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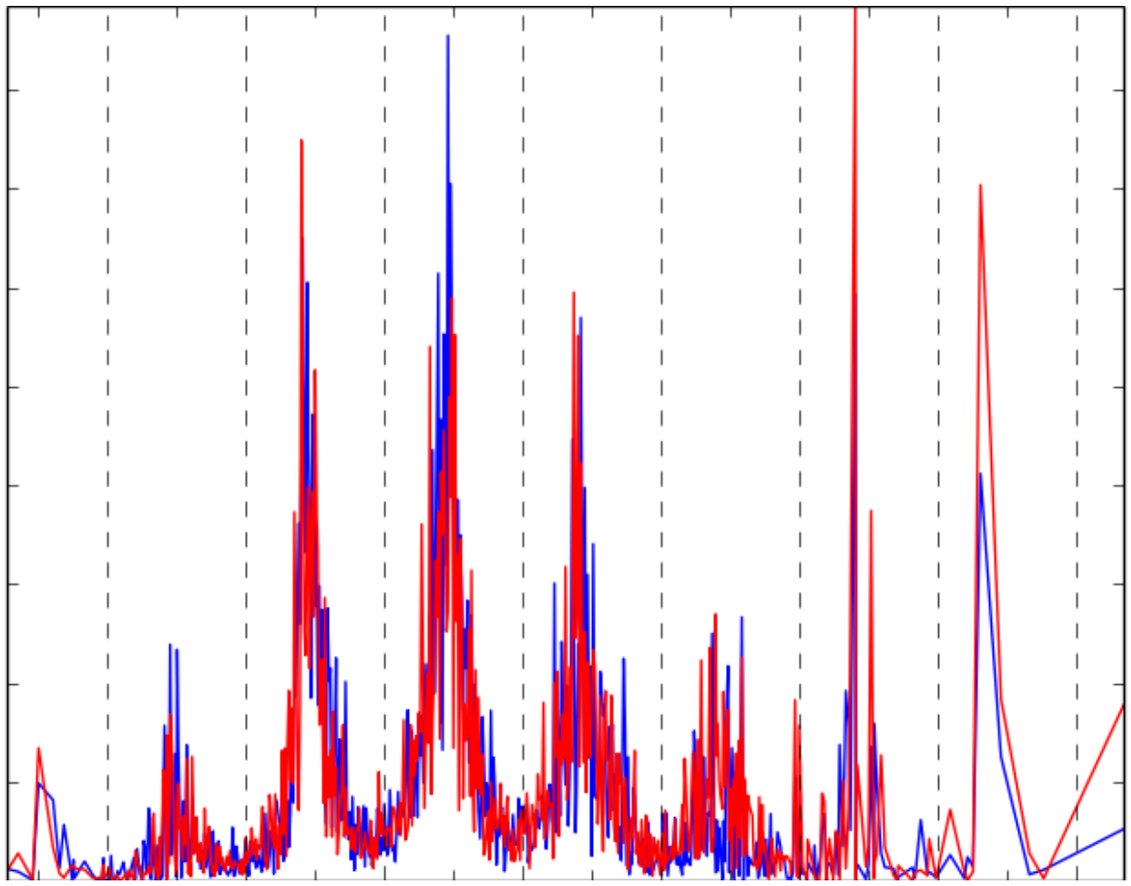
(Pre-)thermalization in Periodically-Driven Systems



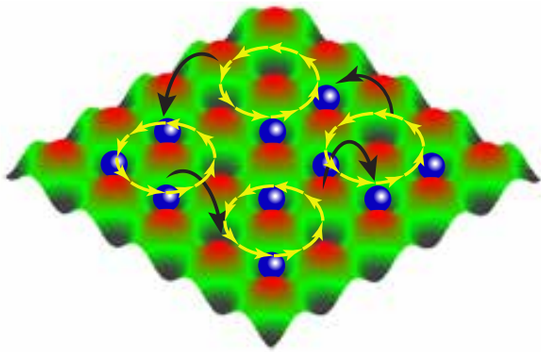
quantum or classical?

M.B., Heyl, Huse, Polkovnikov, PRB 93, 155132 (2016)

Howell, Weinberg, Sels, Polkovnikov, and M.B., PRL 122, 010602 (2019)

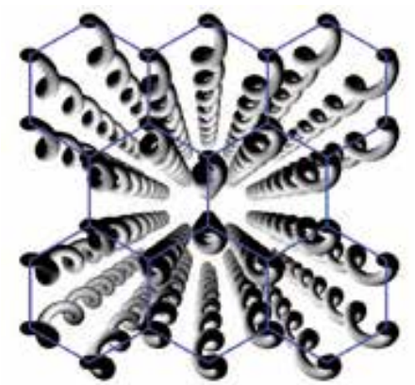
Why study periodically-driven systems?

→ practically: Floquet engineering



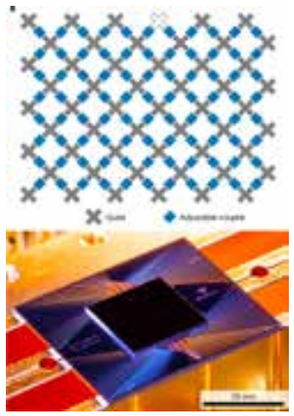
ultracold atoms

PRA, 90:043613, 2014



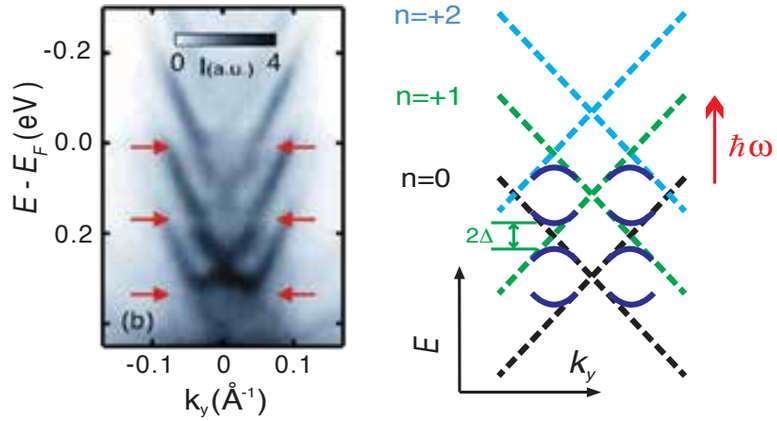
photonic insulators

Nature, 496:196, 2013



superconducting qubits

Nature, 574:505 2019



quantum materials: Bi_2Se_3

Nature Physics, 12(4):306, 2016

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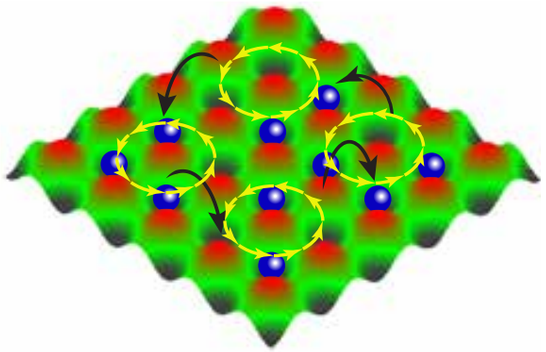

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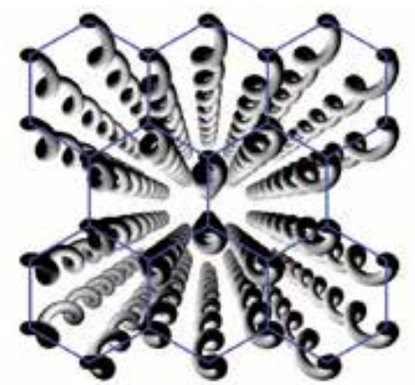

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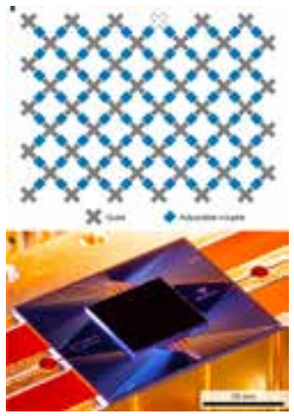

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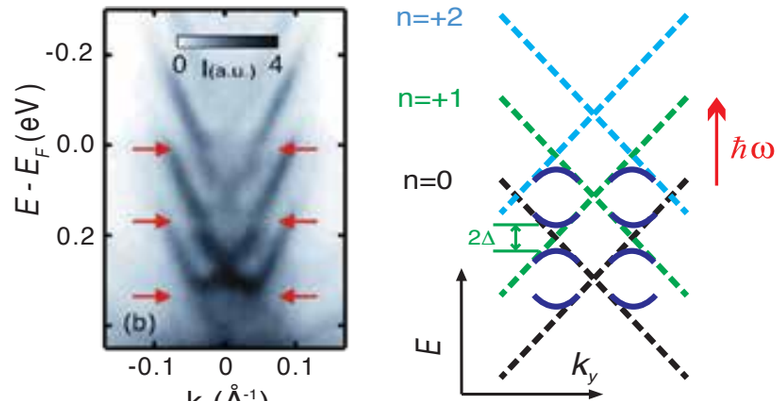

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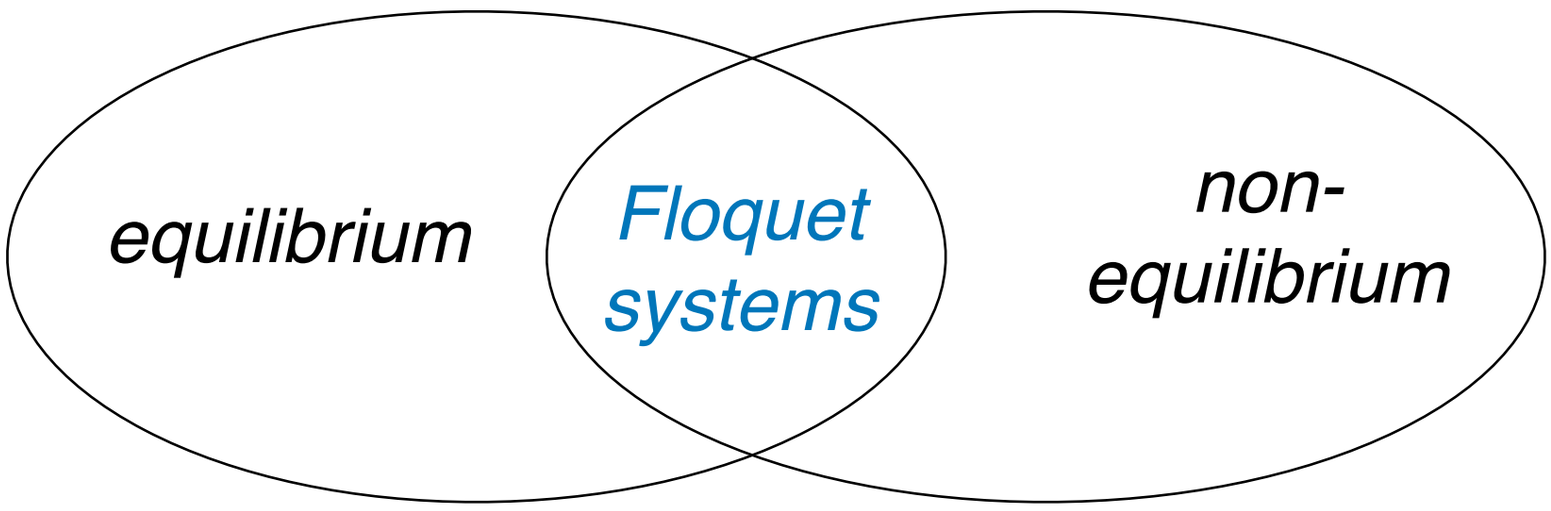
→ fundamentally:



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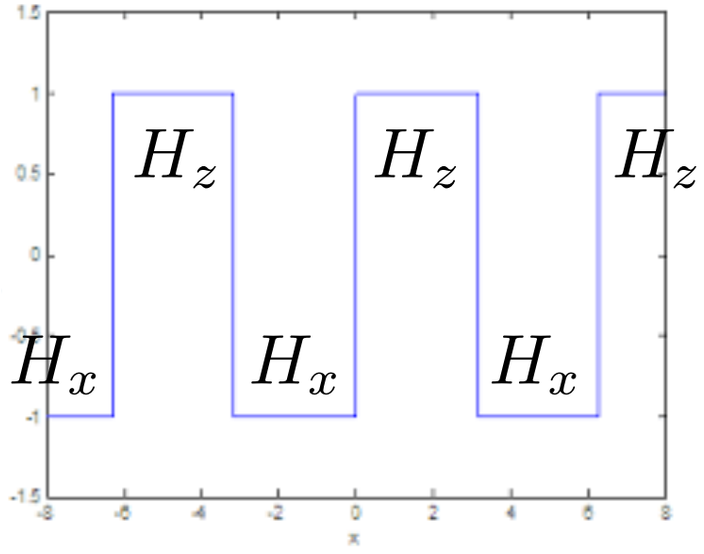
Energy absorption in quantum Floquet systems

$$\Omega = \frac{2\pi}{T}$$

- generic local interacting driven system:

$$H(t) = \begin{cases} \sum_{j=1}^{L} J S_j^z S_{j+1}^z + h S_j^z & \text{for } t \in [0, T/2] \bmod T \\ \sum_{j=1}^{L} g S_j^x & \text{for } t \in [T/2, T] \bmod T \end{cases}$$

$$[S_i^\alpha, S_j^\beta] = i\delta_{ij}\varepsilon^{\alpha\beta\gamma} S_j^\gamma$$



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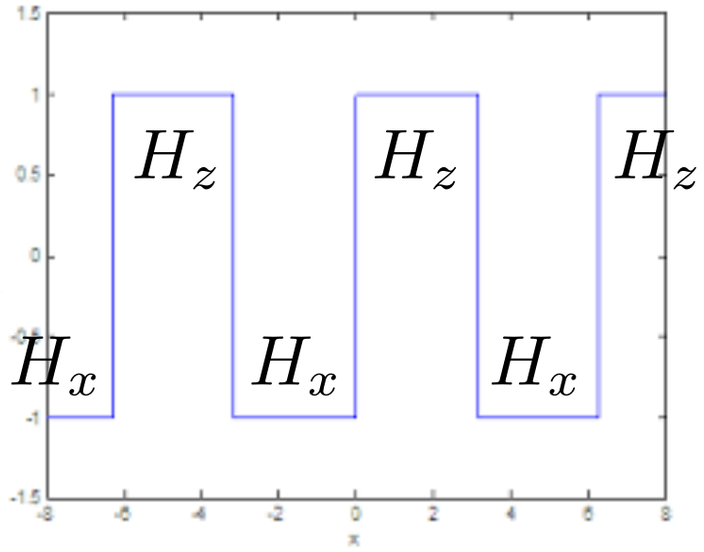

- **large** switching frequency: $\Omega \gg J, h, g$

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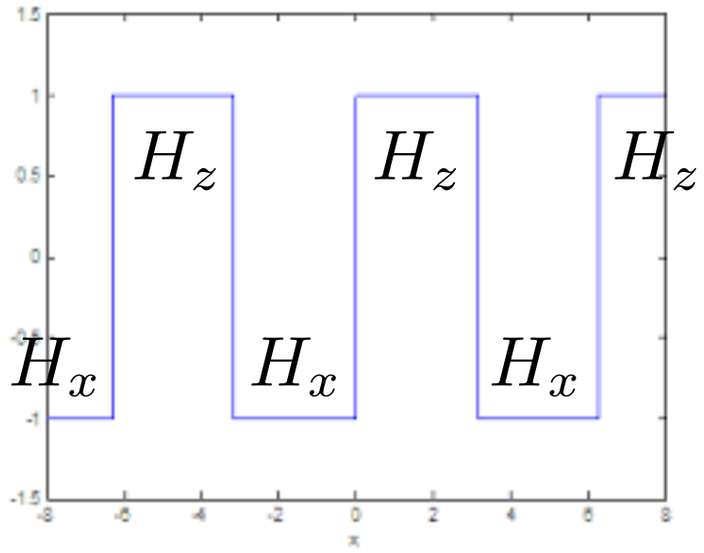

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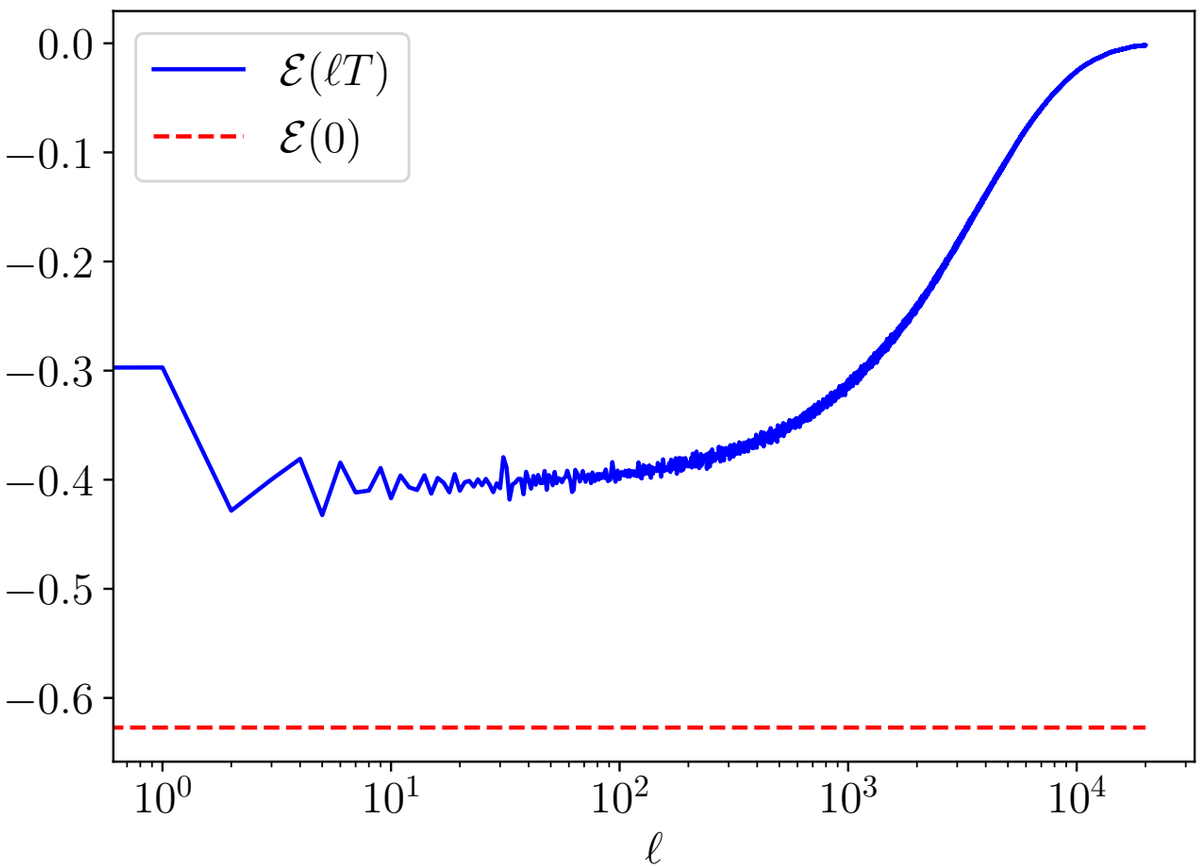

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$$|\psi(t=0)\rangle = |\text{GS}(H_{\text{ave}})\rangle$$

$$\mathcal{E}(\ell T) = L^{-1}\langle\psi(\ell T)|H_{\text{ave}}|\psi(\ell T)\rangle$$

D'Alessio and Rigol, PRX 4, 041048 (2014)

Lazarides et al, PRL 112, 150401 (2014), PRE 90, 012110 (2014).

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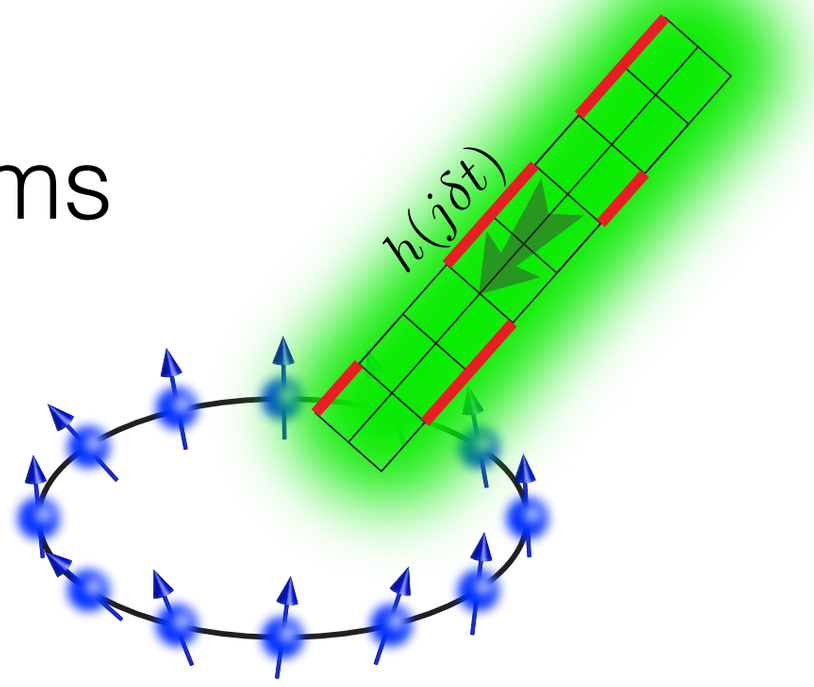

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Abanin et al, PRB 95, 014112 (2017)
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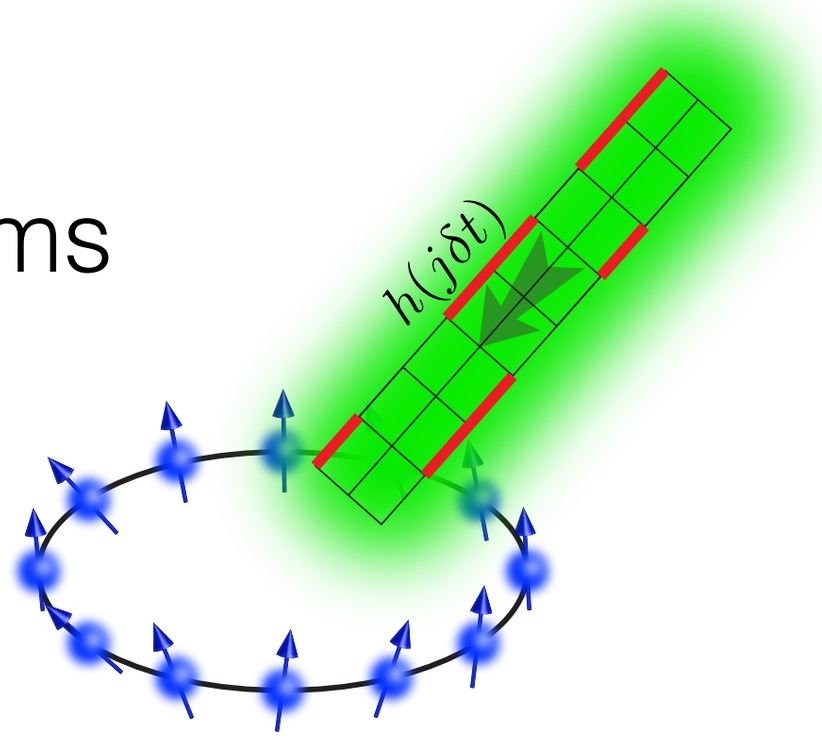

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→ absorption of a single driving quantum requires multiple simultaneous local processes

