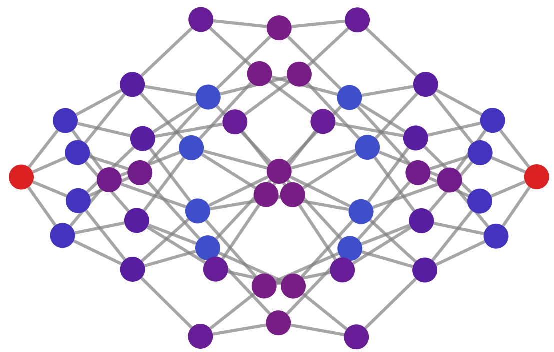
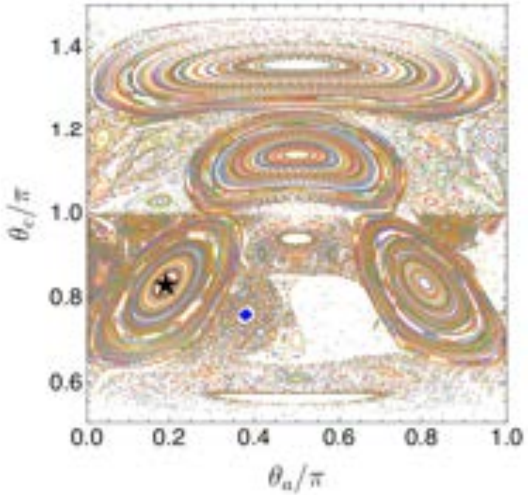
Quantum many-body scars, mixed phase spaces & non-universal thermalization



Maksym Serbyn

IST Austria



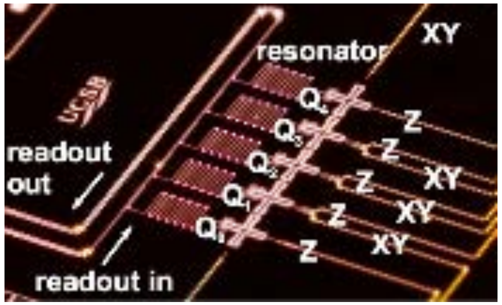
Nature Physics 14, 745–749 (2018)
Phys. Rev. B. 98, 155134 (2018)
Phys. Rev. Lett. 122, 220603 (2019)
arXiv: 1905.08564

IST AUSTRIA

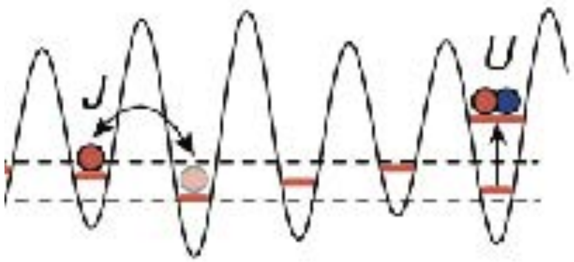
ICTS 2019

Isolated interacting quantum systems

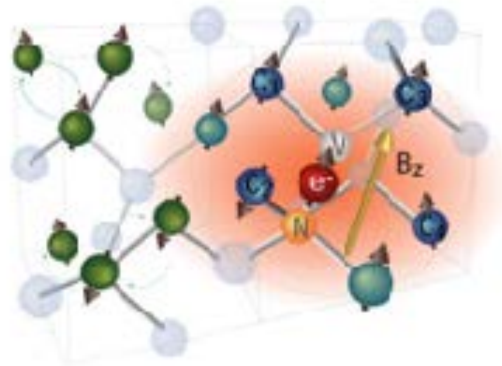
mutli-qubit systems



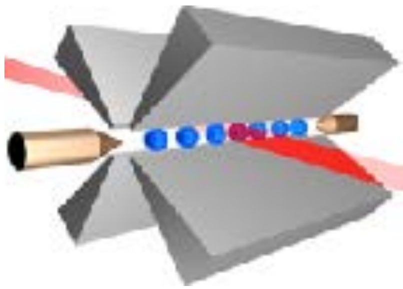
cold atoms



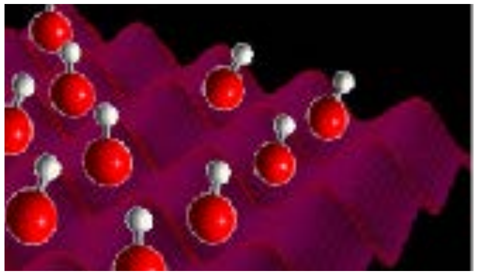
NV centers in diamond,



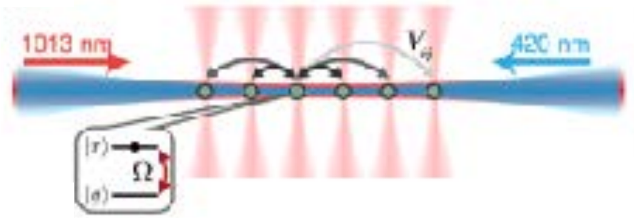
trapped ions



polar molecules



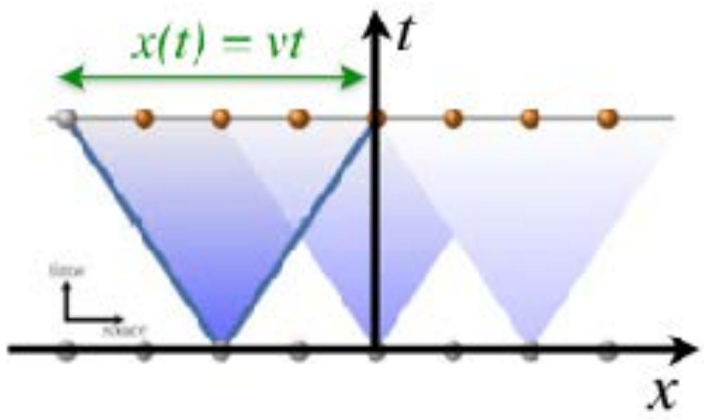
Ry atoms chains



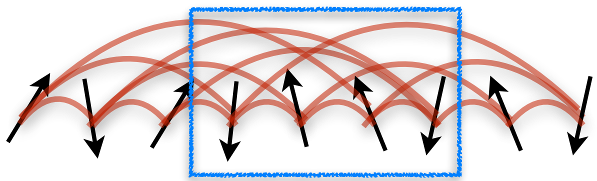
Long coherence times = new dynamical phenomena?

Thermalization vs MBL

$$S_{\text{ent}} \propto vt$$



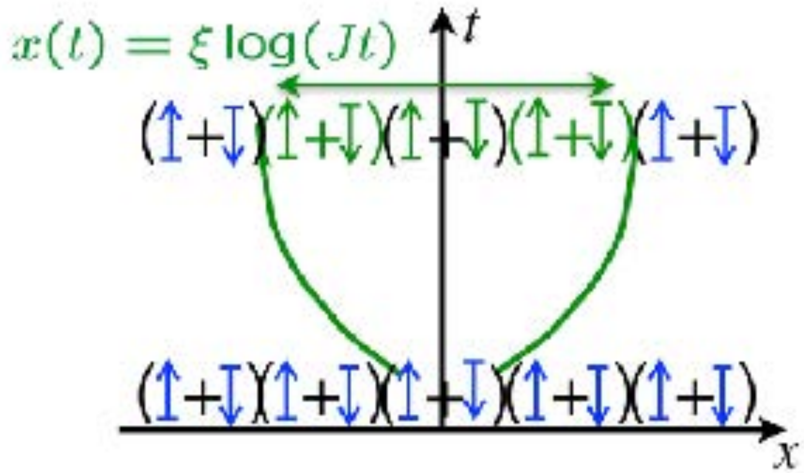
$$S_{\text{ent}}(A) \propto \text{vol}(A)$$



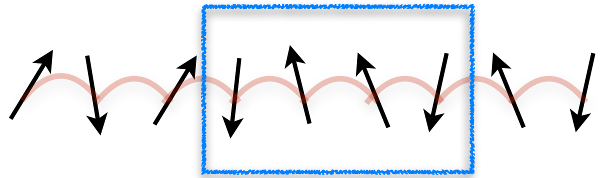
Eigenstate Thermalization Hypothesis

[D'Alessio et al, arXiv:1509.06411]

$$S_{\text{ent}} \propto \xi \log Jt$$



$$S_{\text{ent}}(A) \propto \text{area}(A)$$

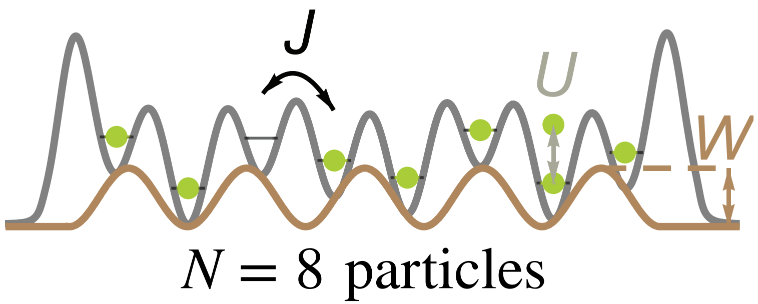


Quasi-Local Integrals of Motion

[Nandkishore &Huse, Annual Rev Cond Mat'15]
[Abanin, Altman, Bloch & MS, RMP'19]

Example: entanglement in MBL phase

- Interacting bosons in quasi-periodic potential [A. Lukin,…,Greiner, Science'19]



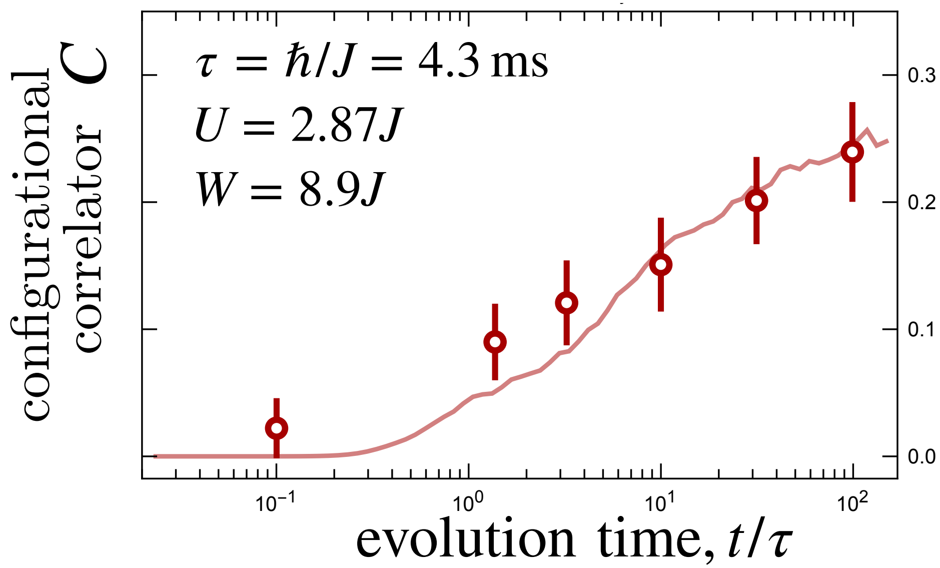
$$\hat{\mathcal{H}} = -J\sum_i(\hat{a}_i^\dagger\hat{a}_{i+1} + h.c.) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + W\sum_i h_i\hat{n}_i$$

- Measure entanglement entropy $S_{\mathrm{ent}}(t)$

$$S_{\mathrm{ent}}(t) = S_n + C$$

S_n: O(1) without transport

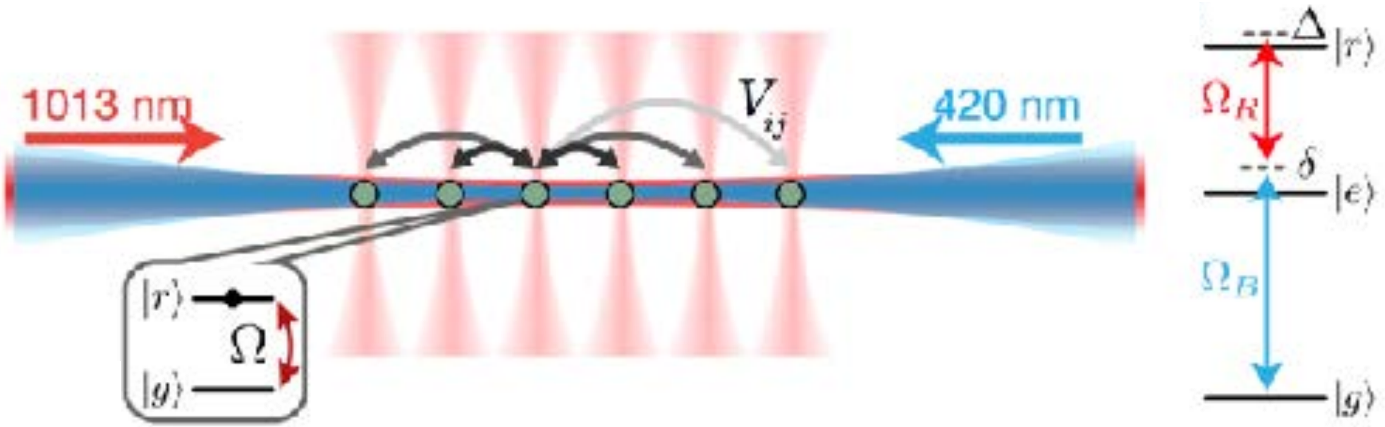
C: $\propto \ln t$



Routes to **long-time coherent dynamics** without disorder?

