Collapse and revival of quantum many-body scars via Floquet engineering

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Program: Thermalization, Many body localization and Hydrodynamics

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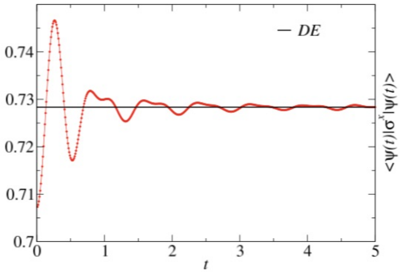
Reference:

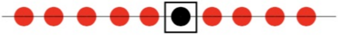
Mukherjee, Nandy, Sen, Sen, Sengupta, arxiv:1907.08212

Outline

- Ergodicity and its breakdown
- Experimental realization of (weak) ergodicity breaking in Rydberg chain
- PXP model/Dipole model
- Quantum scarring
- Floquet version of PXP
- Magnus expansion and Floquet perturbation theory
- Role of scars in Floquet dynamics
- Reentrant transitions between non-ergodic and ergodic regimes as a function of driving frequency

Thermalization in isolated many-body quantum systems

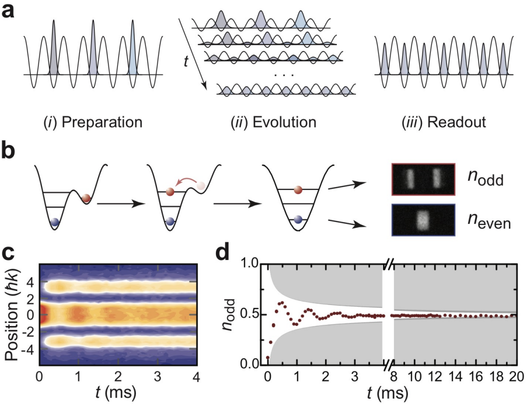




- "Simple" starting wavefunction $|\psi(0)\rangle$ and local $H \rightarrow$ $|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(0)\rangle$
- **Local properties** of $|\psi(t)\rangle$ at late times indistinguishable from the thermal result with temperature determined by energy density of $|\psi(0)\rangle$.

  (review by Alessio, Kafri, Polvovnikov, Rigol, Adv. Phys. 65, 239 (2016))

Thermalization in isolated quantum systems (Experiments)



- Relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas (Trotzky et al., Nat. Phys. 8, 325 (2012))

Eigenstate Thermalization Hypothesis

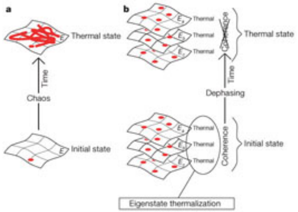
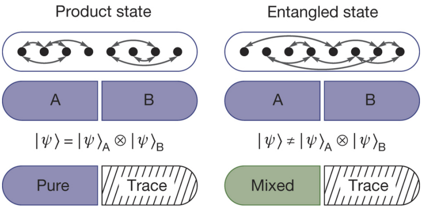


Figure 2 | Thermalization in classical versus quantum mechanics. a, In classical mechanics, time evolution constructs the thermal state from an initial state that generally bears no resemblance to the former. **b,** In quantum mechanics, according to the ETH, every eigenstate of the hamiltonian always implicitly contains a thermal state. The coherence between the eigenstates initially hides it, but time dynamics reveals it through dephasing.



R. Islam et al, Nature (2015)

- $|\psi(0)\rangle = \sum_i c_i |\mathcal{E}_i\rangle$ where $|\mathcal{E}_i\rangle$ denote eigenstates of H.
- $\langle \psi(t)|O|\psi(t)\rangle =$
  $\sum_i |c_i|^2 \langle \mathcal{E}_i|O|\mathcal{E}_i\rangle + \sum_{i \neq j} c_i c_j^* \exp(-i(E_i - E_j)t) \langle \mathcal{E}_j|O|\mathcal{E}_i\rangle$
- High-energy eigenstates in a generic system expected to follow ETH (Deutsch (1991), Srednicki (1994), Rigol+Dunjko+Olshanii (2008))

Violation of ETH

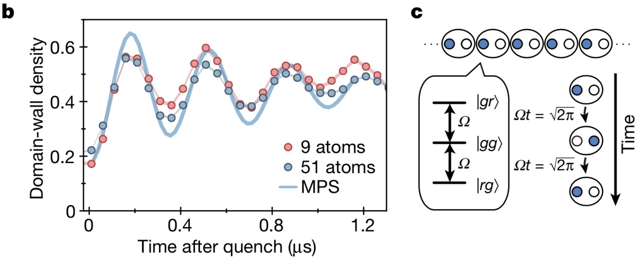
- Integrable systems
- Many-body localization (review by Nandkishore and Huse, Annu. Rev. Condens. Matter. Phys. 6, 15 (2015))

⋮

Examples above show strong violation of ETH.

