Generalized HydroDynamics (GHD) on an Atom Chip

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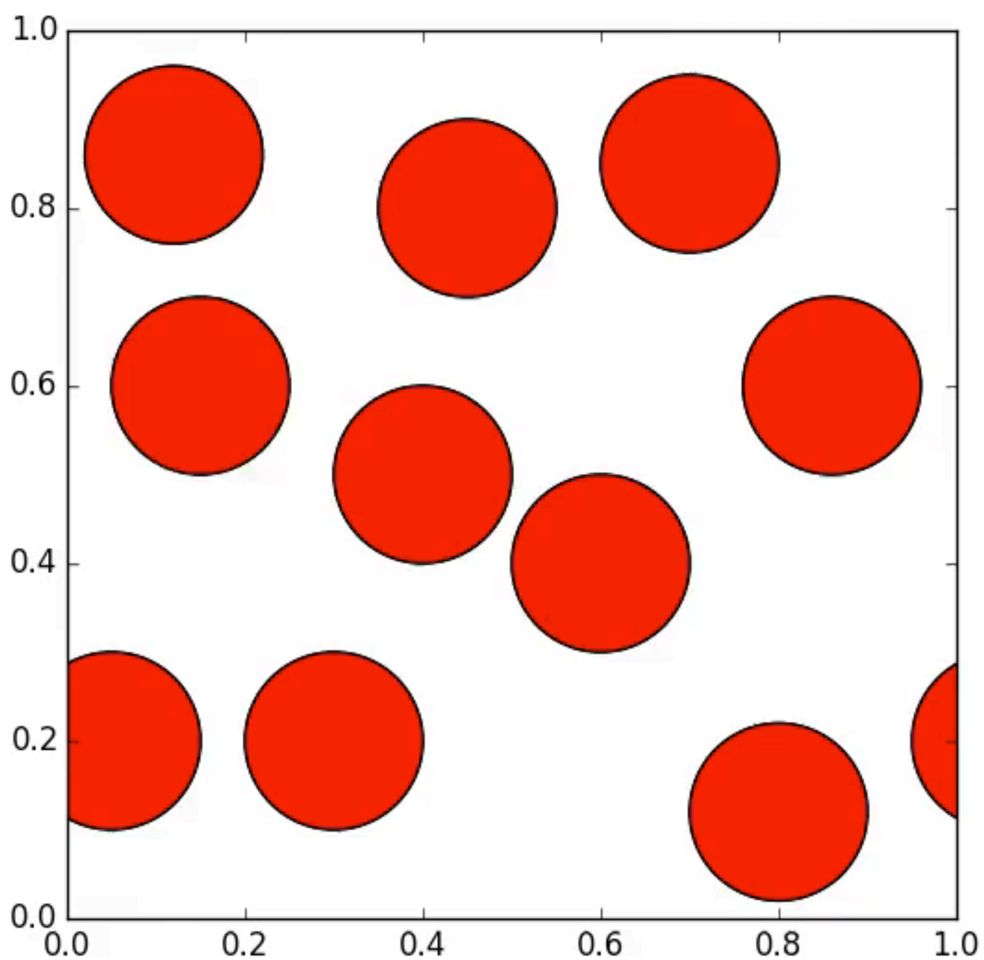
Based on **[PRL 122, 090601 (2019)]** with:

Isabelle Bouchoule (Institut d'Optique, Palaiseau)
Max Schemmer (Institut d'Optique, Palaiseau)
Benjamin Doyon (King's College London)

'Thermalization, Many Body Localization and Hydrodynamics', ICTS Bangalore, November 2019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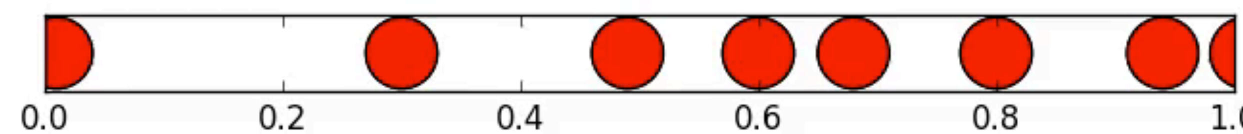
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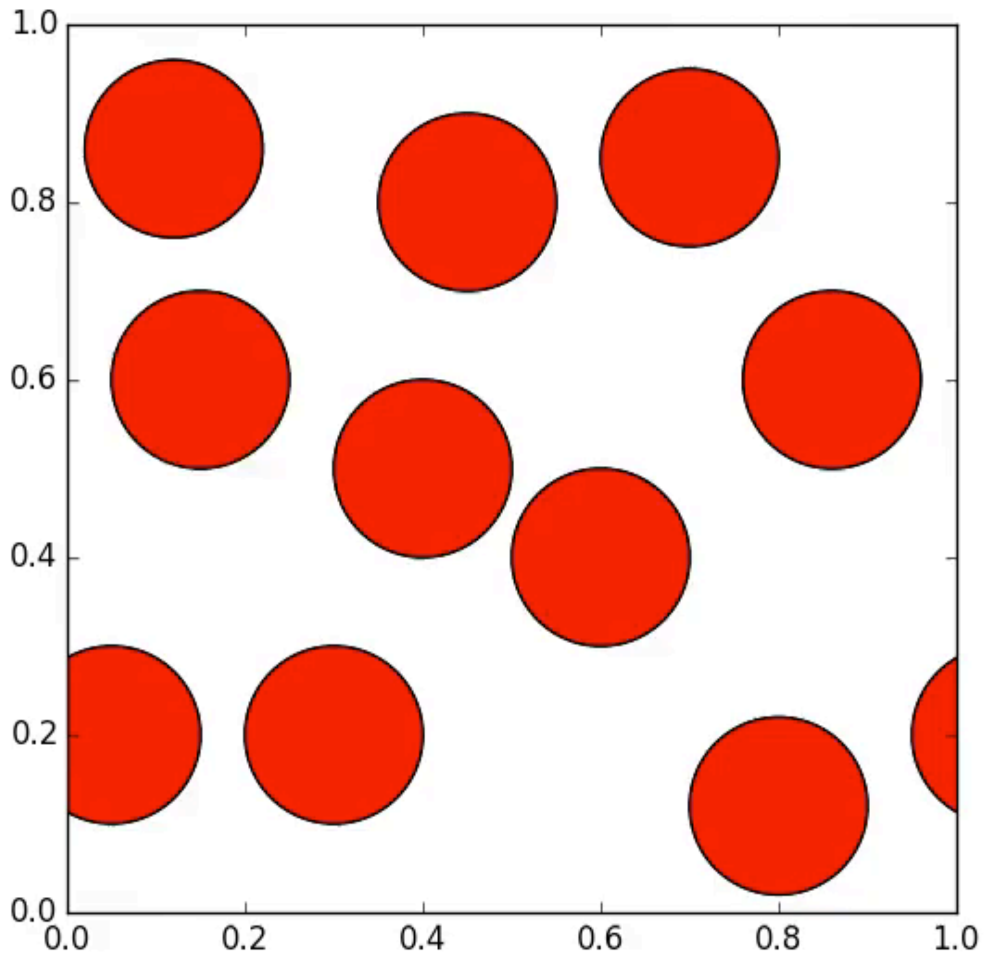
(Classical) Newton's Cradle on YouTube

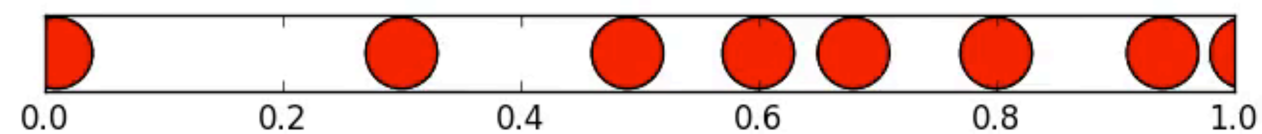
chaotic / ergodic **vs.** **integrable / non-ergodic**



after short relaxation time τ_{relax}, the macrostate in the box is entirely characterized by

$$n,\ u,\ \varepsilon$$

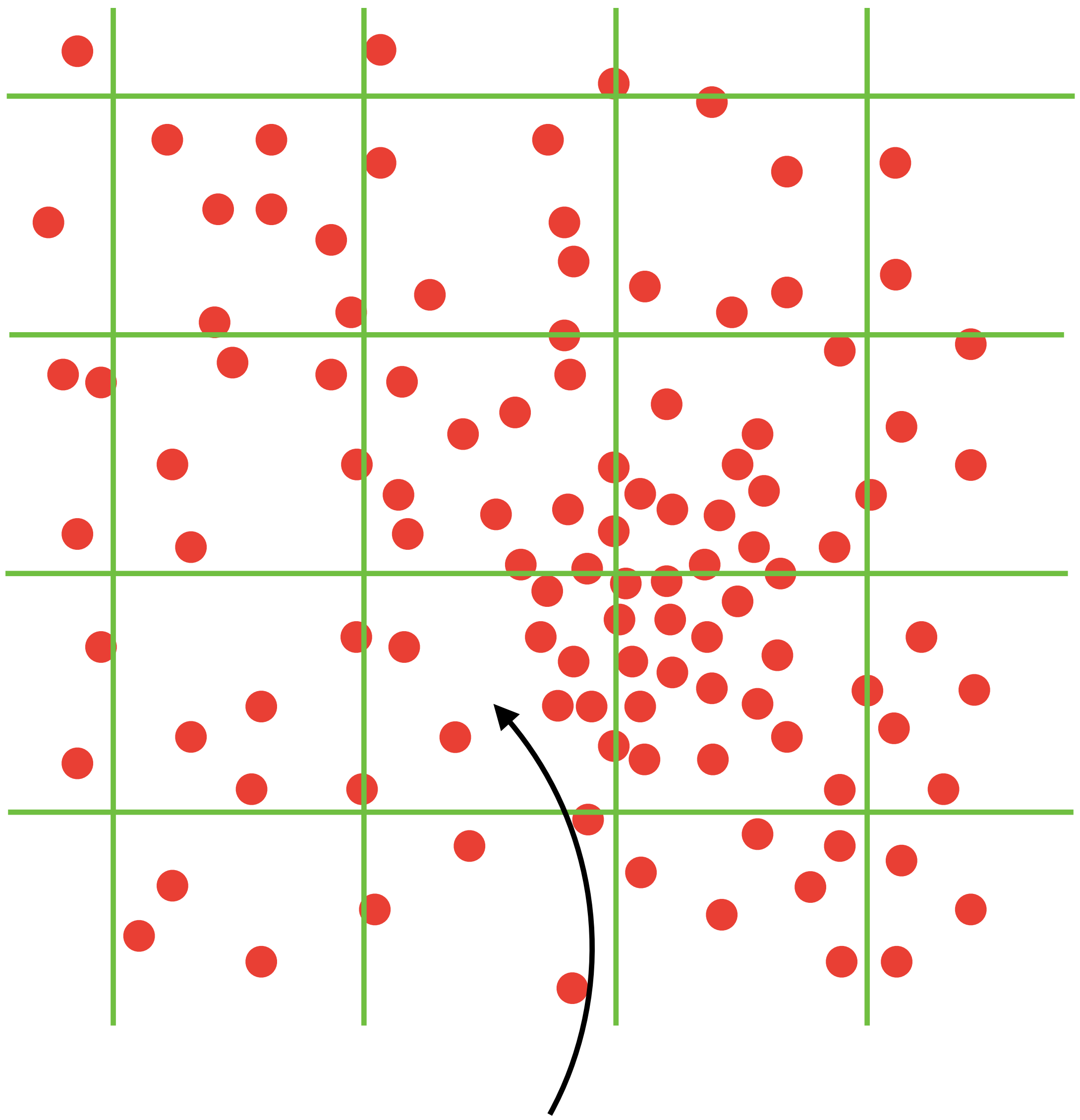
particle density, mean velocity, energy density



to characterize a macrostate, one needs the entire distribution of velocities

$$\rho(v) = \sum_{i=1}^{N} \delta(v - v_i)$$

chaotic / ergodic **vs.** **integrable / non-ergodic**

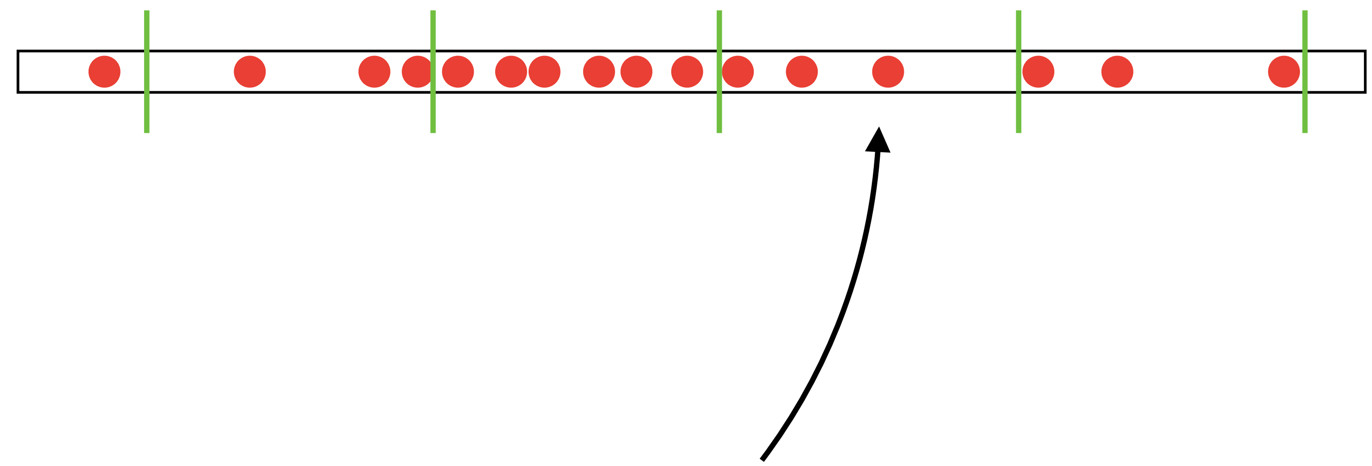


state in each ‘fluid cell’ characterized by

$$n(x,t), u(x,t), \varepsilon(x,t)$$

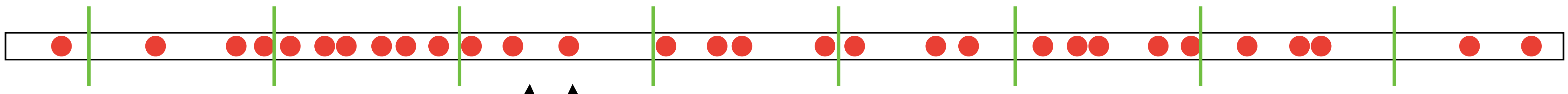
whose evolution is governed by continuity eqns

$$\begin{cases} \partial_t n + \partial_x (nu) &= 0 \\ \partial_t (nu) + \partial_x j_P &= -n\frac{\partial_x V}{m} \\ \partial_t \varepsilon + \partial_x j_E &= 0 \end{cases}$$



is there a coarse-grained or hydrodynamic description?