Hints from Rydberg atoms array

Atom-by-atom assembly of Rydberg chain



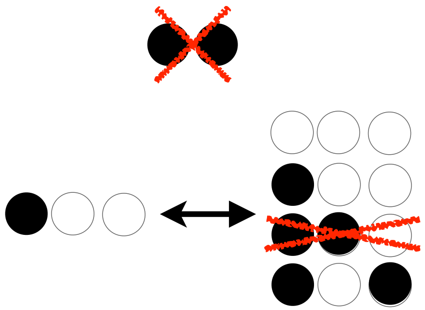
[Bernien et al, Nature 2017, arXiv:1707.04344]

Effective description: two states per atom:

● excited (Rydberg) state

○ ground state

Rydberg blockade

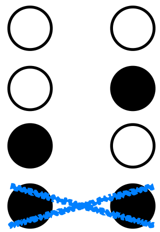


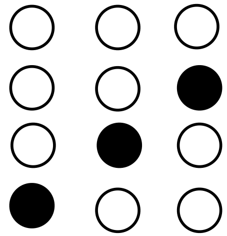
→long-time oscillations

→ rapid relaxation

PXP model as a graph

Hilbert space: $\mathcal{D}_2 = 3$ $\mathcal{D}_3 = 4$ $\mathcal{D}_L = F_{L-1} + F_{L+1}$





sum of Fibonacci #

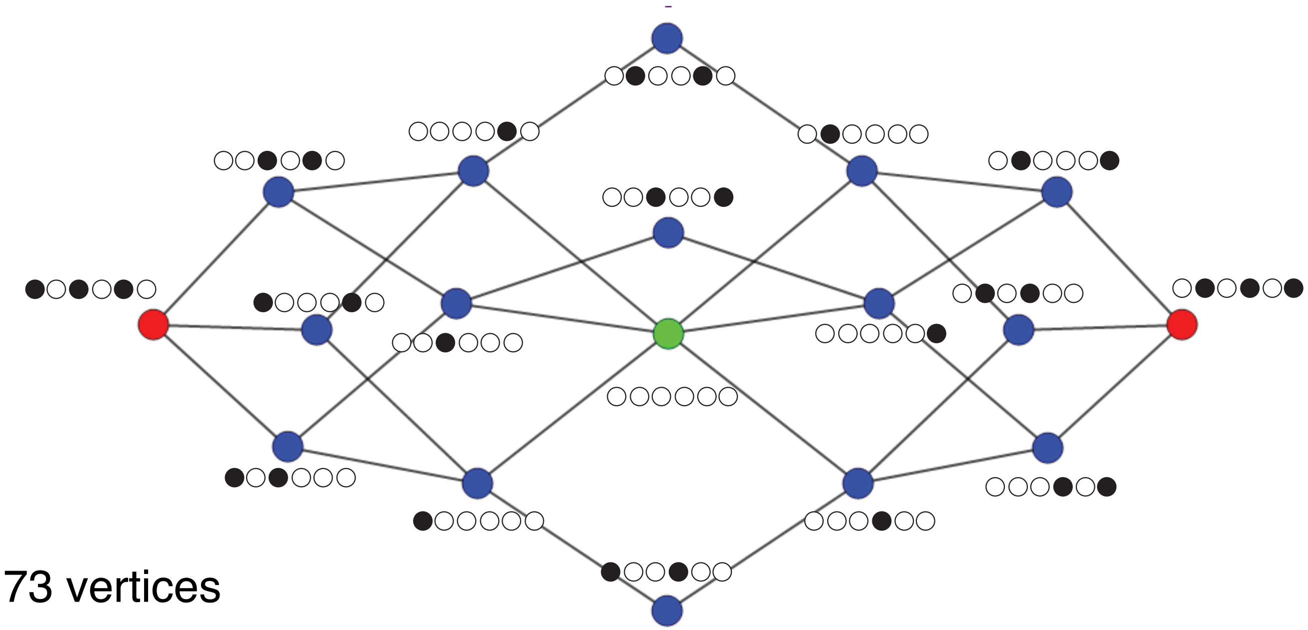
no tensor product structure!

Hamiltonian:

$$H = \sum_i P^{\circ}_{i-1} X_i P^{\circ}_{i+1}$$

[Fendley, Sengupta, Sachdev, PRB'04]
[Fendley, Schoutens, PRL'05]

Hilbert space + Hamiltonian = graph + adjacency matrix



Experiment: L=51

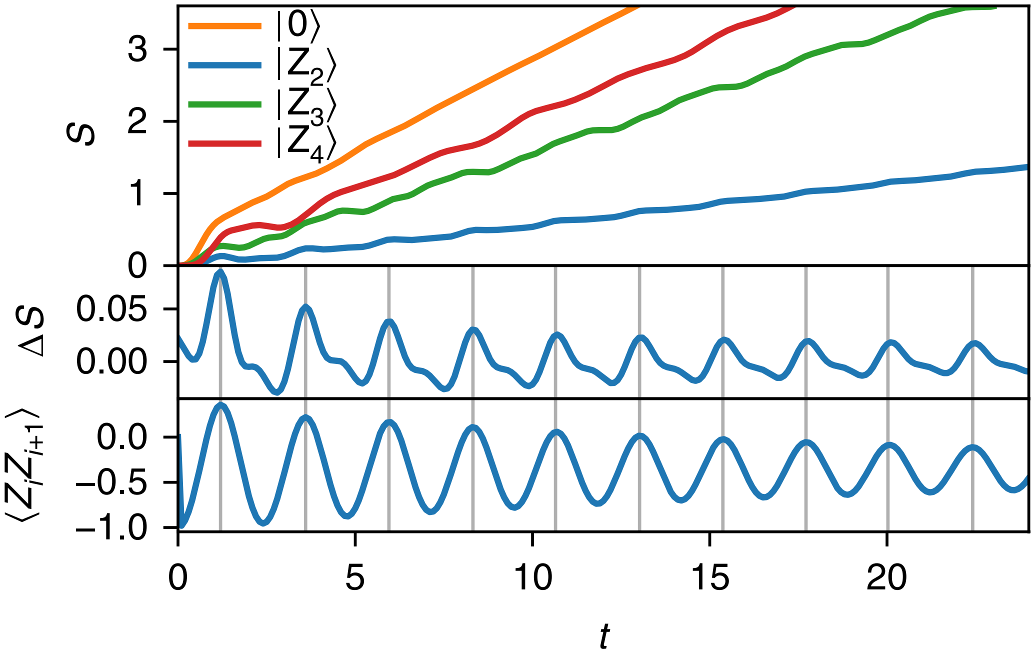
F_{53} =53,316,291,173 vertices

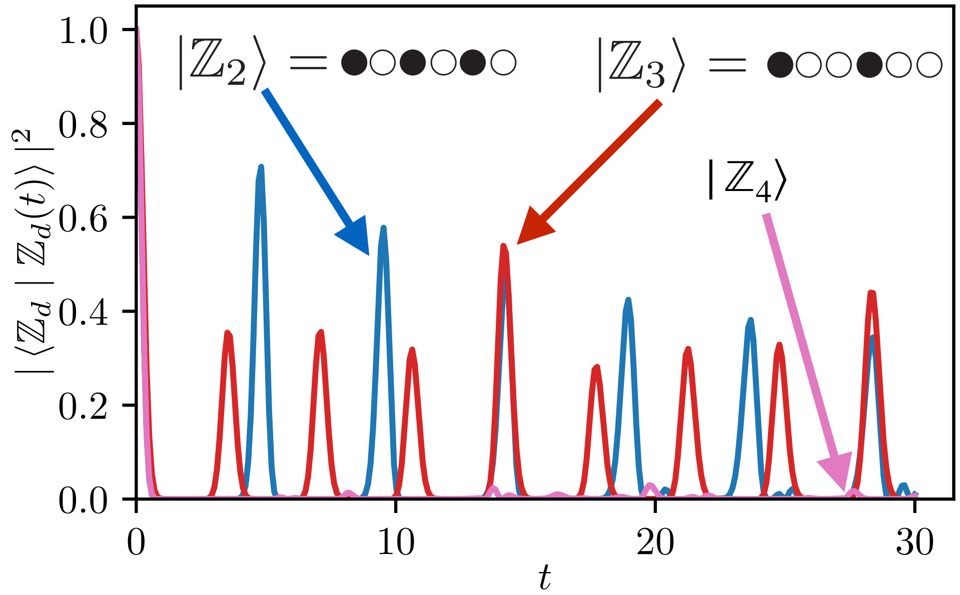
Growth of entanglement vs revivals

$|\mathbb{Z}_2\rangle = $ ●○●○●○ $|\mathbb{Z}_3\rangle = $ ●○○●○○ $|0\rangle = $ ○○○○○○

Bipartite entanglement
Local observables

Return probability L=24





Linear growth → no MBL

Many-body revivals

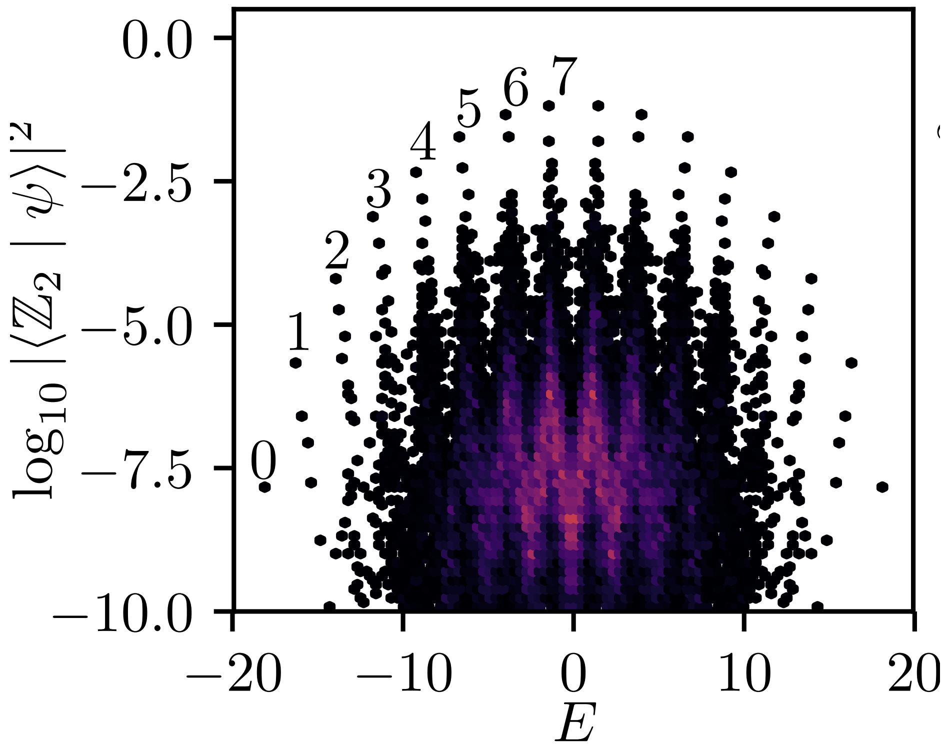
[Turner et al., Nature Physics 2018, arXiv:1711.03528]

Origin of revivals?

Eigenstate picture: Z_2 "special band"