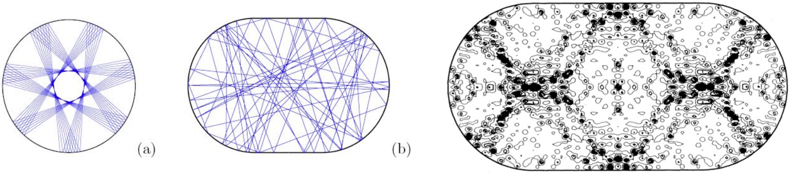
- What about translational invariant non-integrable systems?
- Weak ergodicity breaking from quantum many-body scars
- ETH-violating (non thermal) eigenstates present in an otherwise thermal spectrum
- E.g., 1D AKLT models (Moudgalya, Regnault, Bernevig, PRB 98, 235156 (2018)), PXP model (Turner, Michailidis, Abanin, Serbyn, Papić, Nat. Phys. 14, 745 (2018)), the dipole model (Sachdev, Sengupta, Girvin, PRB 66, 075128 (2002)) etc

Experimental realization of weak ergodicity breaking



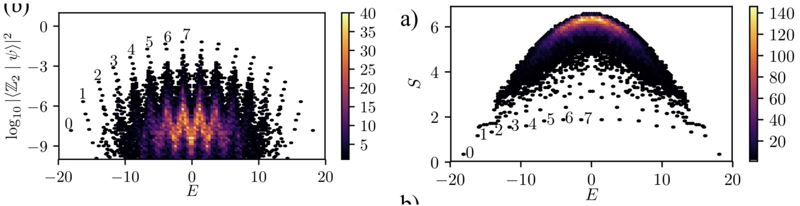
- Bernien et al., Nature 551, 579 (2017) realized programmable Ising-type quantum spin model with tunable interactions $\sim$ 51 qubits.
- Certain initial conditions took much longer to relax towards thermal equilibrium.
- In particular, persistant oscillations in $|\uparrow\downarrow\uparrow\cdots\rangle \to |\downarrow\uparrow\downarrow\cdots\rangle \to |\uparrow\downarrow\uparrow\cdots\rangle$ while $|\downarrow\downarrow\downarrow\cdots\rangle$ rapidly thermalized

Quantum scars



- Classical circular billiard $\rightarrow$ two constants of motion $\rightarrow$ integrable
- Classical stadium billiard $\rightarrow$ only one constant of motion $\rightarrow$ chaotic
- However, unstable periodic orbits still exist (classical scars)
- Quantum version also has certain eigenfunctions that are highly non-uniform in space (quantum scars) Heller, PRL 53, 1515 (1984)

Quantum many-body scars



- $H = -w \sum_i P_{i-1}\sigma_i^x P_{i+1}$ where $P_i = (1 - \sigma_i^z)/2$ and model defined in a constrained Hilbert space where ↑↑ not allowed on neighboring sites (↓↓, ↑↓, ↓↑ okay)

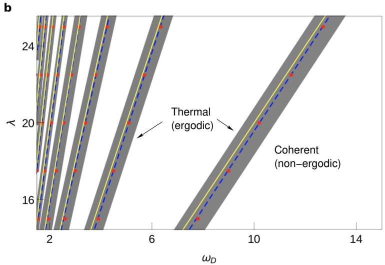
  define $\tilde{\sigma}_i^\alpha = P_{i-1}\sigma_i^\alpha P_{i+1}$ for convenience
- Certain finite-energy density eigenstates athermal ⇒ Violate ETH and have half-chain entanglement $(S \sim \ln L)$ instead of $S \sim L$
- General conditions required for their presence is still unclear

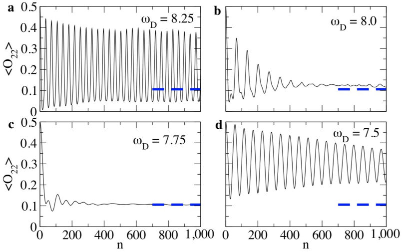
Periodically driven (Floquet) systems

- Consider driving protocol of the form $H(t) = H(t + nT)$ where $T = 2\pi/\omega_D$ is the time period
- Can lead to novel nonequilibrium states like *Floquet time crystals* [Else, Bauer, Nayak (2016), Khemani, Lazarides, Moessner, Sondhi (2016)]
- $U(T) = \exp(-iH_F T)$ where $U(T)$ is the Floquet operator and H_F the Floquet Hamiltonian.
- Nature of eigenstates of H_F (or equivalently $U(T)$)?
- Generic systems heat up to a featureless ITE.
- Not true for many body localized systems (Nandkishore, Huse (2014)) and for *certain* integrable models
- What happens for Floquet version of PXP?