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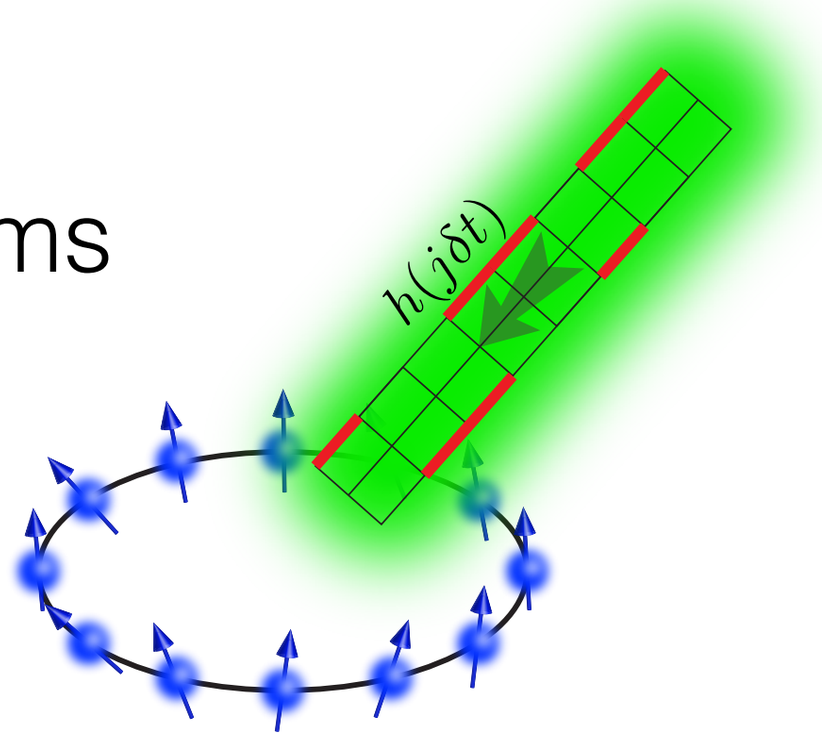

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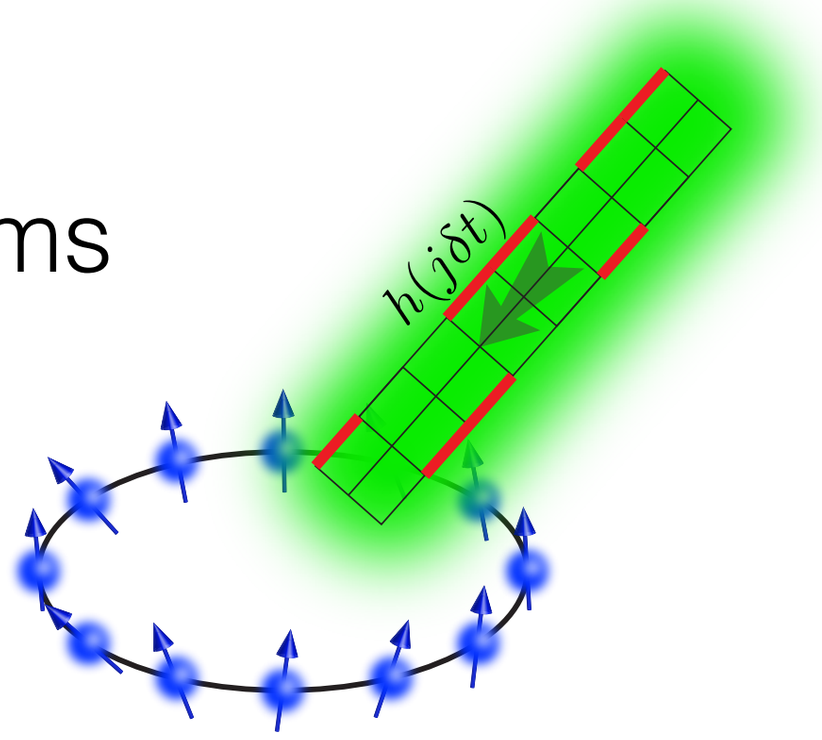

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long pre-thermal window: ideal for Floquet engineering

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Floquet theory

- Floquet's theorem $U(t,0) = P(t)\exp(-itH_F)P^\dagger(0)$

time-periodic $P(t+T) = P(t)$

Floquet Hamiltonian, time-**in**dependent

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$$\mathcal{T}e^{-i\int_0^T \mathrm{d}t H(t)} = e^{-iTH_F} \qquad H_F = \sum_{n=0}^{\infty} H_F^{(n)} \qquad H_F^{(n)} \sim \Omega^{-n}$$

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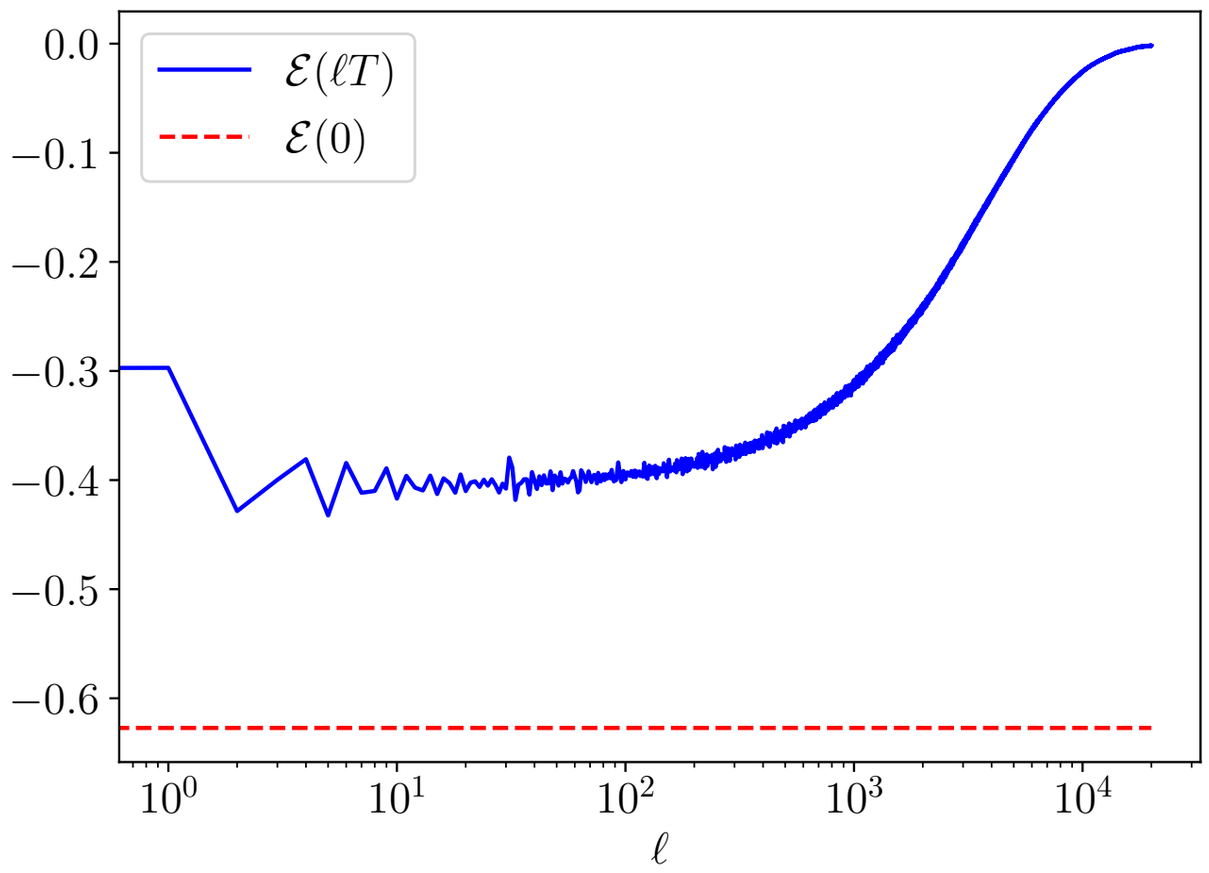
$$H_F^{(0)} = T^{-1}\int_0^T \mathrm{d}t H(t)$$

$$H_F^{(1)} \sim \int_0^T \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 [H(t_1), H(t_2)]$$

Understanding the pre-thermal plateau

$$\mathcal{E}_{\mathrm{ave}}(\ell T) = L^{-1}\langle\psi_0|(U_F^{\dagger})^{\ell} H_{\mathrm{ave}} (U_F)^{\ell}|\psi_0\rangle$$
$$\approx L^{-1}\langle\psi_0|(U_F^{[n]\dagger})^{\ell} H_{\mathrm{ave}} (U_F^{[n]})^{\ell}|\psi_0\rangle$$



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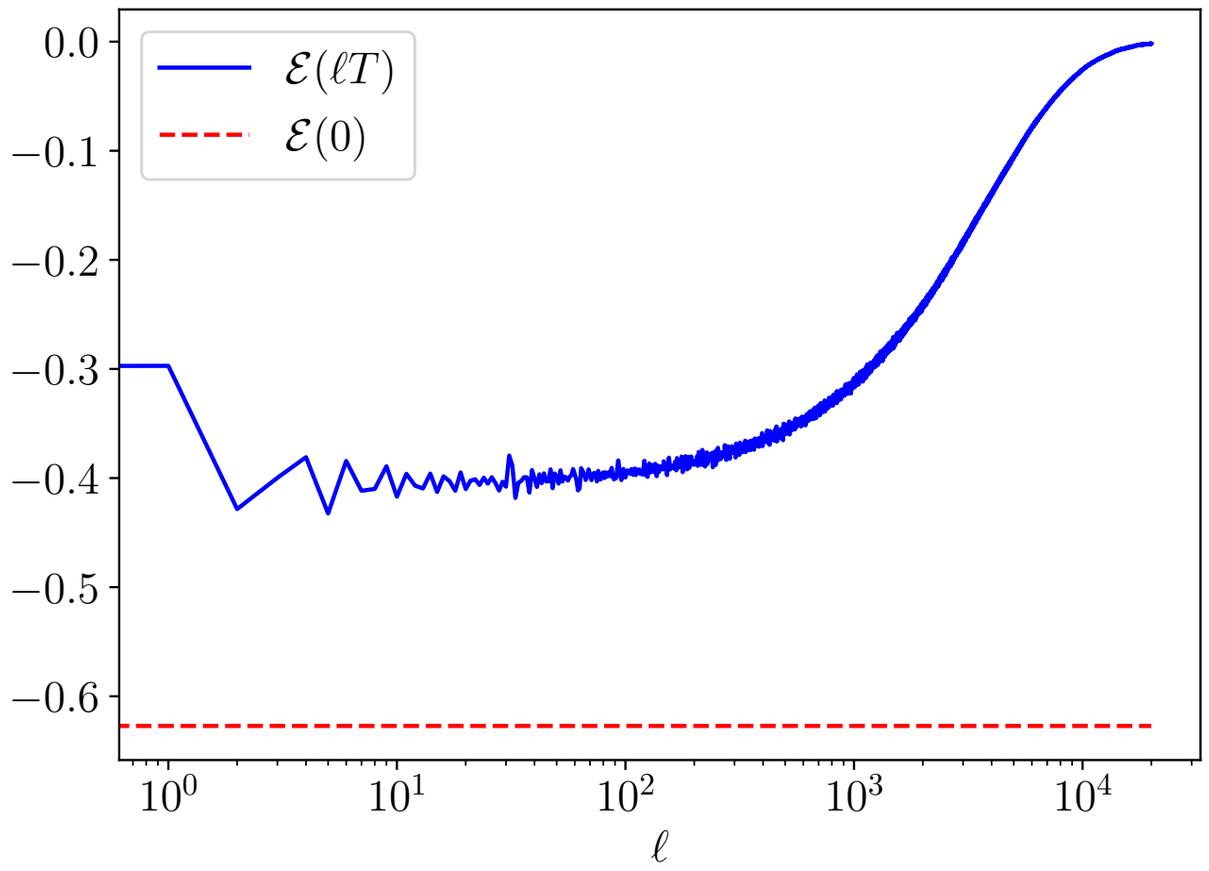

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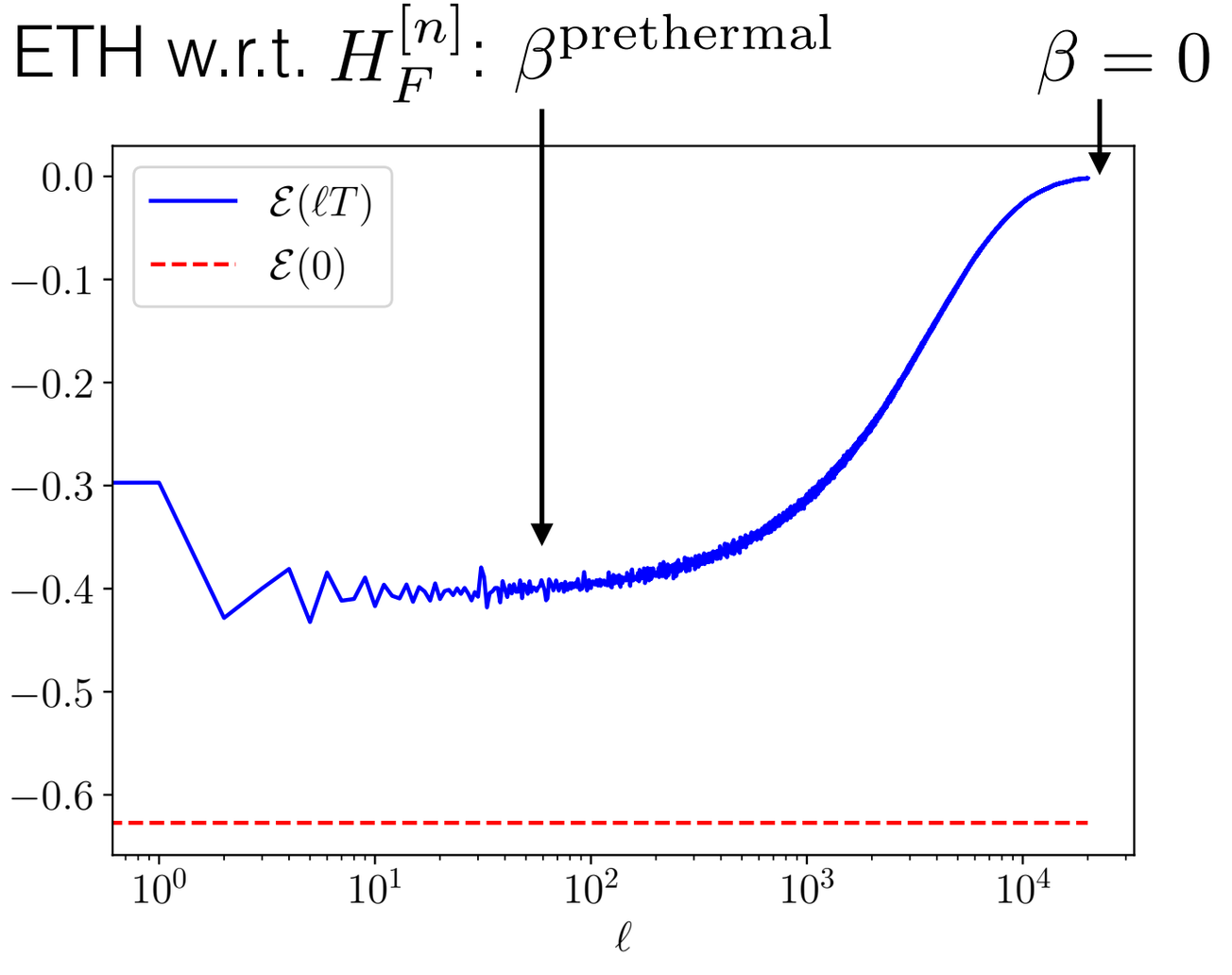

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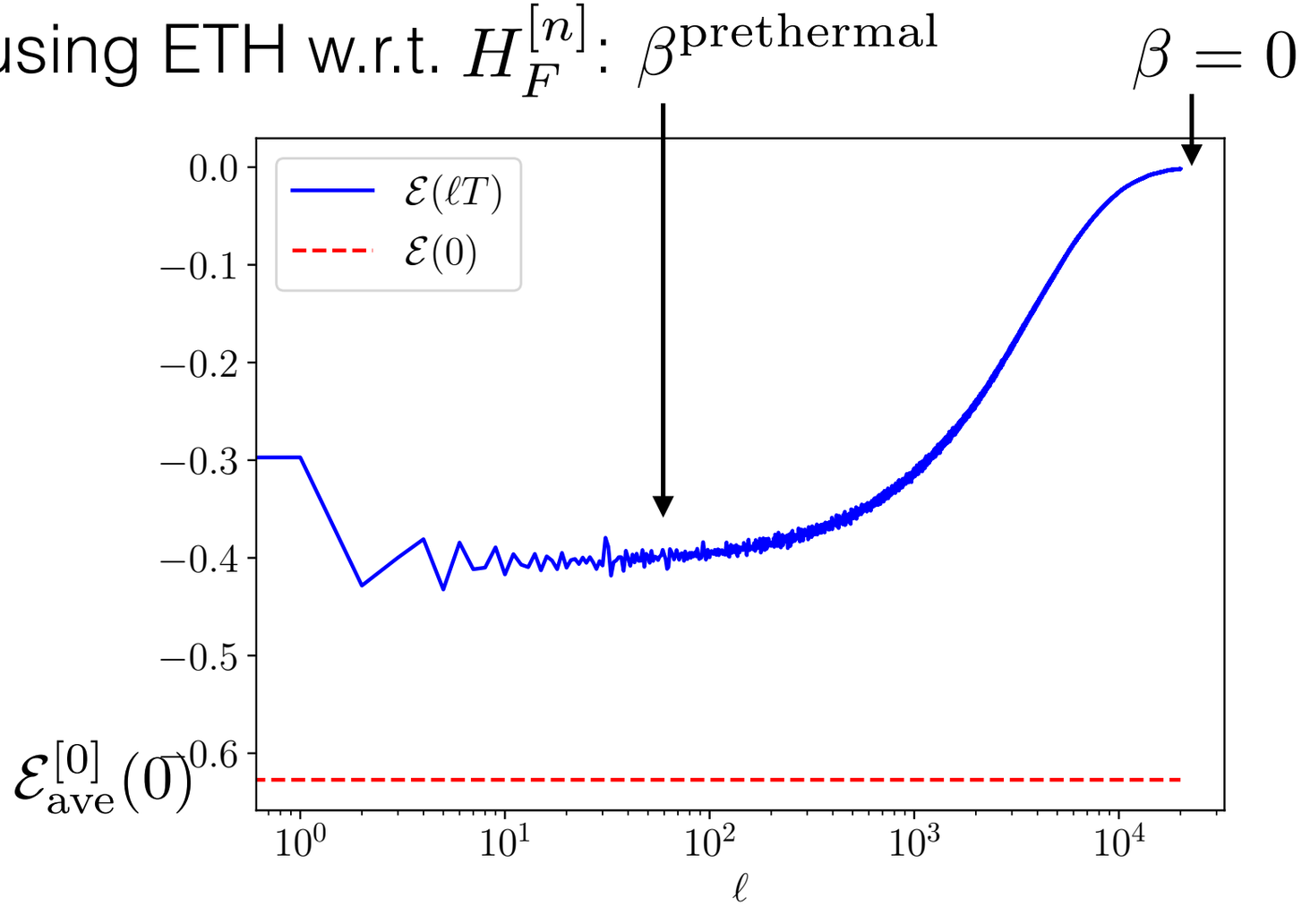

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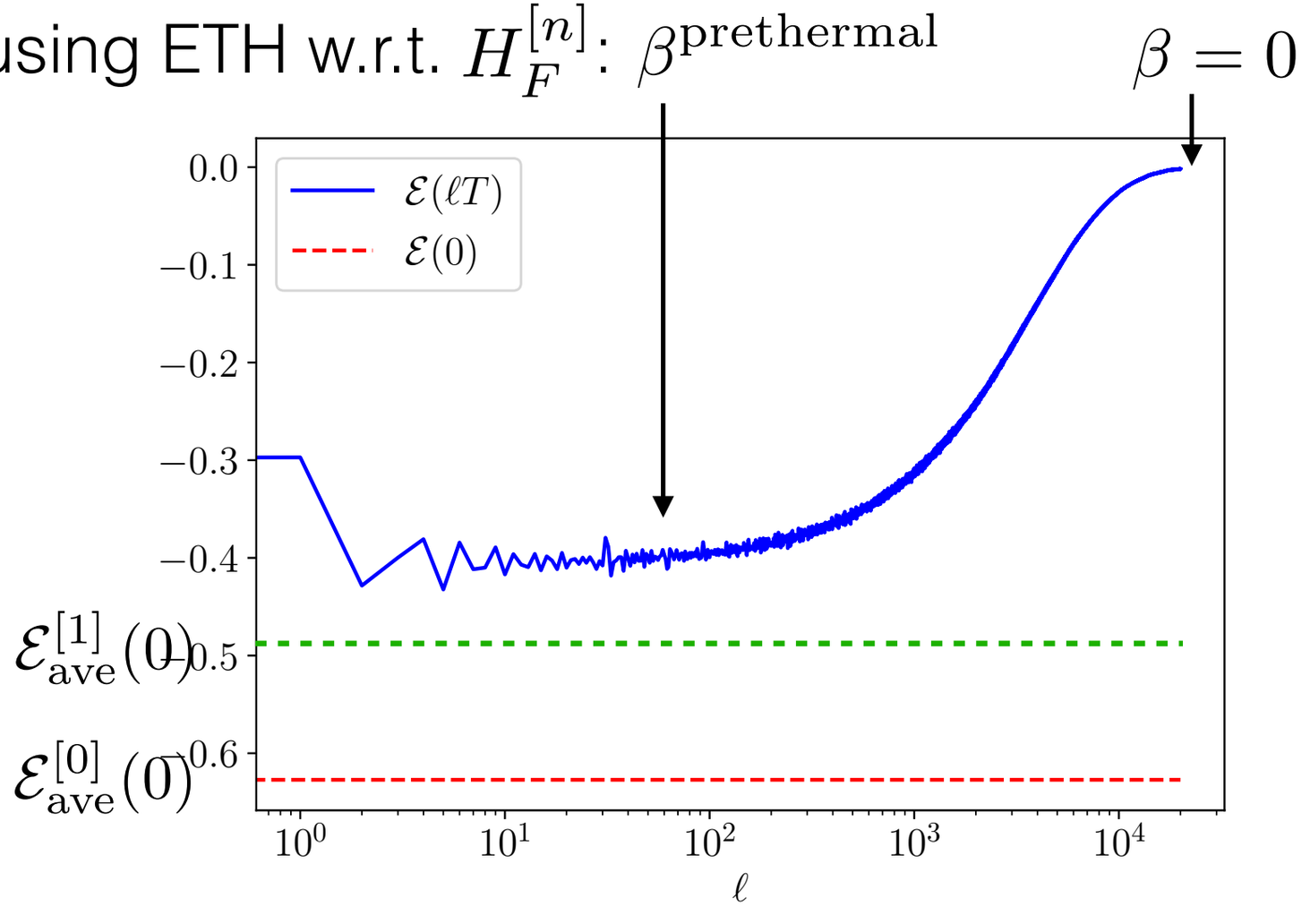