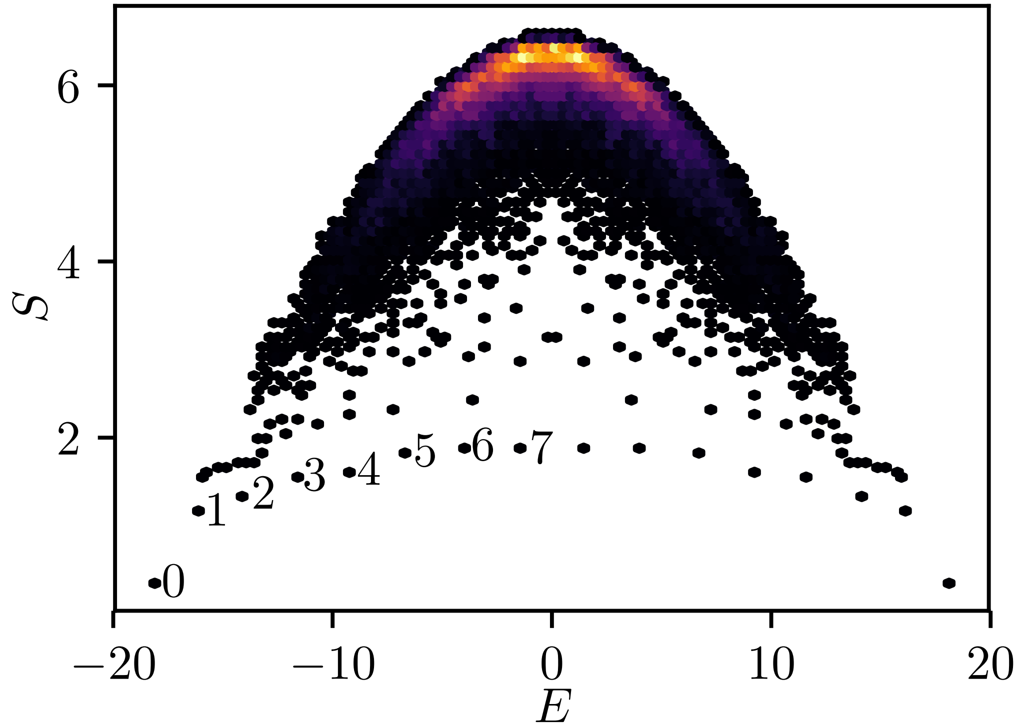
enhanced overlaps with

$$|\mathbb{Z}_2\rangle = \bullet\circ\bullet\circ\bullet\circ$$



Vs. random eigenstates in ergodic systems

anomalously low entanglement entropy



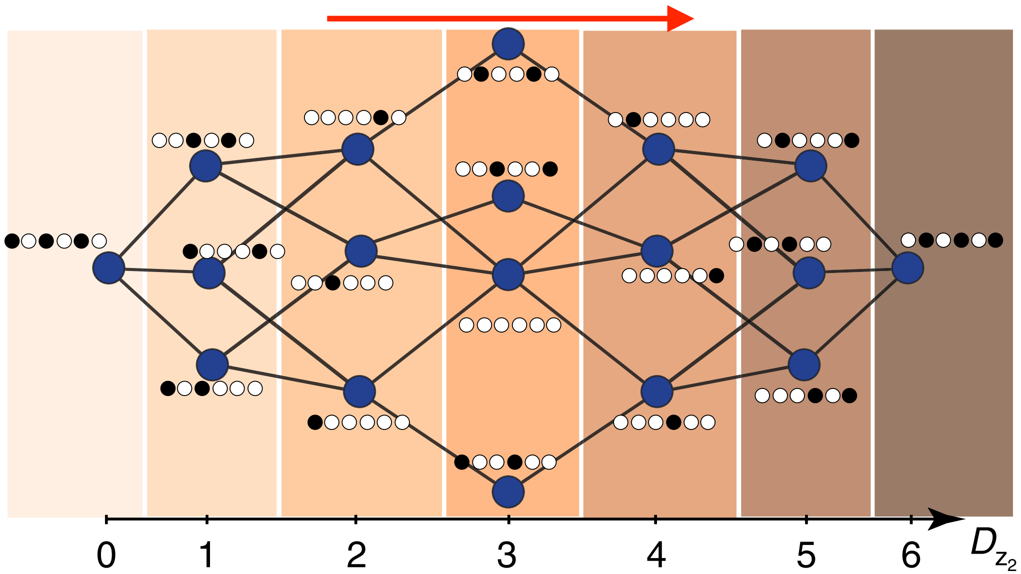
Vs. volume law S_{ent} in ergodic systems area-law in MBL systems

How to understand these L+1 ETH violating eigenstates?

Failed SU(2) algebra

$H = H_+ + H_-$ = forward + backward

$$H^+ = \sum_{i\in \text{ even}} P_{i-1}\sigma_i^+ P_{i+1} + \sum_{i\in \text{ odd}} P_{i-1}\sigma_i^- P_{i+1}$$



$$[H^+, H^-] = H^z = \sum_{i\in \text{ even}} P_{i-1} Z_i P_{i+1} - \sum_{i\in \text{odd}} P_{i-1} Z_i P_{i+1}$$

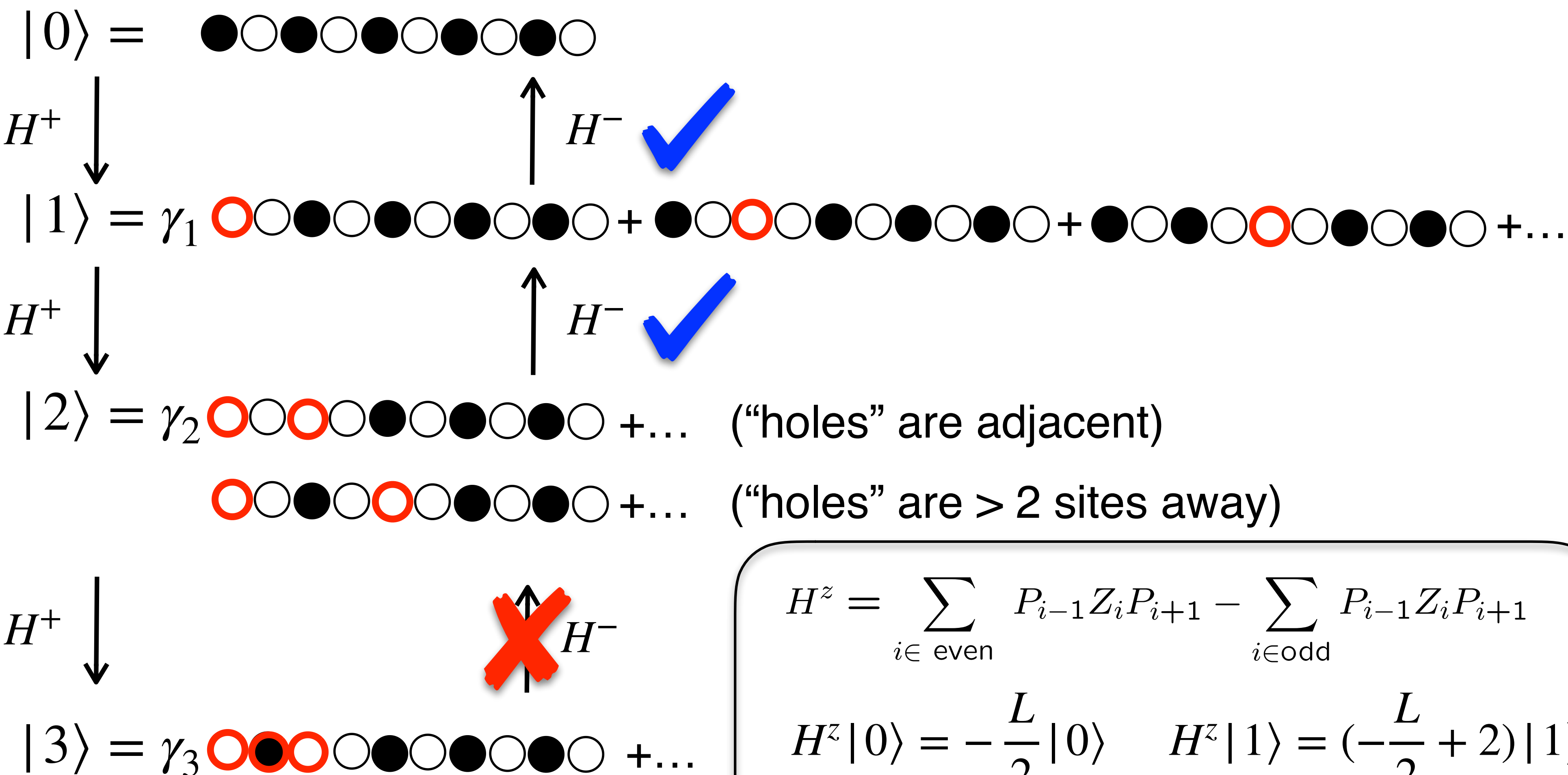
Incomplete spin algebra:

$$[H^z, H^+] = 2H^+ - \sum_{\text{odd}} P_{i-1} S_i^+ P_{i+1}(P_{i-2} + P_{i+2}) - \sum_{\text{even}} \text{h.c.}$$

corrections

From algebra to representation

- Split $H = H^+ + H^-$ with $\{H^+, H^-, H^z\} \approx$ SU(2) generators
- Build SU(2) representation from Neel state:



$$H^z = \sum_{i\in \text{ even}} P_{i-1}Z_iP_{i+1} - \sum_{i\in\text{odd}} P_{i-1}Z_iP_{i+1}$$

$$H^z|0\rangle = -\frac{L}{2}|0\rangle \qquad H^z|1\rangle = (-\frac{L}{2}+2)|1\rangle$$

$$H^z|2\rangle \neq (-\frac{L}{2}+4)|2\rangle$$

Fixing SU(2) representation

- Deform H by effective interaction: $\delta H_2 = h_2 \sum_i P_{i-1} X_i P_{i+1} (Z_{i-2} + Z_{i+2})$

$H^z|2\rangle = C_1$ ○○○○●○●○●○ +… ("holes" are adjacent)

$+C_2$ ○○●○○○●○●○ +… ("holes" are 2 sites away)

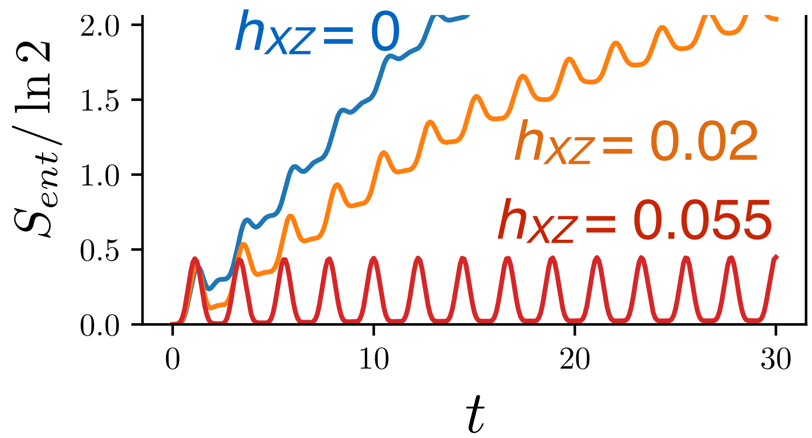
$+C_1$ ○○●○●○○○●○ +… ("holes" are >2 sites away)

- Achieved when: $1 - 20h_2^*(1 - h_2^*) = 0$

$$h_2^* = \frac{1}{2} - \frac{1}{\sqrt{5}} \approx 0.0527$$

Relation to $h_2 \approx 0.02$ max <u>average</u> non-ergodicity? [Khemani et al., PRB 2019]

L=24, quench from Neel



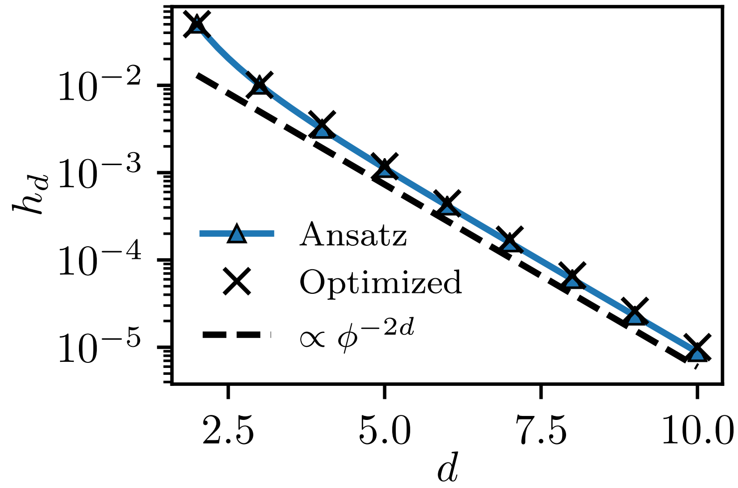
- Problem with NNN holes
  → longer range deformation $\delta H_R = \sum_i P_{i-1} X_i P_{i+1} \sum_{d=2}^{R} h_d (Z_{i-d} + Z_{i+d})$

[Choi et al, PRL 2019]

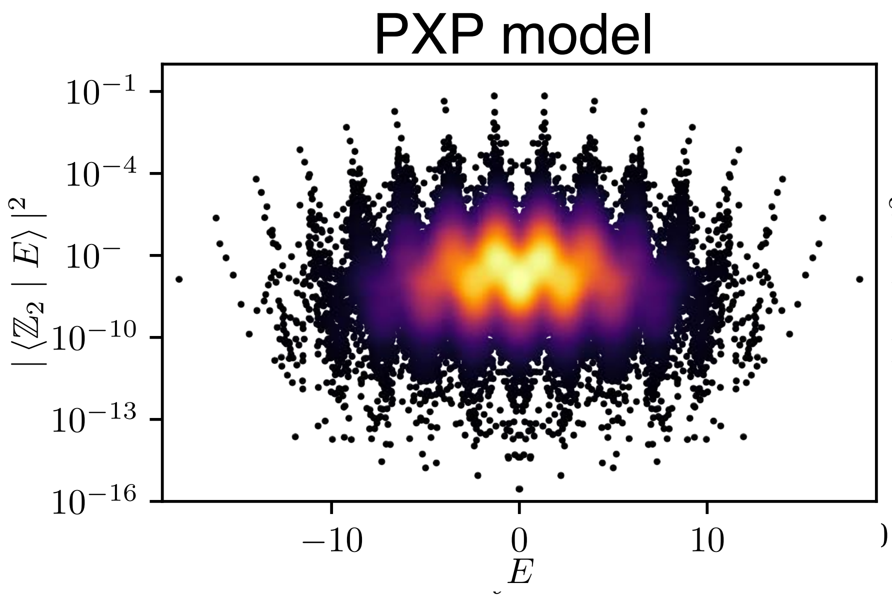
Perfect SU(2) representation

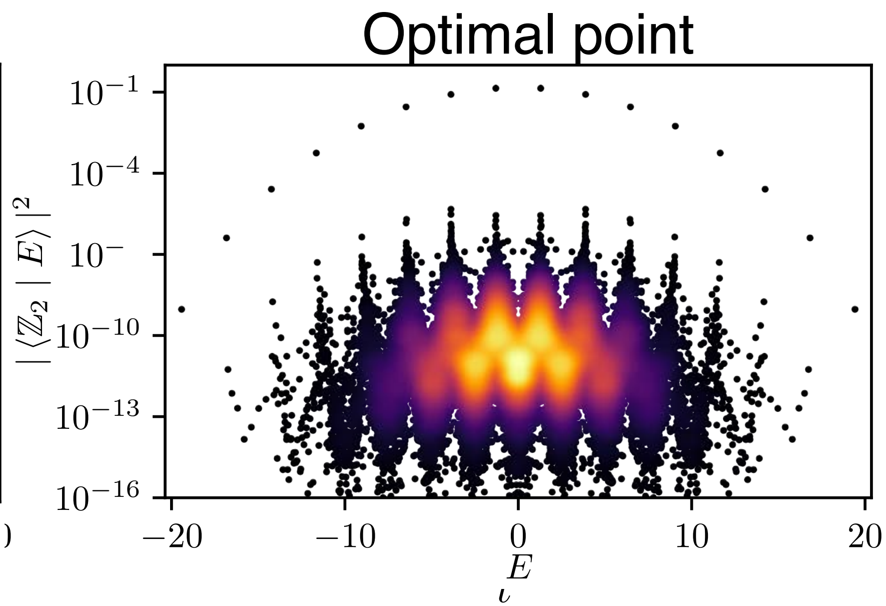
- Optimizing deformation coefficients

$$h_d^{\text{ansatz}} = h_0 \left(\phi^{(d-1)} - \phi^{-(d-1)}\right)^{-2}$$
$$\phi = \left(1+\sqrt{5}\right)/2$$



