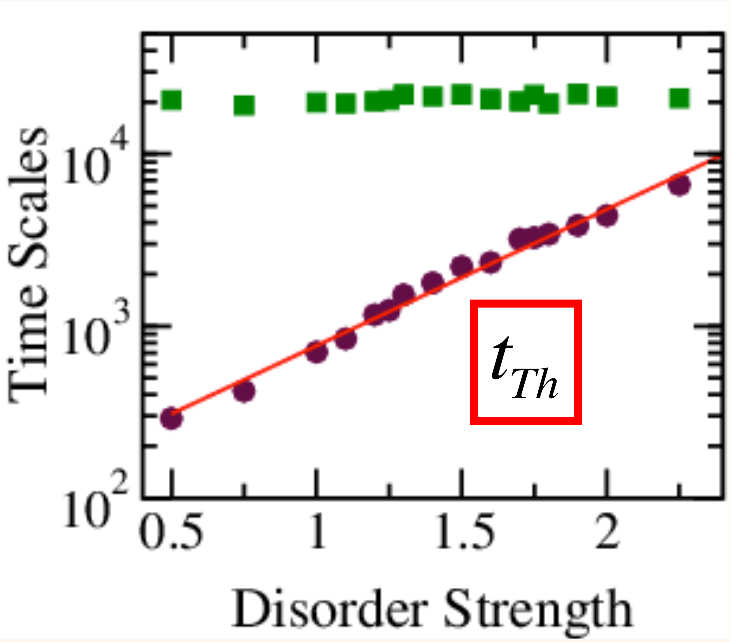
$$\frac{t_R}{t_{Th}} \propto e^{(L\ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \to 1$$

SPIN AUTOCORRELATION

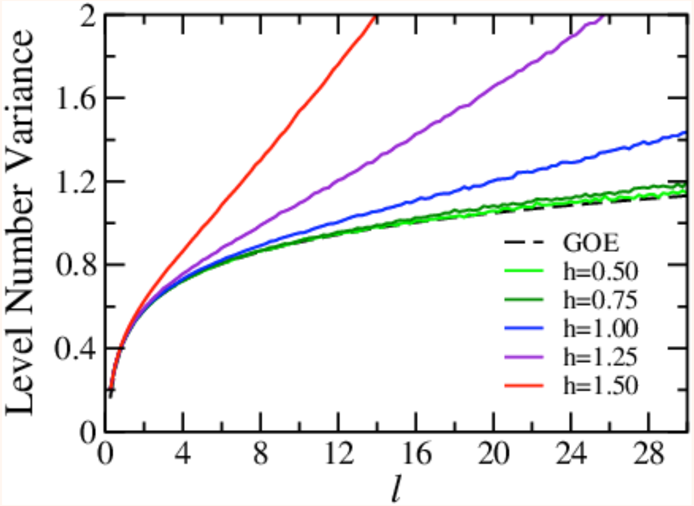
$$\boxed{t_R \propto \frac{D}{\Gamma\sqrt{\delta}}}$$



Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Why Thouless?
Relation with the Thouless Energy

Level number variance



Bertrandt & García-García
Phys. Rev. B **94**, 144201 (2016).

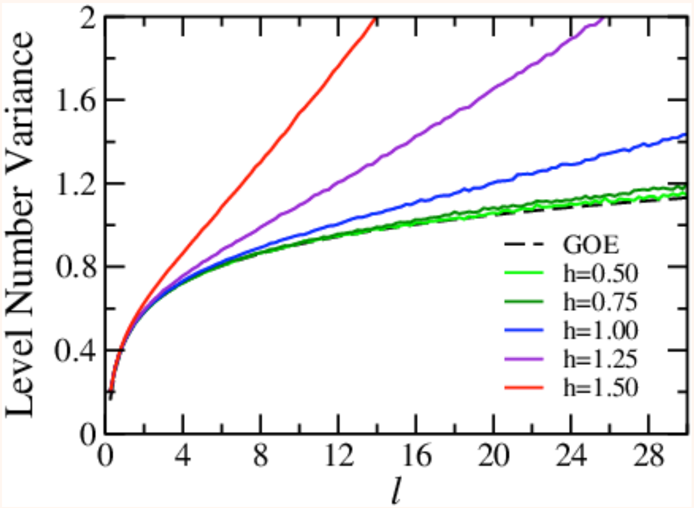
The level statistics of disordered systems follows that of full random matrices up to the Thouless energy