Mukherjee, Nandy, Sen, Sen, Sengupta, arxiv:1907.08212

- $H(t) = \sum_i \left(-w\tilde{\sigma}_i^x + \frac{\lambda(t)}{2}\sigma_i^z\right)$ where $\lambda(t)$ is a symmetric square pulse of the form $\lambda(t) = -(+)\lambda$ for $t \leq (>) T/2$.
- Focus on $\lambda \gg w$. Instantaneous $H(t)$ has no scars in this limit but H_F may still have them.
- Signatures in $\langle\psi(n)|O|\psi(n)\rangle$ where $|\psi(n)\rangle = U(T)^n|\psi(0)\rangle$ with initial state being $|\mathbb{Z}_2\rangle$ = $|\cdots\downarrow\uparrow\downarrow\uparrow\cdots\rangle$





Magnus expansion for H_F (I)

- $U = e^{-iH_+ T/(2\hbar)} e^{-iH_- T/(2\hbar)} = e^{X_+} e^{X_-}$ where $X_\pm = (-iT/2\hbar) H_\pm$
- Using Baker-Campbell-Hausdorff formula, $\ln[e^{X_+} e^{X_-}] = X_+ + X_- + \frac{1}{2}[X_+, X_-] + \frac{1}{12}[X_+ - X_-, [X_+, X_-]] - \frac{1}{24}[X_-, [X_+, [X_+, X_-]]] + \cdots$
- $H^F_{\text{Magnus}} = H_0 + H_1$ upto $\mathcal{O}(1/\omega_D^3)$ gives
  $H_0 = -w\left\{\left[1 - \frac{2\gamma^2}{3}\right]\sum_l \tilde{\sigma}_l^x + \gamma\left(1 - \frac{\gamma^2 - 4\delta^2}{3}\right)\sum_l \tilde{\sigma}_l^y\right\} - \frac{4\lambda\delta^3}{3}\sum_l \tilde{\sigma}_l^y$
  (Renormalized PXP)
  $H_1 = -\frac{2\lambda\delta^3}{3}\sum_l[\tilde{\sigma}_{l-1}^y \tilde{\sigma}_l^z + \tilde{\sigma}_l^z \tilde{\sigma}_{l+1}^y + \frac{1}{2}(\tilde{\sigma}_l^y \tilde{\sigma}_{l+1}^y + \tilde{\sigma}_l^x \tilde{\sigma}_{l+1}^x)\tilde{\sigma}_{l+1}^y]$
  (non-PXP)
- Dimensionless quantities $\gamma = \lambda T/(4\hbar)$ and $\delta = wT/(4\hbar)$

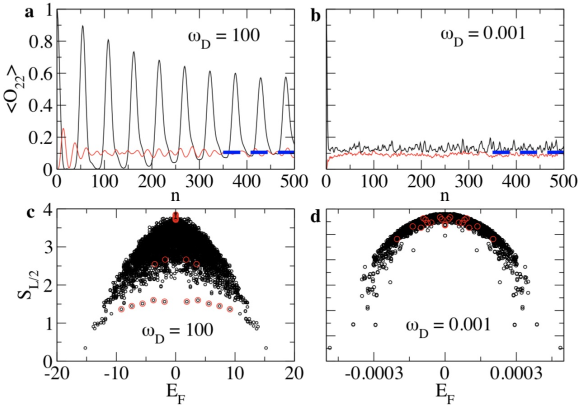
Magnus expansion for H_F (II)