Hydrodynamic approach to integrable system?



Two approaches:

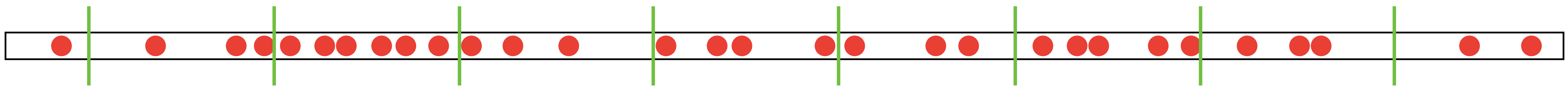
1. pretend that the fluid cells are locally at thermal equilibrium, characterized by $n(x,t), u(x,t), \varepsilon(x,t)$ and write

$$\left\{ \begin{array}{rcl} \partial_t n + \partial_x (nu) & = & 0 \\ \partial_t (nu) + \partial_x j_P & = & -n \frac{\partial_x V}{m} \\ \partial_t \varepsilon + \partial_x j_E & = & 0 \end{array} \right.$$

2. keep track of the full distribution $\rho(v)$ in each 'fluid cell' at (x,t), and find an evolution equation for

$$\rho(x,v,t)$$

Hydrodynamic approach to integrable system?



Two approaches:

'conventional' hydrodynamics (CHD)

1. pretend that the fluid cells are locally at thermal equilibrium, characterized by $n(x,t), u(x,t), \varepsilon(x,t)$ and write

$$\left\{ \begin{array}{rcl} \partial_t n + \partial_x (nu) & = & 0 \\ \partial_t (nu) + \partial_x j_P & = & -n \frac{\partial_x V}{m} \\ \partial_t \varepsilon + \partial_x j_E & = & 0 \end{array} \right.$$

2. keep track of the full distribution $\rho(v)$ in each 'fluid cell' at (x,t), and find an evolution equation for

$$\rho(x,v,t)$$

Generalized HydroDynamics (GHD)

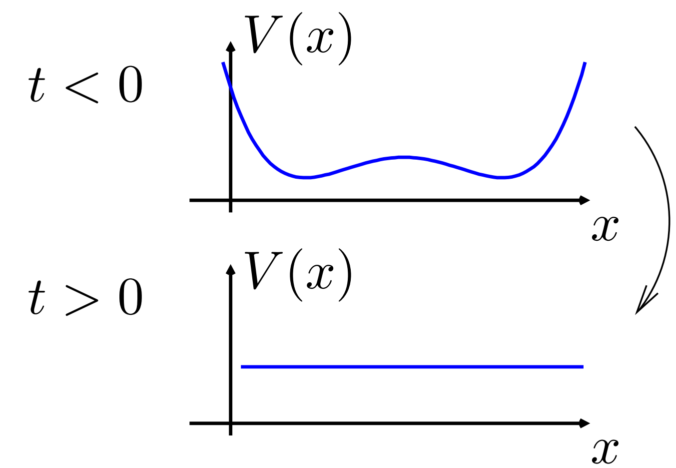
This talk

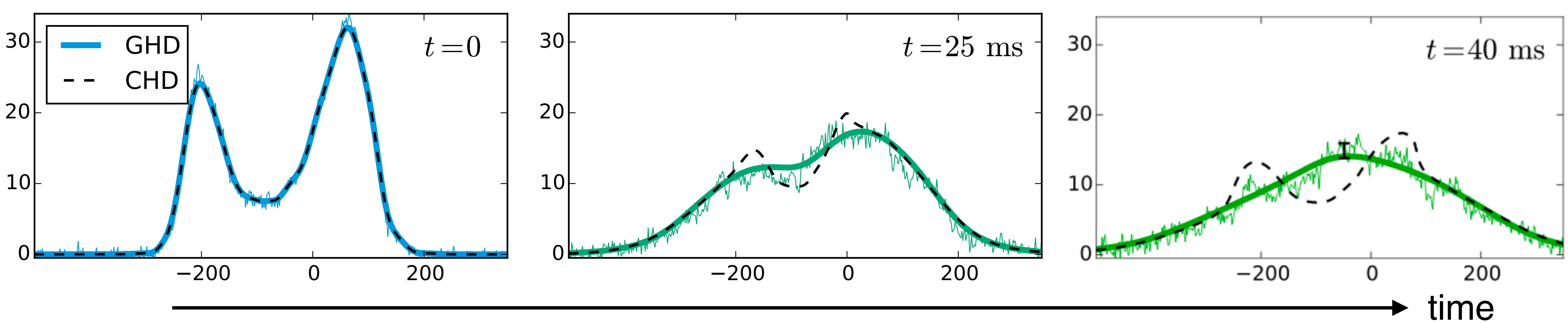
Generalized HydroDynamics (GHD) works

while ’conventional’ hydrodynamics doesn’t.

We demonstrate this **experimentally** in a 1d **quantum** gas of Rb atoms that is (approximately) integrable.

The atomic cloud is initially at equilibrium in a double-well potential. At $t = 0$ the potential is suddenly switched off, and the gas expands freely in 1d.





Plan of the rest of the talk

More on:

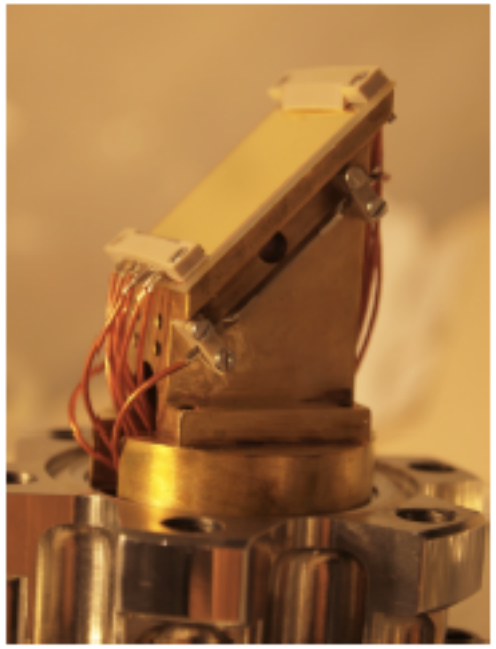
1. the **atom chip experiment** of Isabelle Bouchoule + Max Schemmer

2. the **2016 breakthrough** of Generalized HydroDynamics (GHD)

3. the **results**

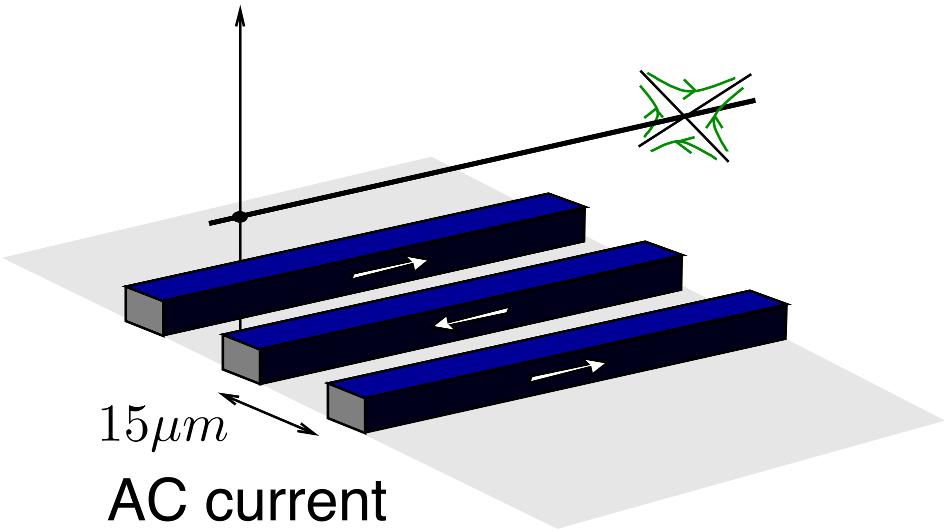
1. The experimental setup

Trapping potential created by the atom chip

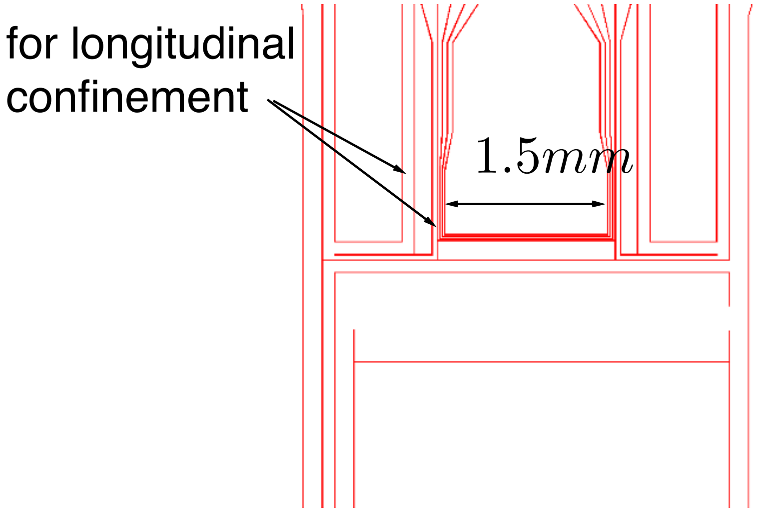


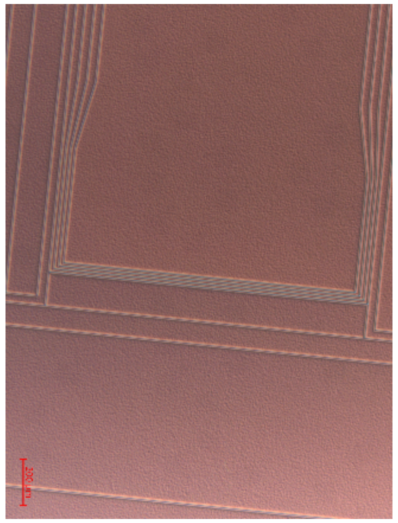
Magnetic confinement created by **micro-wires**.

-Transverse confinement:



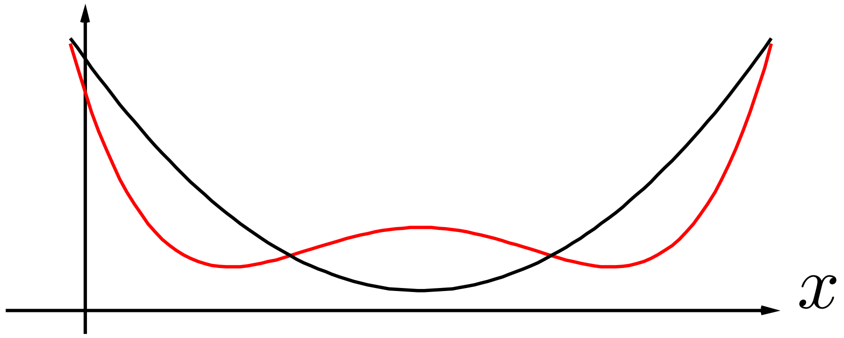
AC current



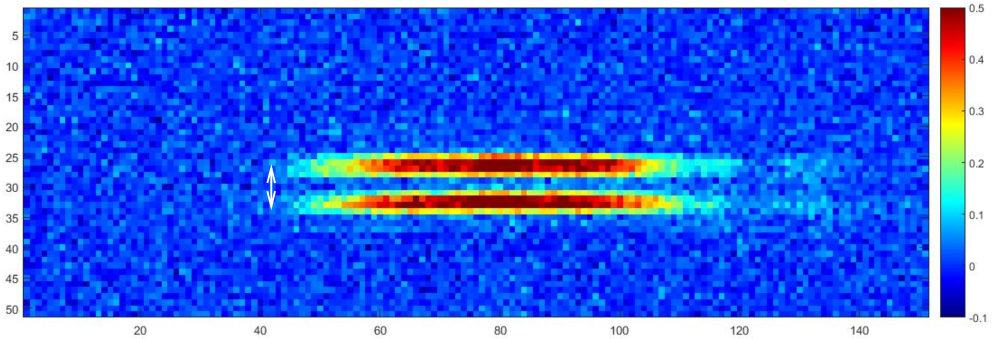


-Longitudinal confinement: four wires allow to create potentials

$$V(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x$$



-Imaging with CCD camera. Typical absorption image:



1. The experimental setup

Regime

$$\mu/k_{\mathrm{B}} \simeq 50 - 100nK$$
$$T \simeq 50 - 300nK$$
$$\omega_\perp \simeq 2\pi \times 5 - 8\mathrm{kHz} \quad (\rightarrow 250 - 400nK)$$

$\mu, T < \hbar\omega_\perp \ \Rightarrow$ 1d regime, well described by the Lieb-Liniger model

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\partial_{x_i}^2\psi + g\sum_{i<j}\delta(x_i - x_j)\psi + \sum_{i=1}^{N}V(x_i)\psi$$

2. Crash course on GHD

The people who made the discovery

Breakthrough from 2016!