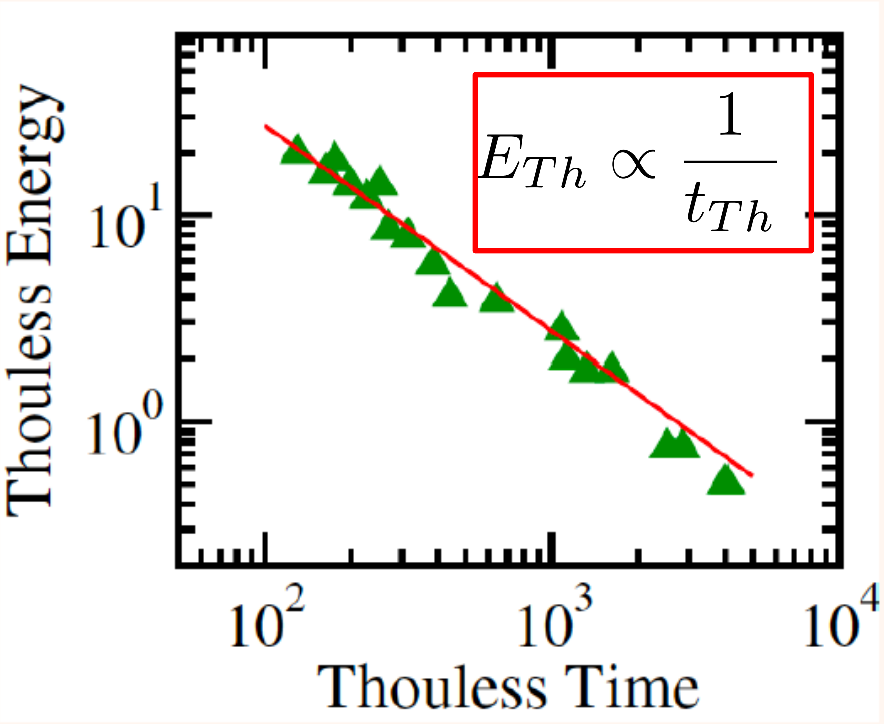
$$E_{Th} \propto \frac{1}{t_{Th}}$$

Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

Why Thouless?
Relation with the Thouless Energy

Level number variance





The level statistics of disordered systems follows that of full random matrices up to the Thouless energy

Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Summary

- **Time scales:** **Chaotic Many-Body Quantum System**

$$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$$

Thouless time: minimum of the correlation hole (signature of spectral correlations)
spread in the MANY-BODY HILBERT SPACE

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$

Relaxation time: end of the hole, fluctuations

Analytical estimates match the results from **exact** quantum dynamics

Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

Summary

➢ **Time scales: Chaotic Many-Body Quantum System**

$$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$$

Thouless time: minimum of the correlation hole (signature of spectral correlations)
spread in the MANY-BODY HILBERT SPACE

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Relaxation time: end of the hole, fluctuations

Analytical estimates match the results from **exact** quantum dynamics

➢ **Correlation hole** is an indicator of the delocalized-localized transition.

Toward **localization**

$$\boxed{\frac{t_R}{t_{Th}} \rightarrow 1}$$

$$\boxed{E_{Th} \propto \frac{1}{t_{Th}}}$$

Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

SELF-AVERAGING

Self-averaging in many-body quantum systems out of equilibrium
ArXiv: 1906.11856

Self-averaging behavior at the metal-insulator transition of many-body quantum systems out of equilibrium
ArXiv: 1910.11332

Self-Averaging

Disordered Systems

A quantity O is self-averaging when its relative variance
(the ratio between its variance and the square of its mean)
goes to zero as the system size increases

$$\mathcal{R}_O(t) = \frac{\sigma_O^2(t)}{\langle O(t)\rangle^2} = \frac{\langle O^2(t)\rangle - \langle O(t)\rangle^2}{\langle O(t)\rangle^2}$$

By increasing the system size, one can **reduce** the number of samples used in

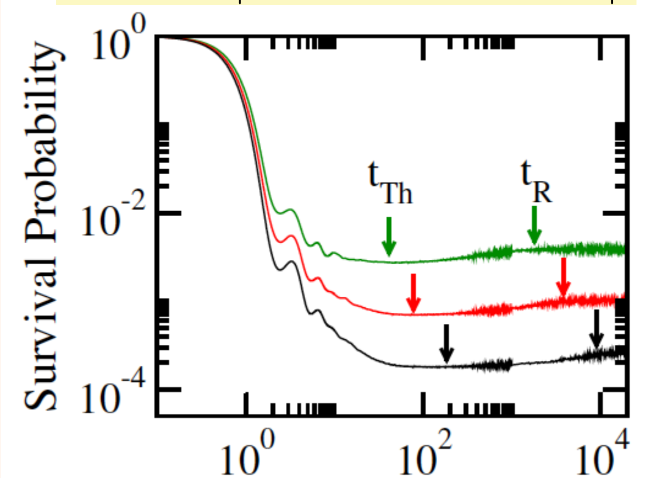
- experiments
- statistical analysis.

If the system exhibits self-averaging, its physical properties are independent of the specific realization.

ArXiv: 1906.11856
ArXiv: 1910.11332

Non-Local in Time

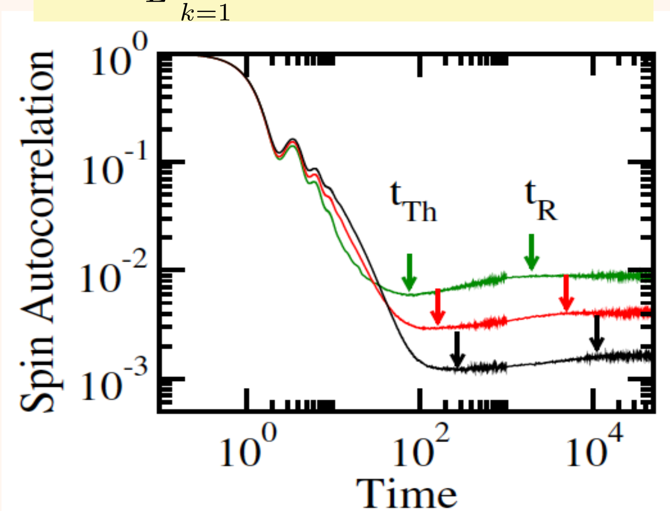
$$SP(t) = \left|\langle \Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$



$L=$
12
14
16

Non-Local in Space

$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle \Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$



$L=$
12
14
16

Local in Space

Autocorrelation functions:

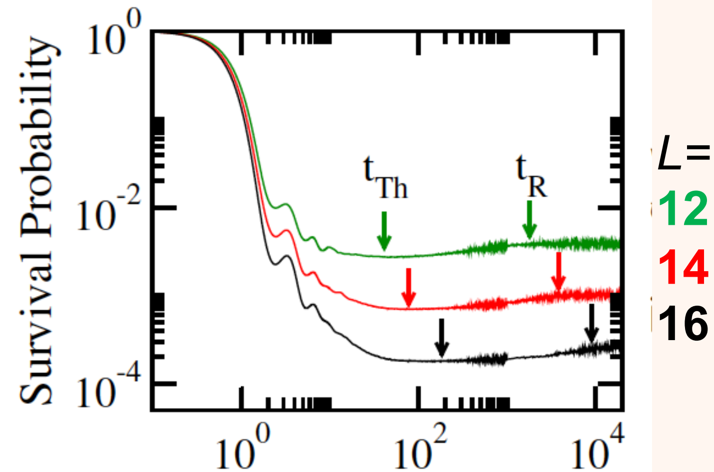
* Dynamical manifestations of spectral correlations

* Relaxation time grows exponentially with system size

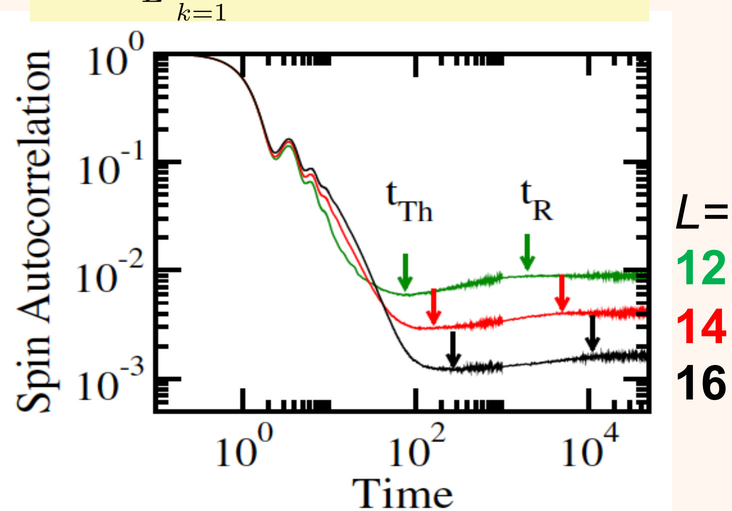
* **NOT self-averaging** at long-time

Non-Local in Time

$$SP(t) = \left|\langle\Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$



$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$

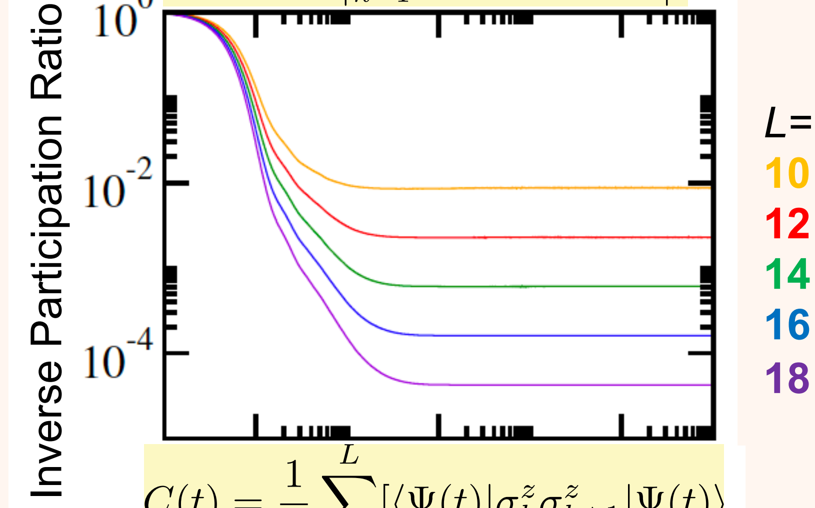


Non-Local in Space

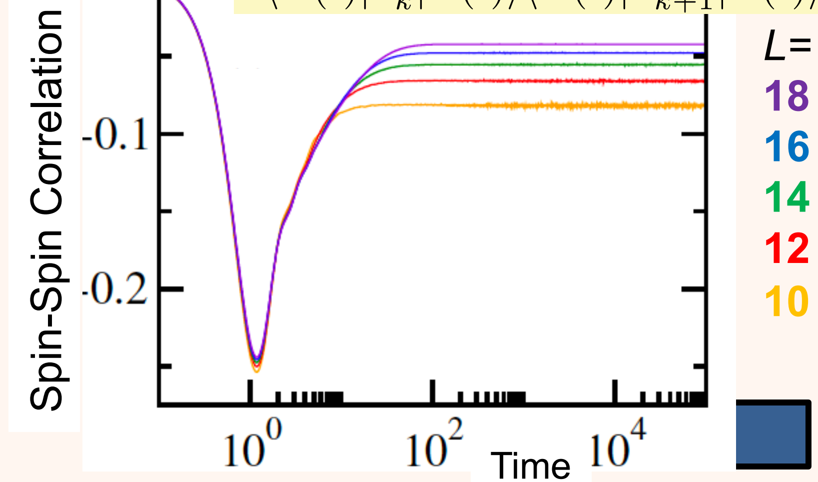
Local in Space

Local in Time

$$IPR(t) = \left|\sum_{n=1}^{D}\langle n|e^{-iHt}|\Psi(0)\rangle\right|^4$$



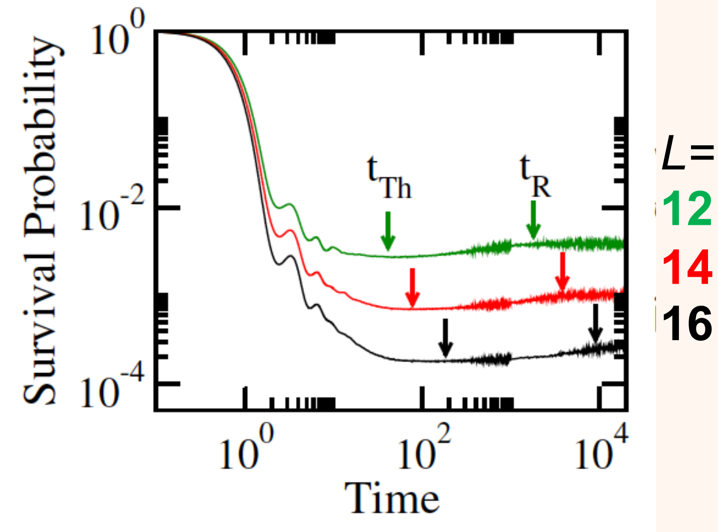