- For $\gamma \gg \delta$, the expression for PXP can be computed to much higher order
- $X_\pm = X_1 \pm X_2$ where $X_{1[2]} = \left(\frac{i\hbar}{2T}\right) w \left[-\frac{\lambda}{2}\right] \sum_l \tilde{\sigma}_l^x [\sigma_l^z]$
- In nth order, leading contribution to PXP terms from commutators with $n-1$ X_2 and one X_1. E.g., $[X_2, [X_2, [X_2, \cdots [X_2, X_1]]] \cdots]$
- $H_2 =$
$$-w \left(\left[1 - \frac{2\gamma^2}{3} + \frac{2\gamma^4}{15} - \frac{4\gamma^6}{315} + \frac{2\gamma^8}{2835} - \frac{4\gamma^{10}}{155925} + \cdots \right] \sum_l \tilde{\sigma}_l^x \right)$$
$$-w\gamma \left(\left[1 - \frac{\gamma^2}{3} + \frac{2\gamma^4}{45} - \frac{\gamma^6}{315} + \frac{2\gamma^8}{14175} - \frac{2\gamma^{10}}{467775} + \cdots \right] \sum_l \tilde{\sigma}_l^y \right)$$
- Coefficients in H_2 can be resummed to yield $H_F = -\left(w\frac{\sin\gamma}{\gamma}\right) \sum_j (\cos\gamma \tilde{\sigma}_j^x + \sin\gamma \tilde{\sigma}_j^y)$

Simple proof for $O(w)$ H_F

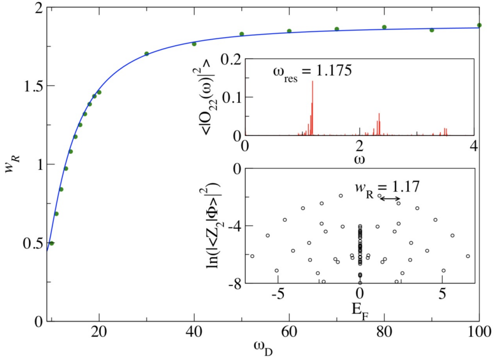
- Note that $[-w\tilde{\sigma}_j^x \pm \frac{\lambda}{2}\sigma_j^z, -w\tilde{\sigma}_{j'}^x \pm \frac{\lambda}{2}\sigma_{j'}^z] \sim \mathcal{O}(w^2)$ when $j \neq j'$
- Thus, to $\mathcal{O}(w)$, $U_\pm = \prod_j e^{-iT(-w\tilde{\sigma}_j^x \pm \frac{\lambda}{2}\sigma_j^z)/2\hbar}$
- Gathering $\mathcal{O}(w)$ terms,
  $U_\pm = \prod_j(\mathbb{I}_j \cos(\gamma) \mp i\sin(\gamma)\sigma_j^z \pm i\frac{2w}{\lambda}\sin(\gamma)\tilde{\sigma}_j^x)$
- Compute $U = U_+ U_-$ to $\mathcal{O}(w)$
  $U = \prod_j(\mathbb{I}_j + i\frac{4w\sin\gamma}{\lambda}(\cos\gamma\tilde{\sigma}_j^x + \sin\gamma\tilde{\sigma}_j^y))$ and re-exponentiate to get
  $H_F = -w\frac{\sin\gamma}{\gamma}\sum_j[\cos\gamma\tilde{\sigma}_j^x + \sin\gamma\tilde{\sigma}_j^y] + \mathcal{O}(w^3)$
- Can be more formally derived using Floquet perturbation theory where $H(t) = H_0(t) + V$ where V is a perturbation and $H_0(t)$ commutes with itself at different times [A. Soori and D. Sen, PRB 82, 115432 (2010)]

Limiting cases

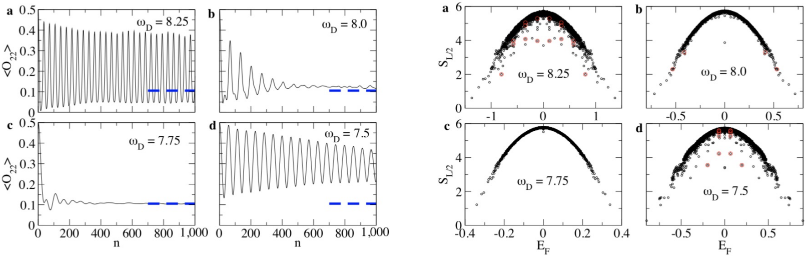


- Here $\hbar\omega_D \gg [\ll] \lambda, w$.
- Upper two panels–black line corresponds to initial $|\mathbb{Z}_2\rangle = |\cdots\downarrow\uparrow\downarrow\uparrow\rangle$ and red line corresponds to initial state $|0\rangle = |\downarrow\downarrow\downarrow\cdots\rangle$.