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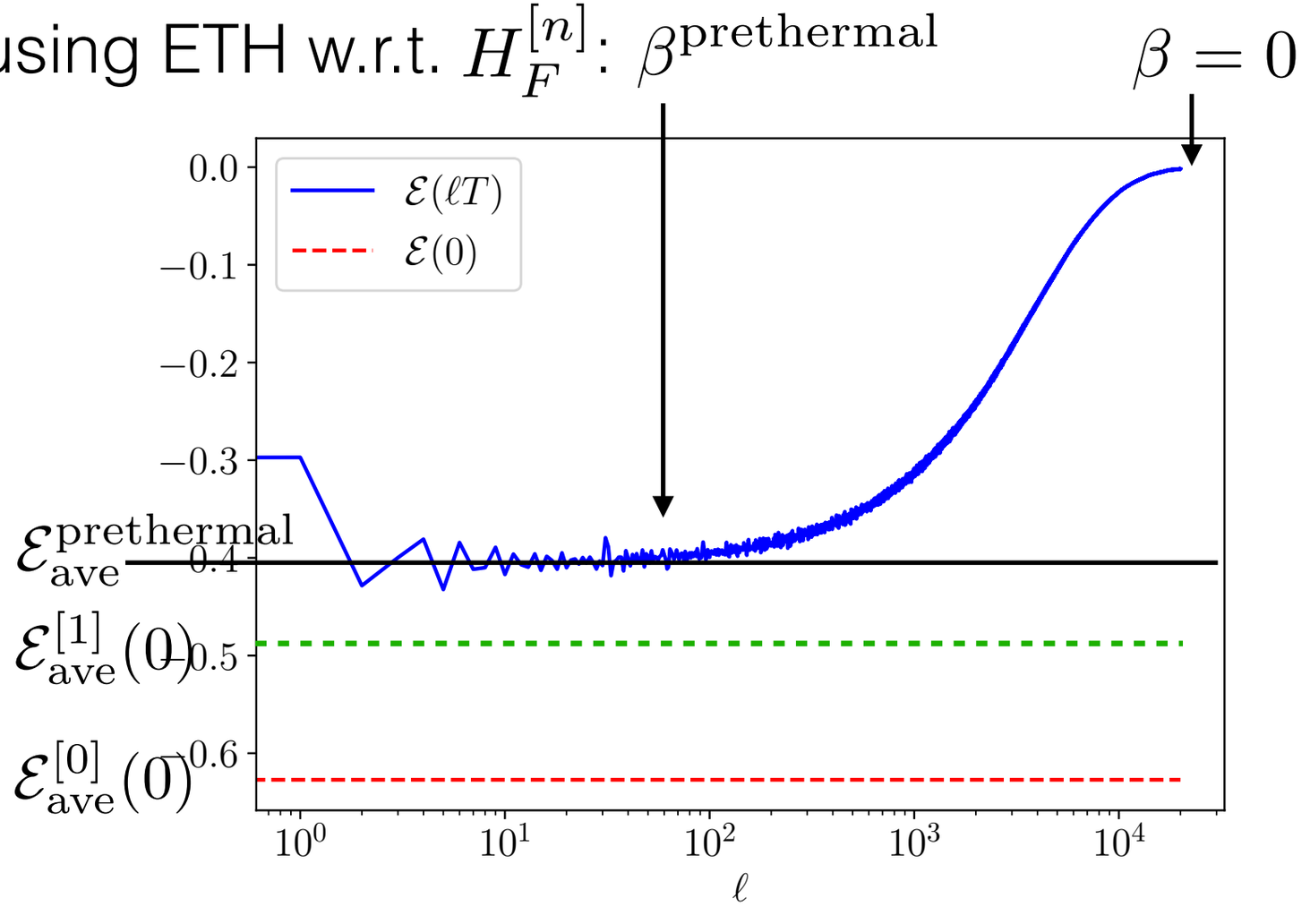

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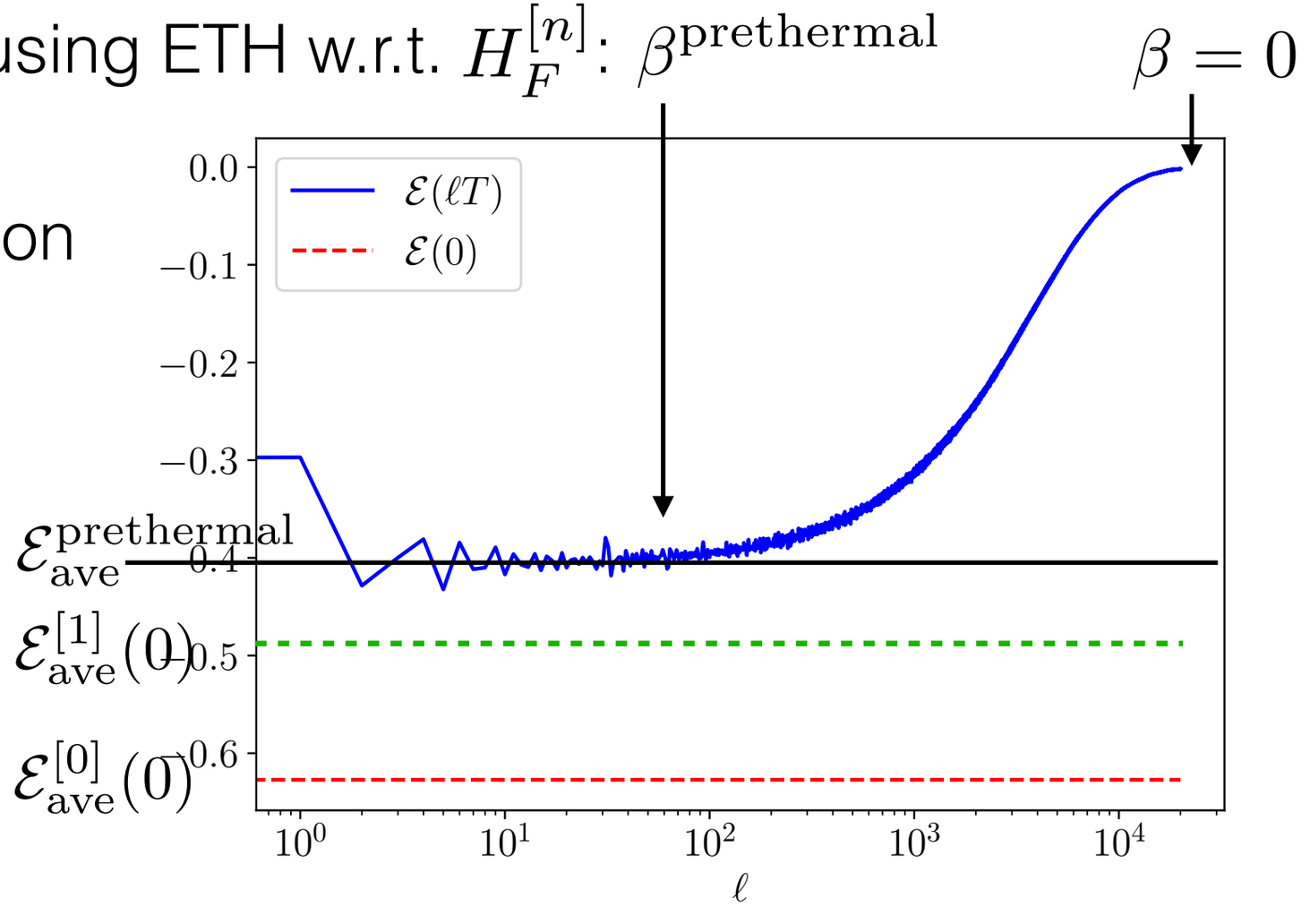
→ four stages of thermalization (high-frequency driving)



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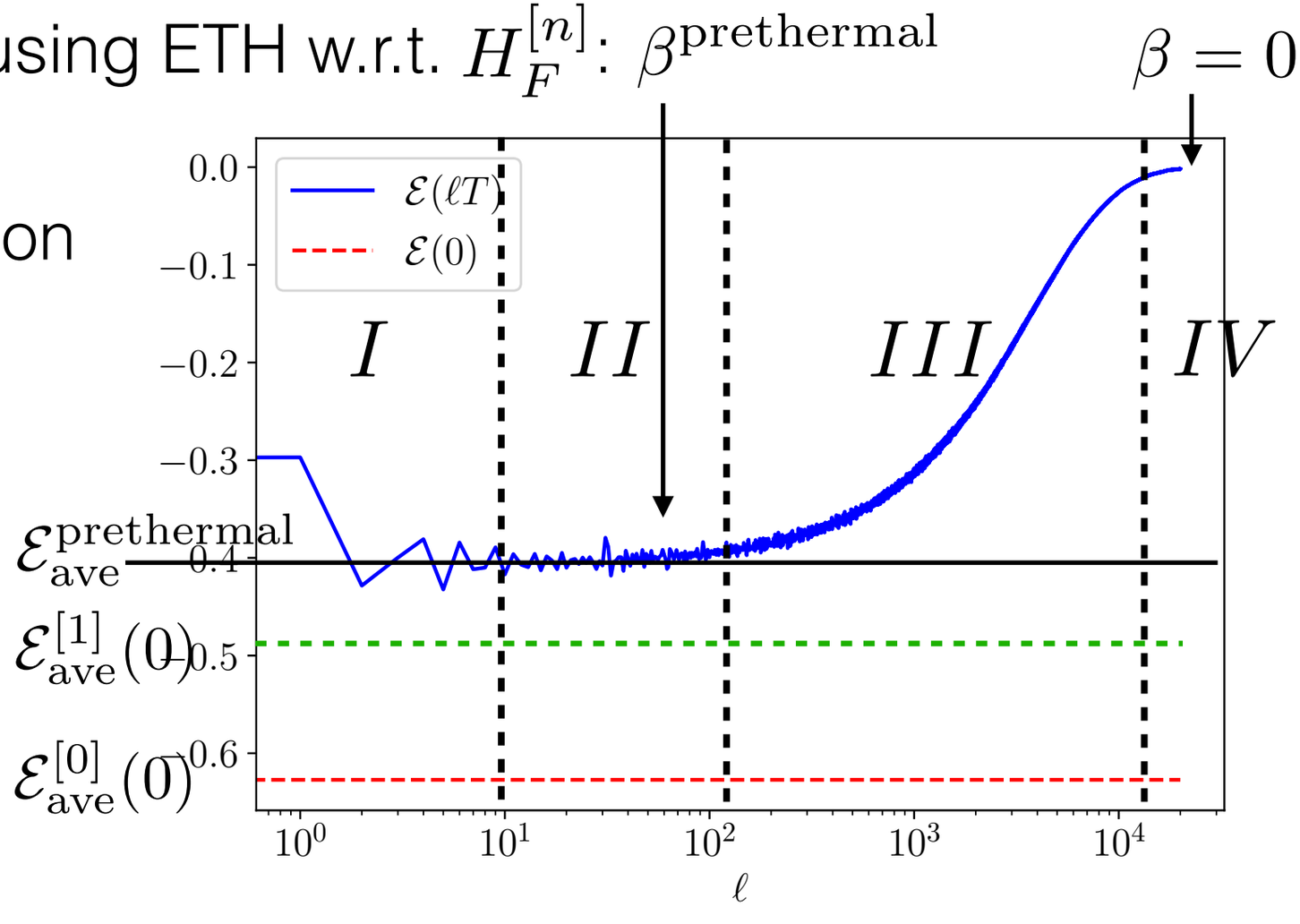

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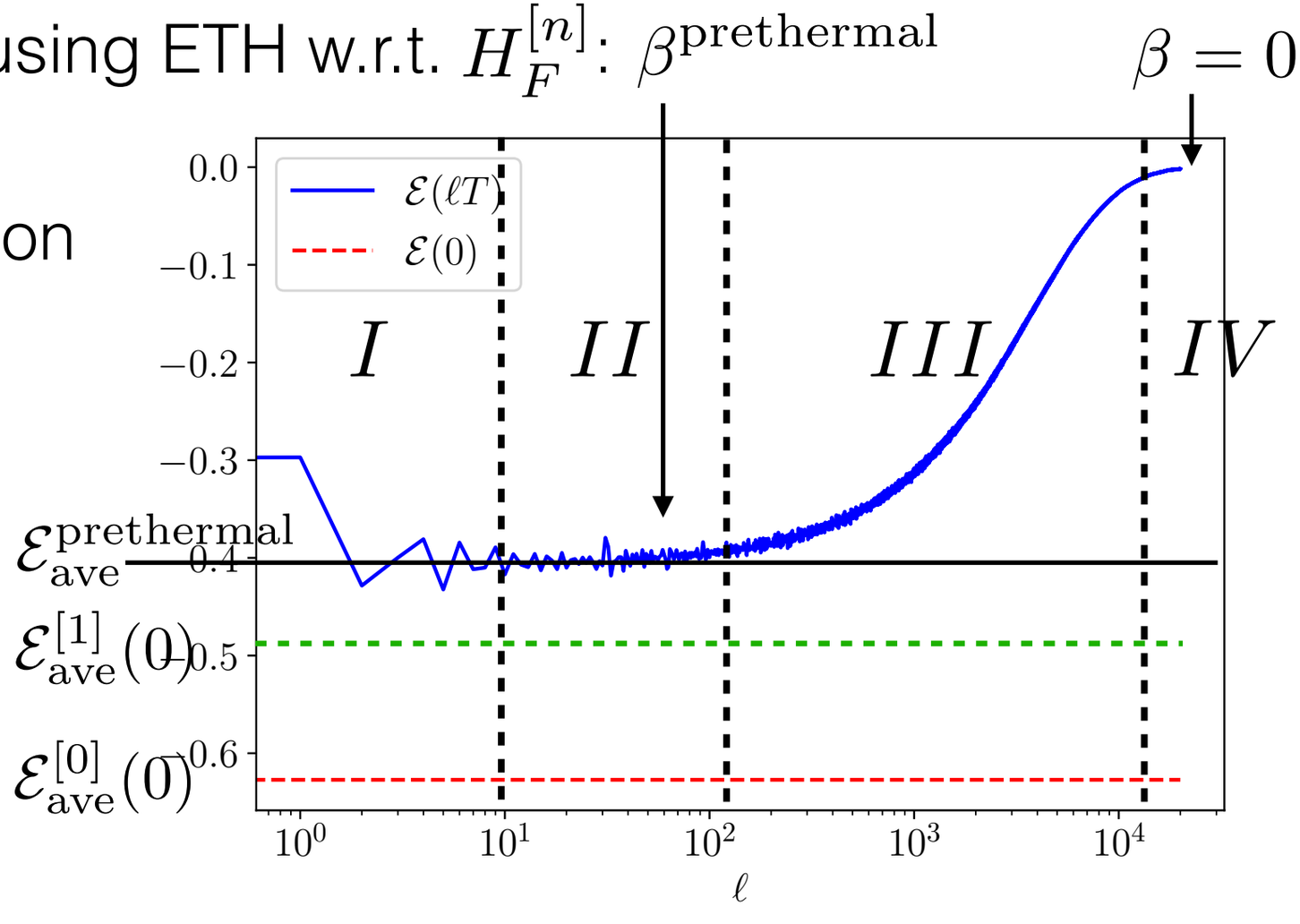
→ associate a temperature using ETH w.r.t. $H_F^{[n]}$: $\beta^{\rm prethermal}$ $\beta = 0$

→ four stages of thermalization (high-frequency driving)

I+II: quench to $H_F^{[n]}$

I+…+IV: quench to H_F



Understanding the pre-thermal plateau

$$\mathcal{E}_{\rm ave}(\ell T) = L^{-1}\langle\psi_0|(U_F^\dagger)^\ell H_{\rm ave}(U_F)^\ell|\psi_0\rangle$$
$$\approx L^{-1}\langle\psi_0|(U_F^{[n]\dagger})^\ell H_{\rm ave}(U_F^{[n]})^\ell|\psi_0\rangle$$

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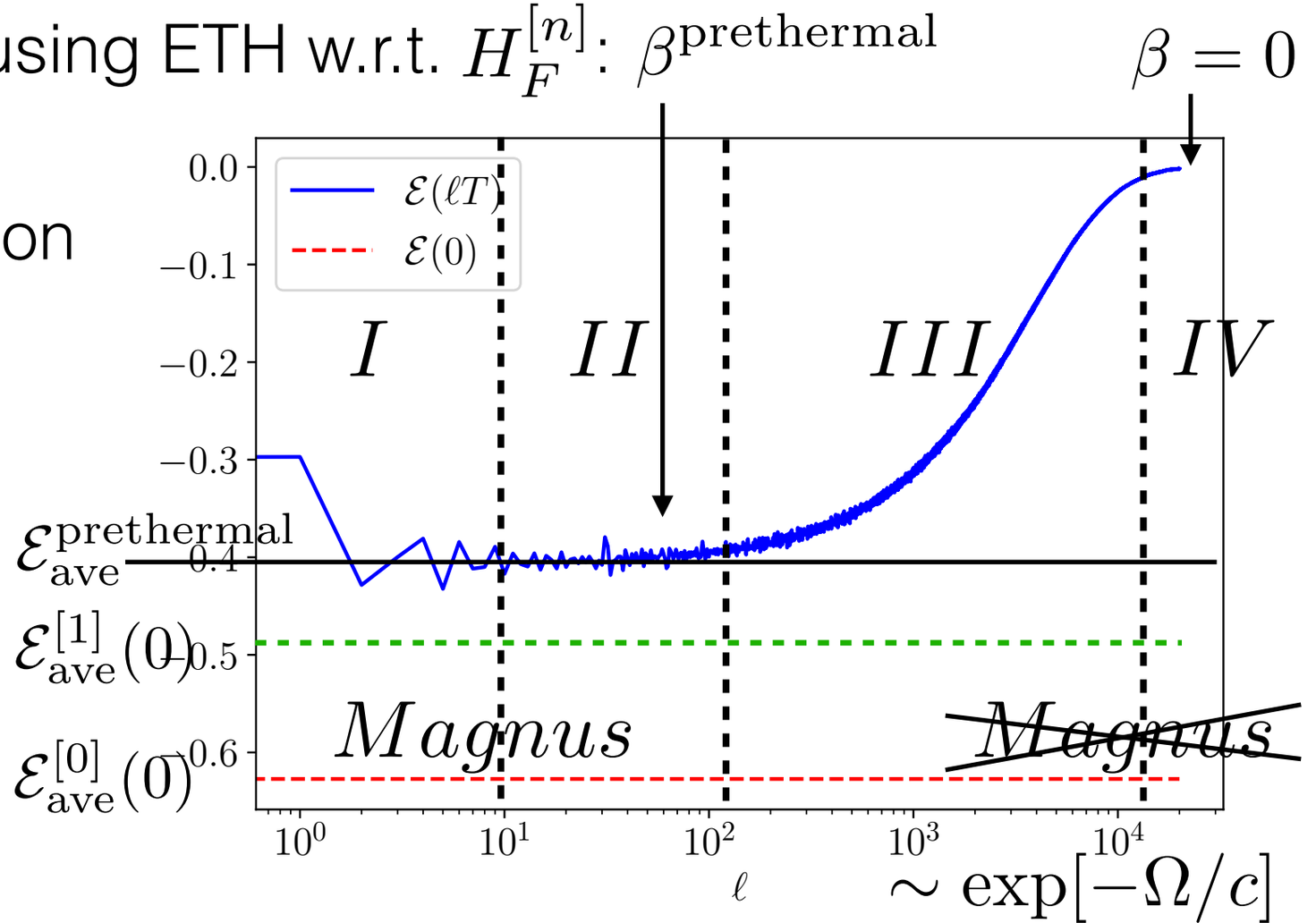
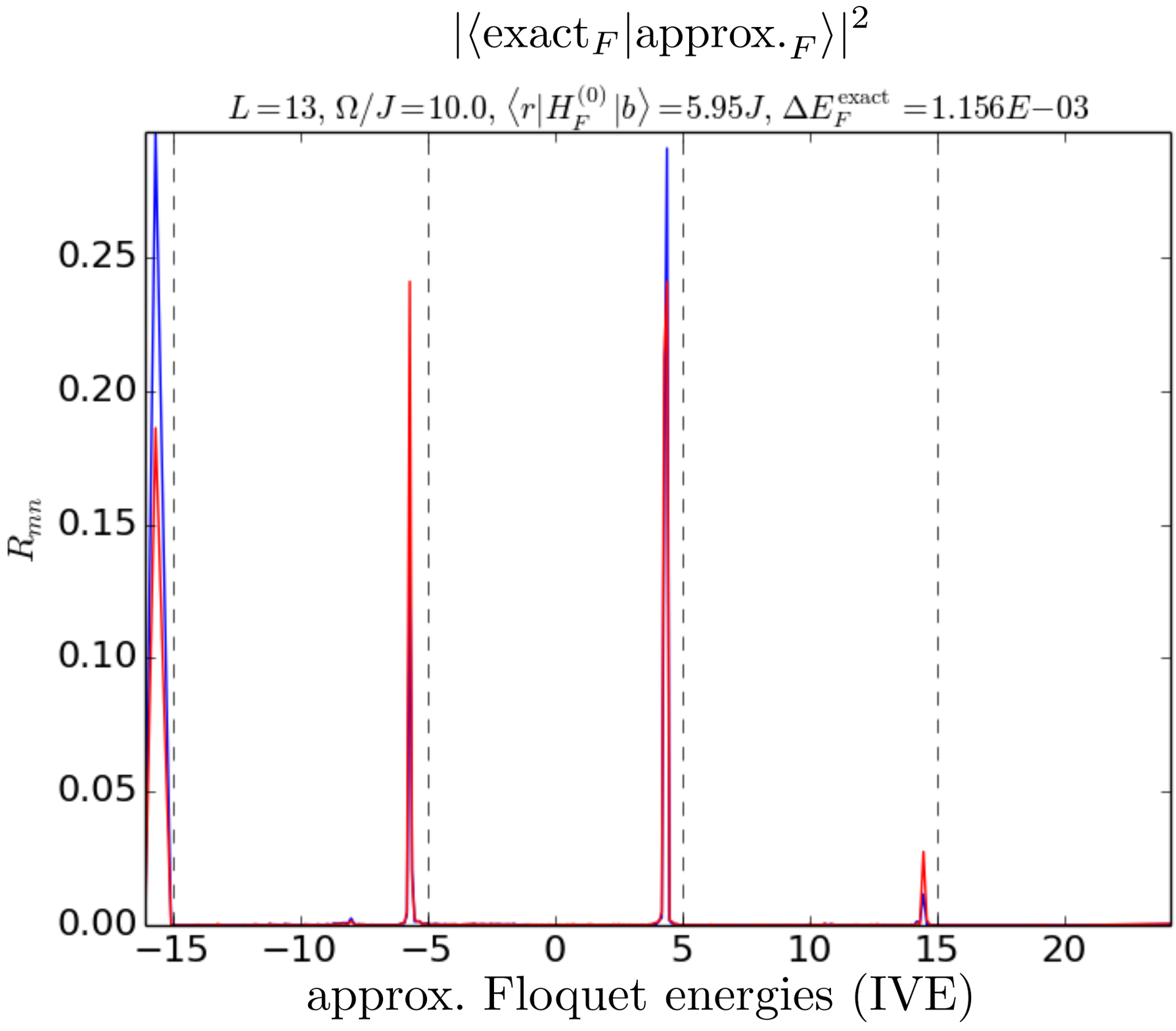


Stage III: the onset of heating: many-body resonances (**not** captured by inverse-frequency expansion)



M.B., Heyl, Huse, Polkovnikov, PRB 93, 155132 (2016)

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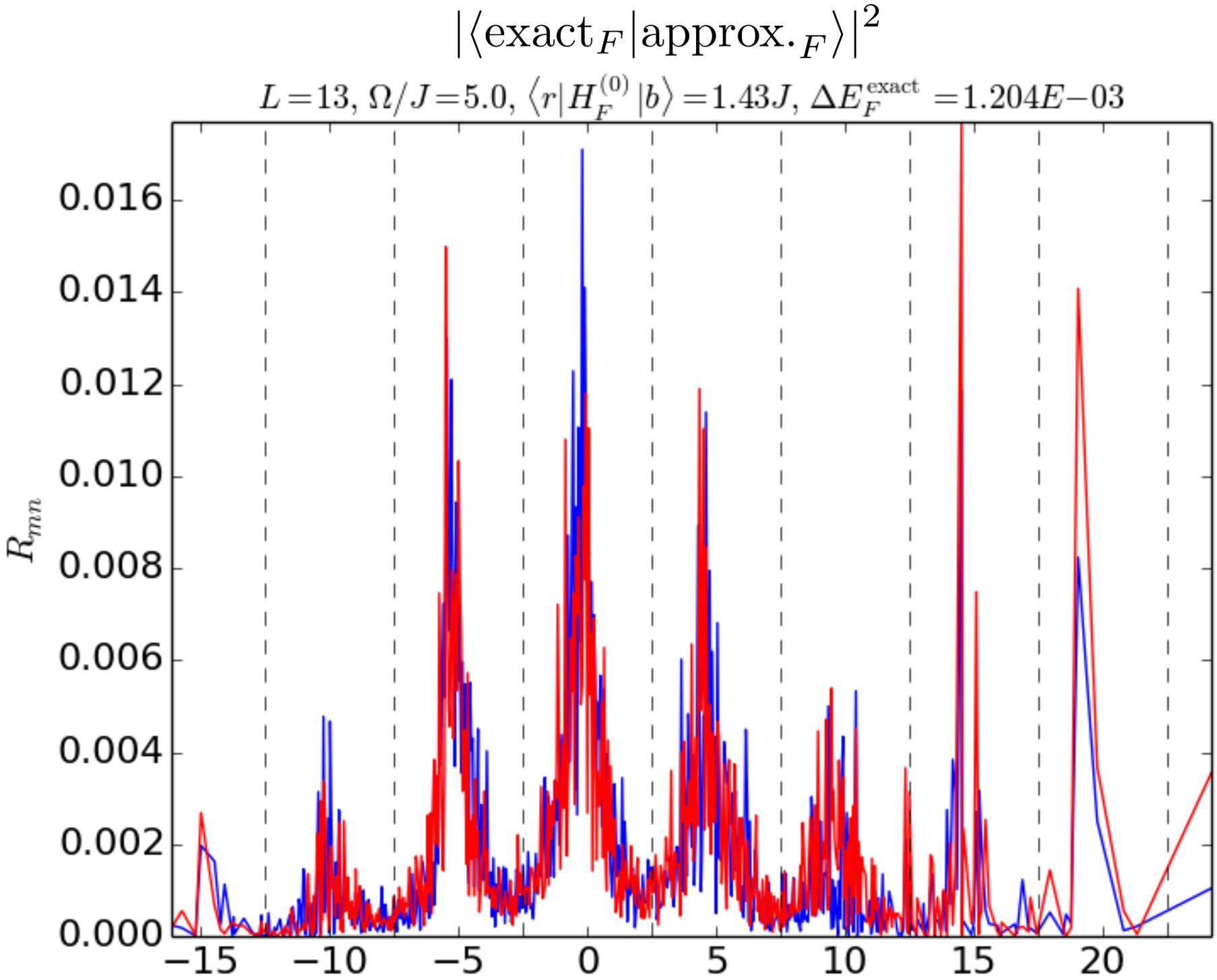