$$C(t) = \frac{1}{L}\sum_{k=1}^{L}\left[\langle\Psi(t)|\sigma_k^z\sigma_{k+1}^z|\Psi(t)\rangle - \langle\Psi(t)|\sigma_k^z|\Psi(t)\rangle\langle\Psi(t)|\sigma_{k+1}^z|\Psi(t)\rangle\right]$$



Non-Local in Time

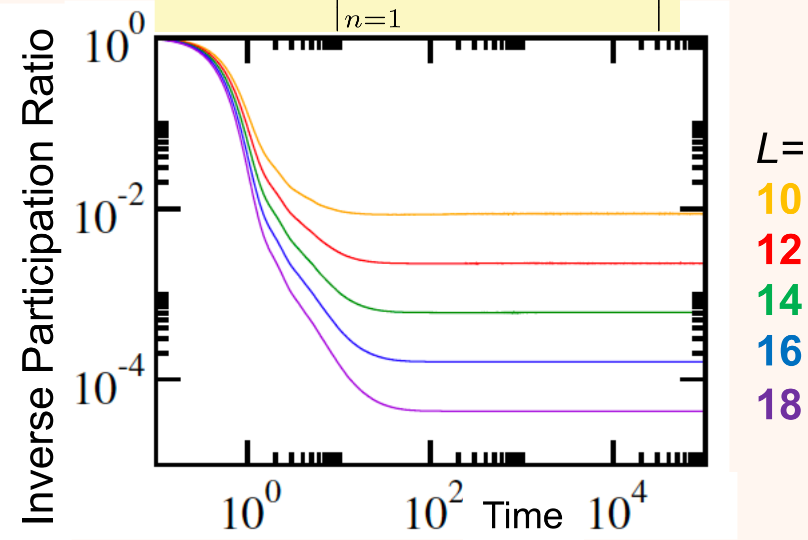
Local in Time

$$SP(t) = \left| \langle \Psi(0) | e^{-iHt} | \Psi(0) \rangle \right|^2$$



Non-Local in Space

$$IPR(t) = \left| \sum_{n=1}^{D} \langle n | e^{-iHt} | \Psi(0) \rangle \right|^4$$



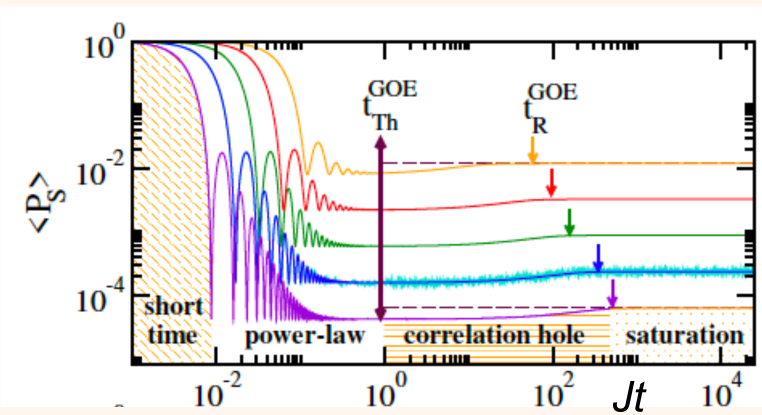
Autocorrelation functions:

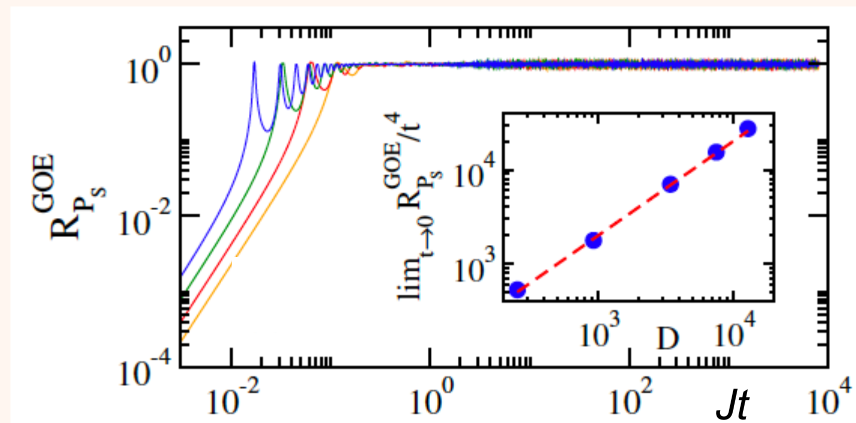
* Dynamical manifestations of spectral correlations
* Relaxation time grows exponentially with system size
* Relative temporal fluctuations do NOT decrease with L