PRL **117,** 207201 (2016) PHYSICAL REVIEW LETTERS week ending 11 NOVEMBER 2016

Transport in Out-of-Equilibrium *XXZ* Chains: Exact Profiles of Charges and Currents

Bruno Bertini,[1] Mario Collura,[1,2] Jacopo De Nardis,[3] and Maurizio Fagotti[3]
[1]*SISSA and INFN, via Bonomea 265, 34136 Trieste, Italy*
[2]*The Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford, OX1 3NP, United Kingdom*
[3]*Département de Physique, École Normale Supérieure/PSL Research University, CNRS, 24 rue Lhomond, 75005 Paris, France*
(Received 17 June 2016; published 8 November 2016)

We consider the nonequilibrium time evolution of piecewise homogeneous states in the *XXZ* spin-1/2 chain, a paradigmatic example of an interacting integrable model. The initial state can be thought of as the result of joining chains with different global properties. Through dephasing, at late times, the state becomes locally equivalent to a stationary state which explicitly depends on position and time. We propose a kinetic ... and derive a continuity equation which fully characterizes the ... estrict ourselves to the gapless phase and consider cases where the ... mperatures, (2) in the ground state of two different models, and (3) in ... ccellent agreement (any discrepancy is within the numerical error) ... umerical simulations of time evolution based on time-evolving block ... ry, we unveil an exact expression for the expectation values of the ... ry state.

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **6,** 041065 (2016)

Emergent Hydrodynamics in Integrable Quantum Systems Out of Equilibrium

Olalla A. Castro-Alvaredo,[1] Benjamin Doyon,[2] and Takato Yoshimura[2]
[1]*Department of Mathematics, City, University of London, Northampton Square, London EC1V 0HB, United Kingdom*
[2]*Department of Mathematics, King's College London, Strand, London WC2R 2LS, United Kingdom*
(Received 12 July 2016; revised manuscript received 22 September 2016; published 27 December 2016)

Understanding the general principles underlying strongly interacting quantum states out of equilibrium is one of the most important tasks of current theoretical physics. With experiments accessing the intricate dynamics of many-body quantum systems, it is paramount to develop powerful methods that encode the emergent physics. Up to now, the strong dichotomy observed between integrable and nonintegrable evolutions made an overarching theory difficult to build, especially for transport phenomena where space-time profiles are drastically different. We present a novel framework for studying transport in integrable systems: hydrodynamics with infinitely many conservation laws. This bridges the conceptual gap between integrable and nonintegrable quantum dynamics, and gives powerful tools for accurate studies of space-time profiles. We apply it to the description of energy transport between heat baths, and provide a full description of the current-carrying nonequilibrium steady state and the transition regions in a family of models including the Lieb-Liniger model of interacting Bose gases, realized in experiments.

DOI: 10.1103/PhysRevX.6.041065 Subject Areas: Nonlinear Dynamics, Quantum Physics, Statistical Physics

2. Crash course on GHD

The GHD equation

Long story short, the two **GHD** equations are:

$$\partial_t + \partial_x (v^{\rm eff}_{[\rho]}(v)\rho) = \frac{\partial_x V}{m} \partial_v \rho$$

where $\rho(x, v, t)$ is the local density of (quasi-)particles with rapidity v.

The effective velocity is a ``dressed group velocity''. It is a function

$$v^{\rm eff}_{[\rho]}(v)$$

defined by the integral equation

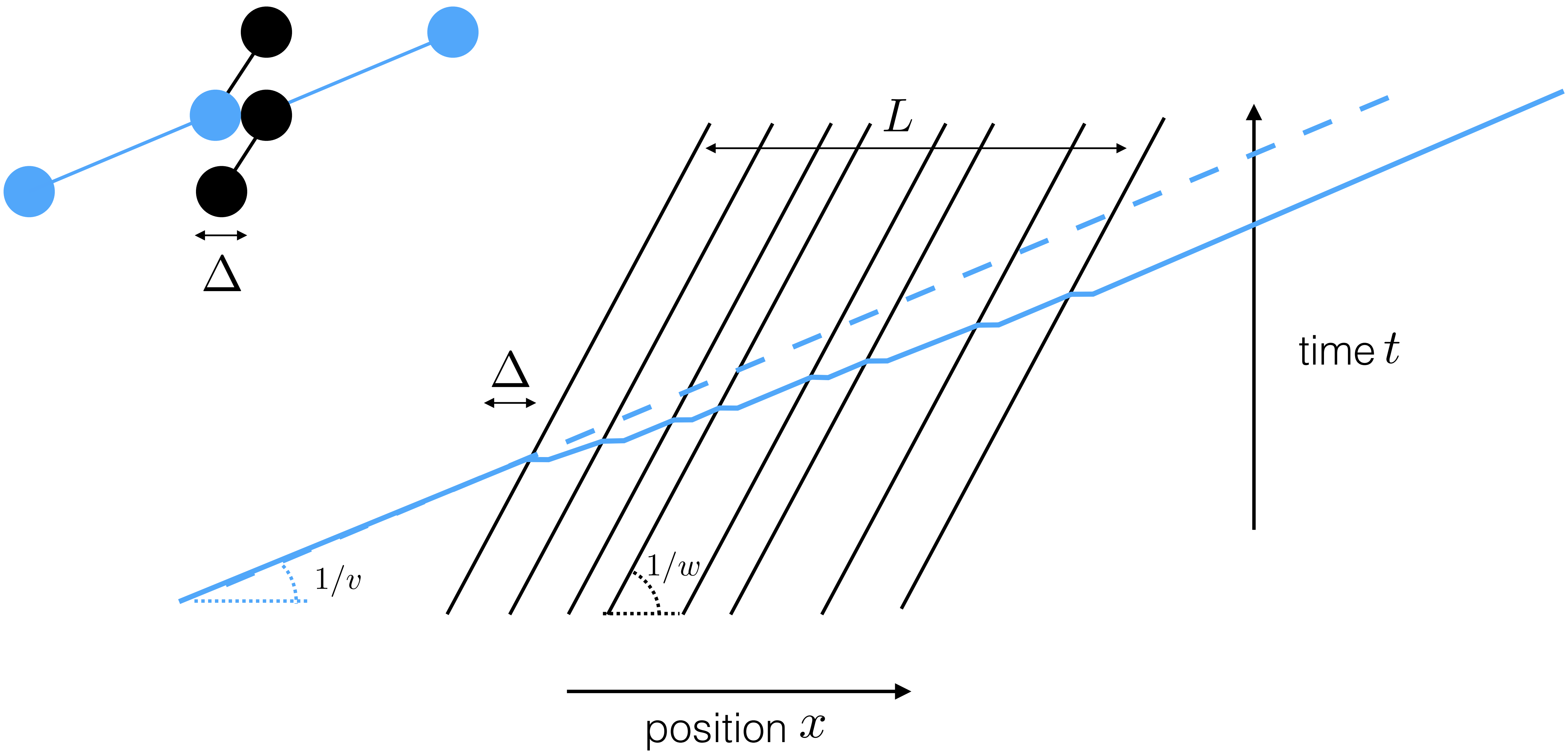
$$v^{\rm eff}_{[\rho]}(v) = v + \int dw \, \Delta(v,w)\rho(w) \left(v^{\rm eff}_{[\rho]}(v) - v^{\rm eff}_{[\rho]}(w) \right)$$

This effective velocity had appeared previously in **[Bonnes, Essler, Läuchli 2014]**, and was known for the 1d billiard or hard rod gas in **[Percus, 1969]**, **[Boldrighini, Dobrushin, Sukhov 1983]**, or in H. Spohn's textbook **[Spohn 1991]**.

2. Crash course on GHD

Meaning of the effective velocity in GHD

$$v^{\rm eff}_{[\rho]}(v) = v + \int dw\, \Delta\, \rho(w) \left(v^{\rm eff}_{[\rho]}(v) - v^{\rm eff}_{[\rho]}(w) \right)$$



2. Crash course on GHD

GHD for the 1d Bose gas

The **quantum 1d Bose gas**: main changes with respect to the classical 1d billiard:

1.a the **macrostate** in each fluid cell is represented by an **eigenstate** of the Lieb-Liniger model **[Yang and Yang, 1969]** (+ many works on GETH-GGE)

1.b those eigenstates are obtained by the **Bethe Ansatz**. In the thermodynamic limit they are **labeled by a distribution of rapidities** $\rho(v)$ **[Yang and Yang, 1969]** (+ many recent works on GETH-GGE)

2. the 'diameter of the balls' Δ is replaced by the **Wigner time delay** caused by the **two-body scattering phase**,

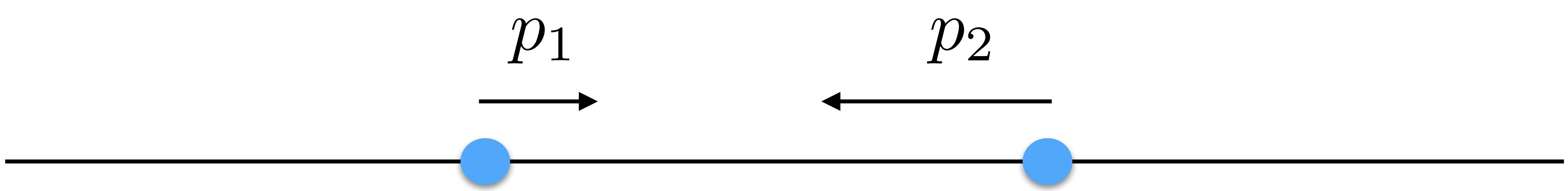
$$\Delta = -\hbar \frac{d\varphi}{dp}$$

in particular, it depends on the relative rapidities of two quasi-particles

$$\Delta(v-w) = -\frac{2g/m}{(g/\hbar)^2 + (v-w)^2}$$

2. Crash course on GHD

Scattering phase and time delay



Take two particles on a line with contact interaction:

$$H = -\frac{\hbar^2}{2m}(\partial_{x_1}^2 + \partial_{x_2}^2) + g\,\delta(x_1 - x_2)$$

Then the eigenstates are of the form:

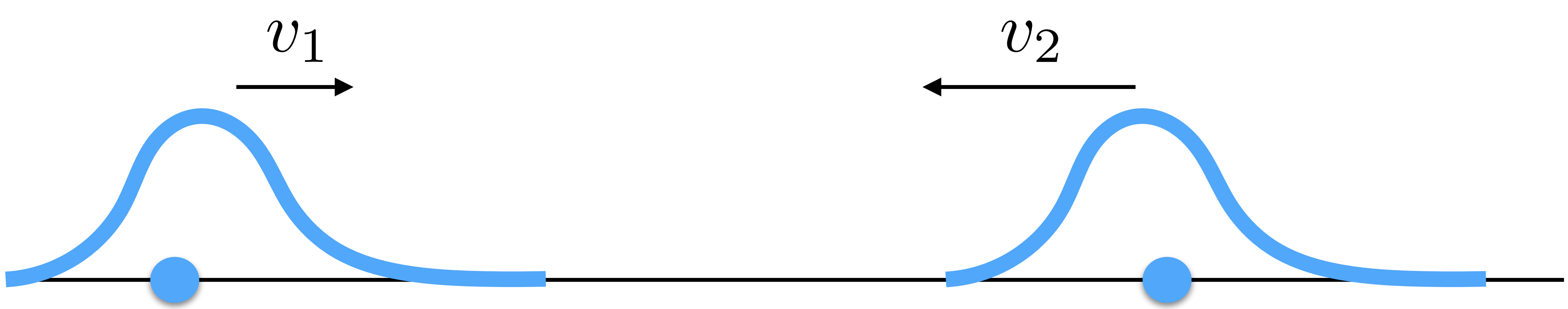
$$\psi(x_1, x_2) = \begin{cases} e^{\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)} - e^{i\varphi} e^{\frac{i}{\hbar}(p_2 x_1 + p_1 x_2)} & \text{if} \quad x_1 < x_2 \\ (x_1 \leftrightarrow x_2) & \text{if} \quad x_2 < x_1 \end{cases}$$

$$e^{i\varphi} = \frac{mg/\hbar - i(p_2 - p_1)}{mg/\hbar + i(p_2 - p_1)}$$

2. Crash course on GHD

Scattering phase and time delay

One physical consequence of this scattering phase is the following. Take two wave packets with semiclassical velocities $v_1 = p_1/m$ and $v_2 = p_2/m$