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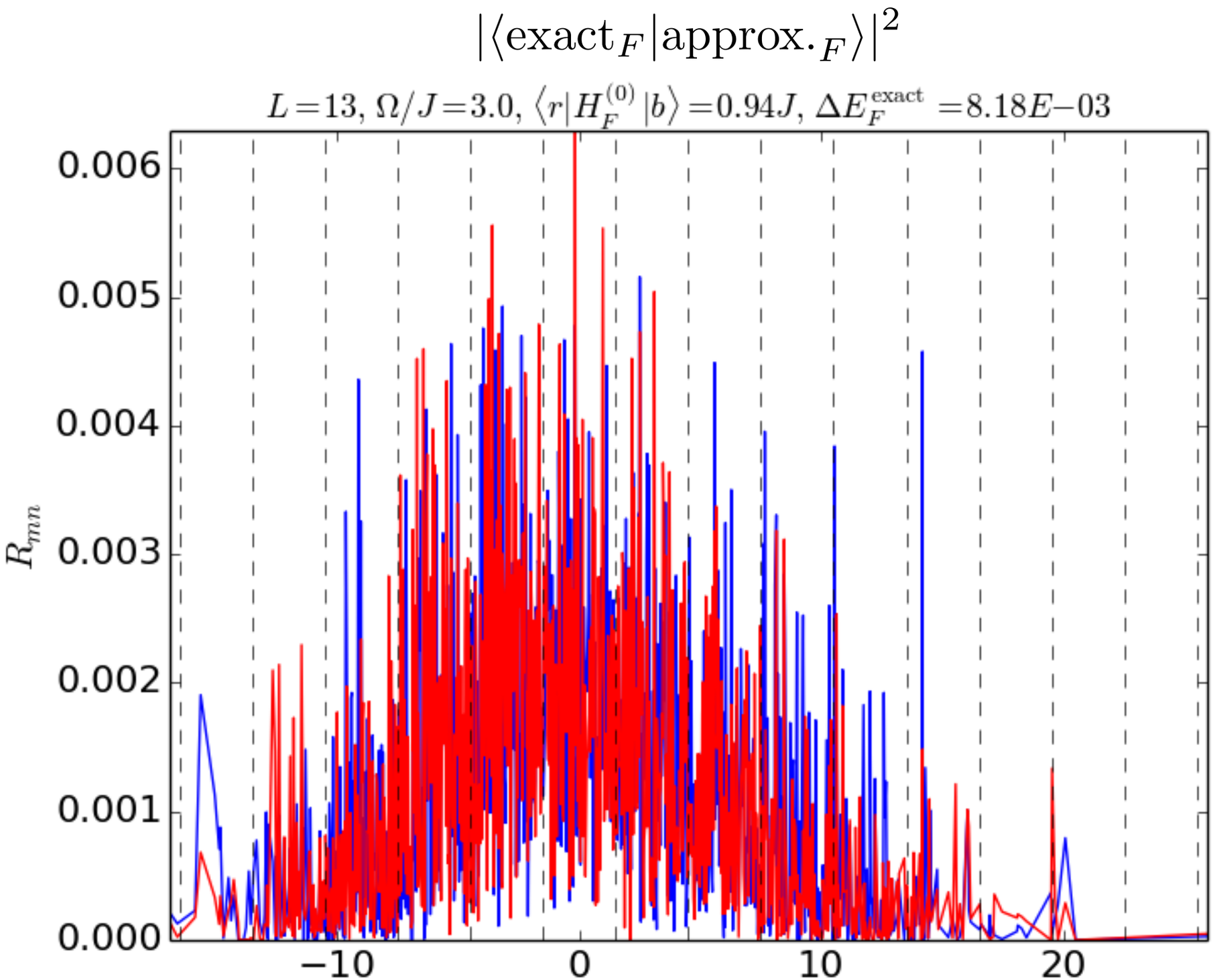

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Finite-size effects limit energy absorption

- only 100% safe method to study thermalization is (still) ED

but see also **Ye et al., arXiv:1902.01859**

- many-body bandwidth scales with system size L

- finite system size limits the number of resonance peaks

- discrete spectrum: finite-size gaps close to GS

- appropriate order of limits: $\lim_{\Omega\to\infty} \lim_{L\to\infty}$ hard to achieve

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can we avoid these issues?

which feature(s) can we sacrifice?

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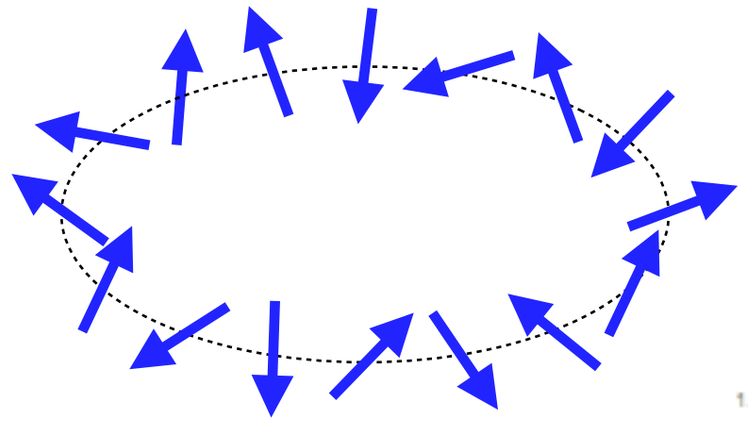
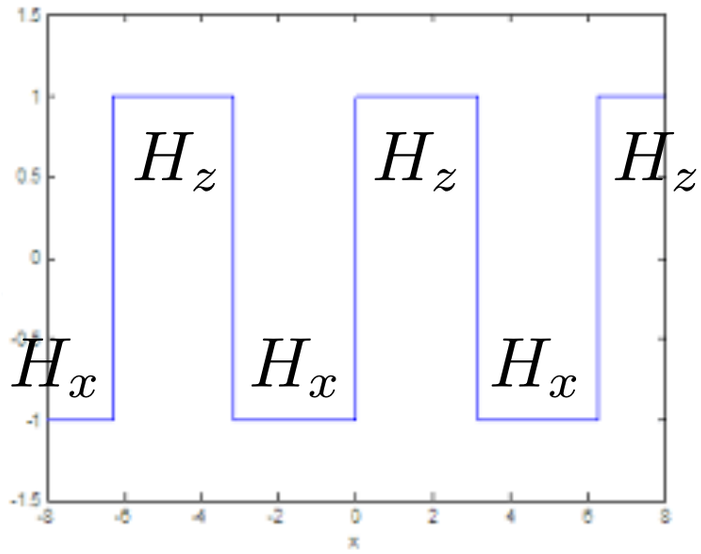
Classical Floquet systems

Owen Howell

- driven coupled rotors

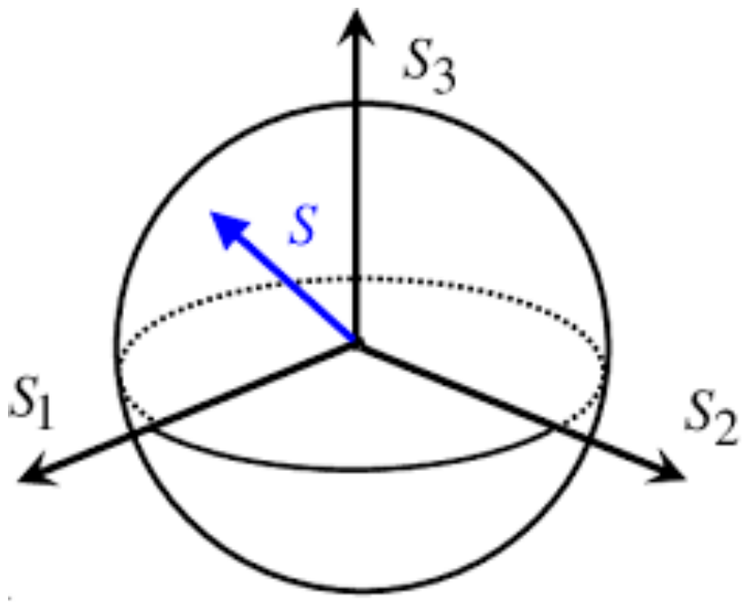


$$H(t) = \begin{cases} \sum_{j=1}^{L} J S_j^z S_{j+1}^z + h S_j^z & \text{for } t \in [0, T/2] \bmod T \\ \sum_{j=1}^{L} g S_j^x & \text{for } t \in [T/2, T] \bmod T \end{cases}$$



$$\{S_i^\alpha, S_j^\beta\} = \delta_{ij} \varepsilon^{\alpha\beta\gamma} S_j^\gamma$$

$$|\vec{S}| = 1$$

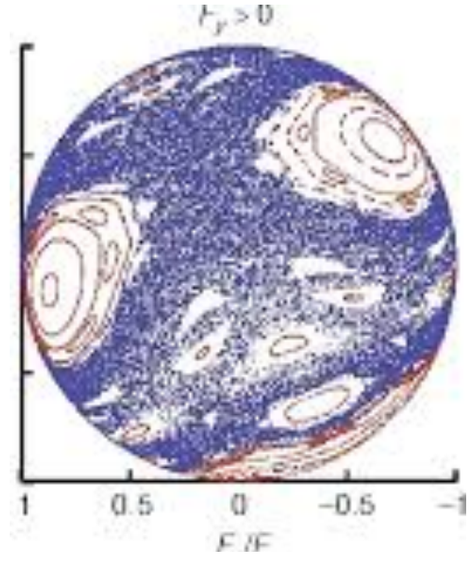


O. Howell, Weinberg, Sels, Polkovnikov, and M.B., PRL 122, 010602 (2019)

A crucial feature of the step drive

- want *exponentially* long times in a **chaotic** system!

$$H(t) = \begin{cases} \sum_{j=1}^{L} JS_j^z S_{j+1}^z + hS_j^z & \text{for } t \in [0, T/2] \bmod T \\ \sum_{j=1}^{L} gS_j^x & \text{for } t \in [T/2, T] \bmod T \end{cases}$$



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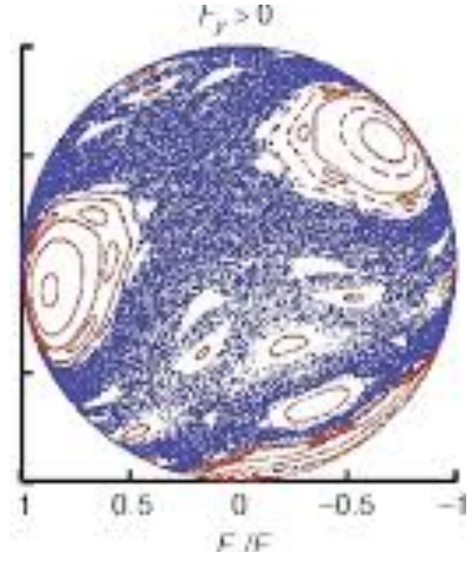

➡ solve Hamilton's EOM **exactly**: $\vec{S}_j(\ell T) = [\tau_2 \circ \tau_1]^{\ell} (\vec{S}_j(0))$

$$\tau_1(\vec{S}_j) = \begin{bmatrix} S_j^x \cos(\kappa_j T/2) - S_j^y \sin(\kappa_j T/2) \\ S_j^x \sin(\kappa_j T/2) + S_j^y \cos(\kappa_j T/2) \\ S_j^z \end{bmatrix} \qquad \tau_2(\vec{S}_j) = \begin{bmatrix} S_j^x \\ S_j^y \cos(gT/2) - S_j^y \sin(gT/2) \\ S_j^y \sin(gT/2) + S_j^z \cos(gT/2) \end{bmatrix}$$

$$\kappa_j = J(S_{j-1}^z + S_{j+1}^z) + h$$

O. Howell, Weinberg, Sels, Polkovnikov, and M.B., PRL 122, 010602 (2019)

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Ensemble pre-thermalization

- time-averaged energy

$$H_{\text{ave}} = \frac{1}{2}\sum_{j=1}^{N} JS_j^z S_{j+1}^z + hS_j^z + gS_j^x$$

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- measure energy absorption and its fluctuations

$$\langle Q(\ell T)\rangle = \frac{\langle H_{\rm ave}[\{\vec{S}_j(\ell T)\}]\rangle - E_{\rm GS}}{\langle H_{\rm ave}\rangle_{\beta=0} - E_{\rm GS}} \in [0,1],$$

$$\langle \delta Q(\ell T)\rangle = \sqrt{\frac{\langle H_{\rm ave}^2[\{\vec{S}_j(\ell T)\}]\rangle - \langle H_{\rm ave}[\{\vec{S}_j(\ell T)\}]\rangle^2}{\langle H_{\rm ave}^2\rangle_{\beta=0} - \langle H_{\rm ave}\rangle_{\beta=0}^2}}$$

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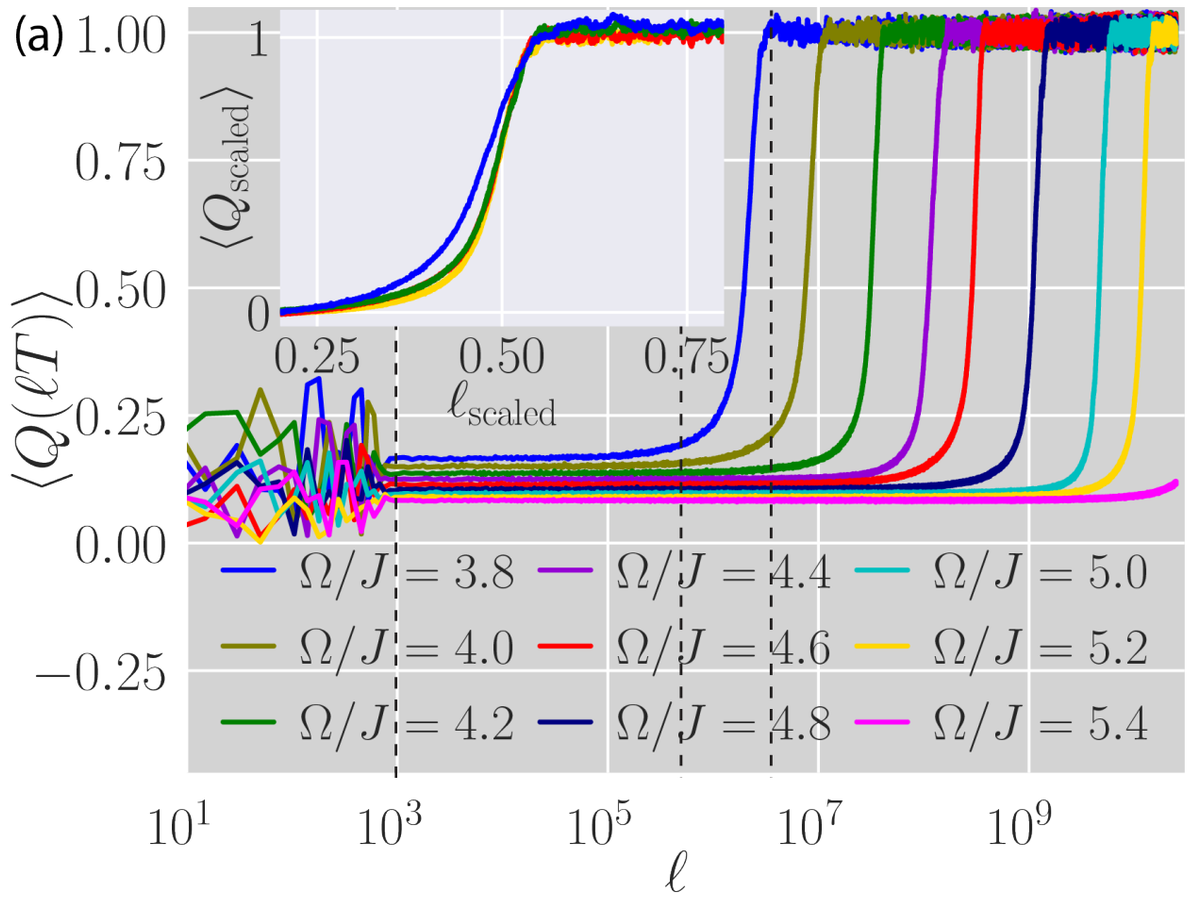

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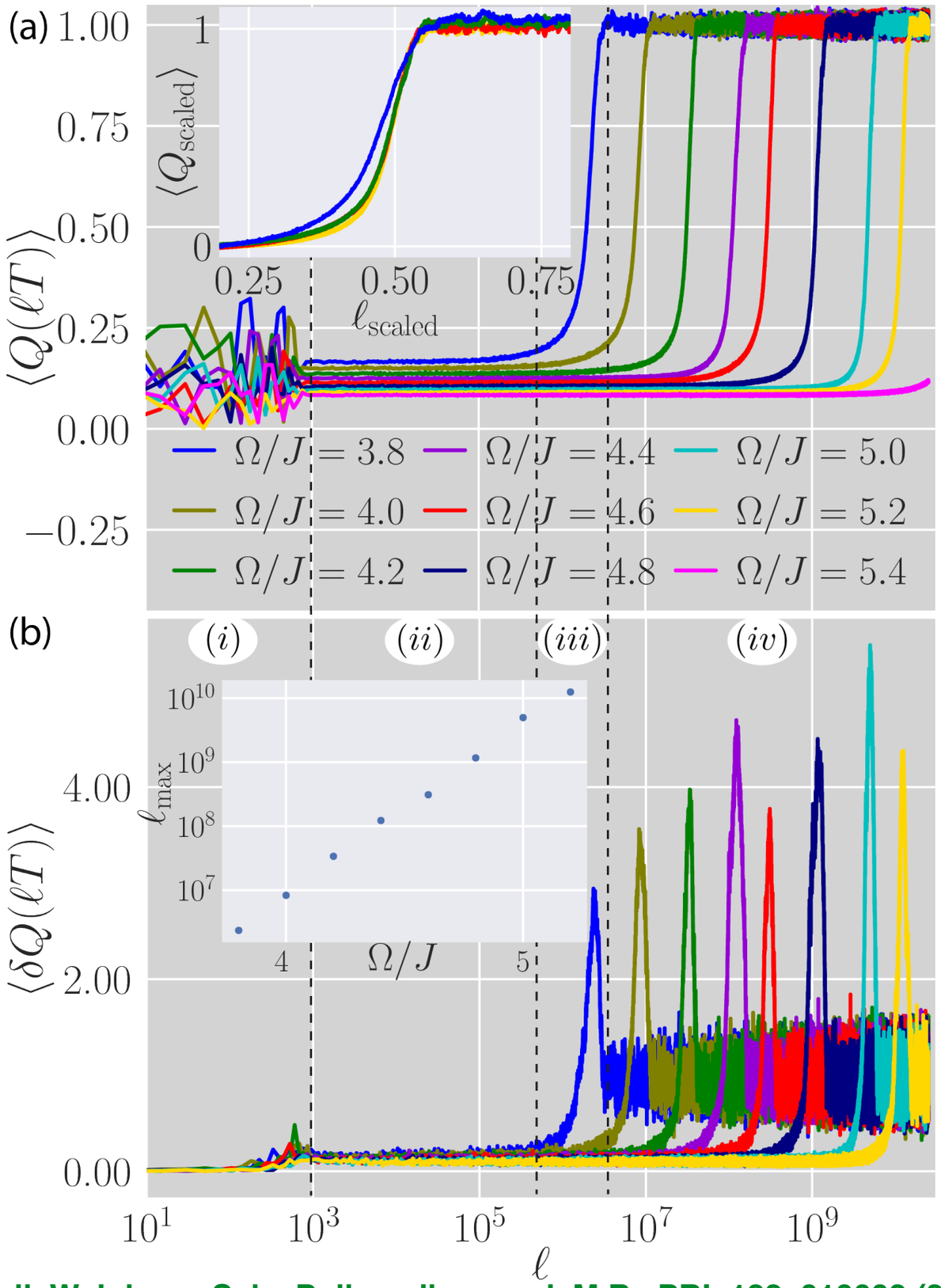

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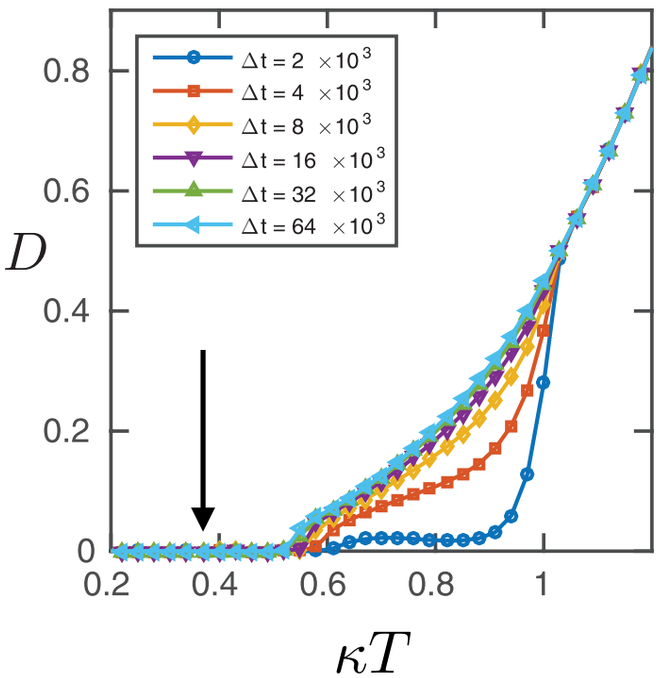
Related phenomenon: subdiffusion

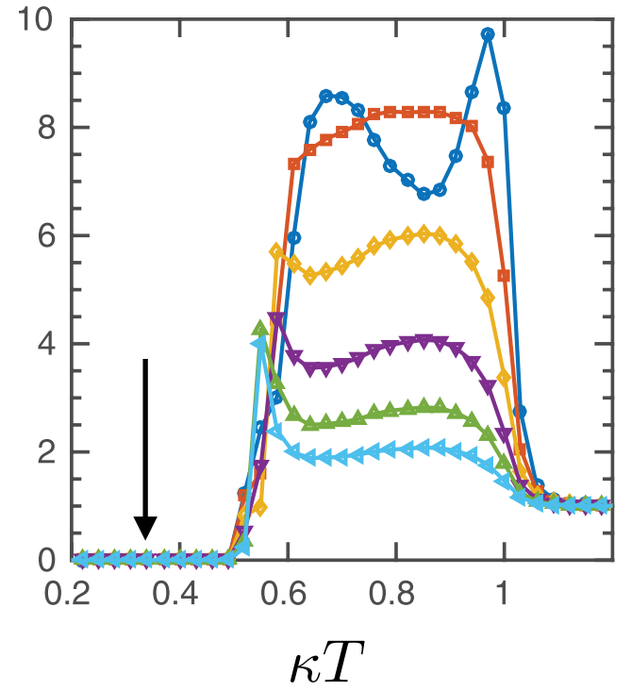
- coupled kicked pendula:

$$H(t) = \sum_{j=1}^{L} \left[\frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{m=-\infty}^{+\infty} \delta(t - mT) \right]$$

$$D = \frac{\langle p^2(t_f) \rangle - \langle p^2(t_i) \rangle}{t_f - t_i} \propto t^{\alpha - 1}$$

diffusion coefficient





Notarnicola, et al, PRE 97, 022202 (2018)