Survival Probability
(non-local in space and time)

$$SP(t) = \left|\langle\Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$





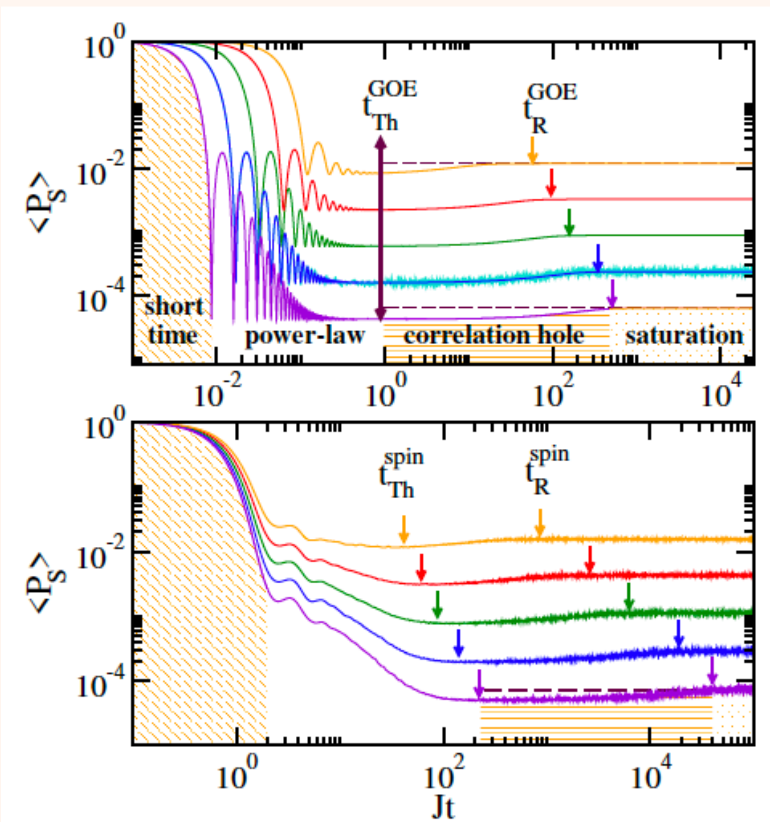
D=
252
924
3432
12870
48620

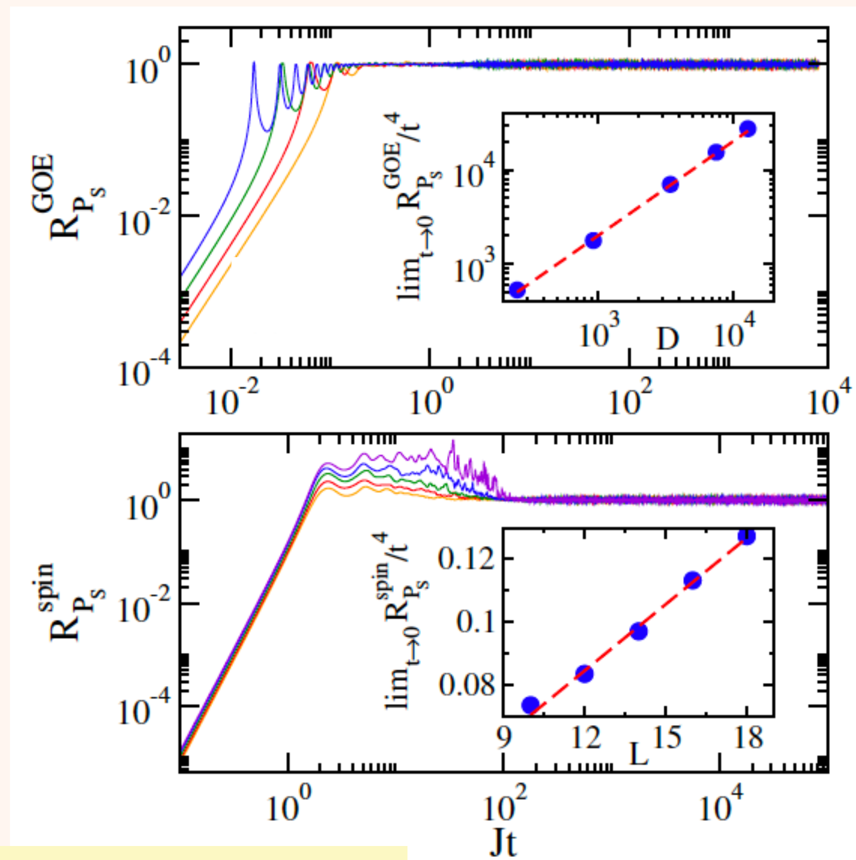
NOWHERE self-averaging

Survival Probability
(non-local in space and time)