- Nearly perfect SU(2) representation, fidelity revivals, etc…



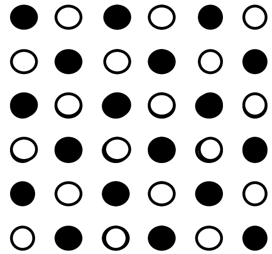


[Choi et al, PRL 2019]

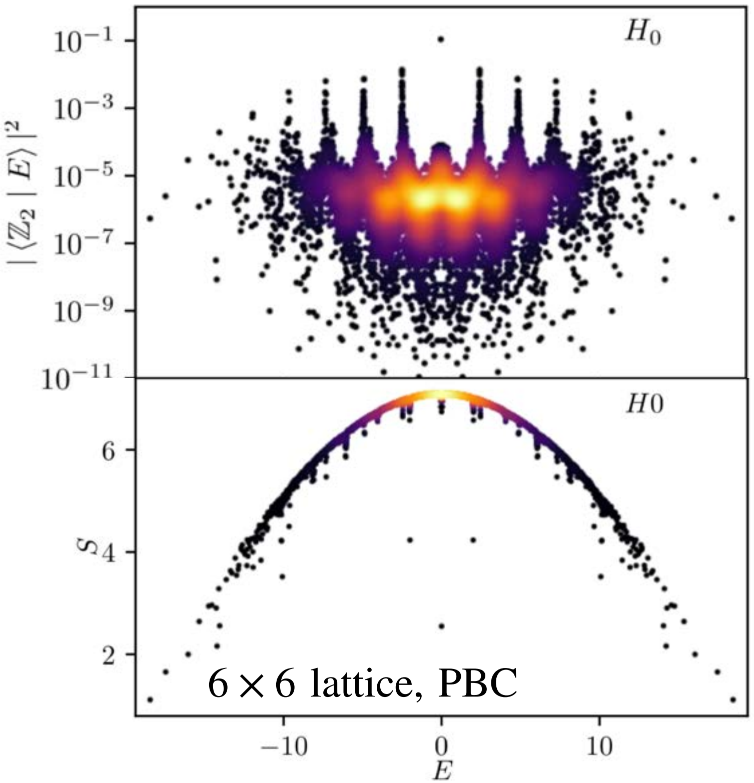
Generalization to Rydberg lattices

PXP model on a square lattice

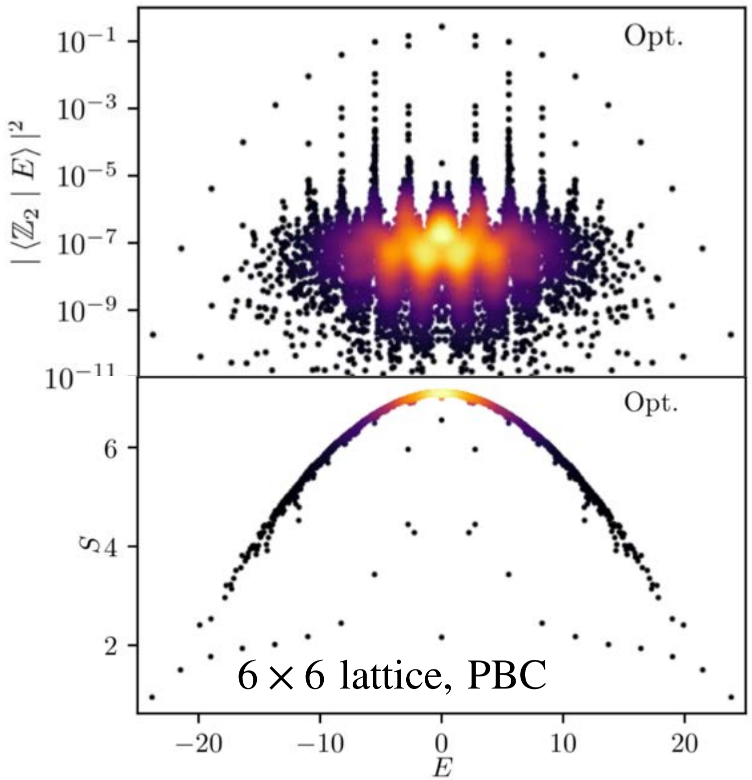
$$H = \sum P^{\circ}_{i-1,j} P^{\circ}_{i,j-1} X_{i,j} P^{\circ}_{i+1,j} P^{\circ}_{i,j+1}$$

$|Z_2\rangle =$ 

PXP model



PXP model + NN deformation



Quantum scars on **arbitrary bipartite lattices** [A. Michailidis *et al.*, in preparation]

Summary I: quantum scars

- Low entanglement, ETH-violating eigenstates

[Nat. Phys. 14, 745–749 (2018)]
[Phys. Rev. B. 98, 155134 (2018)]

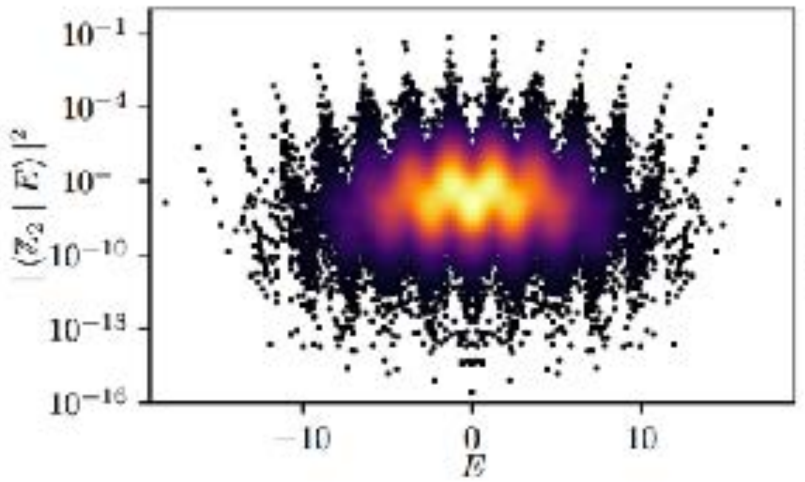
- Quasi-local deformation → emergent SU(2) representation

$$\delta H_R = \sum_i P_{i-1} X_i P_{i+1} \sum_{d=2}^{R} h_d (Z_{i-d} + Z_{i+d})$$

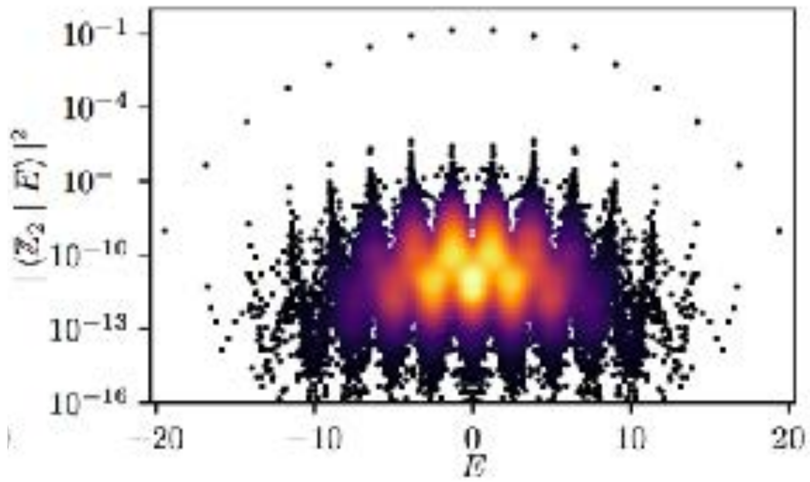
[Phys. Rev. Lett. 122, 220603 (2019)]

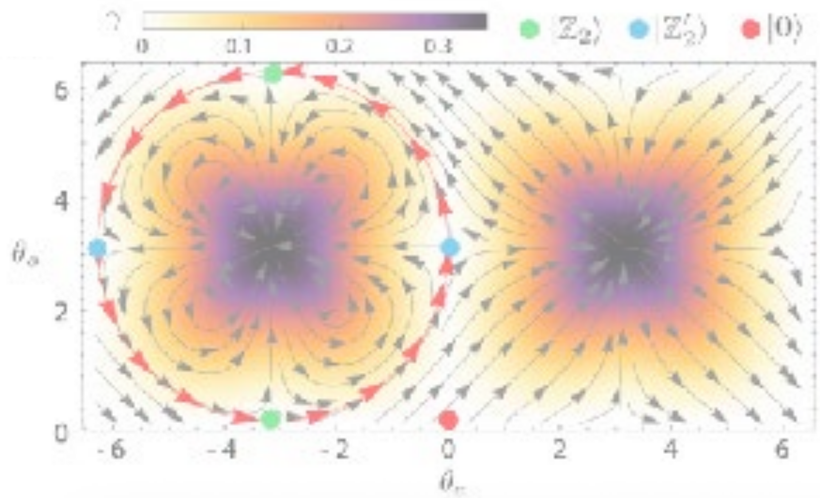
Variational picture of dynamics?