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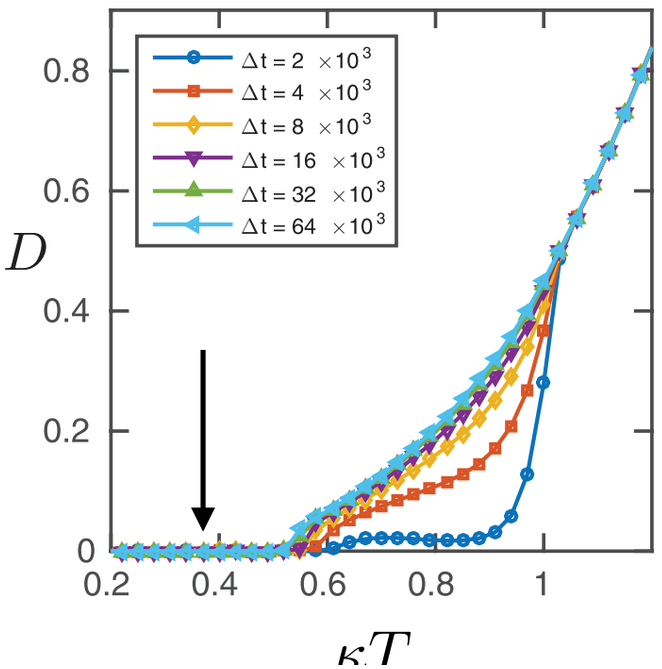

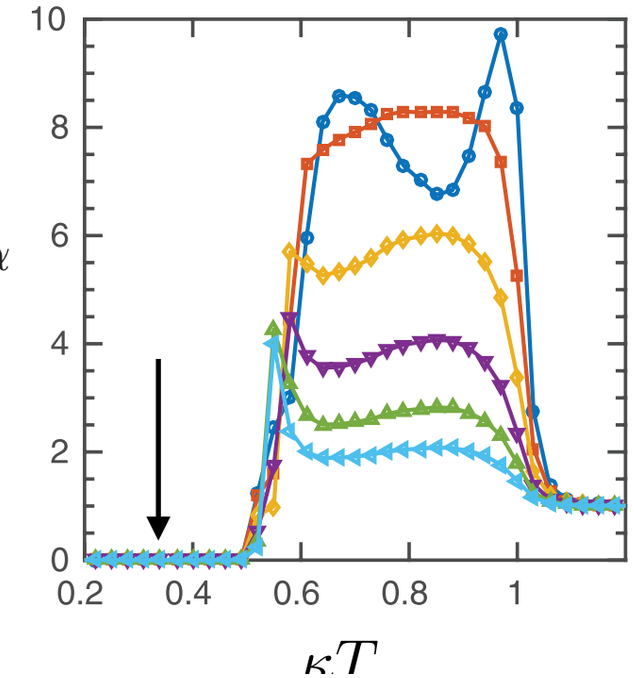

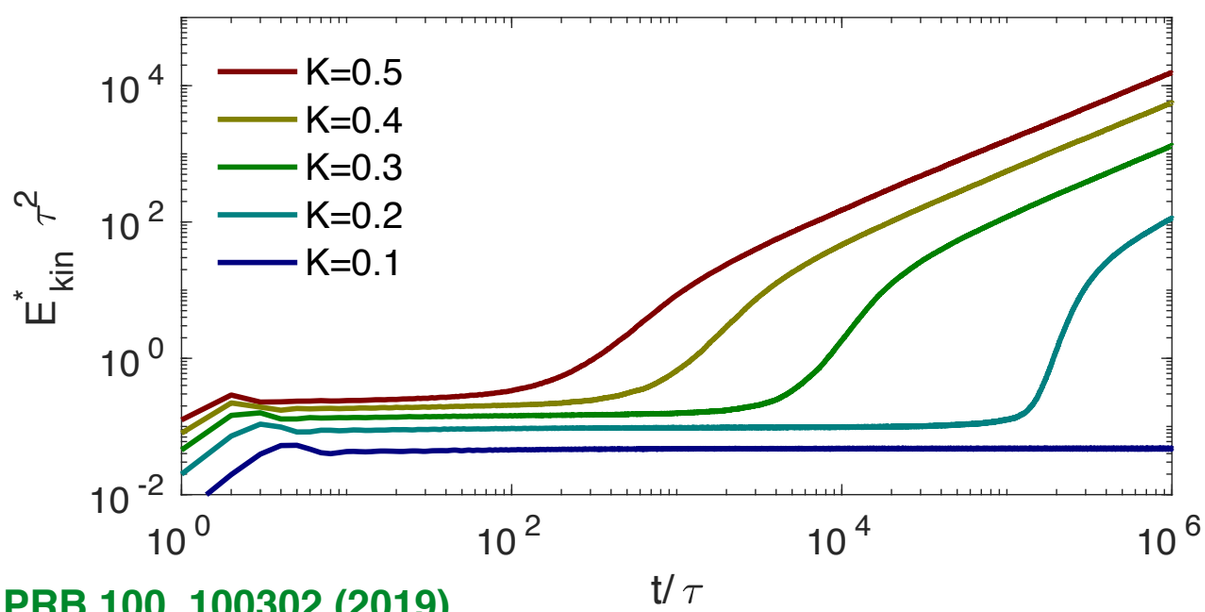

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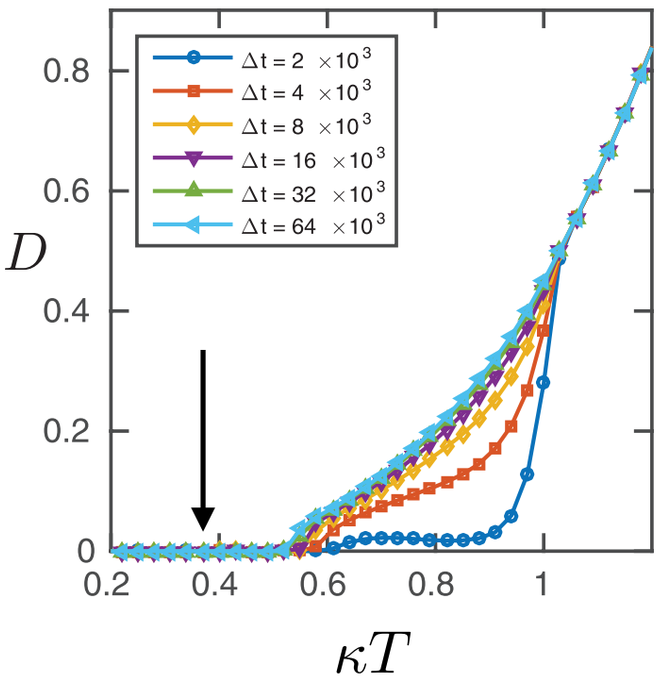
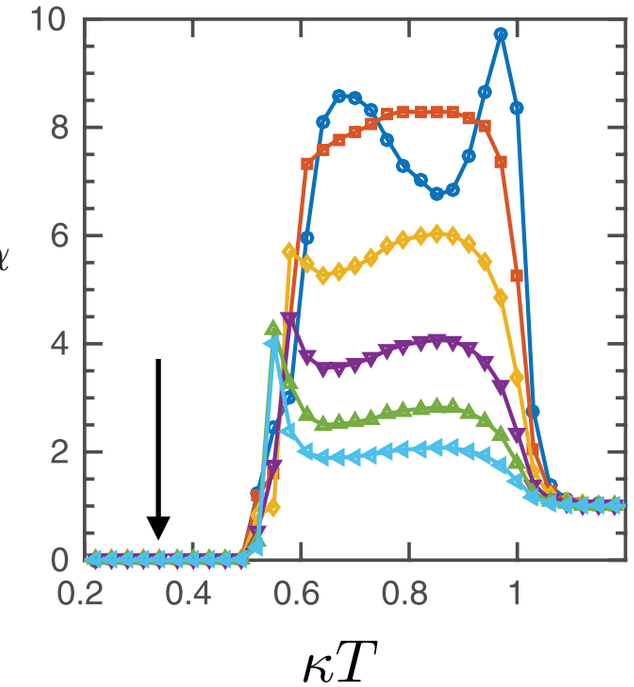
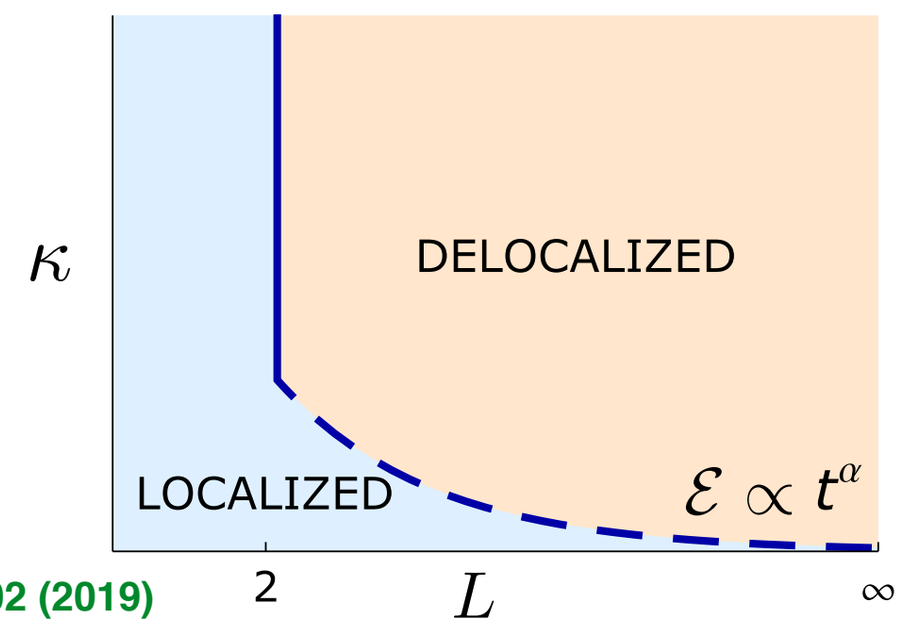




- schematic stability diagram



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Can we use the inverse-frequency expansion?

- apply classical Magnus expansion: $-i[\cdot,\cdot] \longrightarrow \{\cdot,\cdot\}$

$$H_F^{(0)} = T^{-1}\int_0^T \mathrm{d}t H(t)$$

$$H_F^{(1)} \sim \int_0^T \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 [H(t_1), H(t_2)]$$

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O. Howell, Weinberg, Sels, Polkovnikov, and M.B., PRL 122, 010602 (2019)

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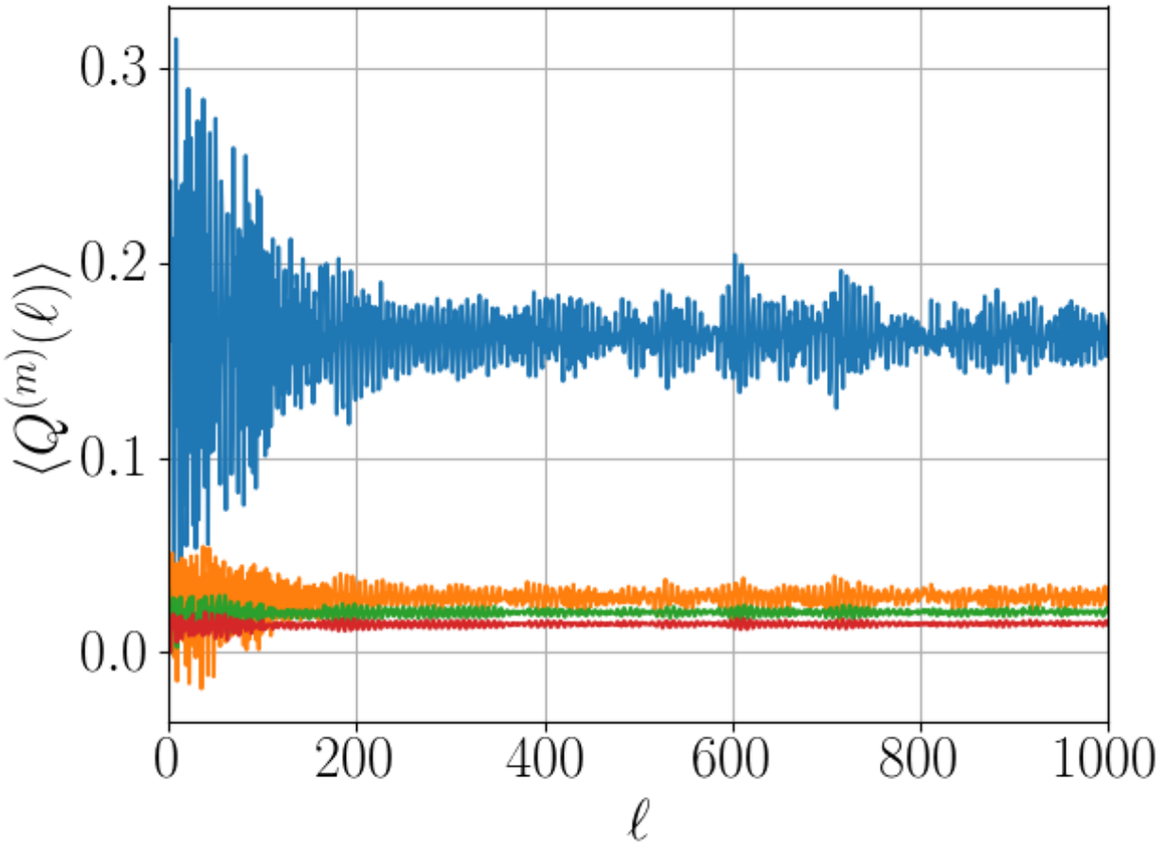

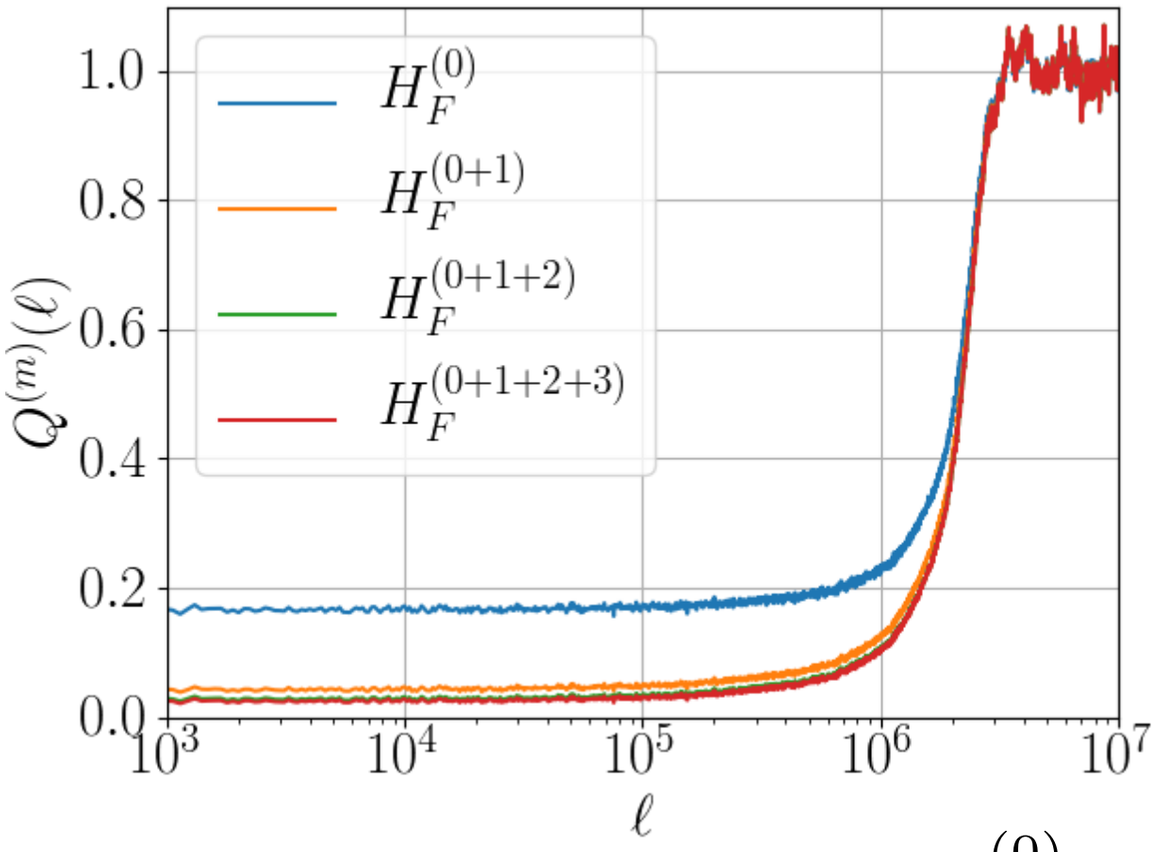

initial state : GS of $H_F^{(0)}$

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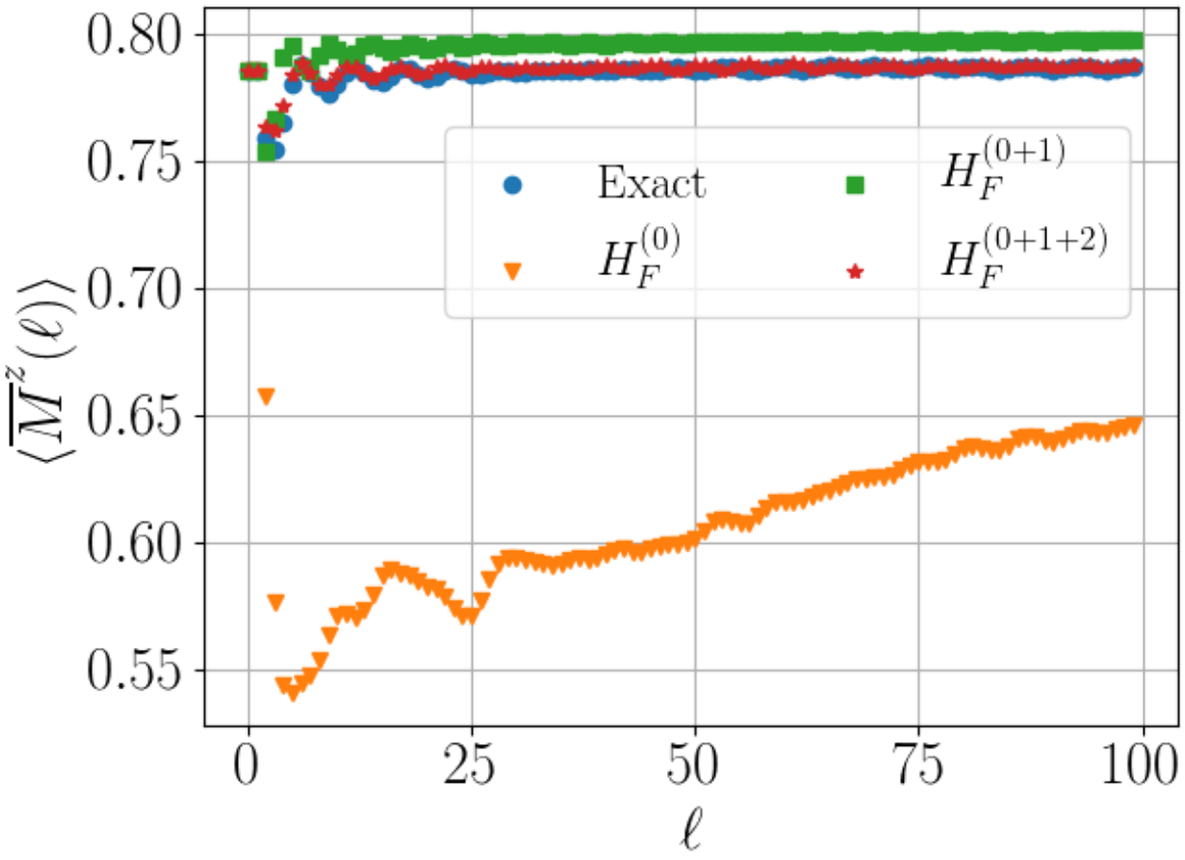
Ensemble-averaged observables captured by Magnus expansion

- time-averaged magnetization

$$M^{\alpha}(\ell T)=\frac{1}{N}\sum_{j=1}^{N}(-1)^{j}S_{j}^{\alpha}(\ell T);\ \overline{M^{\alpha}}=\frac{1}{N_T}\sum_{\ell=N_T}^{N_T+10^3}M^{\alpha}(\ell T)$$



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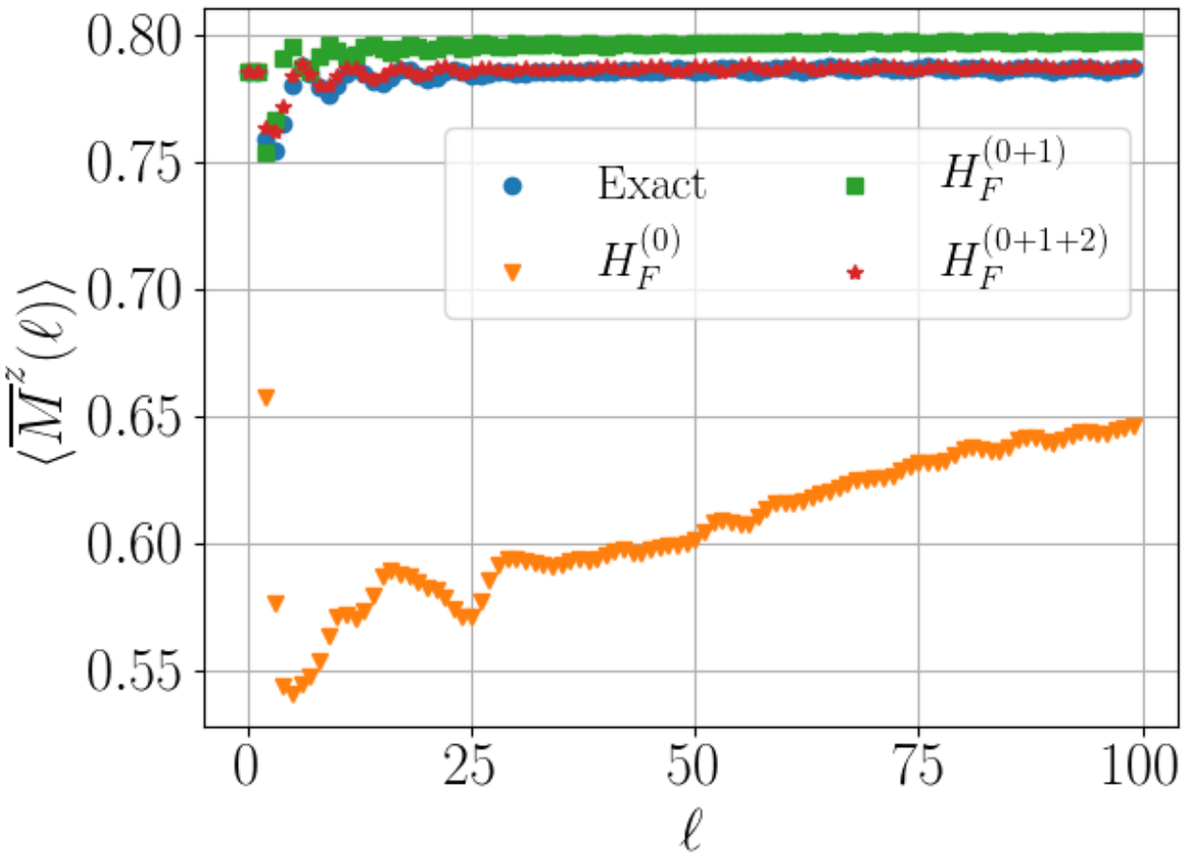