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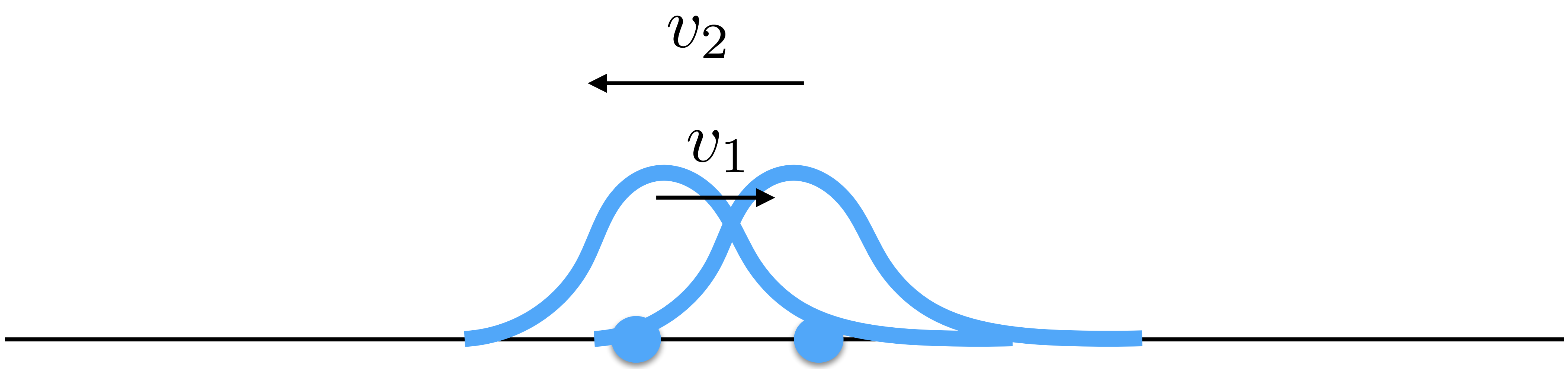

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2. Crash course on GHD

Scattering phase and time delay

One physical consequence of this scattering phase is the following. Take two wave packets with semiclassical velocities $v_1 = p_1/m$ and $v_2 = p_2/m$

After they have scattered, the two packets are not quite where you would expect them. Compared to the non-interacting case, they are shifted by a distance

$$\Delta = \hbar \frac{d\varphi(p_2 - p_1)}{d(p_2 - p_1)}$$

In the Lieb-Liniger model this is a lorentzian

$$\Delta(v_2 - v_1) \; = \; \frac{2g/m}{(g/\hbar)^2 + (v_2 - v_1)^2}$$

2. Crash course on GHD

Scattering phase in the N-body problem

For N particles, the structure of eigenstates mimics the one we've seen for 2 particles:

$$\psi(\{x_j\}) = \begin{cases} \sum_{\text{perm. } \sigma} (-1)^{|\sigma|} e^{i\Phi_\sigma} e^{\frac{i}{\hbar}(p_{\sigma(1)}x_1 + \cdots + p_{\sigma(N)}x_N)} & \text{if} \quad x_1 < x_2 < \cdots < x_N \\ (x_i \leftrightarrow x_k, x_j \leftrightarrow x_l, etc.) & \text{otherwise} \end{cases}$$

(the total phase breaks down into a sum of 2-body scattering phases)

$$e^{i\phi_\sigma} = \prod_{\text{transp. } \tau_{ij}} e^{i\varphi(p_j - p_i)}$$

That is the Bethe wave function **[Bethe, 1931]** for the delta Bose gas **[Lieb, Liniger, 1963]**.

2. Crash course on GHD

The distribution of quasi-particle rapidities

For N particles, the structure of eigenstates mimics the one we've seen for 2 particles:

$$\psi(\{x_j\}) = \begin{cases} \sum_{\text{perm. } \sigma} (-1)^{|\sigma|} e^{i\Phi_\sigma} e^{\frac{i}{\hbar}(p_{\sigma(1)}x_1 + \cdots + p_{\sigma(N)}x_N)} & \text{if} \quad x_1 < x_2 < \cdots < x_N \\ (x_i \leftrightarrow x_k, x_j \leftrightarrow x_l, etc.) & \text{otherwise} \end{cases}$$

Eigenstates are labeled by the sets of (quasi-)momenta $\{p_j\}$

In the thermodynamic limit, a typical eigenstate may be represented by its distribution of (quasi-)momenta, or rapidities $v_j = p_j/m$

$$\rho(v) = \frac{1}{L} \sum_{j=1}^{N} \delta(v - p_j/m)$$

2. Crash course on GHD

The GHD equation

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This effective velocity had appeared previously for the quantum case in **[Bonnes, Essler, Läuchli 2014]**, and it was known for the 1d billiard or hard rod gas in **[Percus, 1969]**, **[Boldrighini, Dobrushin, Sukhov 1983]**, see also Spohn's textbook **[Spohn 1991]**.

Plan of the rest of the talk

More on:

1. the **atom chip experiment** of Isabelle Bouchoule + Max Schemmer

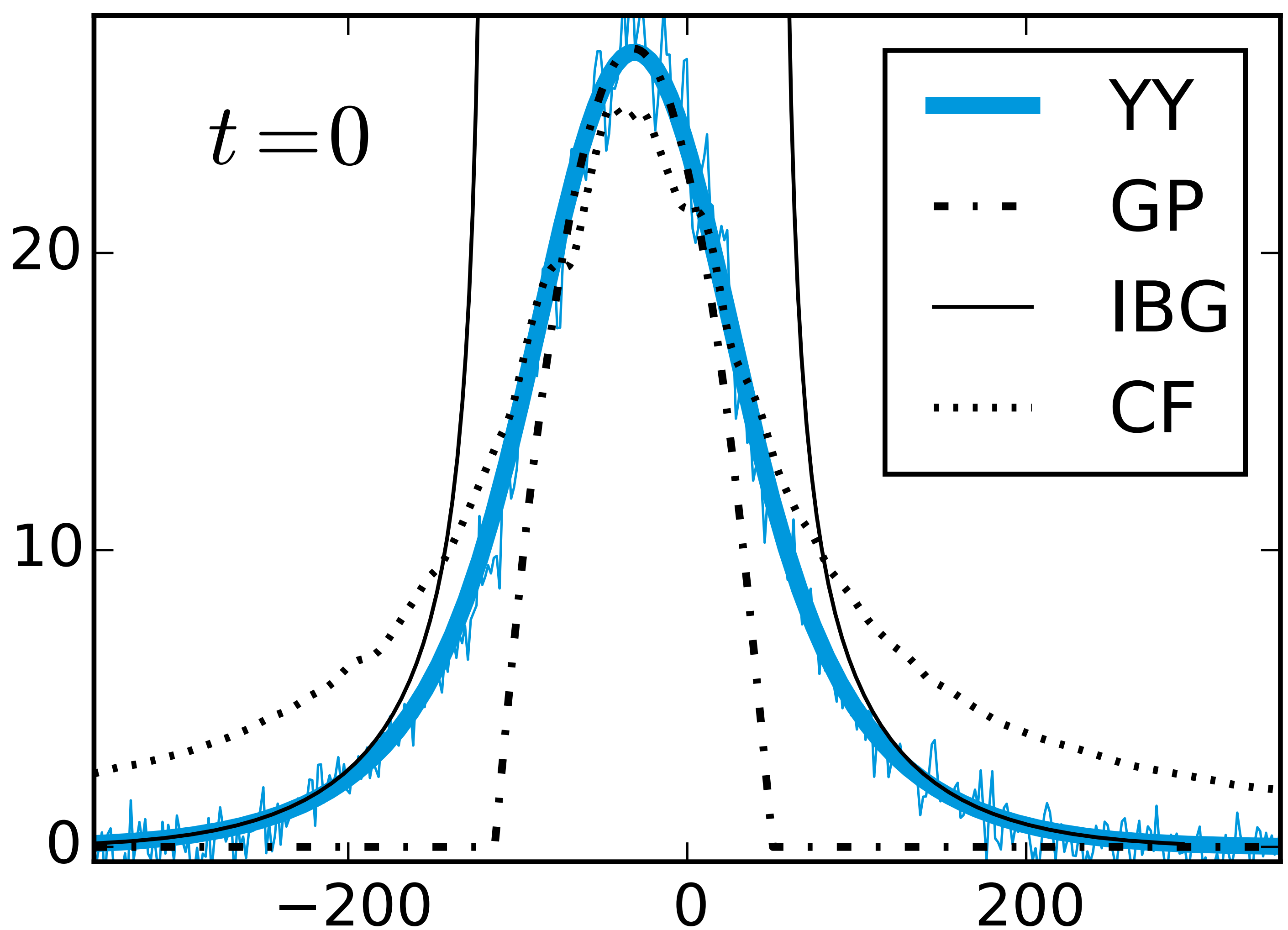
2. the **2016 breakthrough** of Generalized HydroDynamics (GHD)

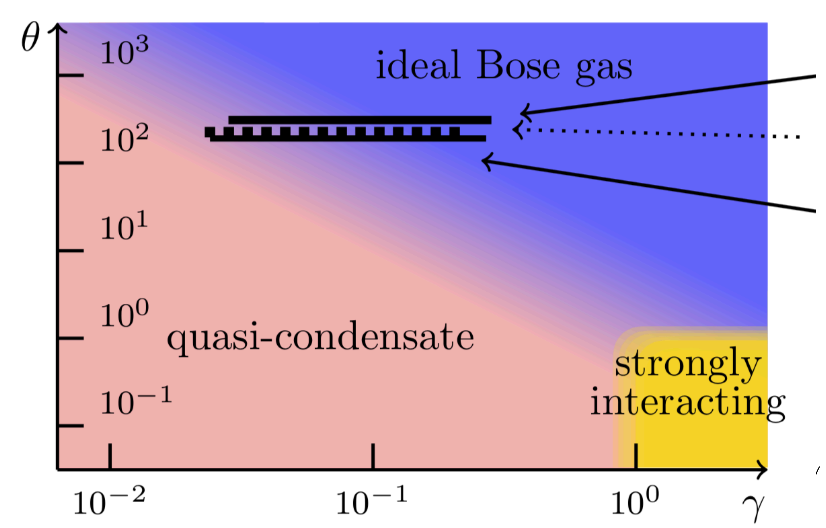
3. the **results**

3. The results

The initial state: LDA + Yang-Yang thermodynamics

The initial density profile is fitted using **LDA + Yang-Yang equation of state [Yang and Yang 1969]**, assuming thermal equilibrium in the initial state. Only **one free parameter: the temperature**.





Method widely used in experiments on 1d Bose gas: **[van Amerongen, van Es, Wicke, Kheruntsyan, van Druten, 2008]**, **[Jacqmin, Armijo, Berrada, Kheruntsyan, Bouchoule, 2011]**, **[Vogler, Labouvie, Stubenrauch, Barontini, Guarrera, Ott, 2013]**.

3. The experimental setup

‘Yang-Yang thermodynamics’

PRL **100,** 090402 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Yang-Yang Thermodynamics on an Atom Chip

A. H. van Amerongen,[1] J. J. P. van Es,[1] P. Wicke,[1] K. V. Kheruntsyan,[2] and N. J. van Druten[1]

[1]*Van der Waals-Zeeman Institute, University of Amsterdam, Valckenierstraat 65-67, 1018 XE Amsterdam, The Netherlands*
[2]*ARC Centre of Excellence for Quantum-Atom Optics, School of Physical Sciences, University of Queensland, Brisbane, Queensland 4072, Australia*

(Received 12 September 2007; revised manuscript received 24 January 2008; published 3 March 2008)

We investigate the behavior of a weakly interacting nearly one-dimensional trapped Bose gas at finite temperature. We perform *in situ* measurements of spatial density profiles and show that they are very well described by a model based on exact solutions obtained using the Yang-Yang thermodynamic formalism, in a regime where other, approximate theoretical approaches fail. We use Bose-gas focusing [I. Shvarchuck *et al.*, Phys. Rev. Lett. **89**, 270404 (2002)] to probe the axial momentum distribution of the gas and find good agreement with the *in situ* results.