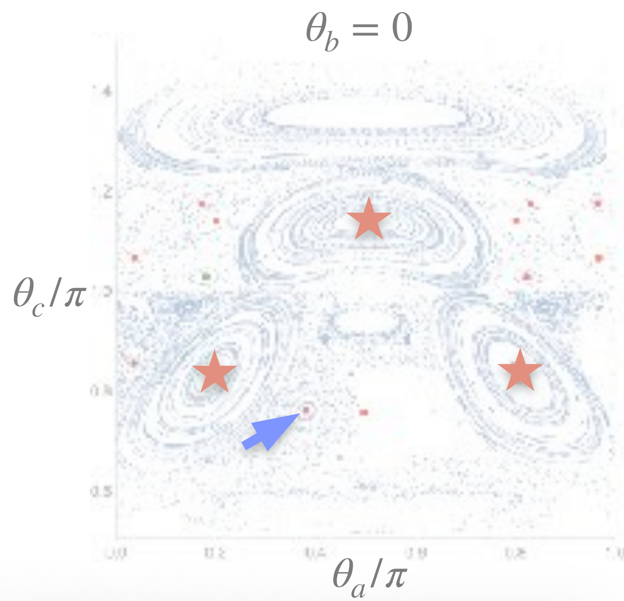
PXP model, *L*=30



deformed model, *L*=30

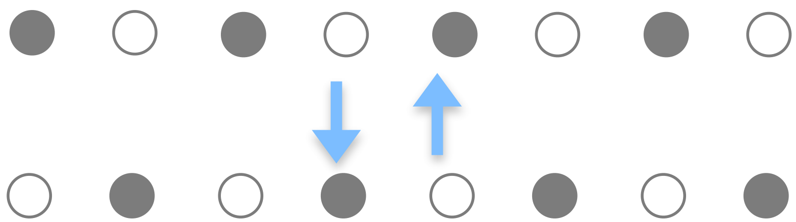






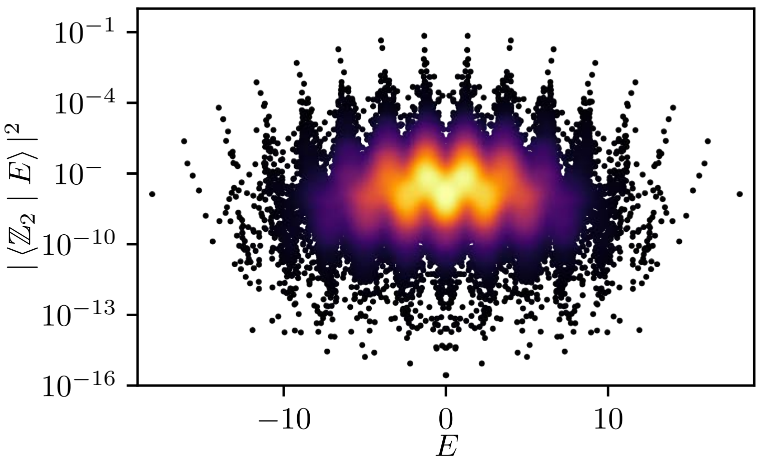
Variational principle and scars

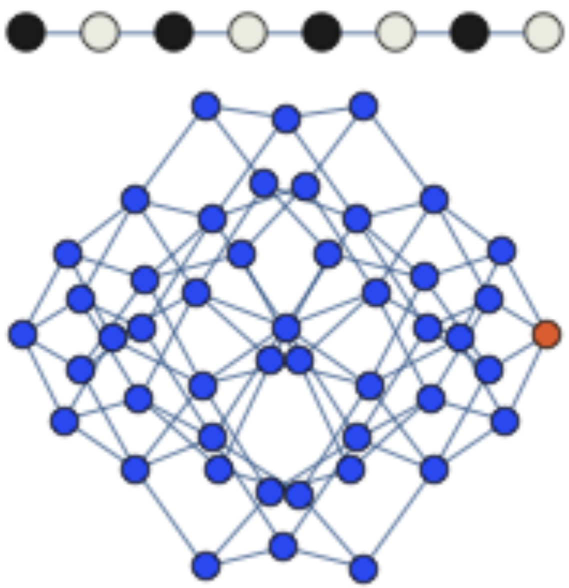
trajectory connecting 2 Néel states



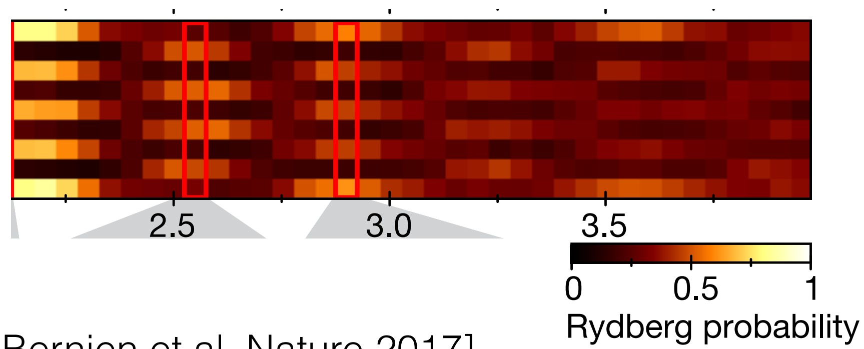
Collective Rabbi oscillations

Hilbert space picture



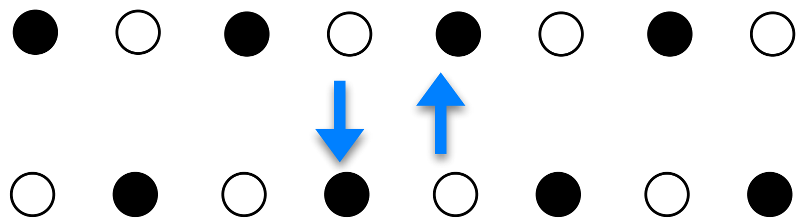


Dynamics picture



[Bernien et al, Nature 2017]

oscillations between 2 Néel states



2-sites unit cell

Variational principle → classical dynamical system

Quantum Hamiltonian

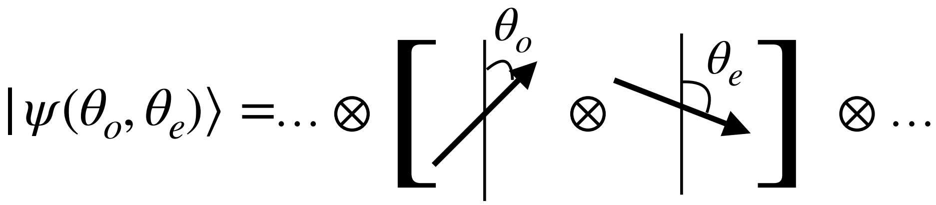
variational state

Classical equations of motion

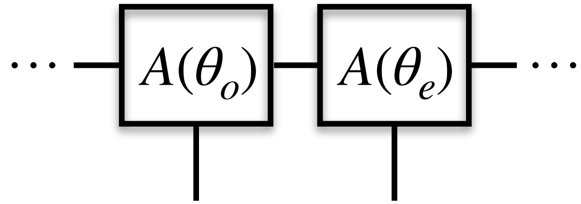
Makes and models:

- Mean field
- Cluster truncated Wigner approximation
- Disentangling & Gaussian states
- TDVP on MPS [Haegeman et al'11]

Time dependent variational principle with MPS

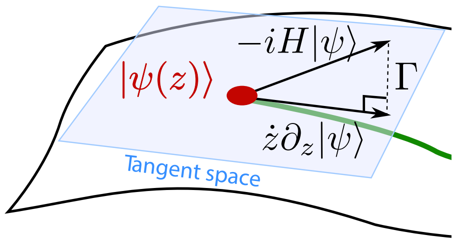


projection



MPS χ=2 state: $A^{\circ} = \begin{pmatrix} \cos\theta & 0 \\ 1 & 0 \end{pmatrix}$ $A^{\bullet} = \begin{pmatrix} 0 & i\sin\theta \\ 0 & 0 \end{pmatrix}$

[Bernien et al, Nature 2017, arXiv:1707.04344]

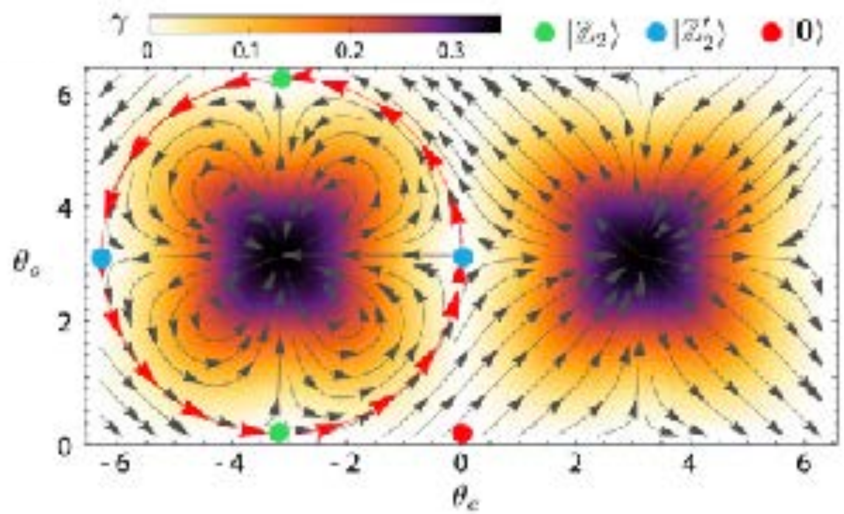


$$-i\dot{z}\langle\bar{\partial}\psi(\bar{z})|\partial\psi(z)\rangle = \langle\bar{\partial}\psi(\bar{z})|H|\psi(z)\rangle$$

[Haegeman et al'11]

$$\dot{\theta}_a = -\frac{1}{\cos\theta_b}(\cos^2\theta_b + \cos^2\theta_a \sin\theta_a \sin\theta_b)$$

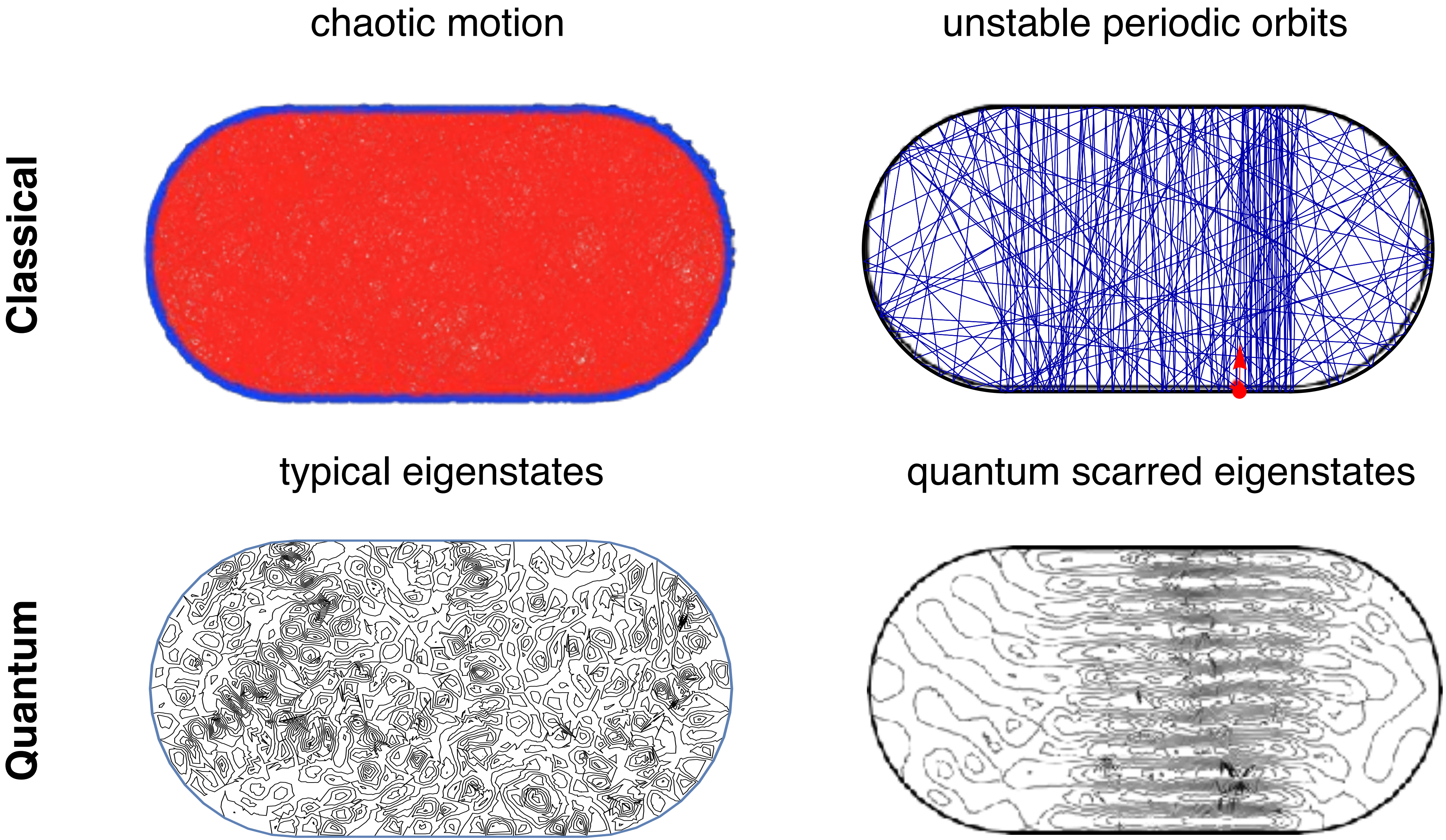
$$\dot{\theta}_b = -\frac{1}{\cos\theta_a}(\cos^2\theta_a + \cos^2\theta_b \sin\theta_b \sin\theta_a)$$



[WW Ho et al, PRL'2018]

Unstable trajectory → "**quantum scarred**" eigenstates

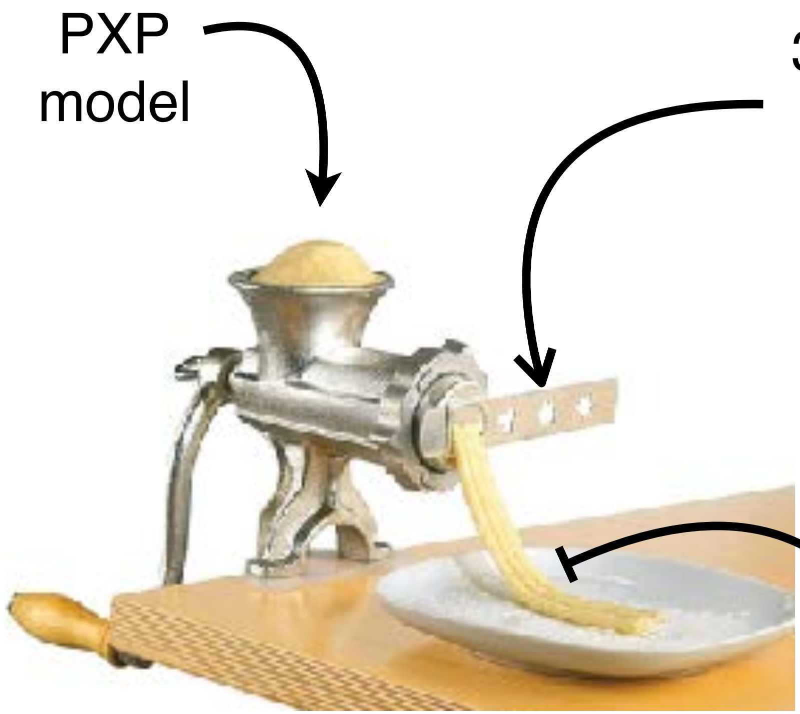
Chaotic Bunimovich stadium

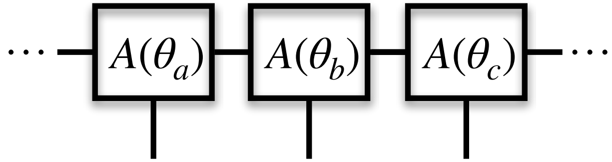


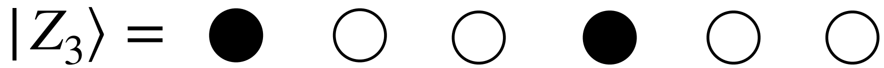
[Heller, PRL'84]

Role of **stable orbits** and **mixed phase space**?