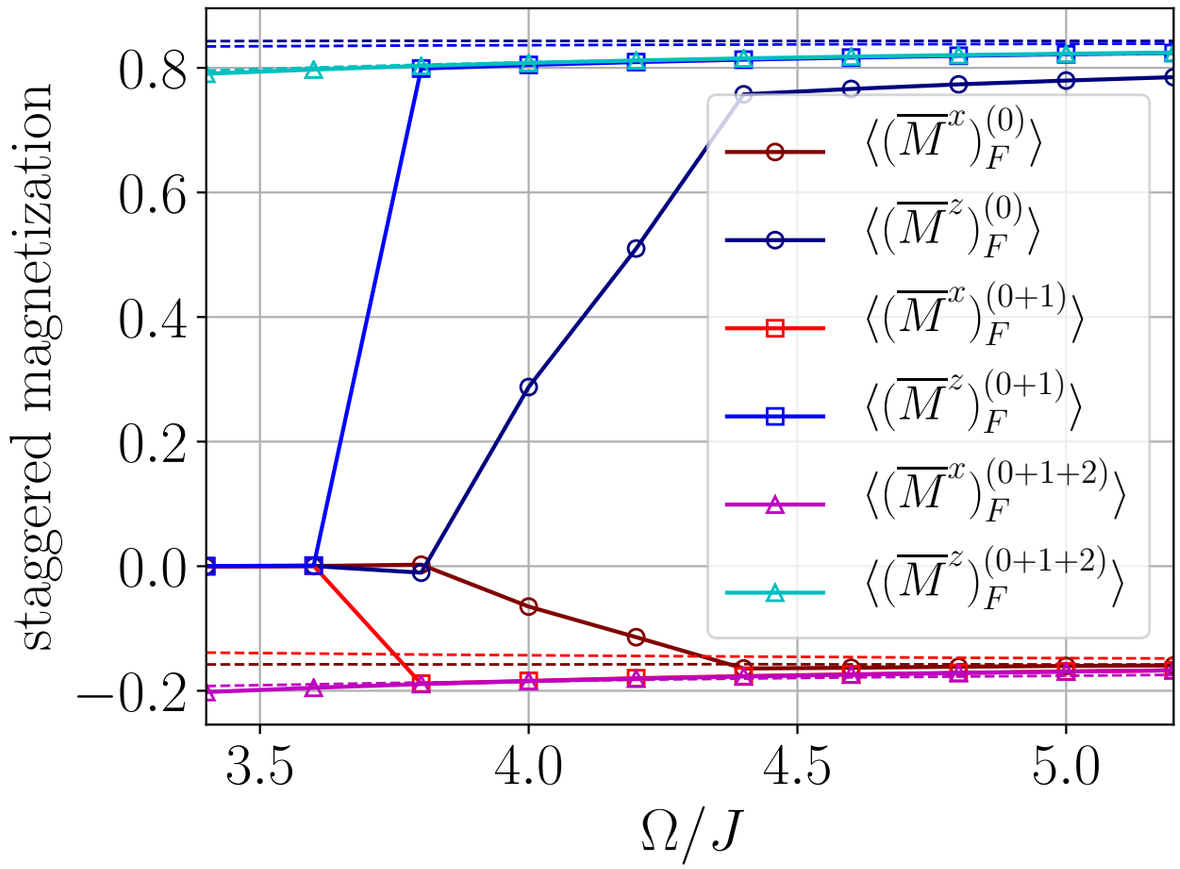

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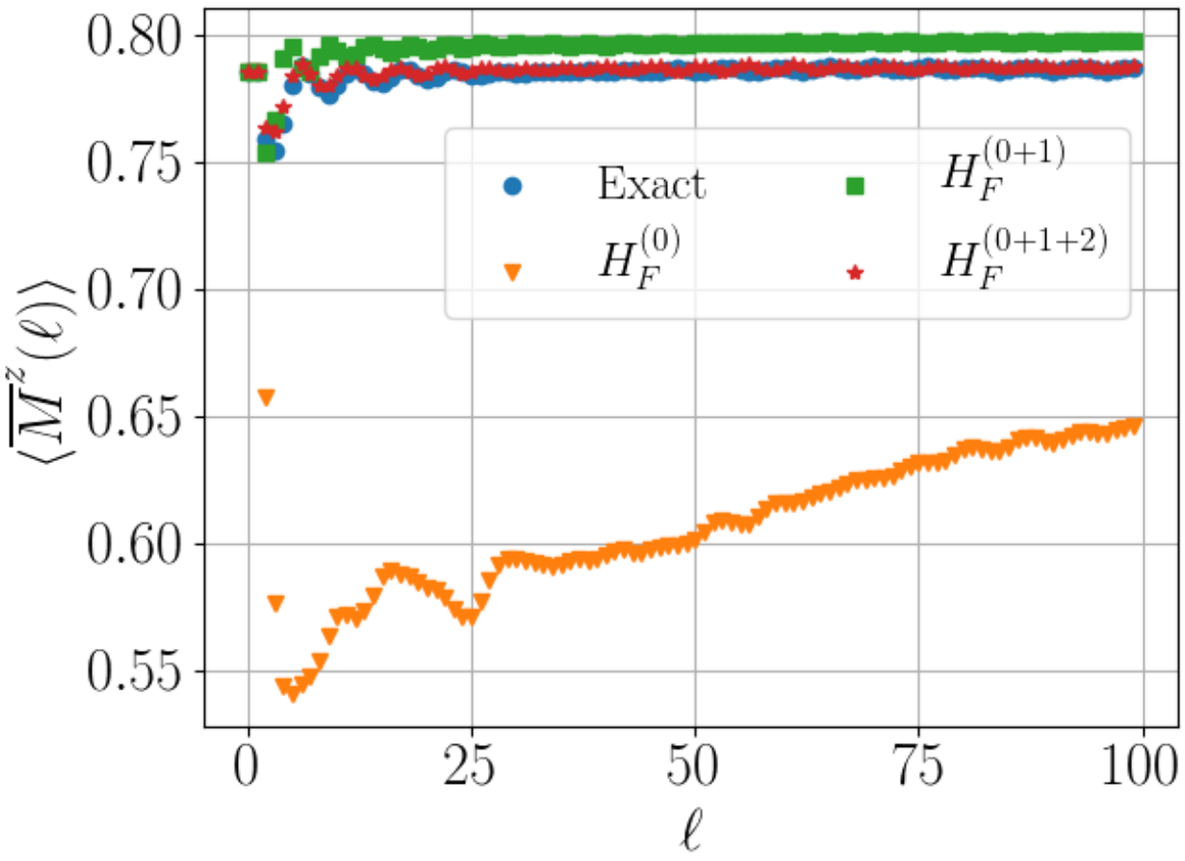
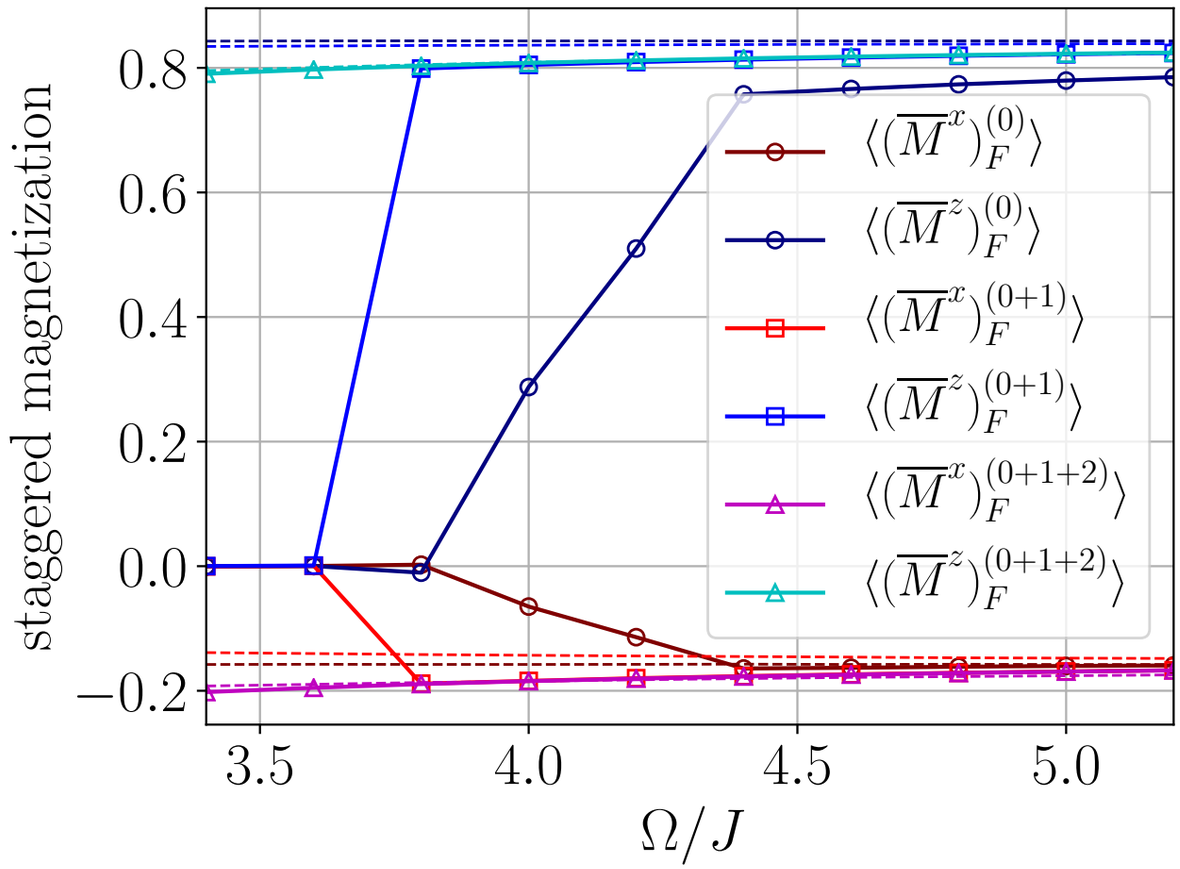




- classical Floquet engineering **Higashikawa, Fujita, Sato, arXiv:1810.01103 (2018)**

O. Howell, Weinberg, Sels, Polkovnikov, and M.B., PRL 122, 010602 (2019)

But why does Magnus work?

classical equations of motion are **nonlinear!**

Higashikawa, Fujita, Sato, arXiv:1810.01103 (2018)

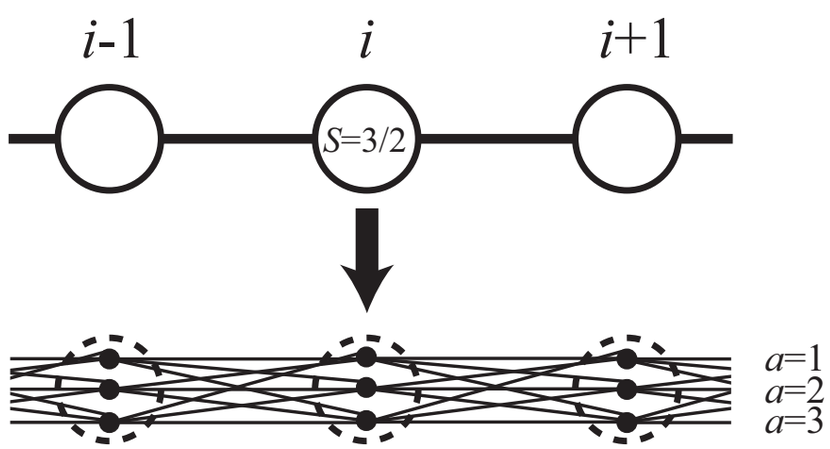
Mori, PRB 98, 104303 (2018)

But why does Magnus work?

- consider proof for quantum systems:

  holds for spin-1/2 particles

  take large-S limit and decompose into many spin-1/2's



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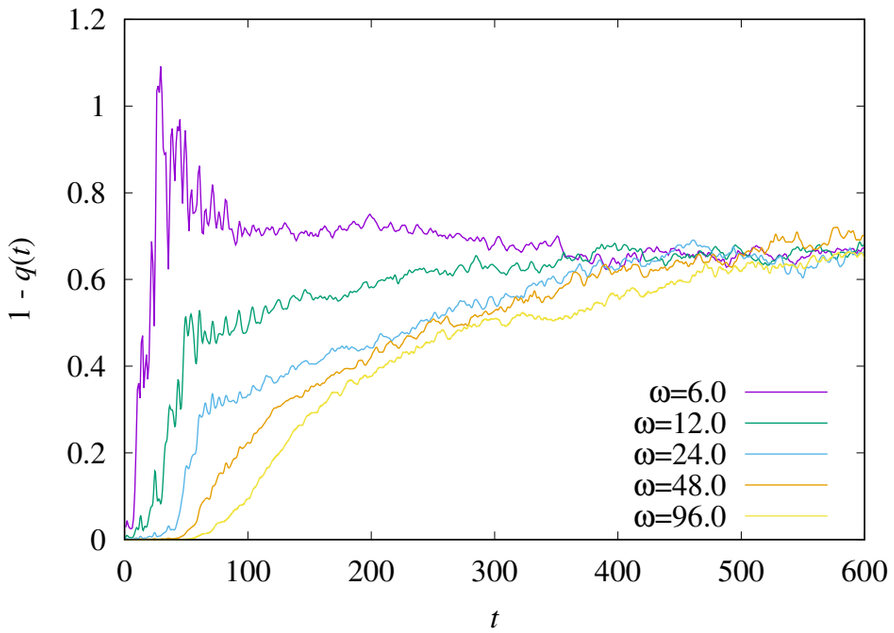
- important **caveat**:

*agreement not exponentially long for **single** trajectories due to chaos*

$$q(t) = \frac{1}{L}\sum_{j=1}^{L} \mathbf{S}_{F,j}^{\text{exact}}(t) \cdot \mathbf{S}_{F,j}^{[0]}(t)$$



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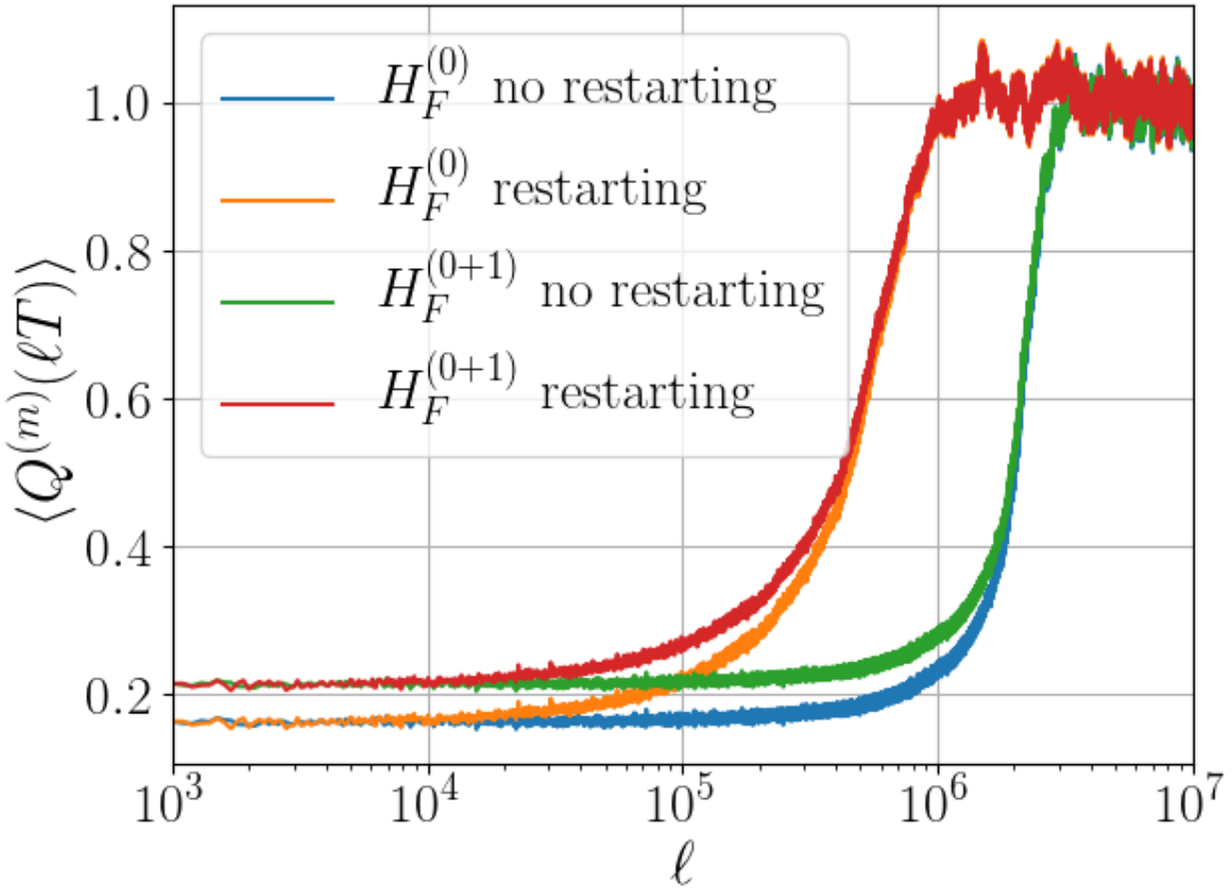
Is the prethermal plateau fully thermal?

- numerical experiment
  - evolve initial ensemble into prethermal phase
  - **restart ensemble:** measure energy mean and width of ensemble and draw new thermal (Gaussian) ensemble every 1000 cycles

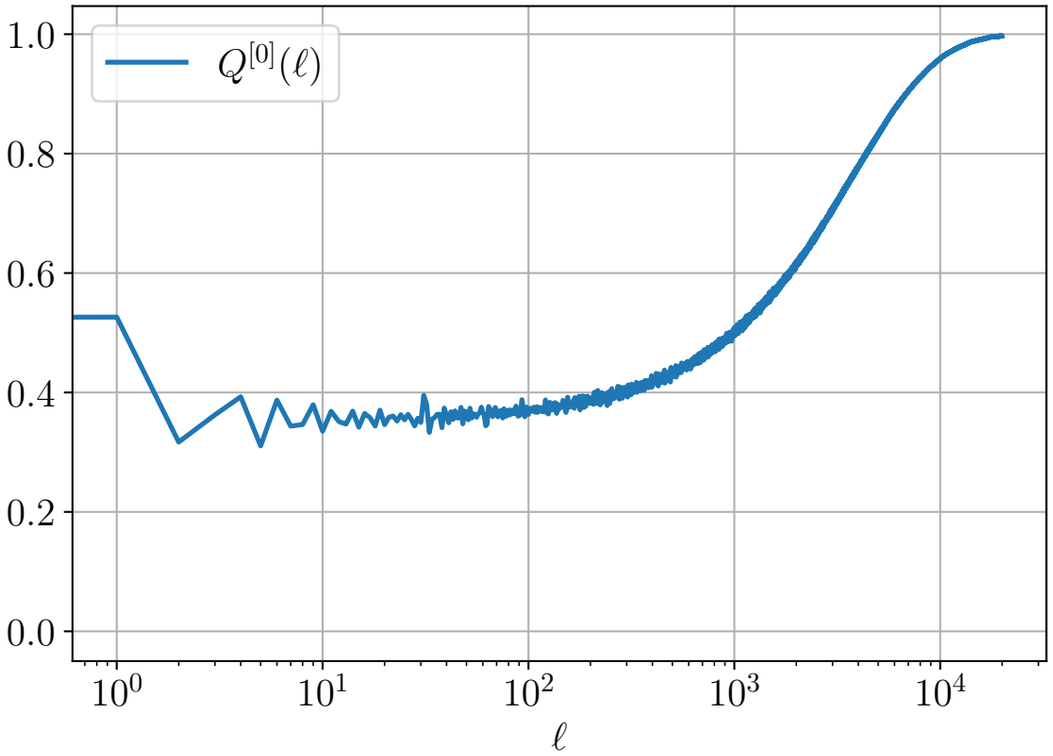


O. Howell, Weinberg, Sels, Polkovnikov, and M.B., PRL 122, 010602 (2019)

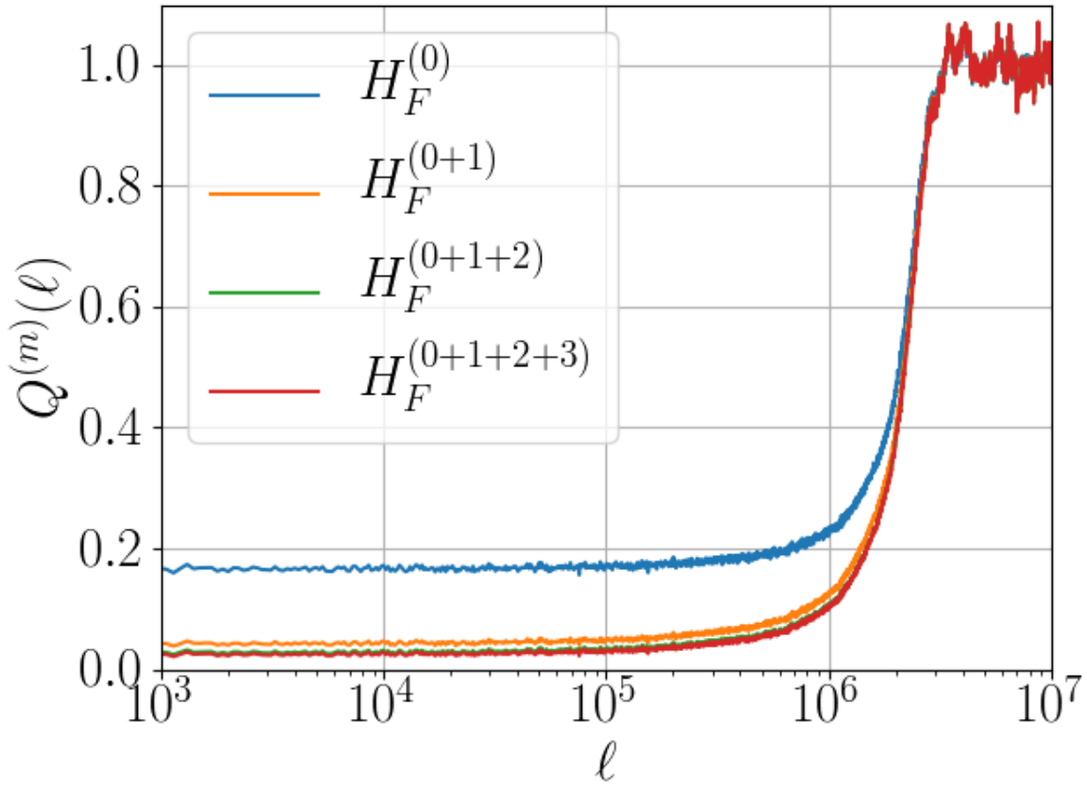
Outlook

- what is quantum about (Floquet) prethermalization?

quantum $L = 22$



classical $L = 100$



PRB 93, 155132 (2016), PRL 122, 010602 (2019)

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- cold atoms: also boson systems — so far no rigorous proof

PRB 93, 155132 (2016), PRL 122, 010602 (2019)