Drive-induced control of scar separation

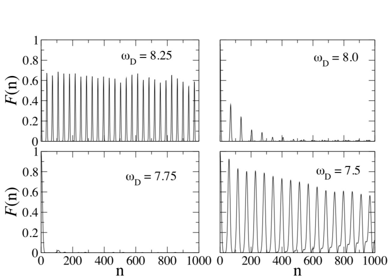


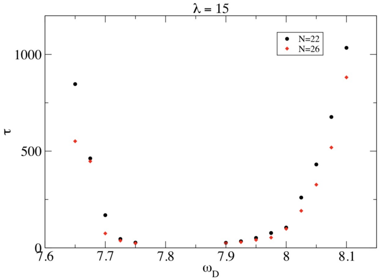
- Quasienergy separation w_R of the scars for $\hbar\omega_D \geq \lambda$ as a function of ω_D
- w_R matches ω_{res} extracted by Fourier transformation of the dynamical response
- Drive-induced control of the oscillation frequency by controlling the scar separation—no quench analog



- Several frequency induced reentrant transitions between regimes with and without scars.
- Regimes distinguished by correlator dynamics; in the former regime, long-lived persistent oscillations starting from $|\mathbb{Z}_2\rangle$
- Latter region shows thermalization consistent with ETH
- note: for numerics shown here, starting state is $(|\mathbb{Z}_2\rangle + |\bar{\mathbb{Z}}_2\rangle)/\sqrt{2}$ and we have further used $k = 0$ and spatial inversion symm. to reach $L = 26$

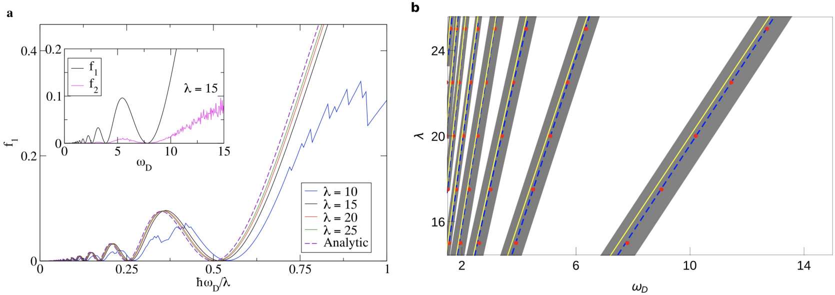
Fidelity etc





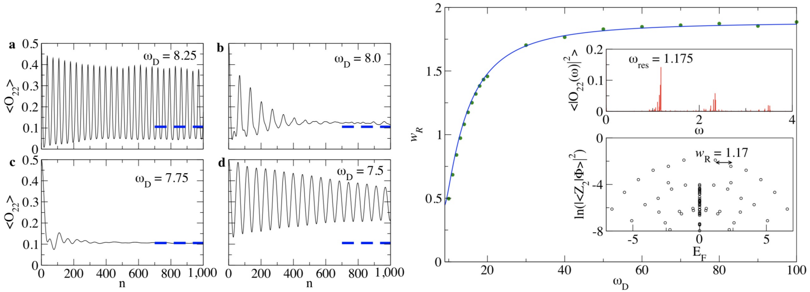
- $\mathcal{F}(n) = |\langle\psi(n)|\psi(0)\rangle|^2$ shows periodic persistent revivals when the density correlators show oscillations
- Consistent with picture of Krylov subspace $\mathcal{K} = span\{|\mathbb{Z}_2\rangle, H|\mathbb{Z}_2\rangle, H^2|\mathbb{Z}_2\rangle, \cdots\}$ that is approximately closed and is much smaller than full Hilbert space dim.
- No persistent revivals in the thermal regimes

PXP versus non-PXP



- Write matrix rep. of H_F in the basis states $|\phi_n\rangle$ of σ^z and identify the matrix elements that have $\langle\phi_n|\sum_l \tilde{\sigma}_l^{x/y}|\phi_m\rangle \neq 0$. Denote this set as $\mathcal{N}_0$ which has $N_0 = 2LF_{L-1}$ elements ($F_n + F_{n+1} = F_{n+2}, F_1 = F_2 = 1$)
- $f_{1[2]} = \frac{1}{N_0}\sum_{n,m\in[\notin]\mathcal{N}_0} |\langle\phi_n|H_F|\phi_m\rangle|^2$ ($f_1 = w^2\frac{\sin^2(\gamma)}{\gamma^2}$, $f_2 = 0$ to $\mathcal{O}(w)$)

Conclusions



- Presence of several non-ergodic and ergodic regimes as a function of the driving frequency ω_D
- Possibility of tuning the quasienergy separation of the scars in the non-ergodic regime as a function of the drive frequency
- Both these features entirely absent in quench studies of PXP model and can be tested by standard experiments using Rydberg chains