$$SP(t) = \left|\langle\Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$





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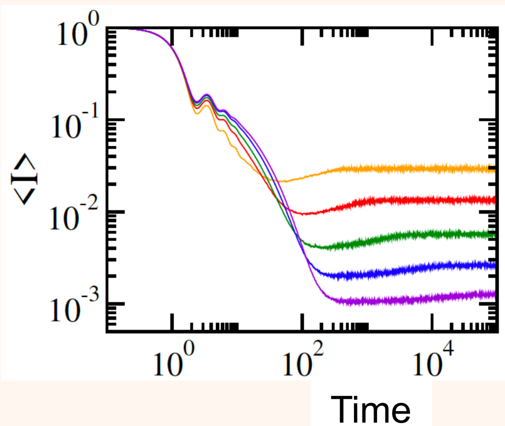
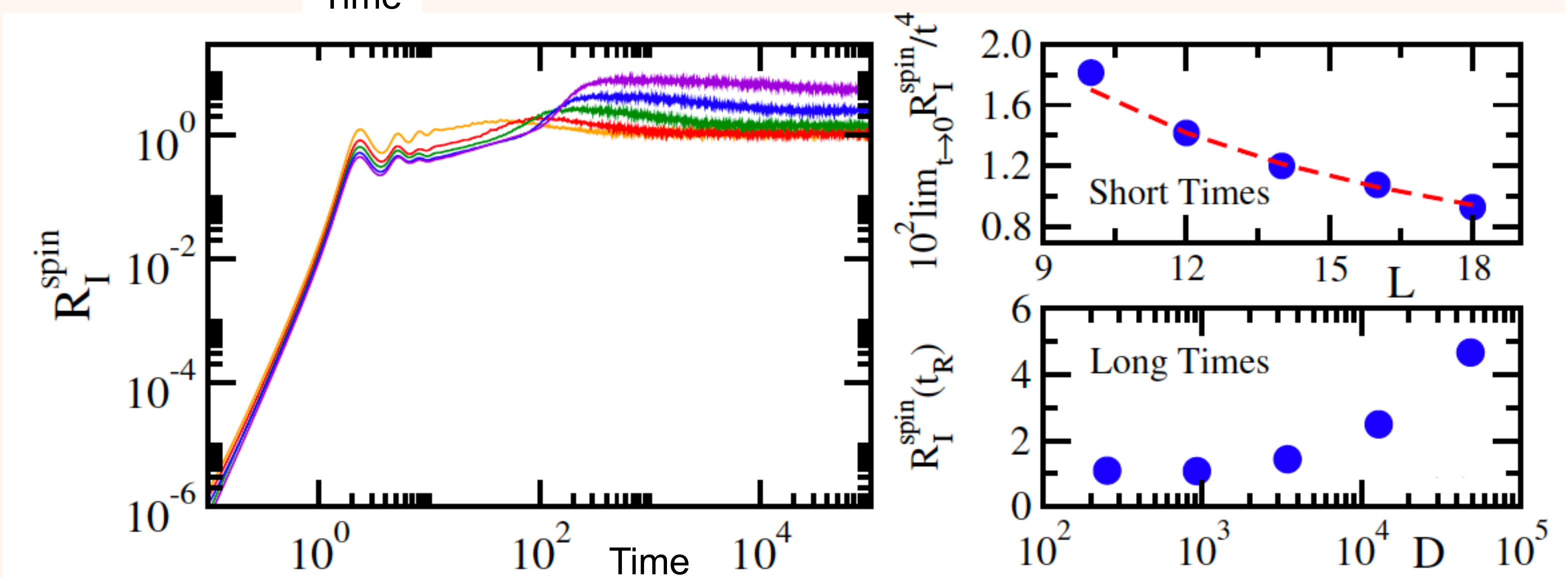
NOWHERE self-averaging

Spin Autocorrelation Function
(local in space BUT non-local in time)



$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$

$$\mathcal{R}_I(t) = \frac{\sigma_I^2(t)}{\langle I(t)\rangle^2} = \frac{\left\langle I^2(t)\right\rangle - \langle I(t)\rangle^2}{\langle I(t)\rangle^2}$$



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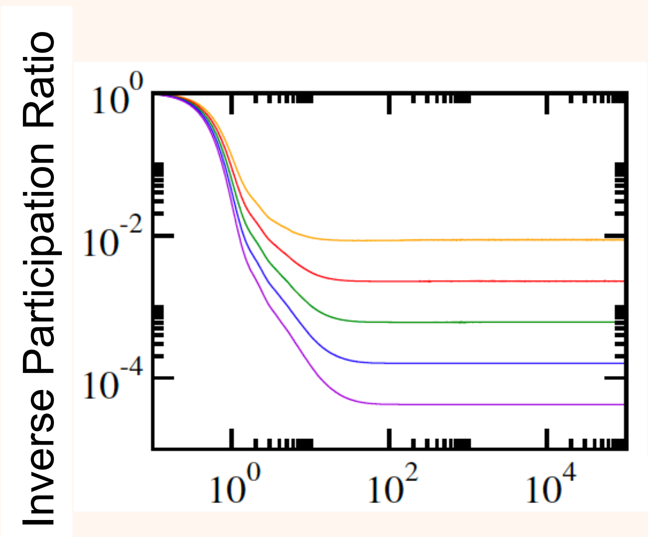
Self-averaging at SHORT times

Inverse Participation Ratio (non-local in space BUT local in time)

$$IPR(t) = \left| \sum_{n=1}^{D} \langle n | e^{-iHt} | \Psi(0) \rangle \right|^4$$

$$\mathcal{R}_{IPR}(t) = \frac{\sigma^2_{IPR}(t)}{\langle IPR(t) \rangle^2} = \frac{\langle IPR^2(t) \rangle - \langle IPR(t) \rangle^2}{\langle IPR(t) \rangle^2}$$