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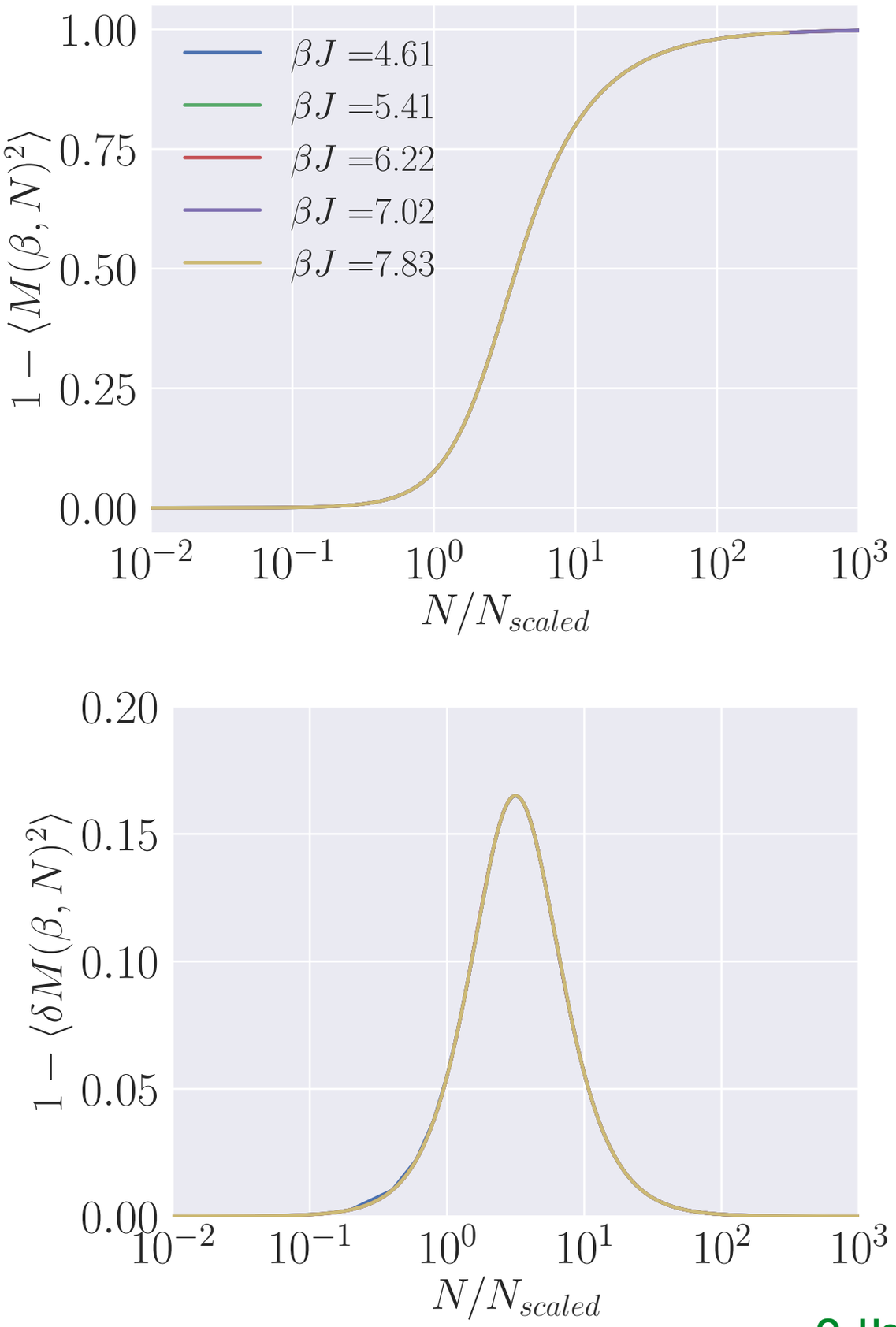
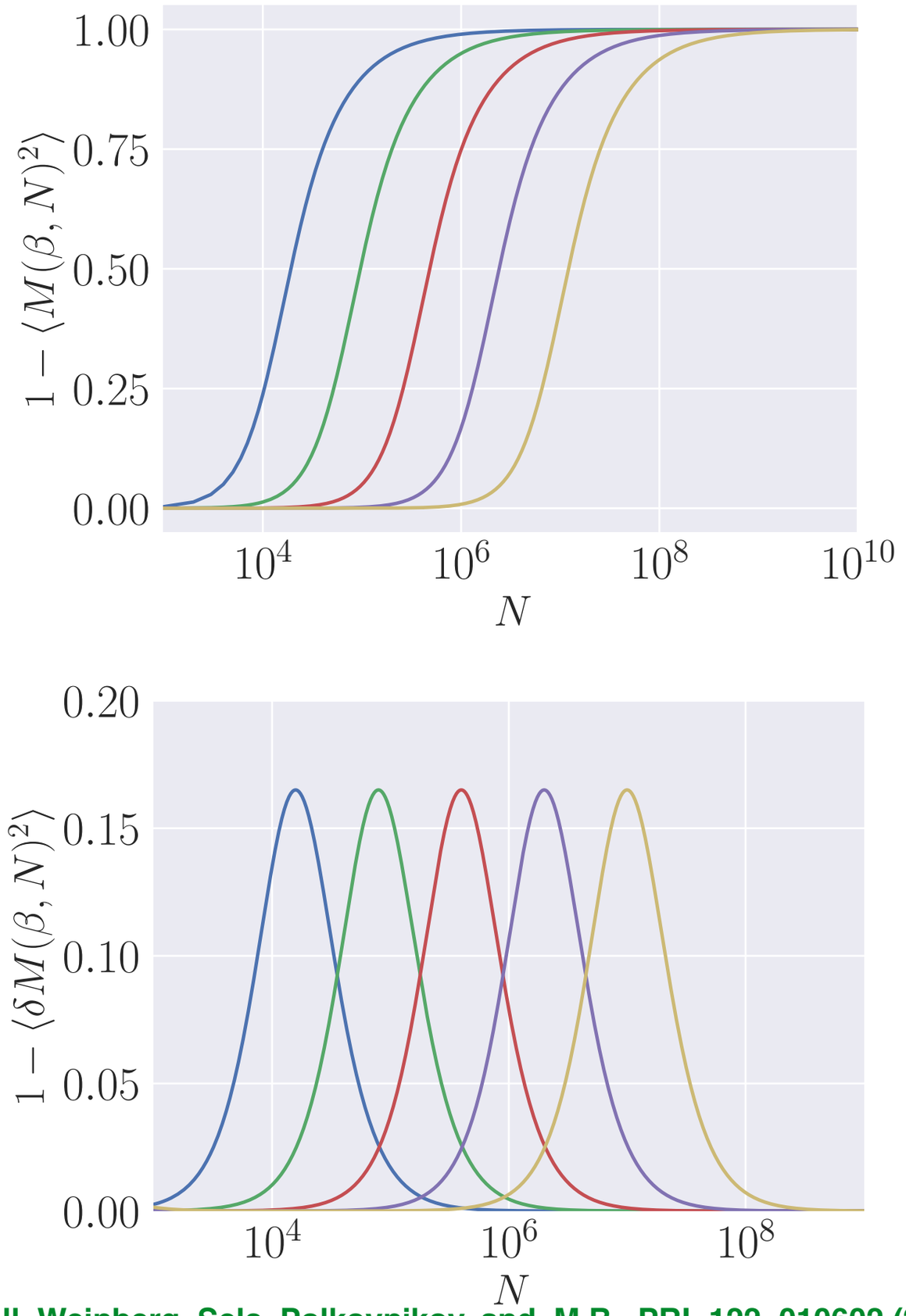
QuSpin: ***http://weinbe58.github.io/QuSpin***

python package for ED & many-body dynamics (with P. Weinberg, BU)

PRB 93, 155132 (2016), PRL 122, 010602 (2019)

Heating transition 1d Ising: $H = -J \sum_j \sigma_j \sigma_{j+1}, \quad \sigma \in \{\pm 1\}$





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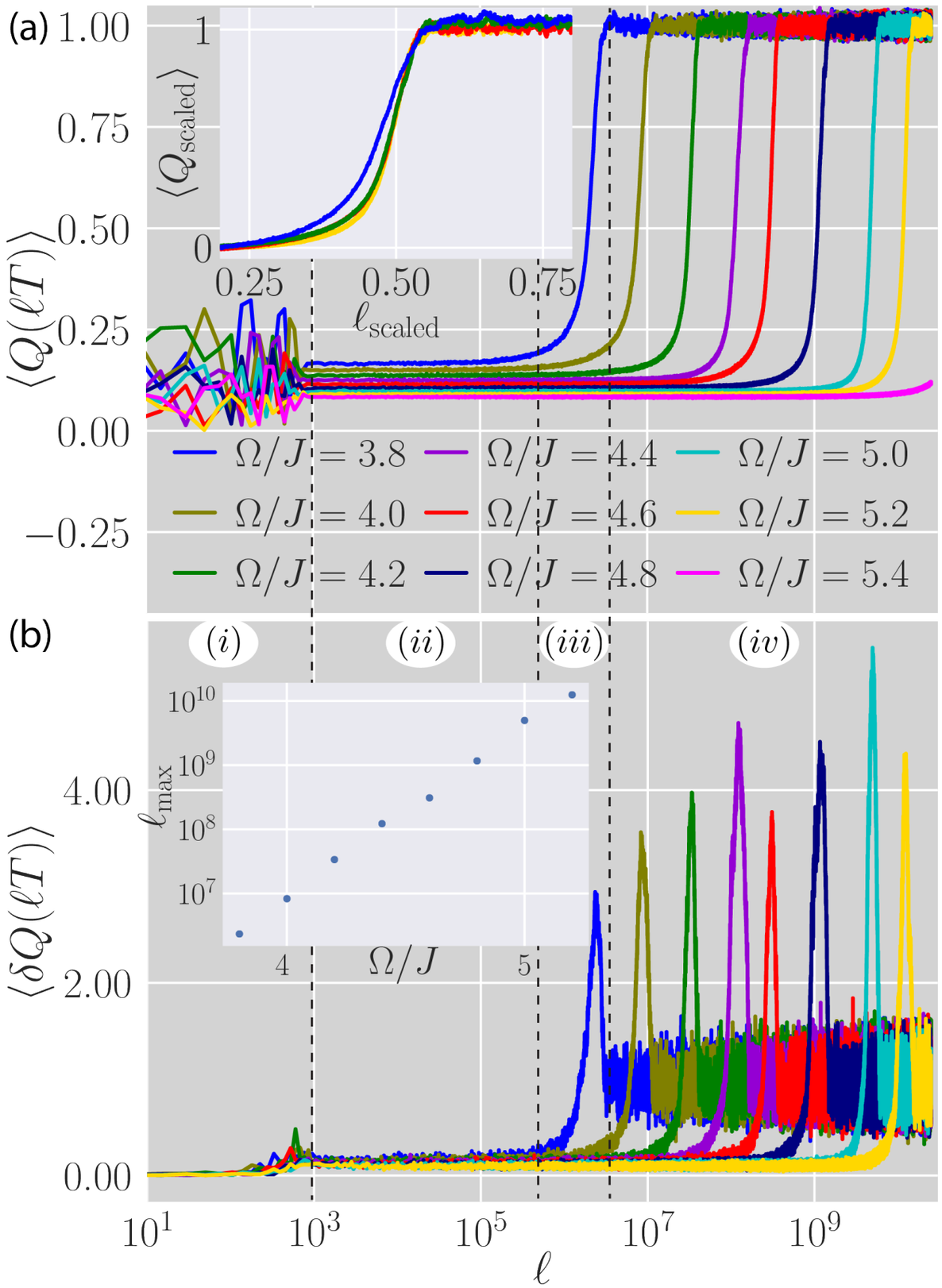

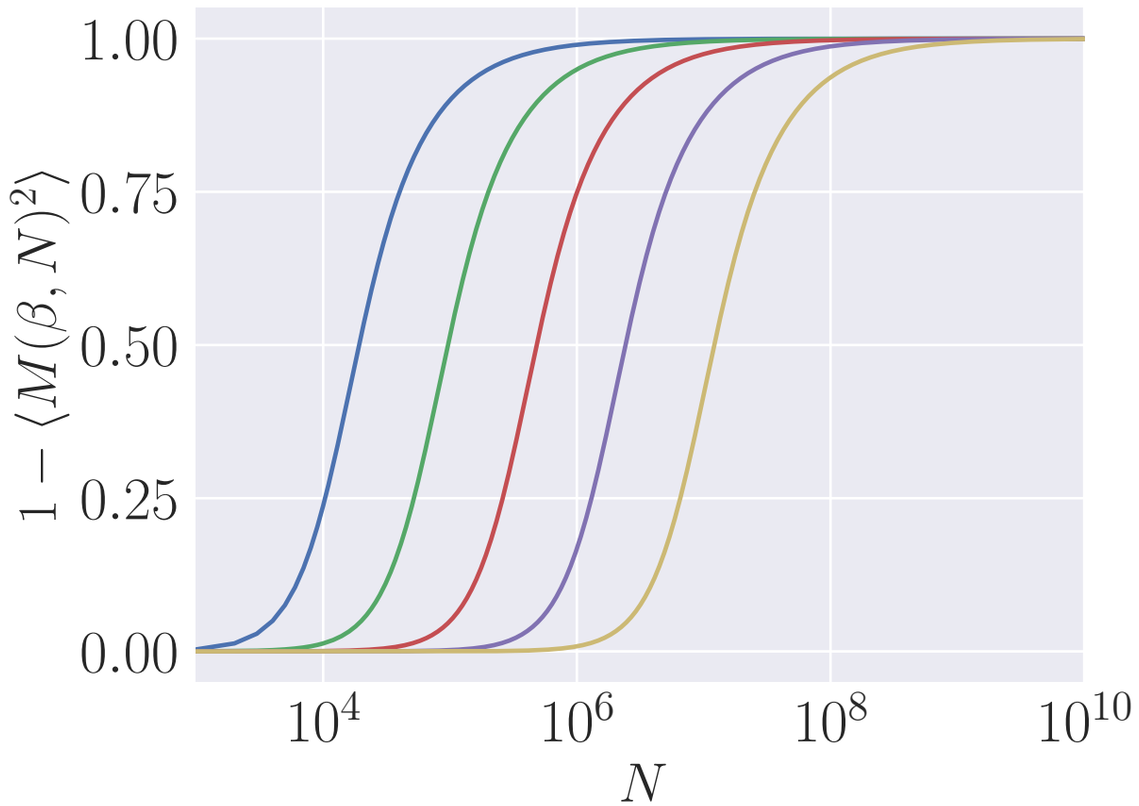

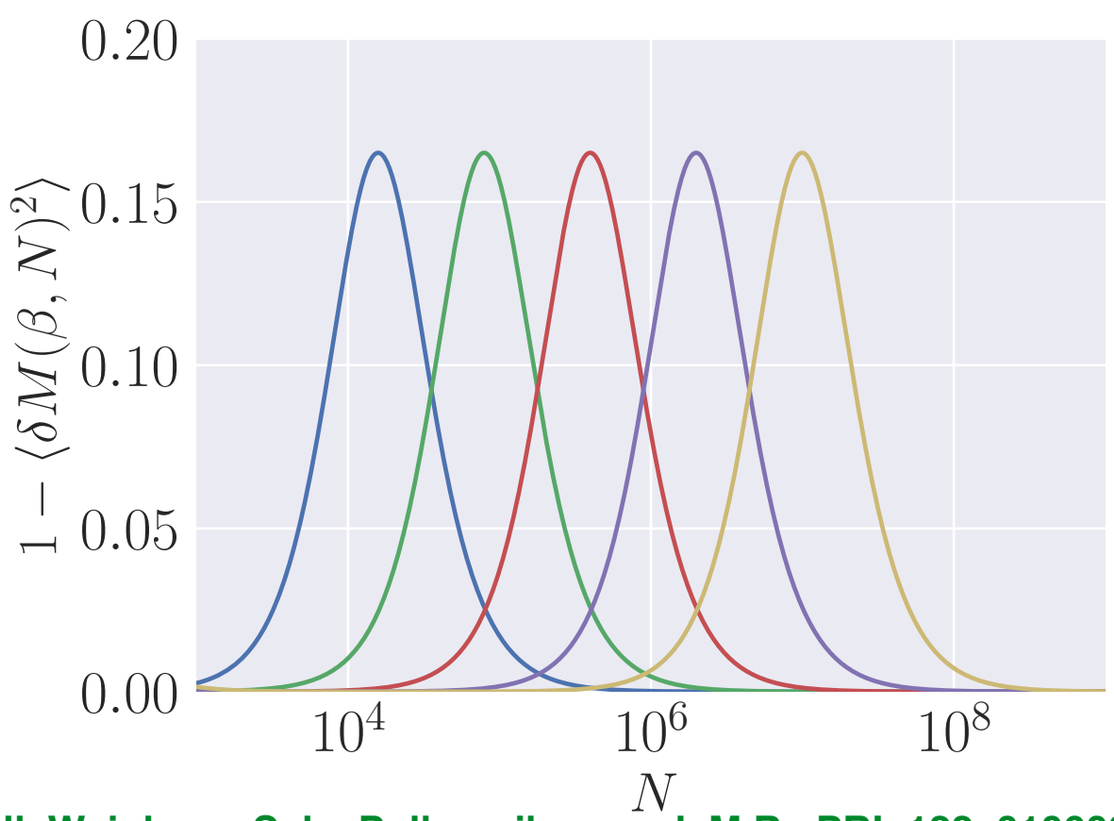

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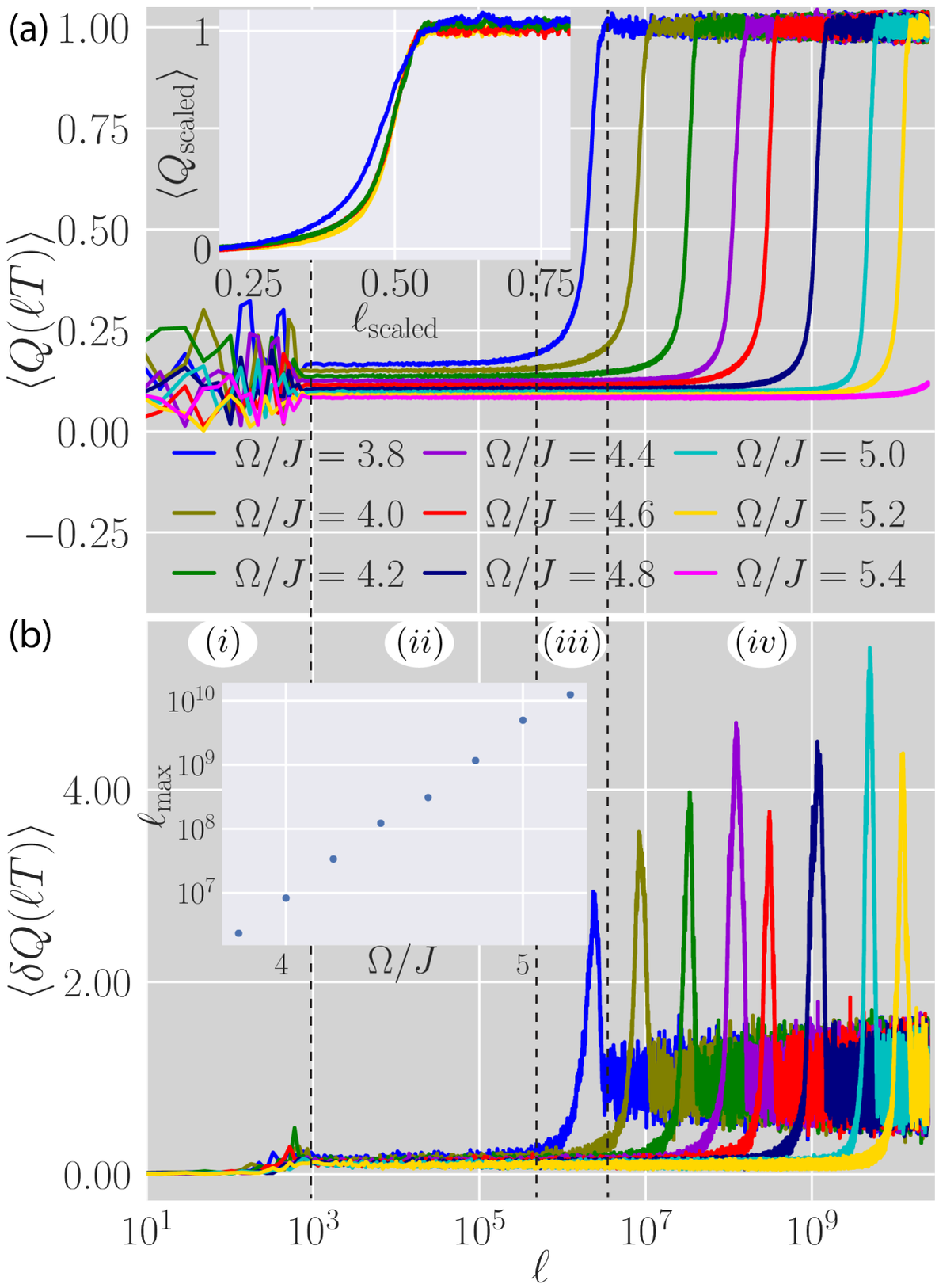

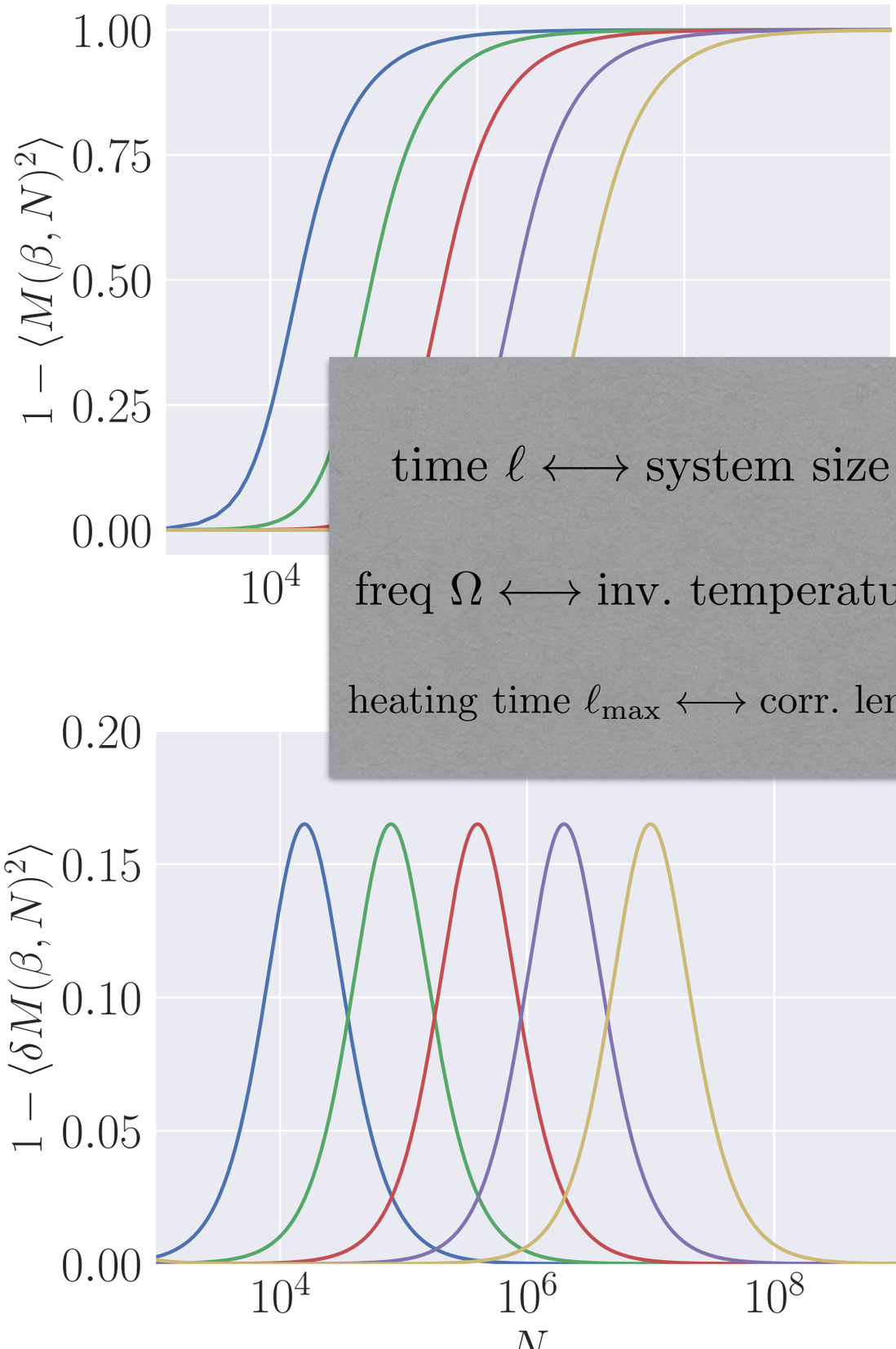

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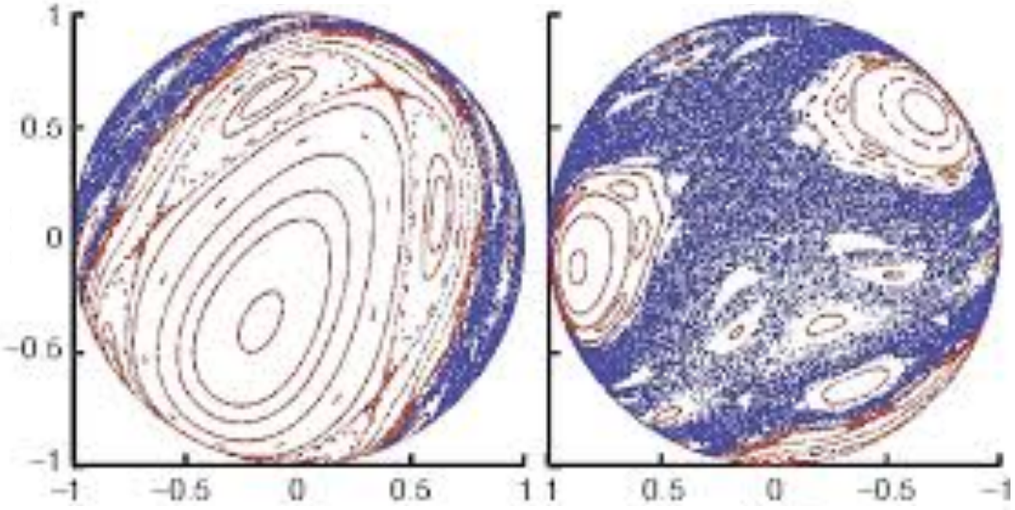
Phase space picture of heating

- KAM theorem

$$H(I,\phi) = H_0(I) + \epsilon H_1(I,\phi)$$

- Arnold diffusion



regular dynamics = localization
chaos = diffusion

KAM: ***many*** solutions to nearly integrable Hamiltonian systems persist under a perturbation for ***all*** time.