TDVP with 3-site unit cell







Classical Hamiltonian system

$$\dot{\theta}_a = \frac{\sin(2\theta_c)\left(\sin(\theta_a)\left(\cos(2\theta_b)+7\right) - 2\sin(3\theta_a)\sin^2(\theta_b)\right) + 8\cos^3(\theta_b)\cos(2\theta_c) + 10\cos(\theta_b) - 2\cos(3\theta_b)}{16\left(\cos^2(\theta_b)\sin^2(\theta_c) - 1\right)}$$

$$\dot{\theta}_b = \frac{-4\sin(2\theta_a)\left(\sin(\theta_b)\left(\cos(2\theta_c)+7\right) - 2\sin(3\theta_b)\sin^2(\theta_c)\right) - 8\cos(\theta_c)\left(4\cos(2\theta_a)\cos^2(\theta_c)+5\right) + 8\cos(3\theta_c)}{64\left(\sin^2(\theta_a)\sin^2(\theta_c) + \cos^2(\theta_a)\right)}$$

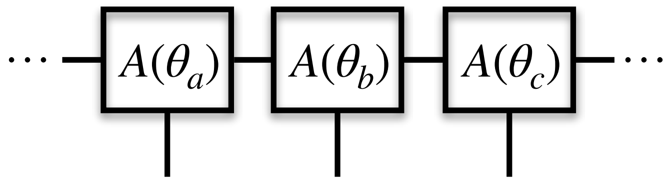
$$\dot{\theta}_c = \frac{\sin(2\theta_b)\sin(\theta_c)\left(-2\cos(2\theta_a)\cos^2(\theta_c) + \cos(2\theta_c) - 3\right) - \cos(\theta_a)\left(3\cos(2\theta_b)+5\right) + 2\cos(3\theta_a)\sin^2(\theta_b)}{2\left(-2\cos(2\theta_a)\sin^2(\theta_b) + \cos(2\theta_b) + 3\right)}$$

[A. Michailidis et al, arXiv: 1905.08564]

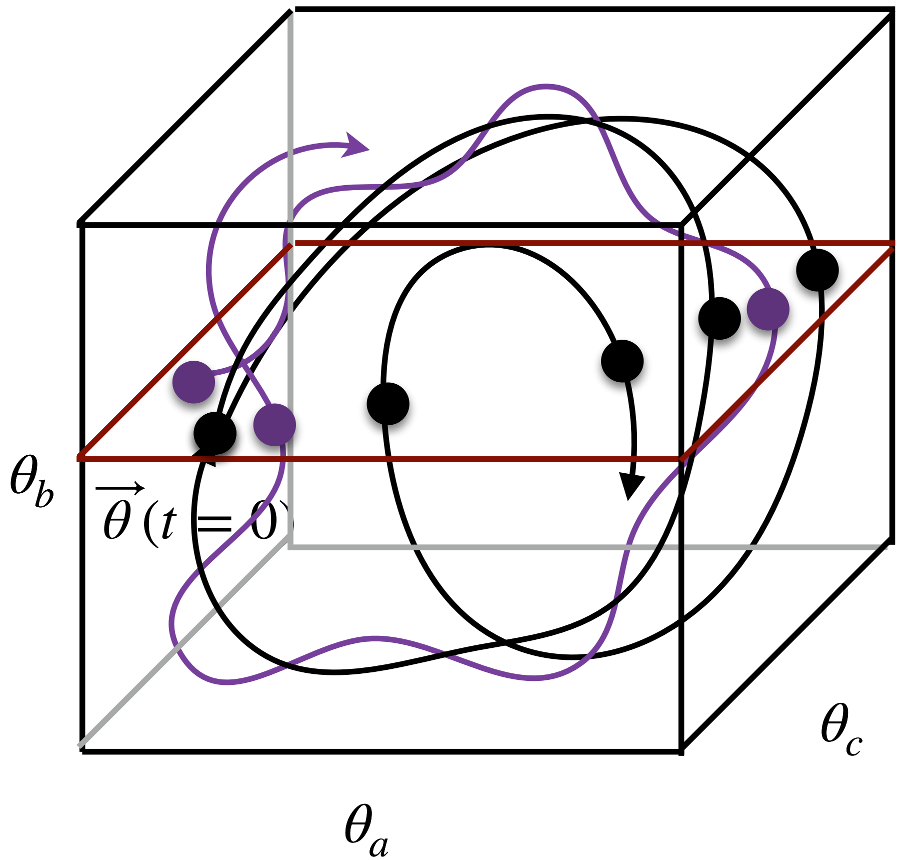
Poincare section: mixed phase space

- Dynamical system for 3 variables

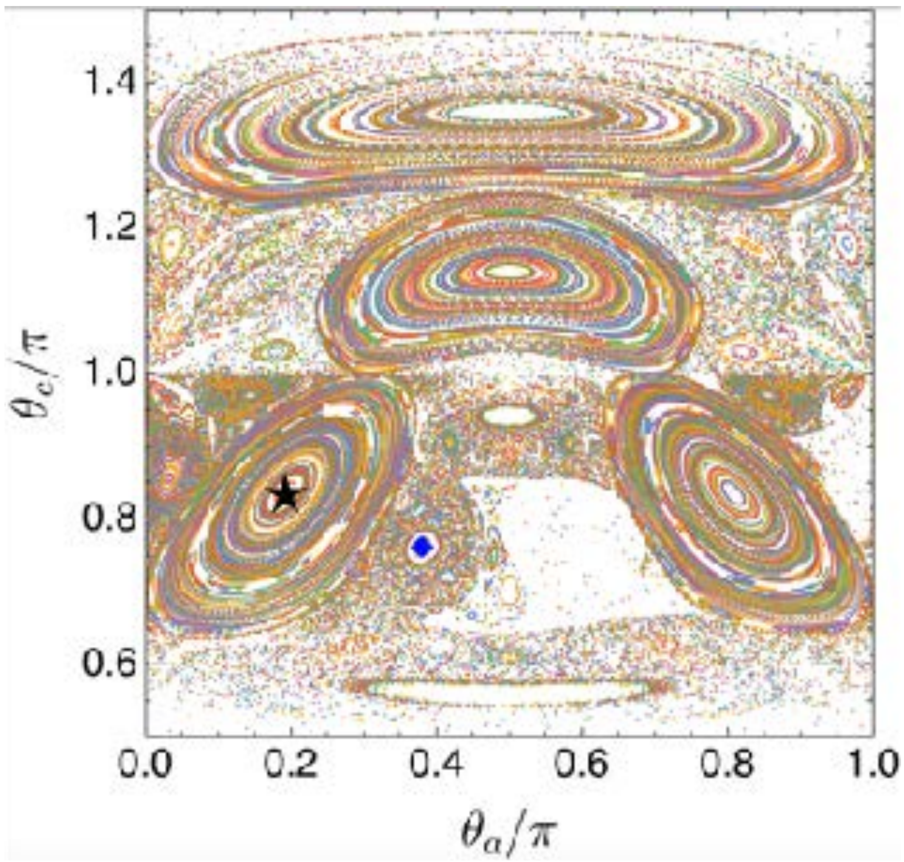
$$\dot{\theta}_i = f_i[\vec{\theta}]$$



Discrete map in the plane



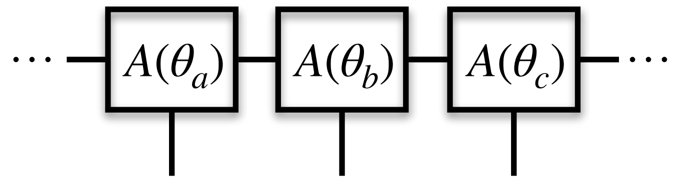
Poincare section $\theta_b = 0$



Stable periodic trajectories & “KAM tori” ; typical in Hamiltonian systems

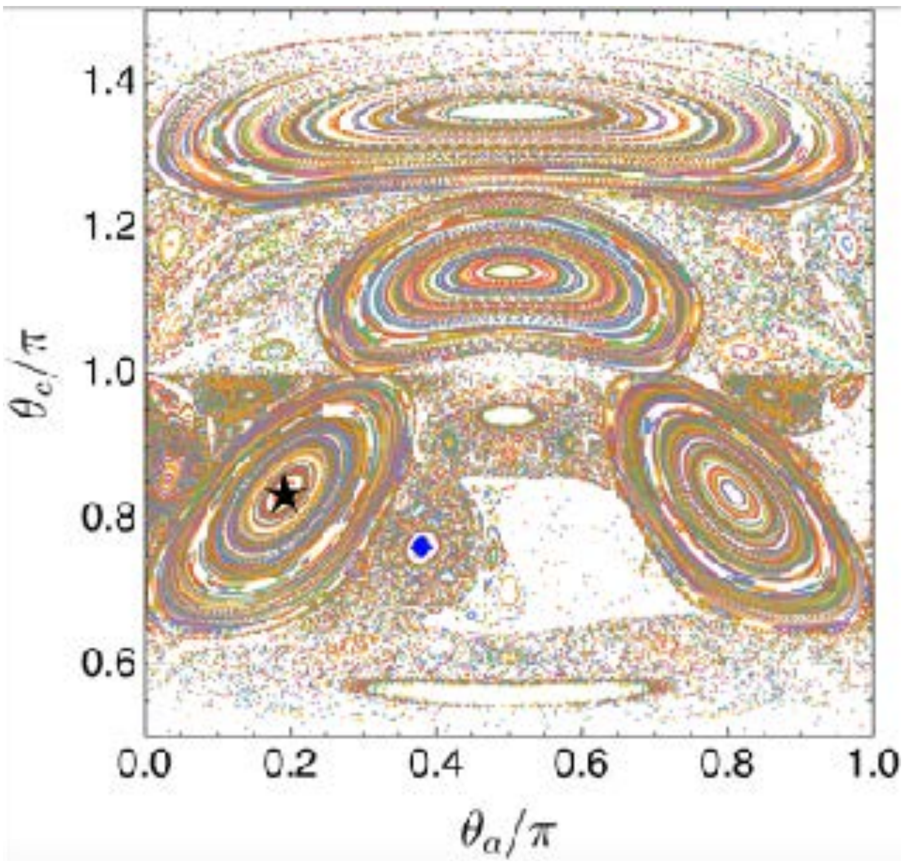
New type of “scars” (regular eigenstates)

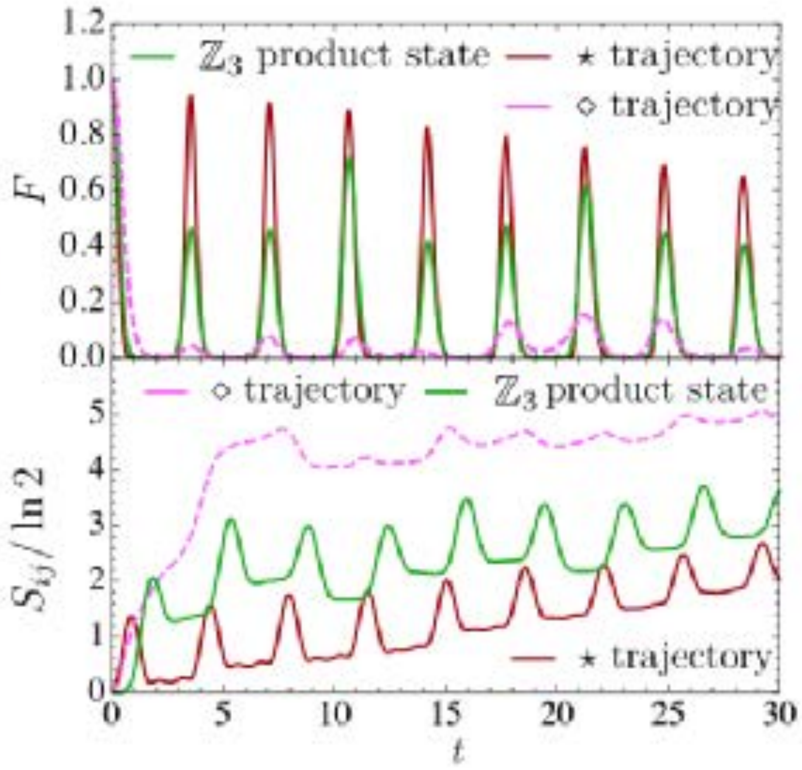
- Most **stable** periodic orbit → best revivals



quantum dynamics

Poincare section $\theta_b = 0$





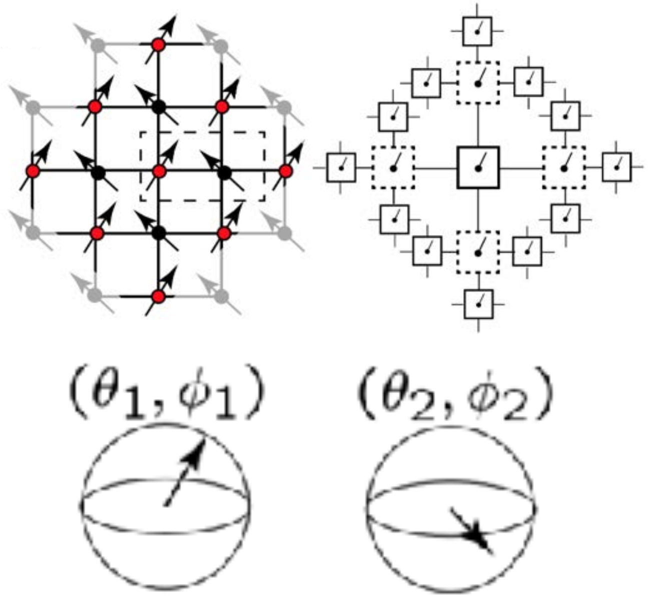
L=18, ED PBC

[Michailidis et al, arXiv: 1905.08564]