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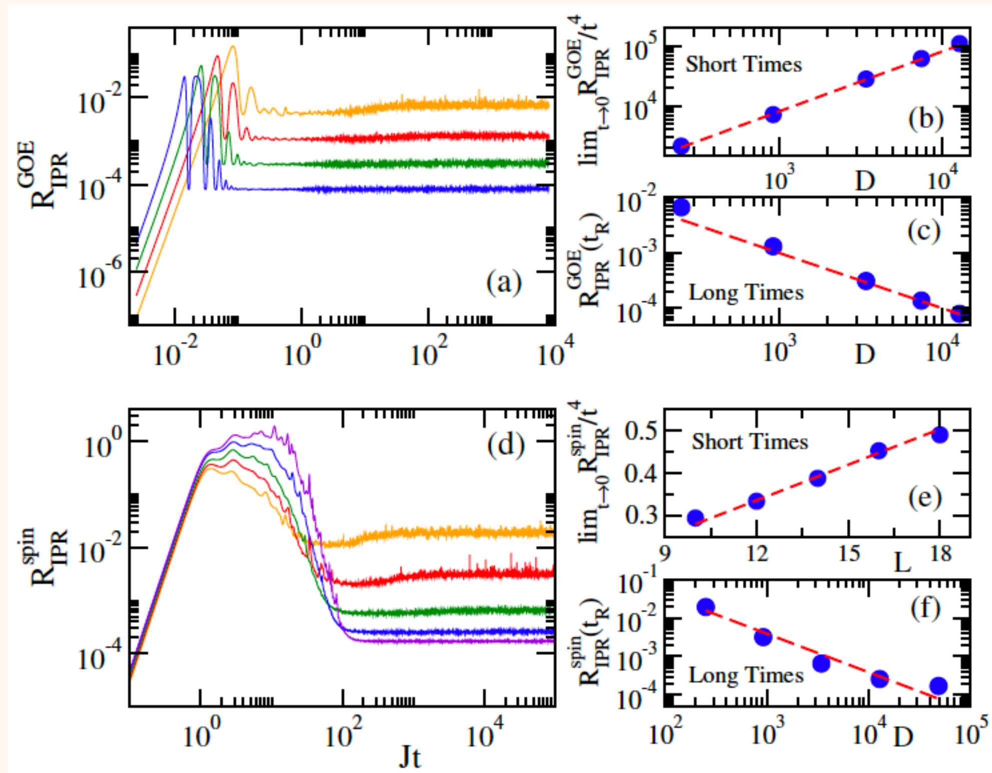
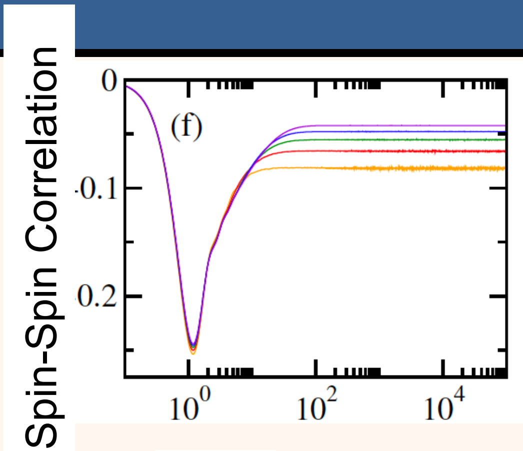


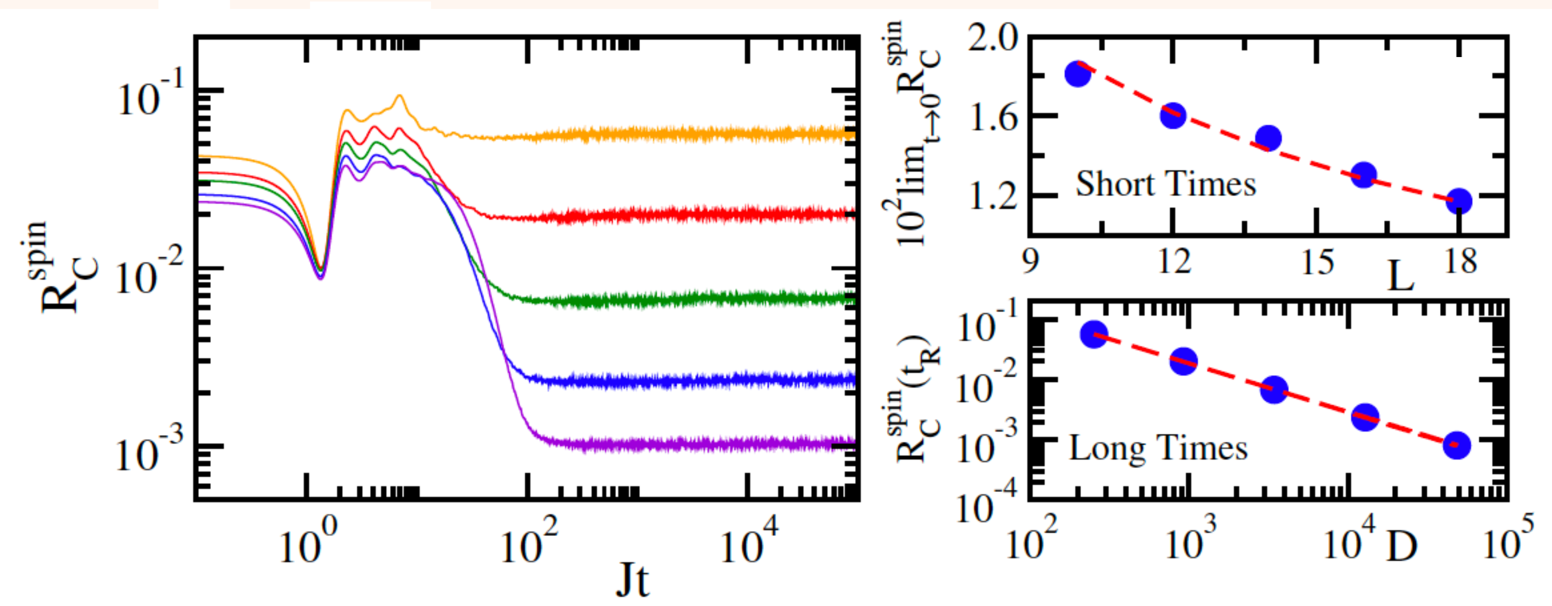
Self-averaging at LONG times

Spin-Spin Correlation Function (local in space and in time)