DOI: 10.1103/PhysRevLett.100.090402 PACS numbers: 05.30.Jp, 03.75.Hh, 05.70.Ce

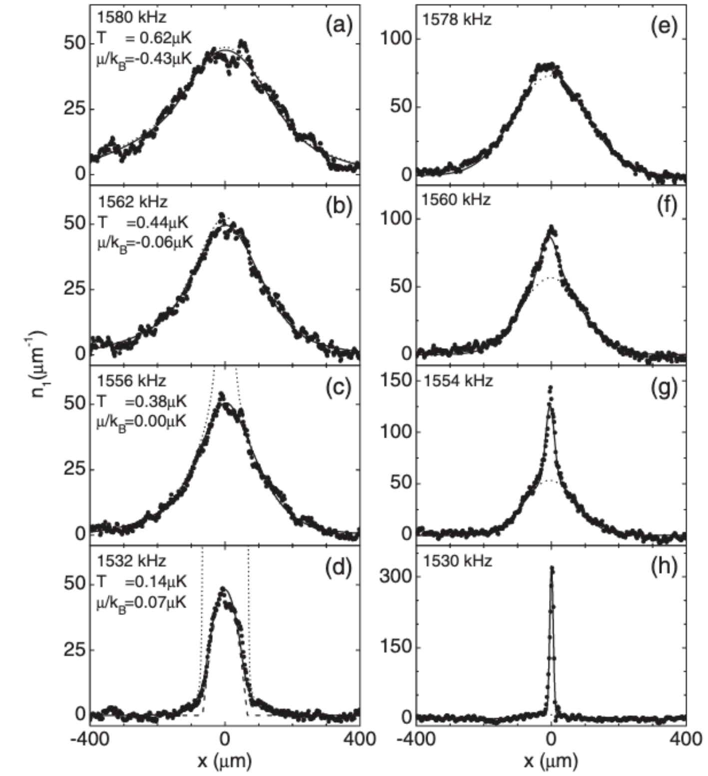


FIG. 1. Linear atomic density from absorption images obtained *in situ* (a)–(d) and *in focus* (e)–(h) by lowering (from top to bottom as indicated) the final rf evaporation frequency. *In situ*: solid lines are fits using Yang-Yang thermodynamic equations (see text). The values of μ and T resulting from the fits are shown in the figure. Dotted line: ideal Bose-gas profile showing divergence for $\mu(x) = 0$. Dashed line in (d): quasicon-

PHYSICAL REVIEW A **88**, 031603(R) (2013)

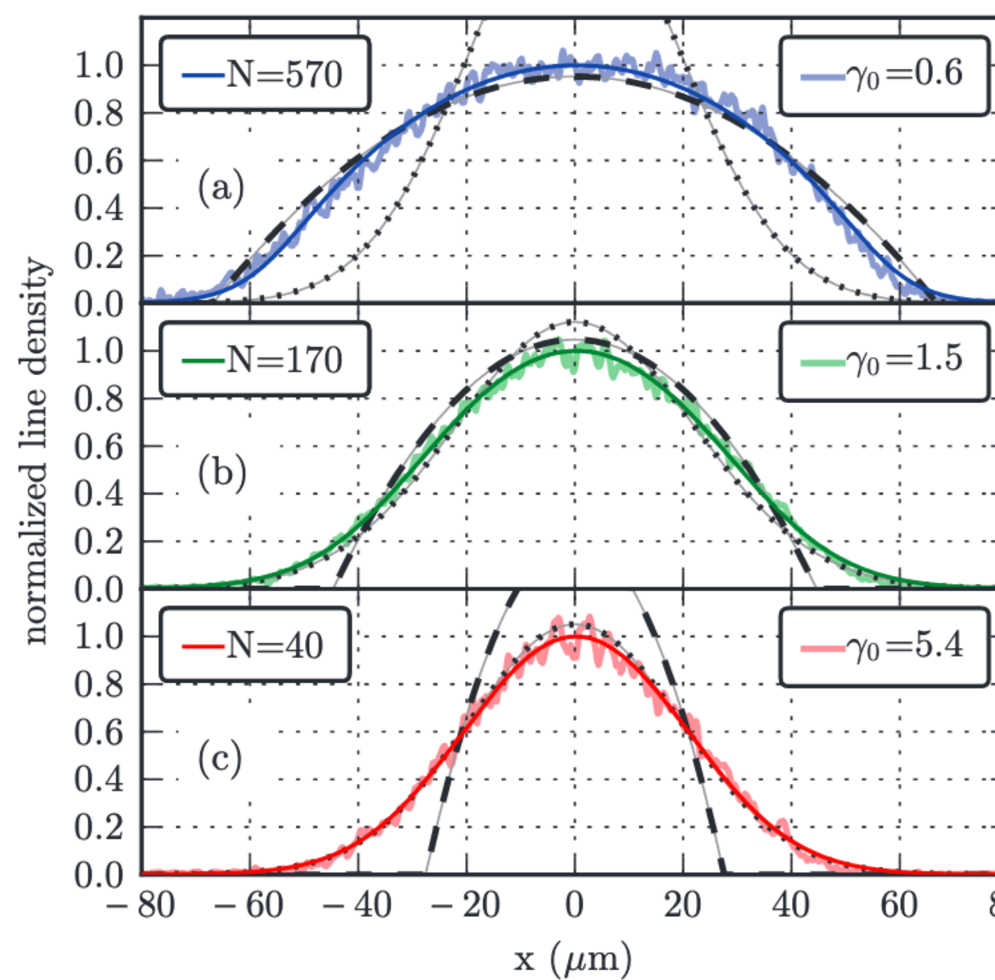
Thermodynamics of strongly correlated one-dimensional Bose gases

Andreas Vogler, Ralf Labouvie, Felix Stubenrauch, Giovanni Barontini, Vera Guarrera, and Herwig Ott*
Research Center OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany

(Received 8 April 2013; revised manuscript received 17 June 2013; published 11 September 2013)

We investigate the thermodynamics of one-dimensional (1D) Bose gases in the strongly correlated regime. To this end, we prepare ensembles of independent 1D Bose gases in a two-dimensional optical lattice and perform high-resolution *in situ* imaging of the column-integrated density distribution. Using an inverse Abel transformation we derive effective one-dimensional line-density profiles and compare them to exact theoretical models. The high resolution allows for a direct thermometry of the trapped ensembles. The knowledge about the temperature enables us to extract thermodynamic equations of state such as the phase-space density, the entropy per particle, and the local pair-correlation function.

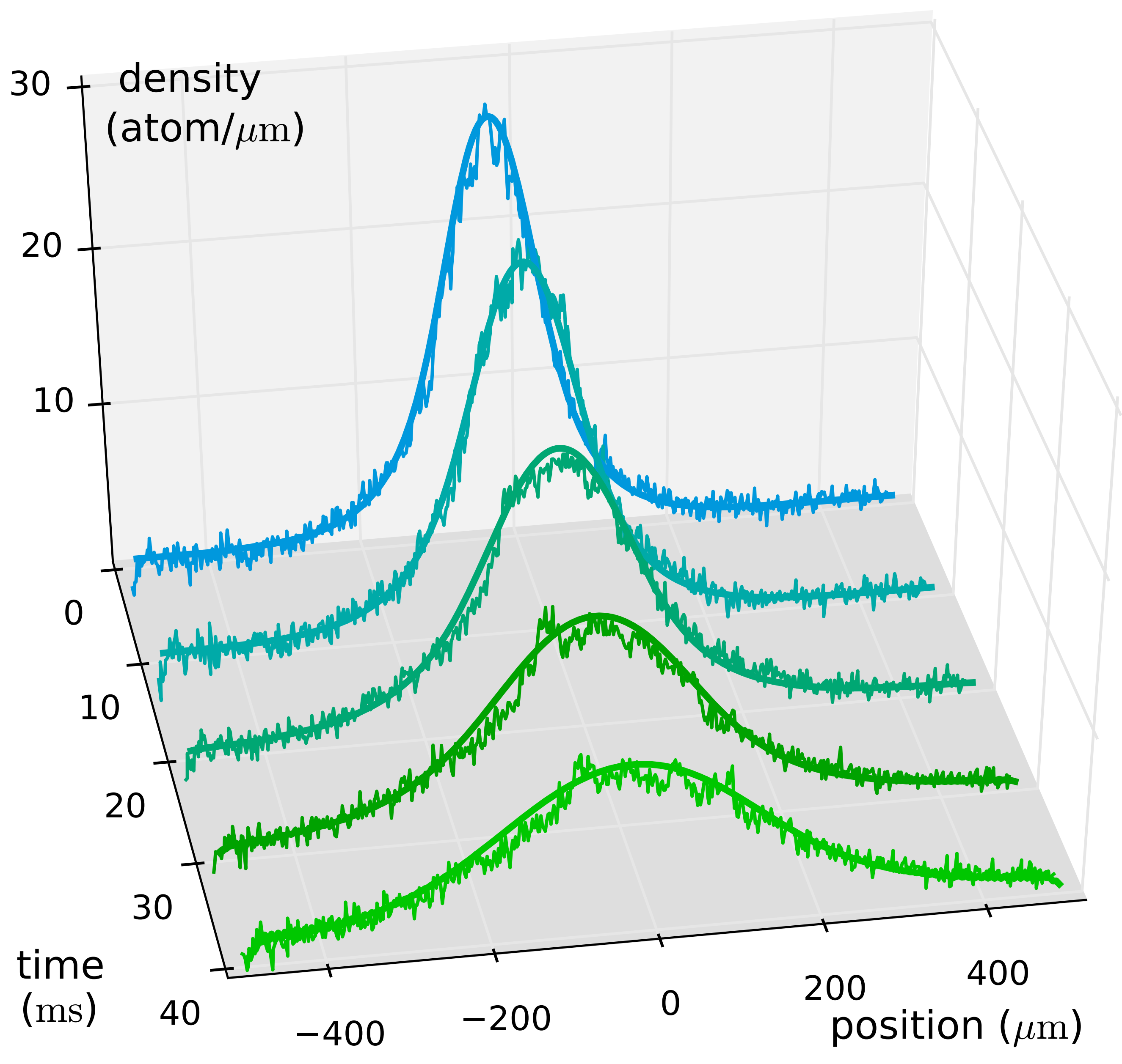
DOI: 10.1103/PhysRevA.88.031603 PACS number(s): 67.85.−d, 03.75.Hh, 05.30.Jp, 37.10.Jk

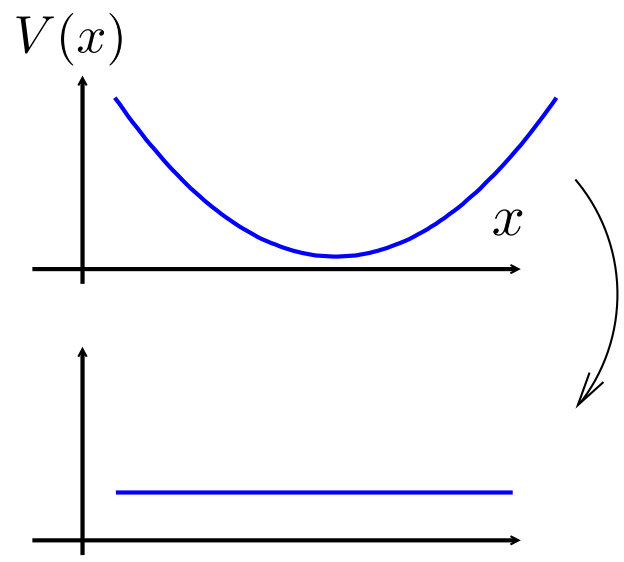


3. The results

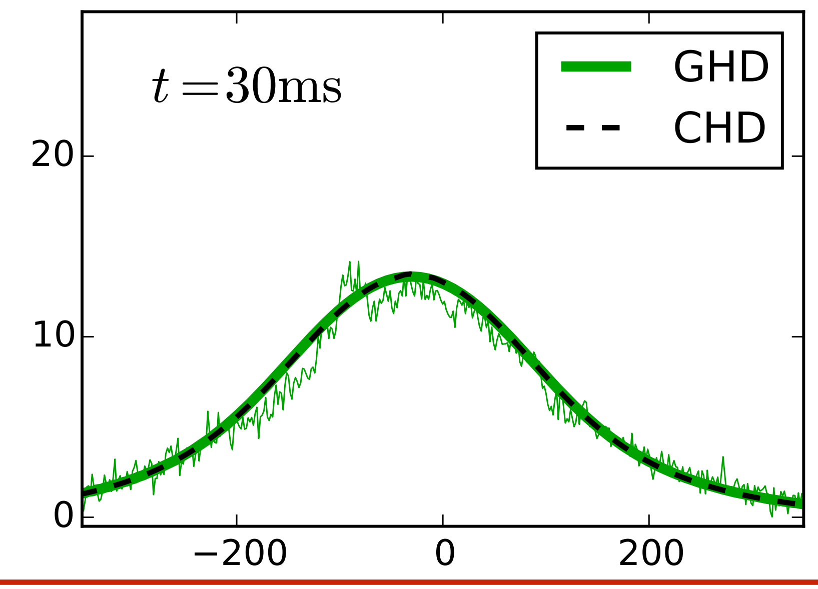
Expansion from harmonic trap

At $t = 0$ the harmonic trap is switched off and the gas expands freely in 1d.





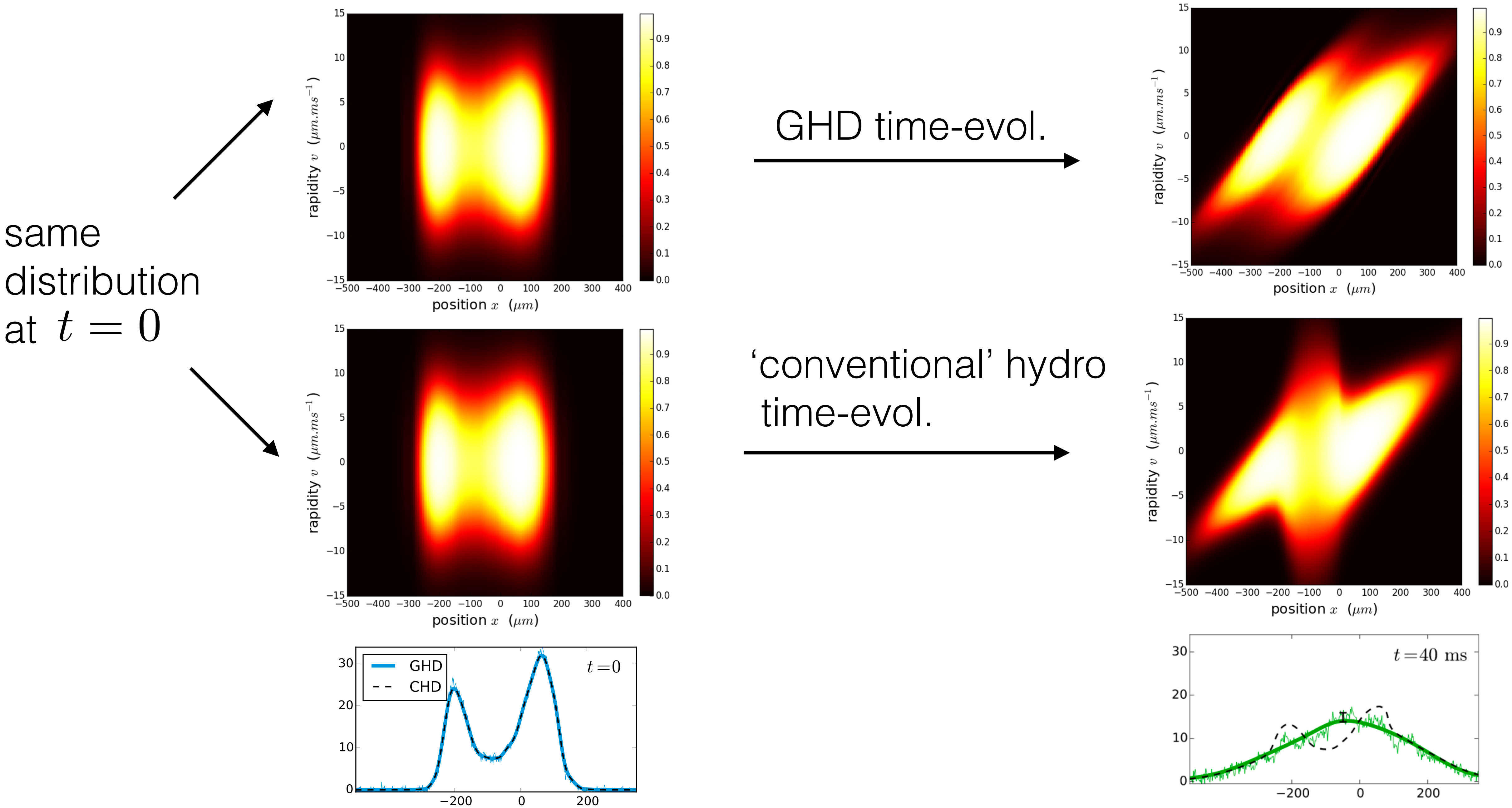
It works perfectly. However, it does not discriminate between GHD and CHD:



3. The results

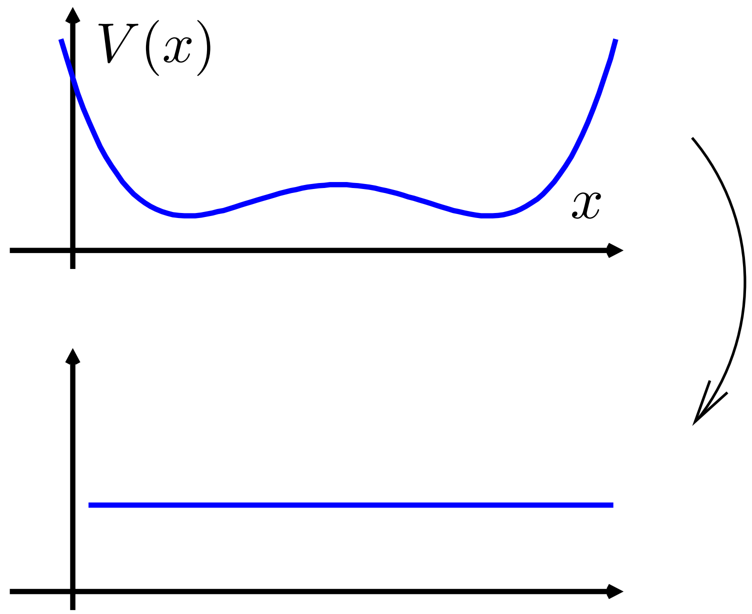
Expansion from double-well potential

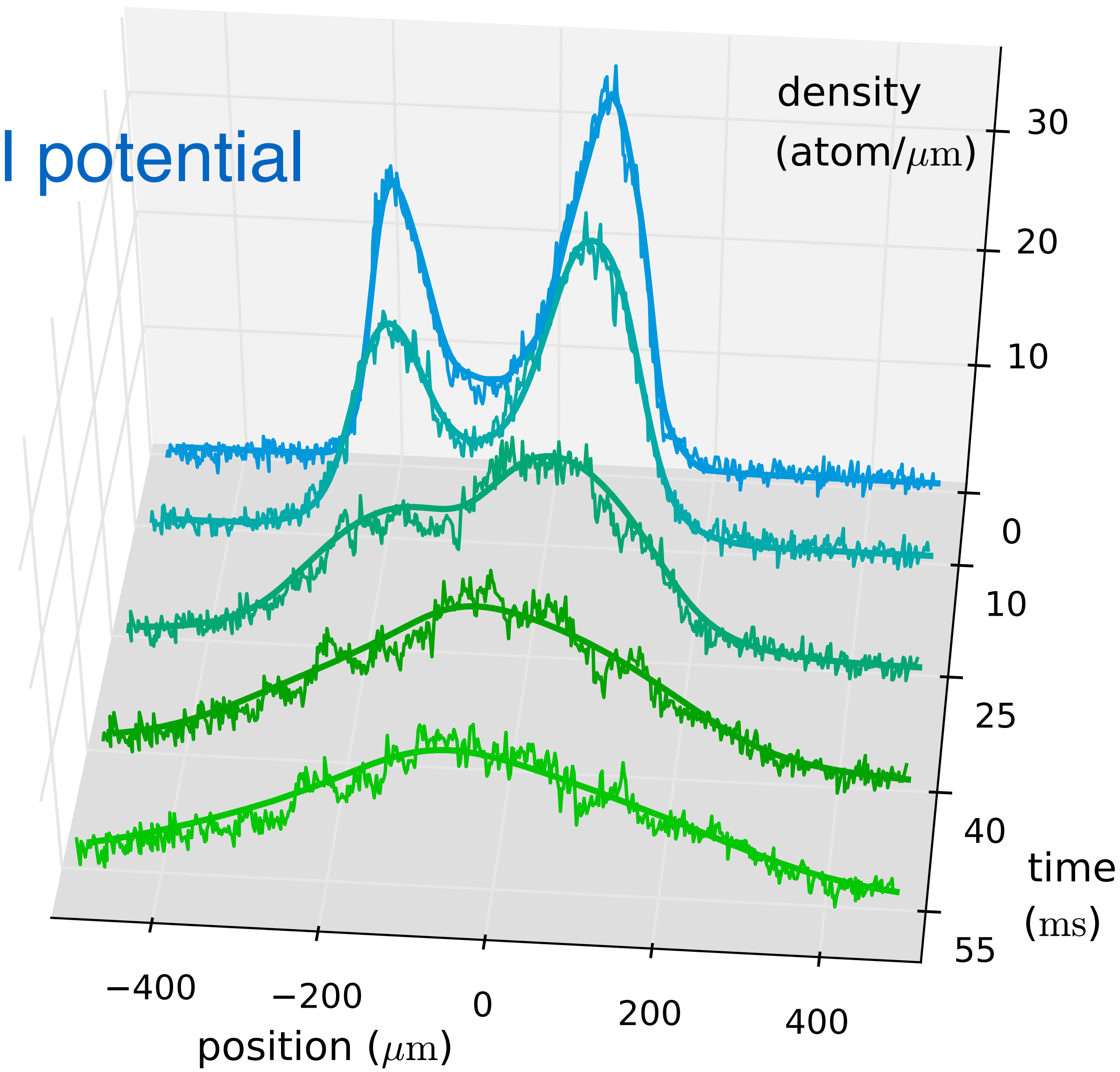
Idea: expand from **double-well potential**, instead of harmonic one



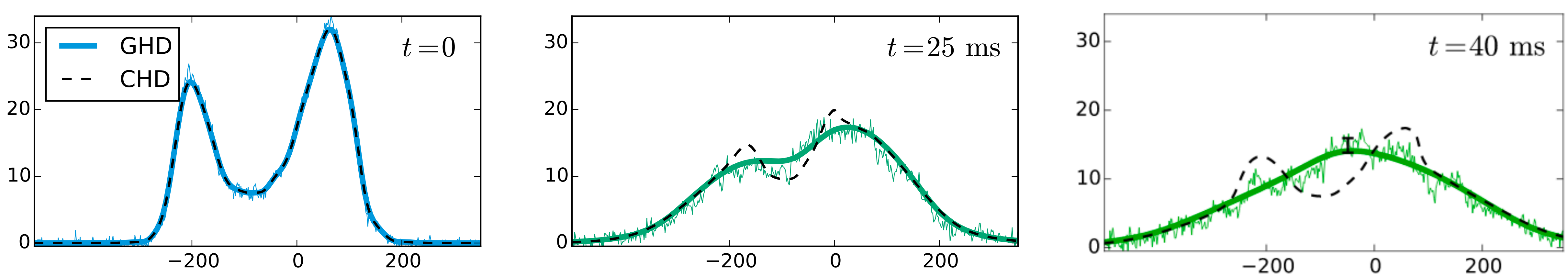
3. The results

Expansion from double-well potential



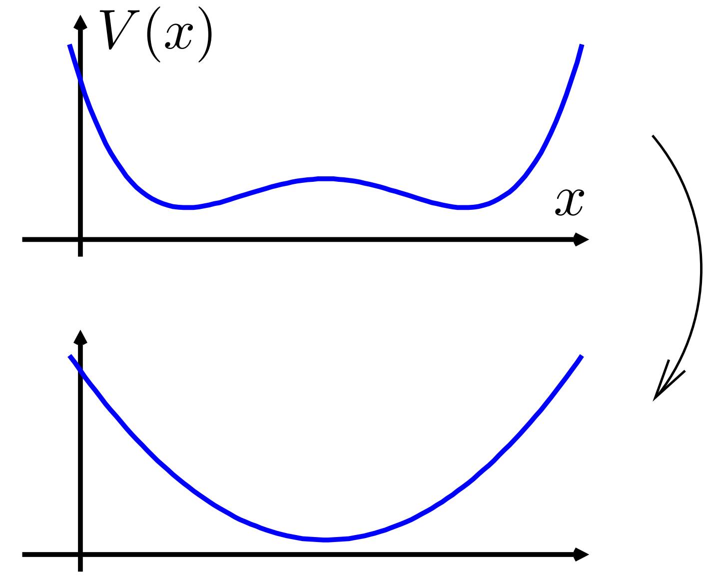


Expanding from the double-well allows to discriminate between GHD and 'conventional' hydrodynamics. The experimental data agree perfectly with GHD.

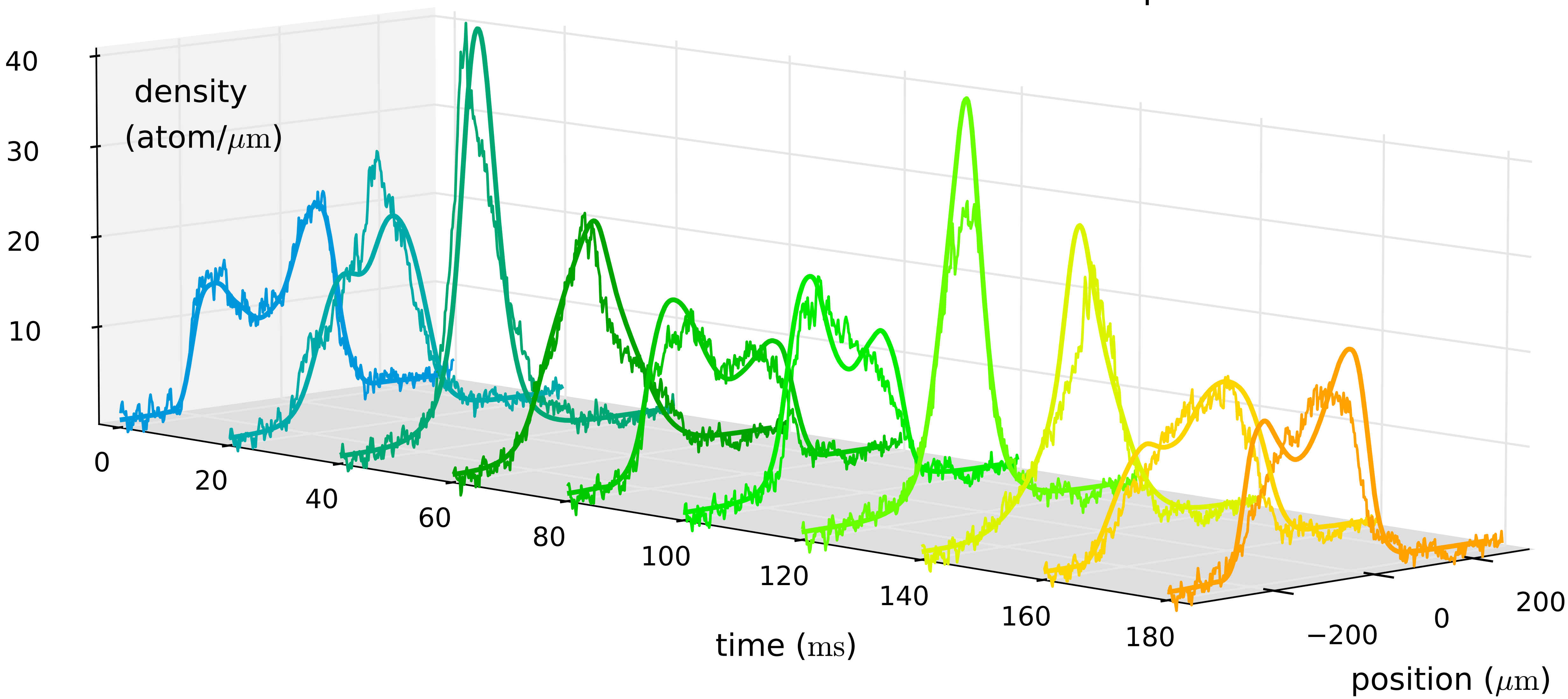


3. The results

Quantum Newton's Cradle



(mimics the famous work **[Kinoshita Wenger Weiss 2006]** but in the weakly interacting regime)



(Conventional hydro not shown, but it completely fails, predicting the formation of a shock at $t \simeq 30ms$.)