TDVP for 2D lattice with chemical potential

$$H = \sum P^{\circ}P^{\circ}X_{i,j}P^{\circ}P^{\circ} + \mu_z \sum Z_{i,j}$$

Tensor tree ansatz *C*=4



$$\dot{\theta}_1 = -\sin\theta_1 \cos^{C}\theta_1 \cos\varphi_1 \tan\theta_2 - \cos^{C-1}\theta_2 \cos\varphi_2$$

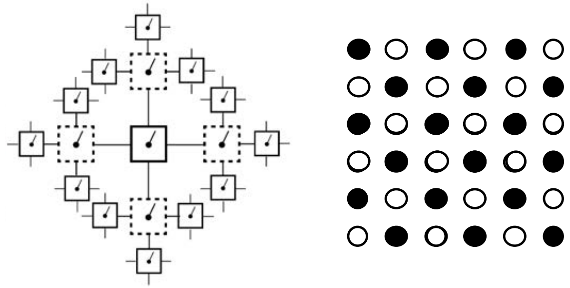
$$\dot{\varphi}_1 = \mu_z - C\tan\theta_1 \cos^{C-1}\theta_2 \sin\varphi_2 + \frac{1}{2}\cos^{C-1}\theta_1 \cot 2\theta_2 \sin\varphi_1 \left(4 + C - (C-1)\cos 2\theta_1\right)$$
$$-(C-1)\cos^{C-1}\theta_1 \sin^2\theta_1 \sin\varphi_1 \sin^{-1} 2\theta_2$$

$$\dot{\theta}_2 = (\theta_1 \leftrightarrow \theta_2, \phi_1 \leftrightarrow \phi_2); \qquad \dot{\phi}_2 = (\theta_1 \leftrightarrow \theta_2, \phi_1 \leftrightarrow \phi_2)$$

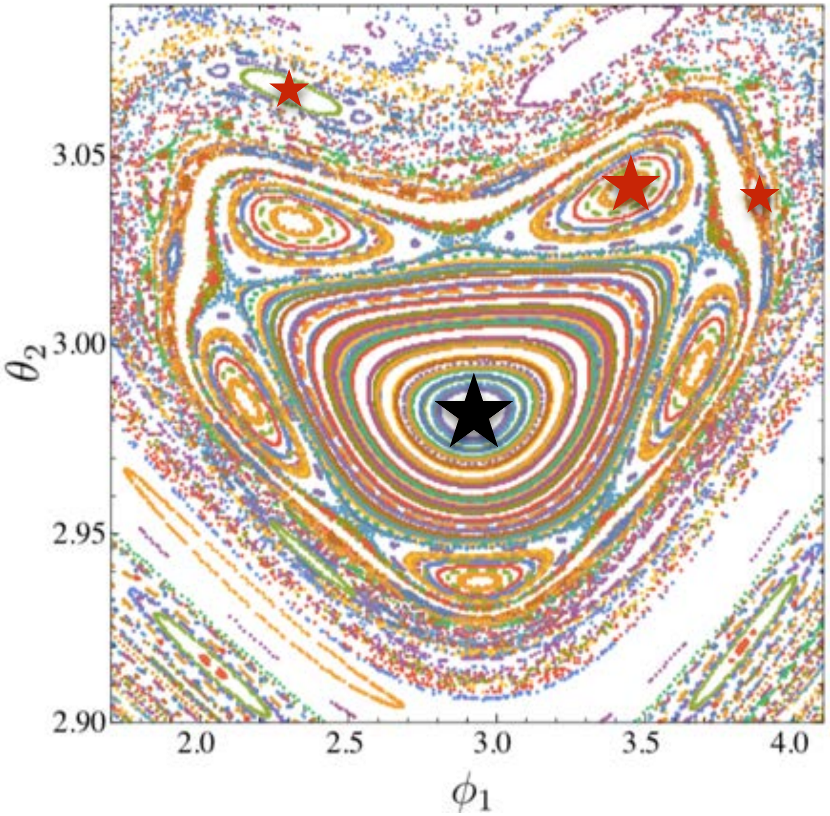
[Michailidis et al, in preparation]

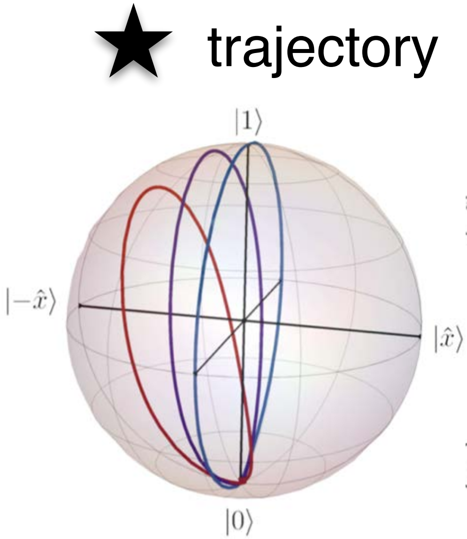
Mixed phase space in two dimensions

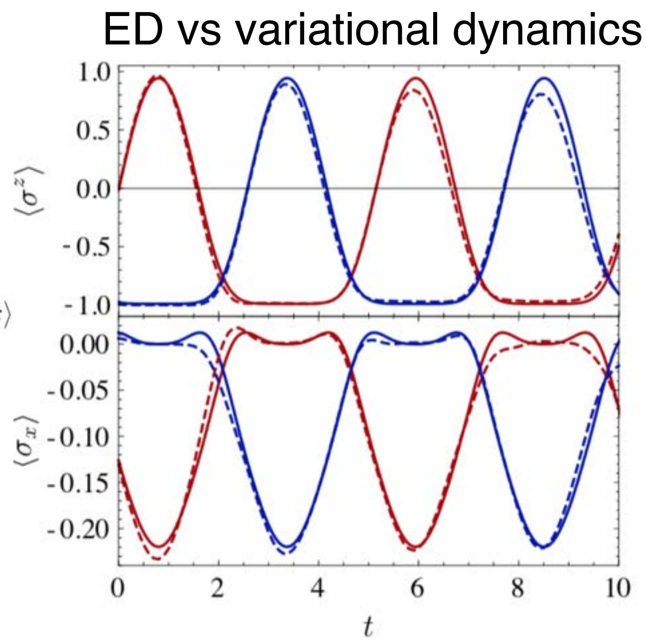
$$H = \sum P^\circ P^\circ X_{i,j} P^\circ P^\circ + \mu_z \sum Z_{i,j}$$



Poincare section $\mu_z = 0.225$







[Michailidis et al, in preparation]

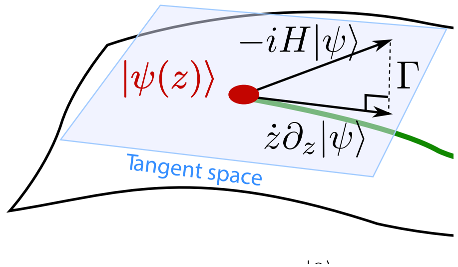
Are all trajectories important?

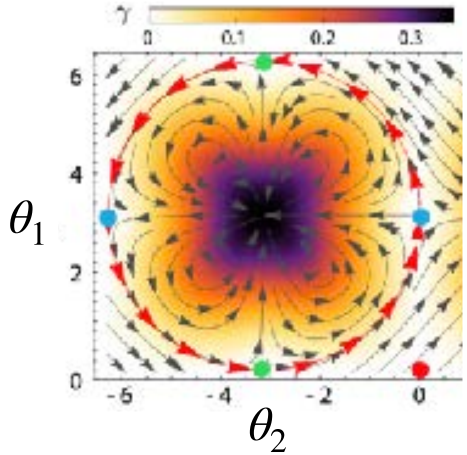
Quantum leakage to classify trajectories

Instantaneous norm loss

$$\Gamma = ||(iH + \dot{x}_b \partial_{x_b})\,|\psi(\{x_a\})\rangle\,||^2$$

[WW Ho et al, PRL'2018]



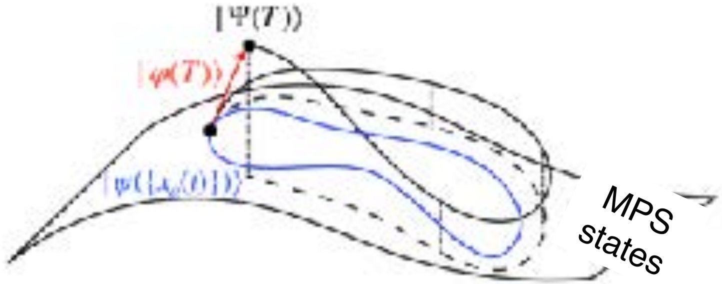


Integrated leakage → **lower** bound on quantum fidelity

$$\Gamma_T = \left(\int_0^T dt\,\Gamma[z(t)]\right)^2$$

$$f_T = -\ln \frac{|\langle\psi|e^{-iHt}|\psi\rangle|^2}{L}$$

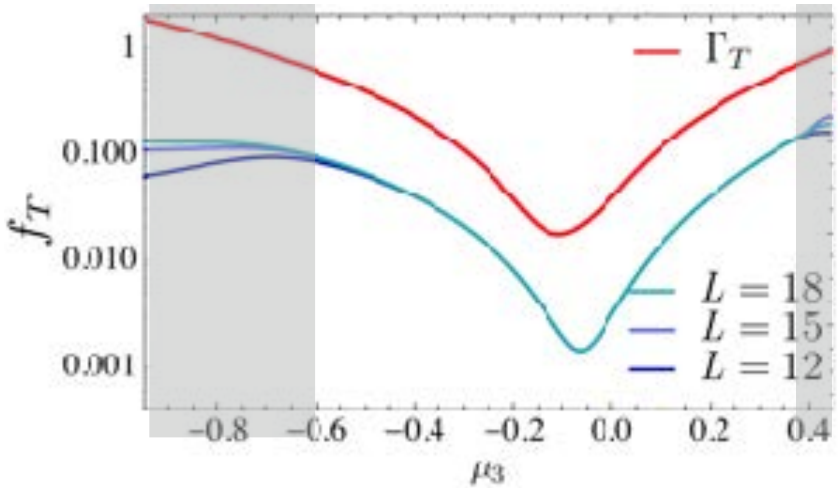
$$f_T \le \Gamma_T$$



Revivals criterion: $\Gamma_T \le 1$

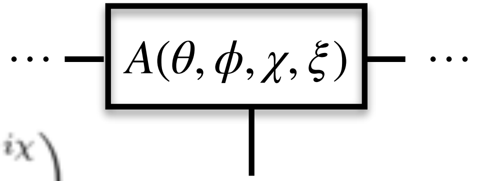
$$H = \sum P_{i-1}X_iP_{i+1} + \mu_3 \sum P_{i-2}X_{i-1}X_iX_{i+1}P_{i+2}$$

[Michailidis et al, arXiv: 1905.08564]



Mixed phase space in chaotic TFIM

- Thermalizing TFIM: $H = \sum 0.4X_i + Z_i + Z_i Z_{i+1}$

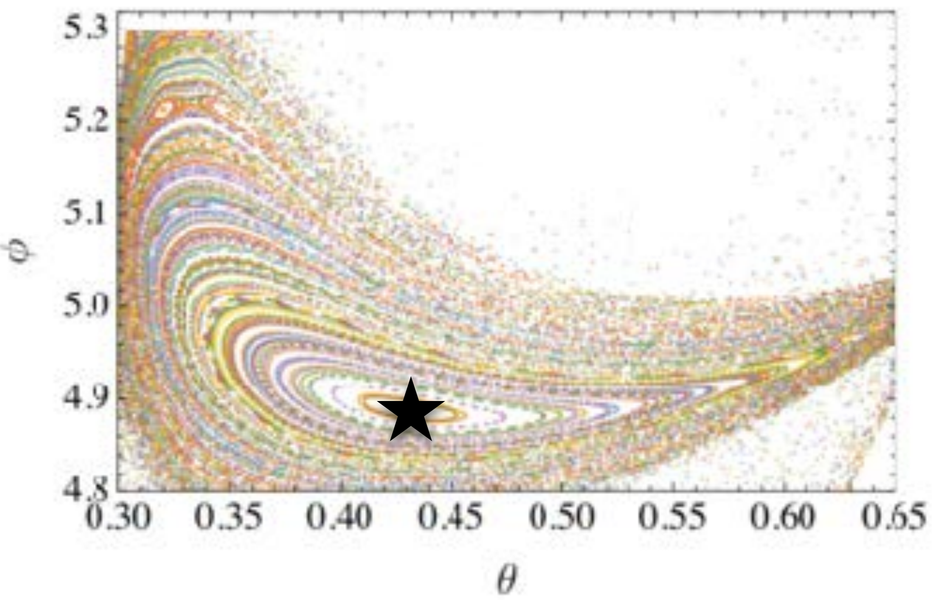


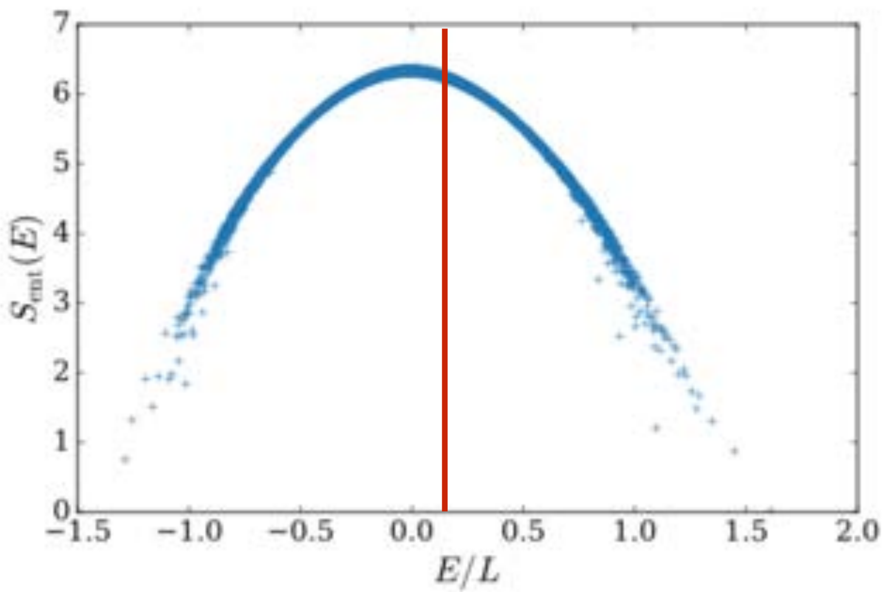
χ = 2 MPS with 4 parameters:

$$A^{\uparrow} = \begin{pmatrix} \cos\theta\cos\xi e^{i\chi} & \cos\theta\sin\xi e^{-i\chi} \\ 0 & 0 \end{pmatrix}$$

$$A^{\downarrow} = \begin{pmatrix} 0 & 0 \\ \sin\theta\sin\xi e^{i(\phi-\chi)} & \sin\theta\cos\xi e^{i(\chi+\phi)} \end{pmatrix}$$

- Mixed phase space but no special states, energy density $\langle H \rangle / L = 0.18$



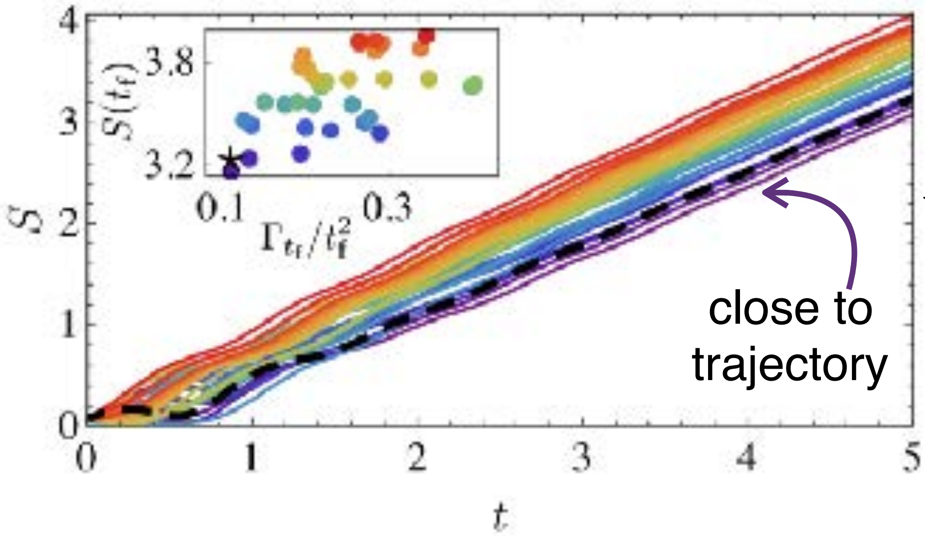


- Strong leakage, TDVP with $\chi = 2$ fails at $t \sim 1$

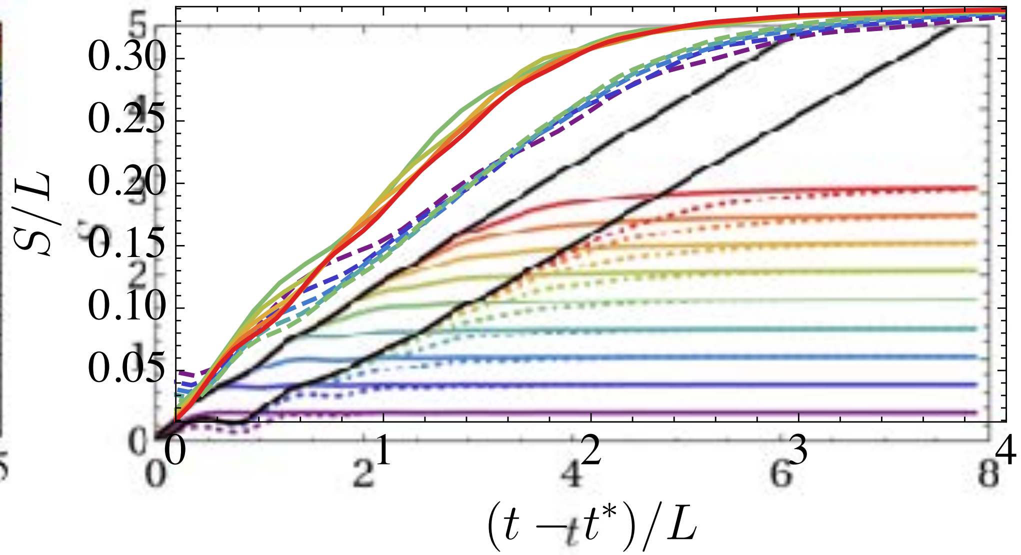
[Michailidis et al, arXiv: 1905.08564]

Influence of trajectories on thermalization

bipartite entanglement



"slowest" vs "fastest" initial states



Absence of "universal" Lieb-Robinson velocity

Relation between leakage at small $\chi \leftrightarrow$ entanglement at late times

↓

Mixed phase space as a source of non-universal thermalization

[Michailidis et al, arXiv: 1905.08564]

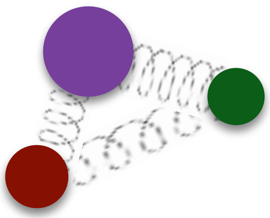
Summary and outlook

Mixed phase space: beyond “plain thermalization”

Classical

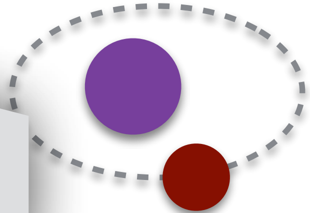
<u>Ergodic systems</u>

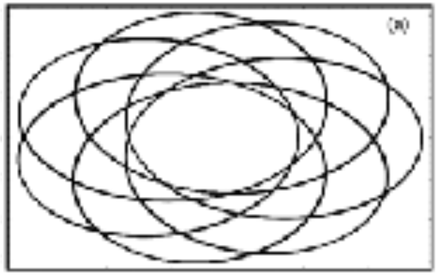
chaos → ergodicity



<u>Integrable systems</u>

stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]

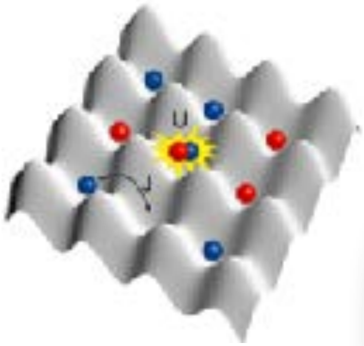


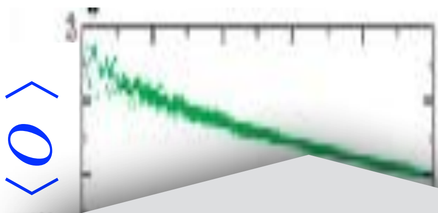


<u>Systems with mixed phase space</u>

Quantum

Thermalizing phases

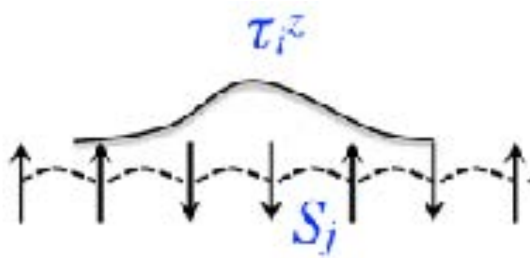




<u>Slow thermalization!</u>

MBL phases

emergent integrability



Possible connections

Quantum scars
(weak ETH breakdown)

zero modes subspace
[Lin & Motrunich, arXiv:1810.00888]

SU(2) symmetric models
[Moudgalya et al., PRB’18]
[Shirashi, arXiv:1904.05182]

connection to integrability
[Khemani et al., PRB’18]

quantum Hall inspired models
[Moudgalya et al., arXiv:1906.05292]

clock models
[Bull et al, PRL’19]

strongly entangled scars

practical uses?

few-body chaos?

2d Rydberg lattices
[Michailidis et al., in preparation]

different sources of weak ETH violation

Znidaric et al,
James, Konik, Robinson, PRL’19

Iadecola&Znidaric PRL’19

Khemani&Nandkishore, arXiv:1904.04815
Sala et al arXiv:1904.04266

From few to many-body chaos

Few-body systems

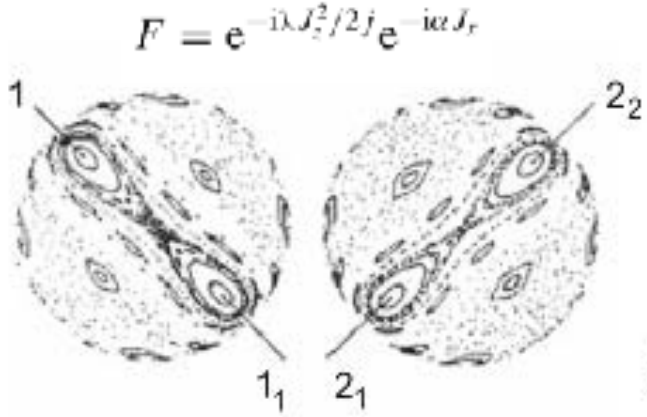
mixed level statistics

$$p(s) = \alpha p_{\mathrm{WD}}(s) + (1-\alpha)p_{\mathrm{Poisson}}(s)$$

different statistics
of wave functions

stability to perturbations

stability to dephasing



Mixed phase space of kicked top
[Haake]

Many-body systems

???

entanglement growth ⟷ Lyapunov exponent

???

TDVP leakage

E. Altman et al, A. Green et al, …

Summary

- Quantum many-body scars: ETH breaking eigenstates, coherent dynamics …

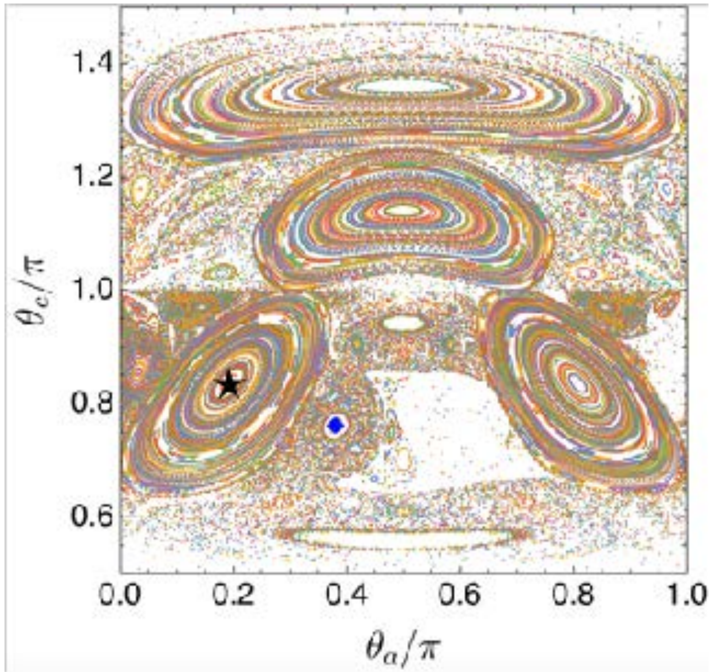
[Nat. Phys. 14, 745–749 (2018)]
[Phys. Rev. B. 98, 155134 (2018)]

- Emergent SU(2) subspace

[Phys. Rev. Lett. 122, 220603 (2019)]

- Mixed phase space in TDVP dynamics:
  - ✳ low leakage → **regular** eigenstates, fidelity revivals
  - ✳ strong leakage → slow thermalization

[arXiv: 1905.08564]

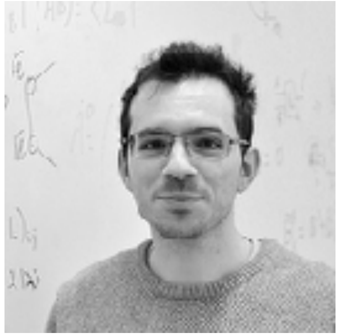


- Deformations to improve generic trajectory?
  Other mechanisms of weak thermalization breakdown?
  Strongly entangled trajectories? TDVP as a route to quantum KAM?

Acknowledgments

Collaborations:

Alex Michailidis (IST)



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