$$C(t) = \frac{1}{L}\sum_{k=1}^{L}[\langle\Psi(t)|\sigma_k^z\sigma_{k+1}^z|\Psi(t)\rangle - \langle\Psi(t)|\sigma_k^z|\Psi(t)\rangle\langle\Psi(t)|\sigma_{k+1}^z|\Psi(t)\rangle]$$

$$\mathcal{R}_C(t) = \frac{\sigma_C^2(t)}{\langle C(t)\rangle^2} = \frac{\langle C^2(t)\rangle - \langle C(t)\rangle^2}{\langle C(t)\rangle^2}$$



L=
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EVERYWHERE self-averaging

Summary

Self-averaging depends on

- Model
- Time scale
- Observable

Self-averaging is **NOT** an intrinsic consequence of quantum **chaos**.

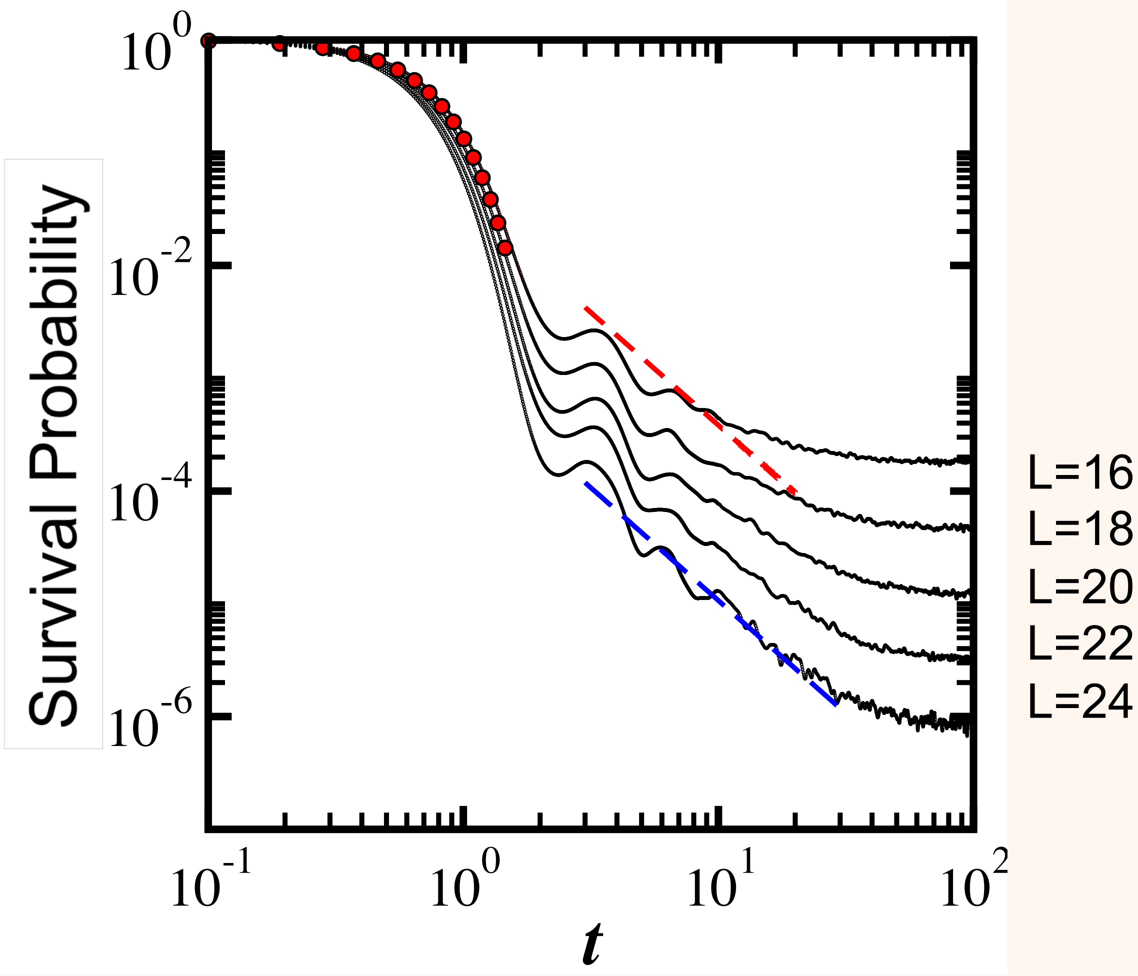
Observables that are **local in space**: **ARE** self-averaging at **short times.**

Observables that are **local in time**: **ARE** self-averaging at **long times.**

Survival probability is NOT self-averaging at ANY time scale (RMT).

Schiulaz, Torres, Bernal & LFS,
ArXiv: 1906.11856
ArXiv: 1910.11332

Power-law Decay: Chaotic Spin Model



Torres & LFS, EJP (2019)