Summary

Main results:

1. **First experiment on Generalized HydroDynamics** (GHD) in a quantum (nearly) integrable system.

2. Experimental demonstration that **GHD supersedes 'conventional' hydrodynamics**.

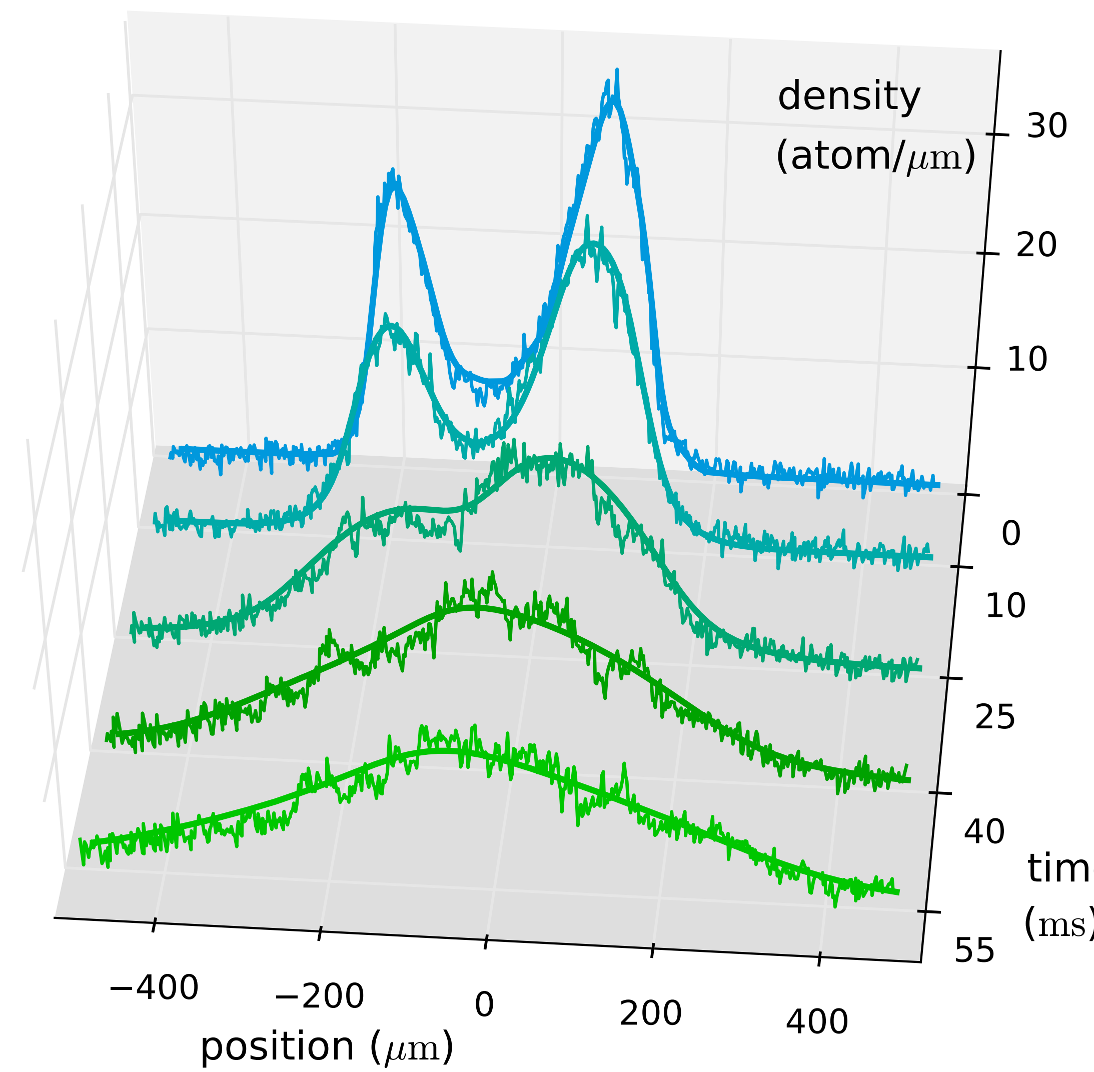
An open question

several experimental effects not taken into account (**three-body losses**, etc.). How to integrate them into the GHD description?

Some other recent theory developments

1. questions about **anomalous transport and diffusion**: **many recent works (2018-19)** by **Agrawal, Bulchandani, de Nardis, Dupont, Gopalakrishnan, Ilievski, Karrasch, Medenjak, Moore, Prosen, Vasseur, Ware, Yoshimura, …**

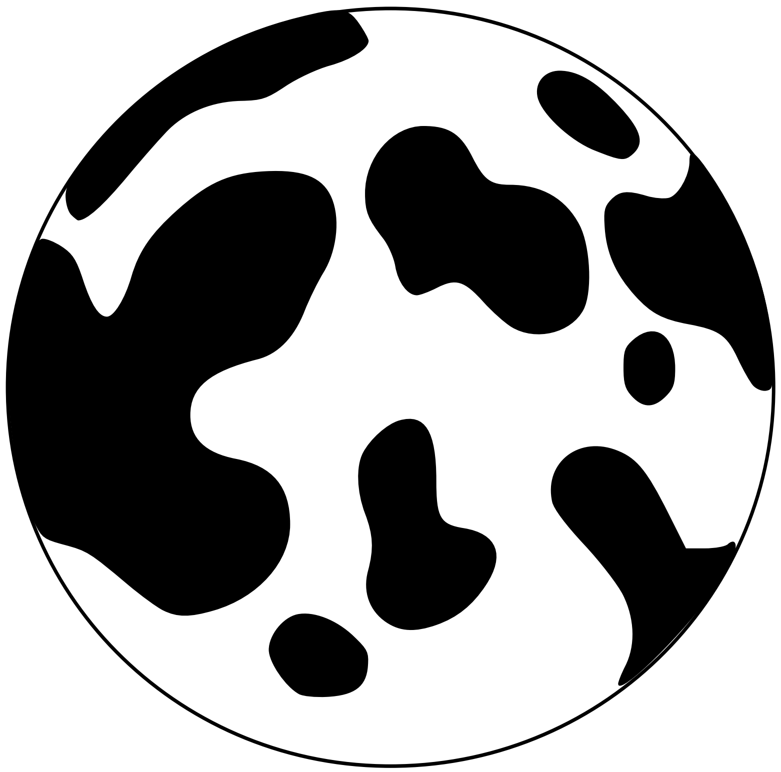
2. GHD is a classical description of a quantum system. What about **quantum fluctuations**? Can we **quantize GHD? [Ruggiero, Calabrese, Doyon, JD 2019]**



Thank you!

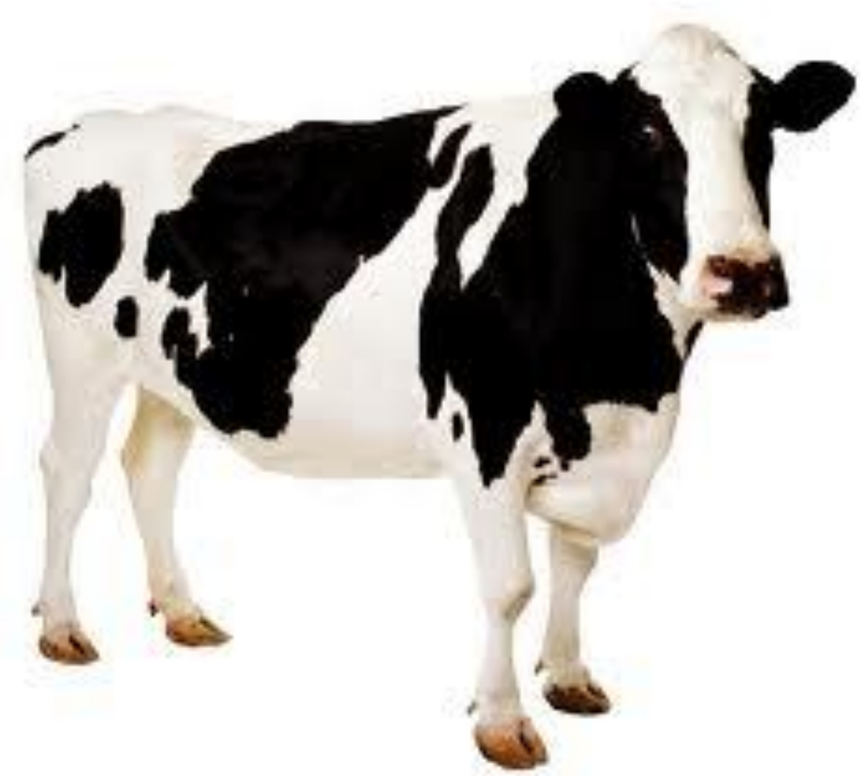
Additional slides

Quantum fluctuations in GHD



$$\partial_t + \partial_x (v^{\mathrm{eff}}_{[\rho]}(v)\rho) = \frac{\partial_x V}{m}\partial_v \rho$$

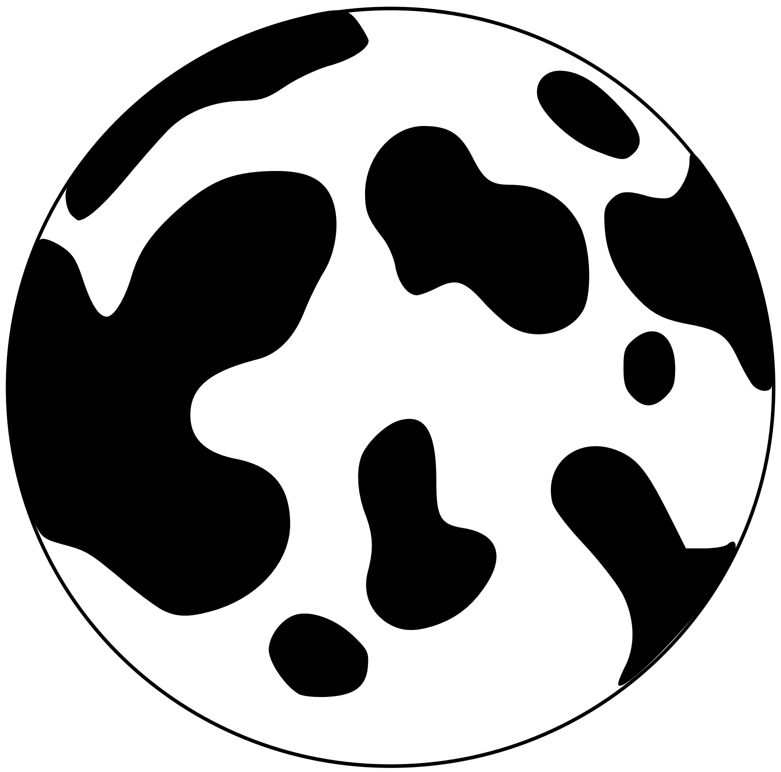
effective classical description

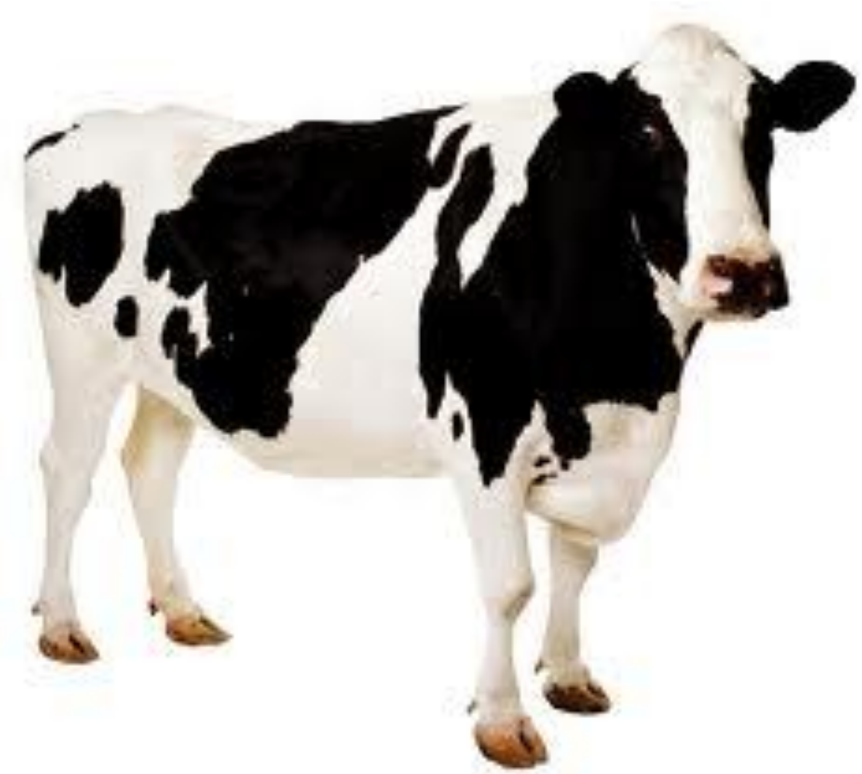


$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\partial^2_{x_i}\psi + g\sum_{i<j}\delta(x_i - x_j)\psi + \sum_{i=1}^{N}V(x_i)\psi$$