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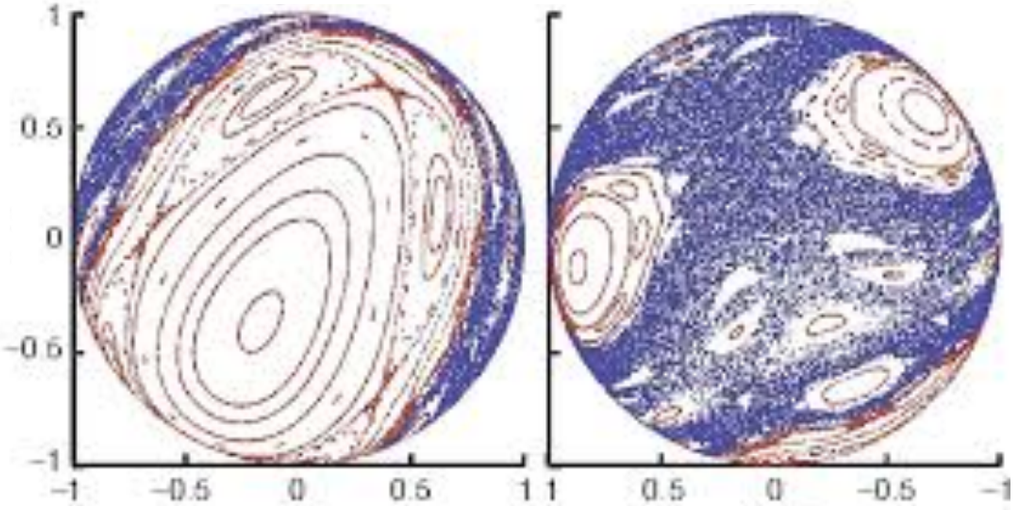

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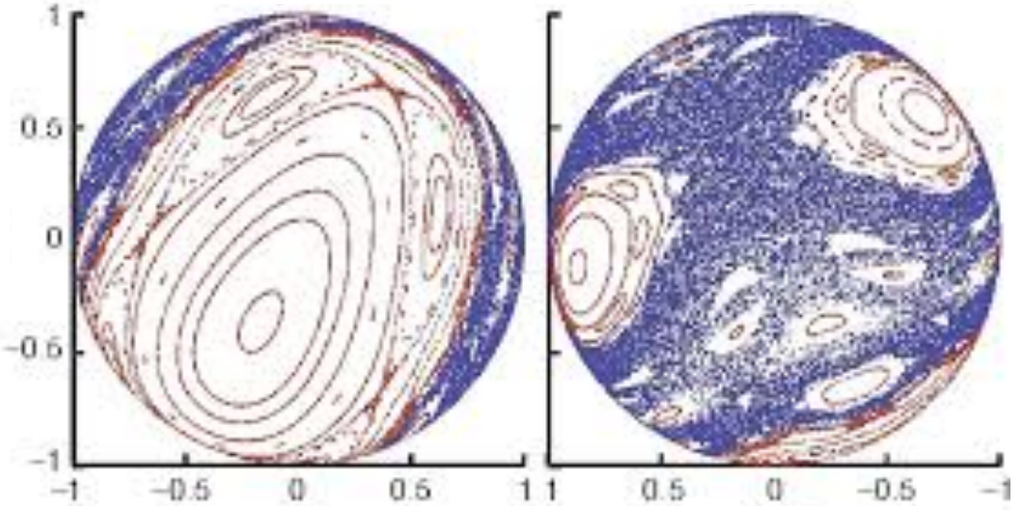

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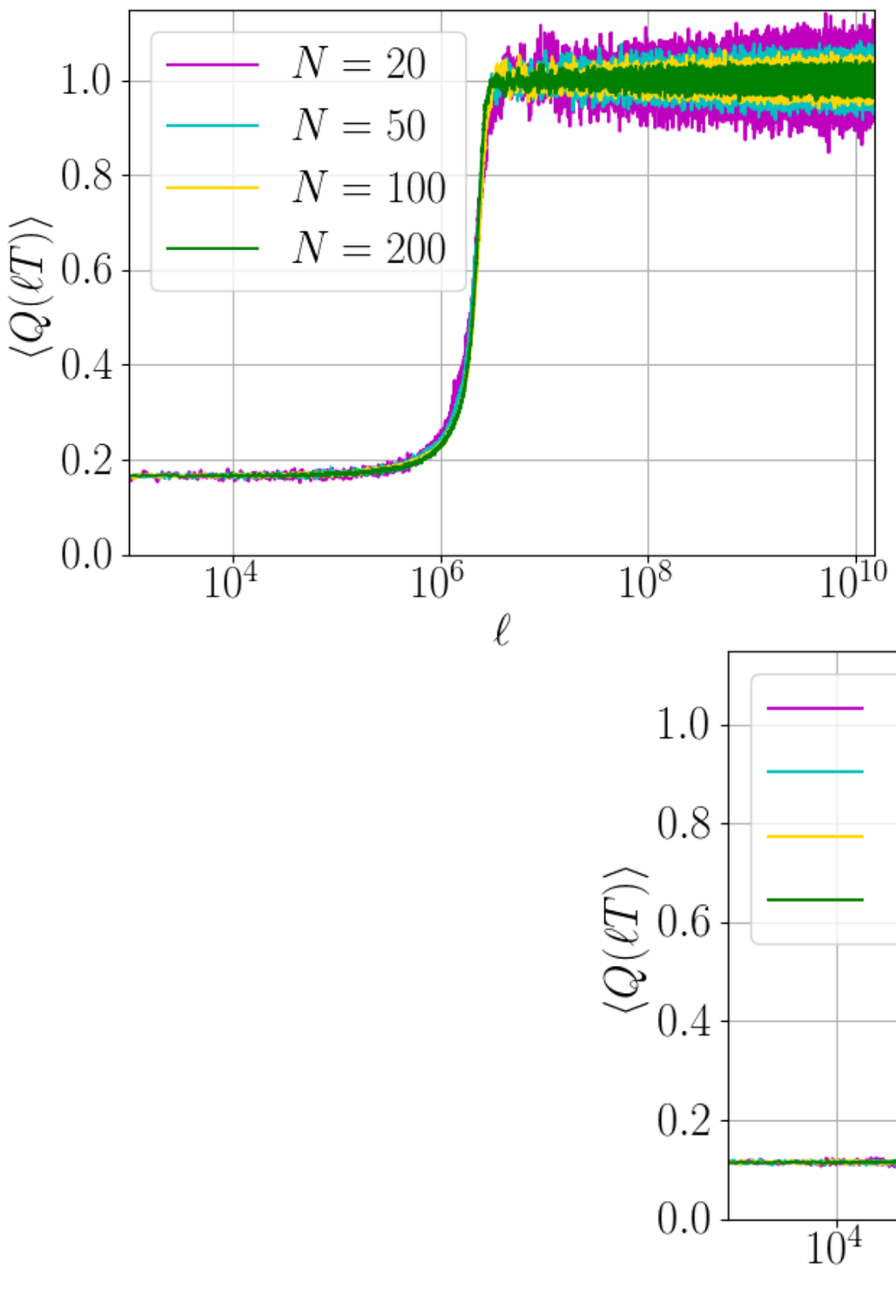
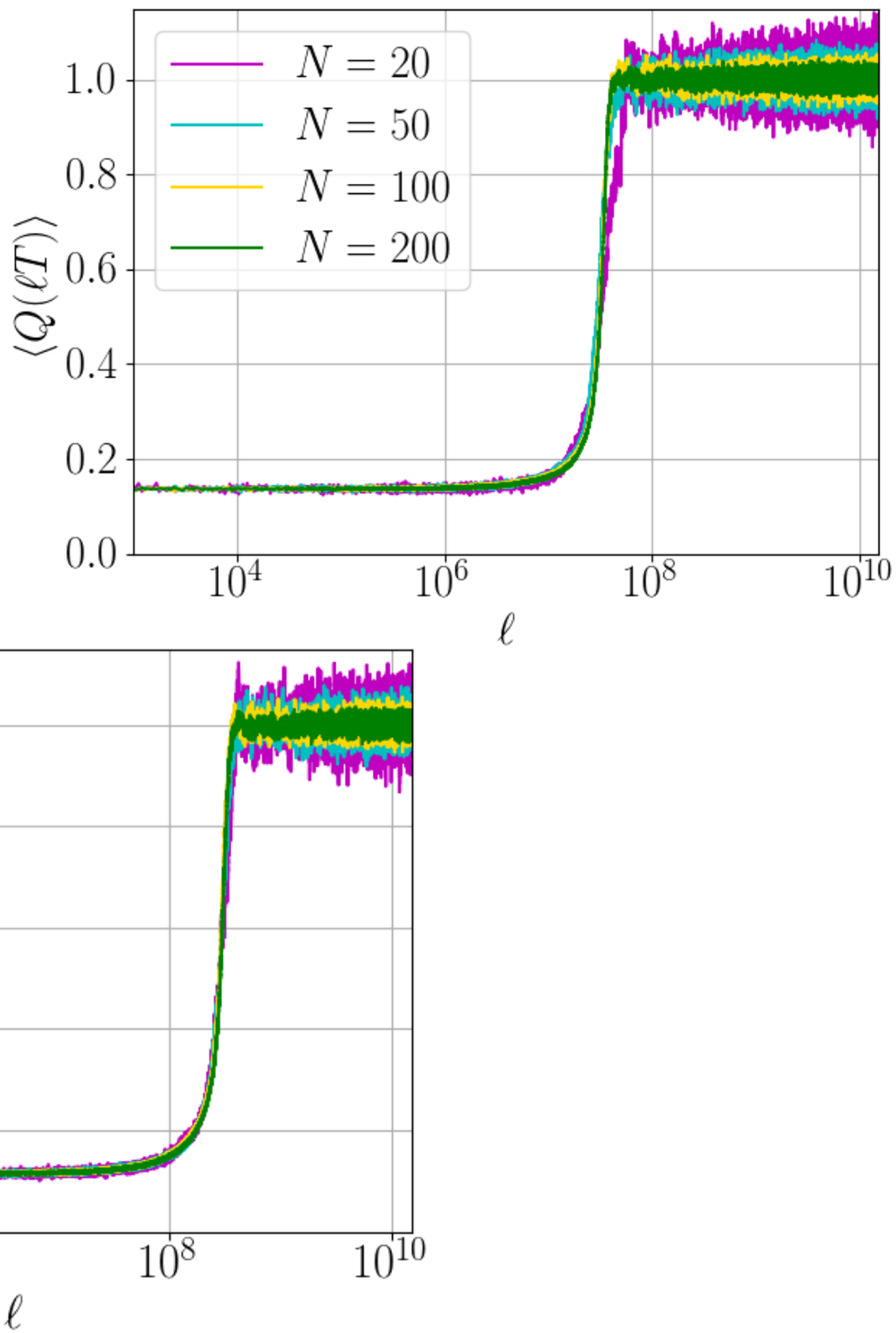
$$|I(t) - I(0)| < \epsilon^{1/(2L)} \text{ for } |t| < \exp\left(c\left(\frac{1}{\epsilon}\right)^{1/(2L)}\right)$$

Finite size dependence





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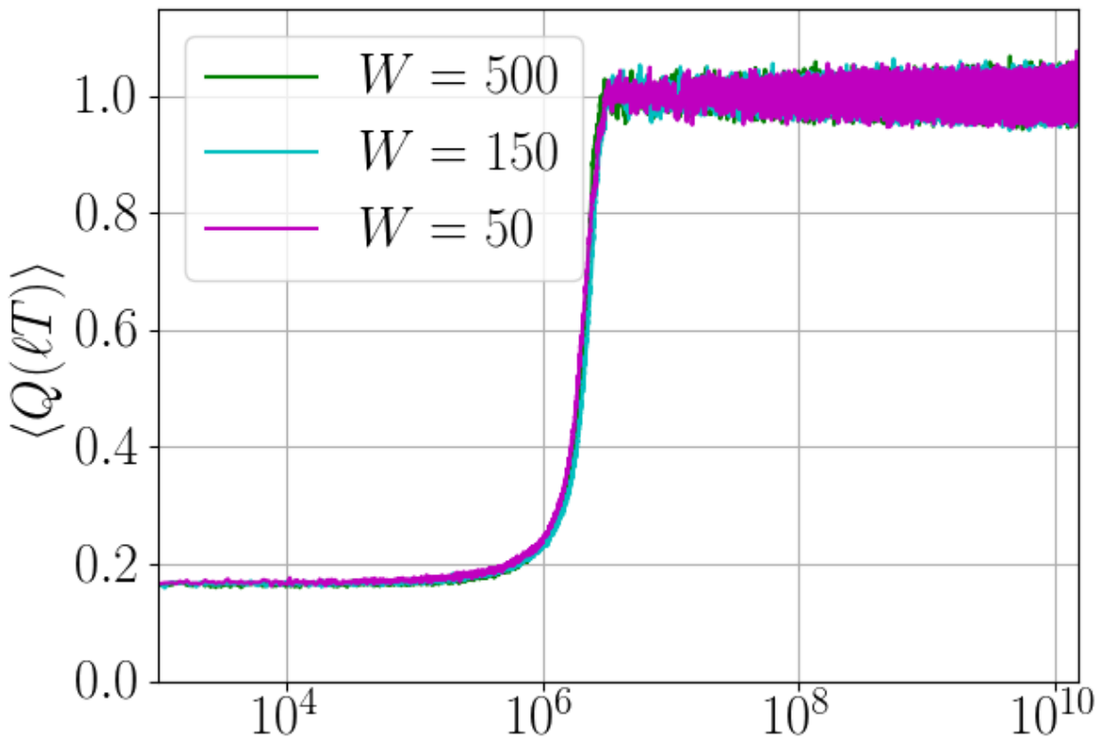
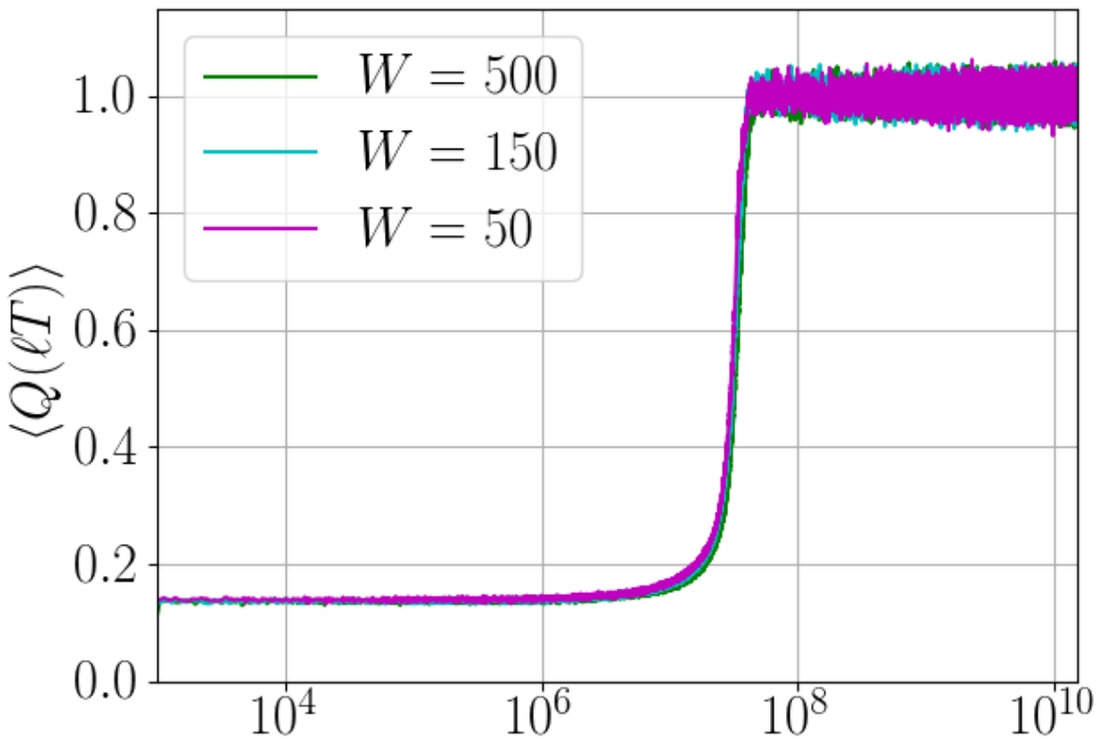
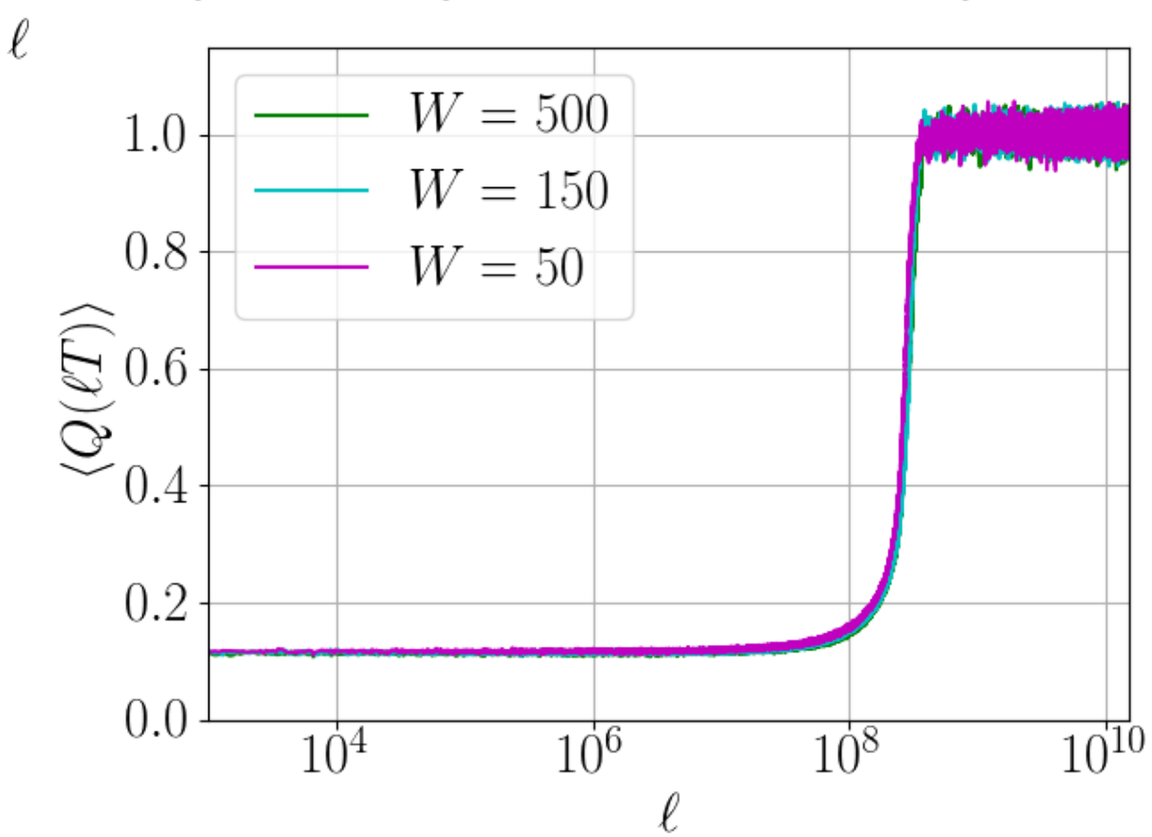
Ensemble noise dependence





$[-\pi/W, \pi/W]$



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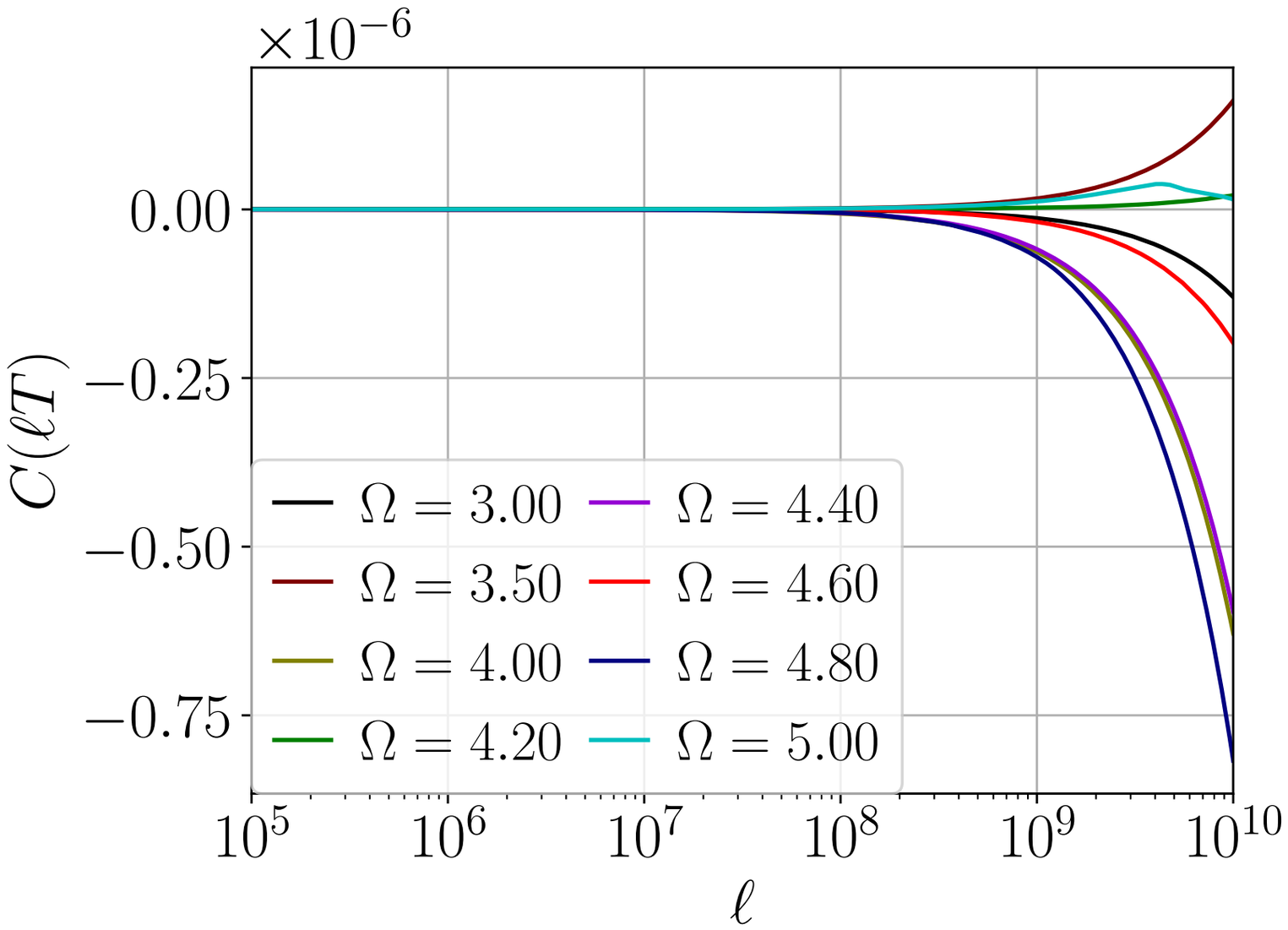
EOM integration error



$$|\vec{S}_j(t)| = 1$$

$$C_j(\ell T) = |\vec{S}_j(\ell T)|^2$$

$$C(\ell T) = \max_j |1 - C_j(\ell T)|$$

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