full quantum description

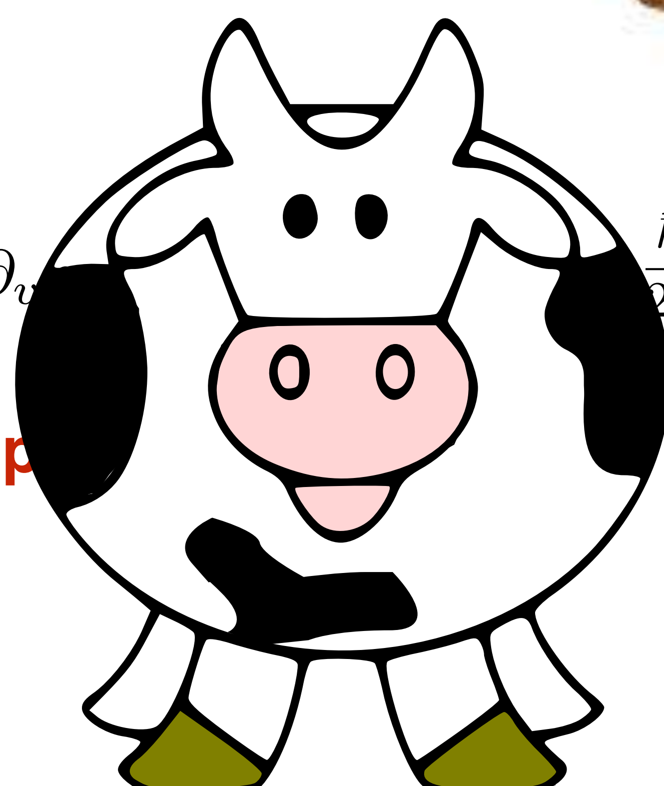
Quantum fluctuations in GHD





$$\partial_t + \partial_x (v^{\mathrm{eff}}_{[\rho]}(v)\rho) = \frac{\partial_x V}{m} \partial_v$$

$$\frac{\hbar^2}{2m} \sum_{i=1}^{N} \partial^2_{x_i} \psi + g \sum_{i<j} \delta(x_i - x_j) \psi + \sum_{i=1}^{N} V(x_i) \psi$$



effective classical descrip

full quantum description

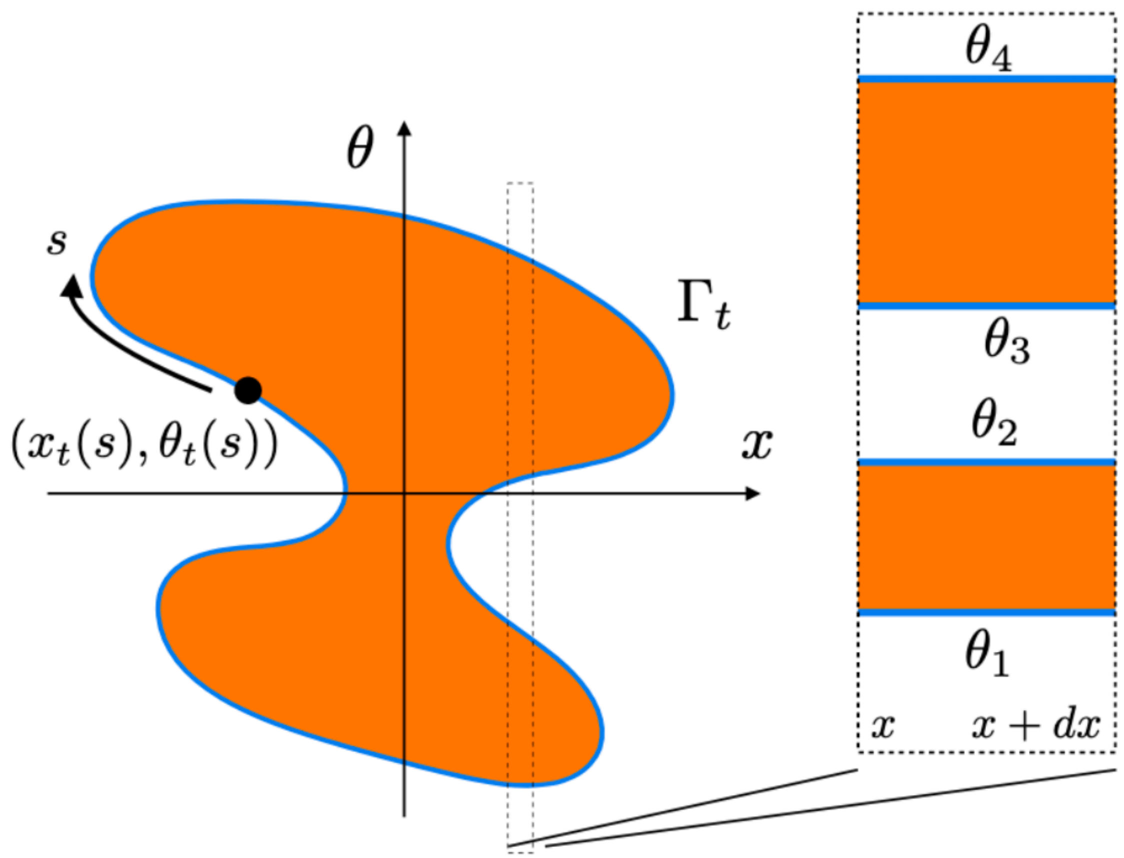
+ quantum linear fluctuations

effective quantum description

Quantum fluctuations in GHD

We start from **GHD at temperature T=0**. There the quasi-particle Fermi factor is either 0 or 1. Phase-space occupation is then encoded by a contour:

$$\Gamma_t = \{(x_t(s), \theta_t(s)), s \in \mathbb{R}/2\pi\mathbb{Z}\}$$



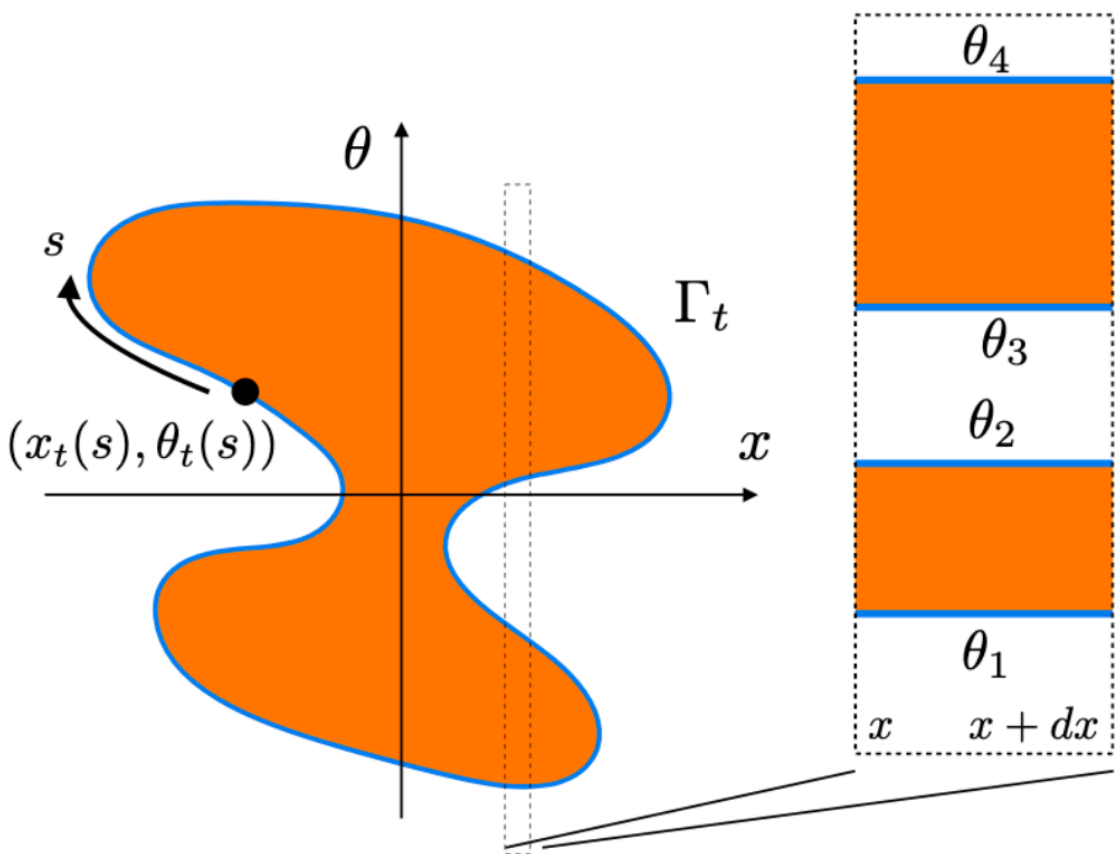
in terms of this contour the first GHD equation reads

$$\frac{d}{dt}\begin{pmatrix} x_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} v^{\text{eff}}(x_t, \theta_t) \\ -\frac{\partial_x V}{m} \end{pmatrix}$$

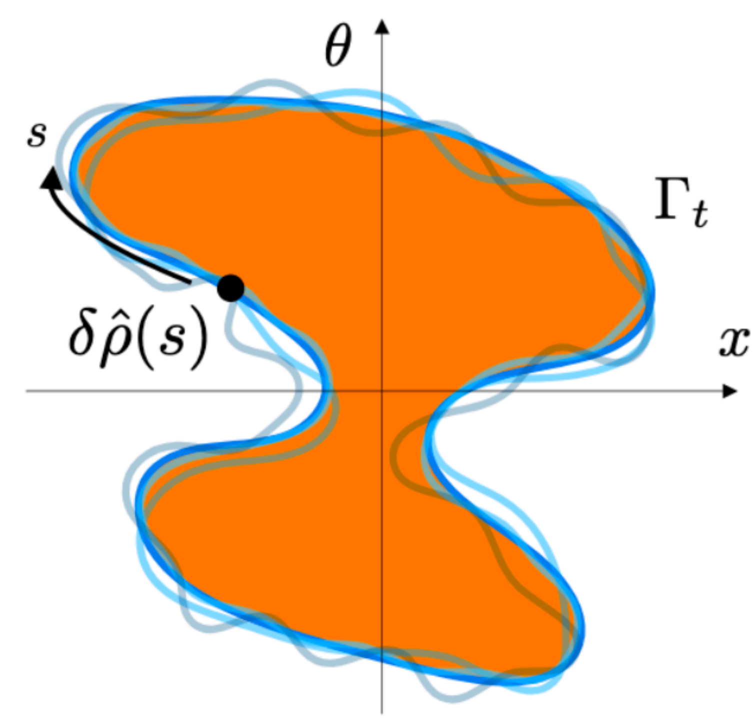
(similar to other recent works on zero-temperature ‘phase-space hydrodynamics’ for non-interacting fermions **[Kulkarni, Mandal, Morita 2018], [Ruggiero, Brun, JD 2019], [Dean, Le Doussal, Majumdar 2019], [Das, Hampton, Liu 2019]**)

Quantum fluctuations in GHD

The idea is that **at zero temperature the relevant degrees of freedom are the fluctuations of the contour** around its classical configuration at time t



GHD (classical)



GHD + quantum fluctuations

The fluctuations of the contour are encoded by an operator $\delta\hat{\rho}(s)$ which measures the excess density of quasi-particles around a point $(x_t(s), \theta_t(s))$

Quantum fluctuations in GHD

Then the **equations governing linear quantum fluctuations** around GHD are:

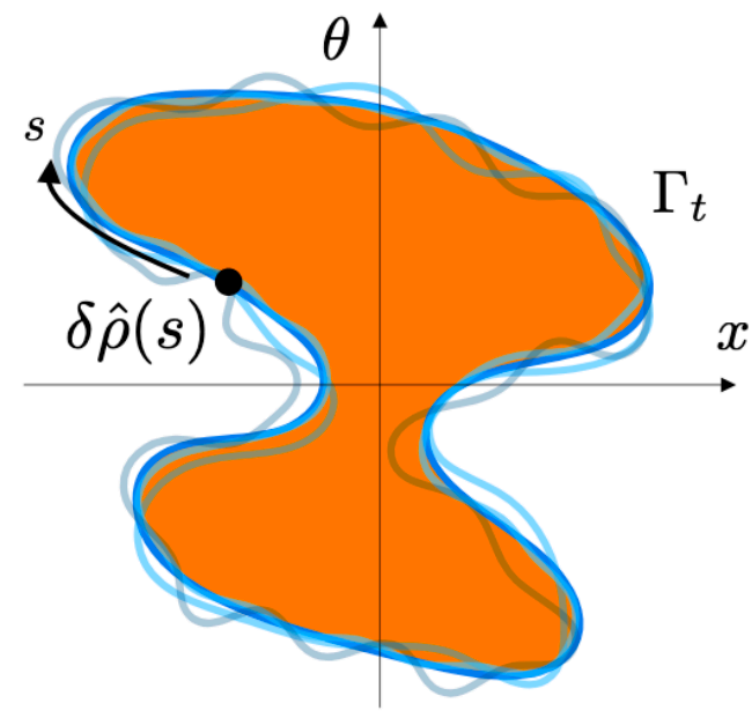
1. commutation relation of the densities (chiral U(1) current algebra):

$$[\delta\hat{\rho}(s), \delta\hat{\rho}(s')] = \frac{1}{2\pi i}\delta(s - s')$$

2. quadratic Hamiltonian:

$$\hat{H}[\Gamma_t] = \int ds\, ds' \delta\hat{\rho}(s) K(s, s') \delta\hat{\rho}(s)$$

where $K(s, s') \propto \delta(x_t(s) - x_t(s'))$ is known exactly and is given by the Thermodynamic Bethe Ansatz.



GHD + quantum fluctuations