Localized and ballistic eigenstates in chaotic spin ladders and the Fermi-Hubbard model

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Thermalization, Many-Body Localization, and Hydrodynamics

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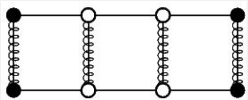
Outline

special eigenstates in chaotic (integrable) models



1. Motivation:
   (a) [1] Thermal vs. non-thermal systems: examples
   (b) [4] “MBL” experiments: role of SU(2)

2. Ladder, special eigenstates:
   (a) [2] Symmetries, quantum chaos
   (b) [2] Invariant subspaces: localized, ballistic,...
   (c) [1] Robustness

Thermalizing phases vs. Non-thermalizing phases

- **Most systems** ...
- Quantum chaotic H

  F. Haake, Quantum Signatures of Chaos (2001).
- Random circuits

  Emerson et al., Science '03; Oliveira et al. '07; Žnidarič, PRA '08; ...

- **Rare**
- Integrable models

  Jordan-Wigner, Bethe,...
- Many-body localized (MBL)

  Review: Alet & Laflorencie, Comp. Rendus Physique '18

How about "intermediate phases"?

- quantum disentangled liquid: some DOF have area-law

  Grover & Fisher '13.
- constrained dynamics: (slow MBL-like dynamics without disorder)
  - dynamical bottlenecks: Schiulaz & Müller AIP Conf. '14; Van Horssen et al. PRB '15; Michailidis et al. PRB '18.
  - scale (energy/mass) separation: Carleo et al. Sci. Reports '12; Yao et al. PRL '16
  - effective initial state disorder: Smith et al. PRL '17, PRB '18; Brenes et al. PRL '18
  - Hilbert space constraint: Turner et al. Nat. Phys. '18; Khemani et al. PRB '19
- non-ETH eigenstates:
  - η-pairing states: C.-N. Yang PRL '89
  - related $S \sim \log L$ states: Luo & Clark PRB '18

    non-integrable SU(2) models: Moudgalya, Regnault, Bernevig PRB '18
  - Conserved $\sum_j S_j^z$ and $\sum_j jS_j^z$: Khemani & Nankishore arXiv '19

1b) Experiments – interacting fermions in 1D

$$\boxed{H = -J \sum_{j\sigma} (c^\dagger_{j+1,\sigma} c_{j,\sigma} + \mathrm{H.c.}) + U \sum_j n_{j\uparrow} n_{j\downarrow} + \Delta \sum_j \epsilon_j n_j}$$

disorder: $\epsilon_j(n_{j\uparrow} + n_{j\downarrow})$, $\quad \epsilon_j = \cos(2\pi\beta j + \phi)$, $\quad$ irrational β.

interacting spin-full fermions in quasiperiodic potential

Localization or no localization?

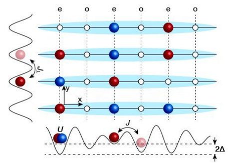
$$H = -J\sum_{j\sigma}(c^\dagger_{j+1,\sigma}c_{j\sigma} + \mathrm{H.c.}) + U\sum_j n_{j\uparrow}n_{j\downarrow} + \Delta\sum_j \epsilon_j n_j$$

Experiment:

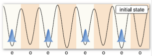
Observation of many-body localization of interacting fermions in a quasirandom optical lattice

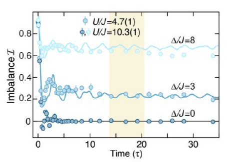
Michael Schreiber,[1,2] Sean S. Hodgman,[1,2] Pranjal Bordia,[1,2] Henrik P. Lüschen,[1,2] Mark H. Fischer,[3] Ronen Vosk,[3] Ehud Altman,[3] Ulrich Schneider,[1,2,4] Immanuel Bloch[1,2]*

Science 2015

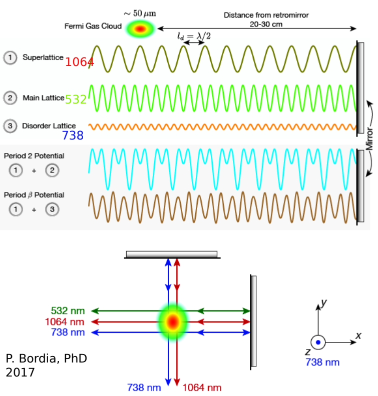


Charge imbalance: $I = \frac{N_e - N_o}{N_e + N_o}$



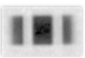


1. Cooling: ultracold fermionic gass of potassium ^{40}K (two lowest hyperfine levels, spin-balanced), $T/T_\mathrm{F} \approx 0.2$, 10^5 atoms ($\approx 100 \times 100$, thin in z).
2. Lattice: 6 lattice lasers, slowly turn on ($\approx 200\,$ms), CDW.
3. Evolve: $\approx 50\,$ms.
4. Measure: ramping 532 nm, band-mapping, time-of-flight.



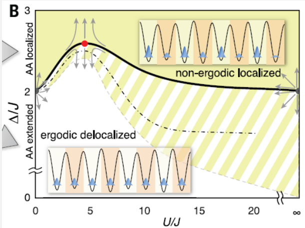
P. Bordia, PhD 2017

Bordia '17



1b) ... exper. cont'd.

$$H = -J \sum_{j\sigma} (c^\dagger_{j+1,\sigma} c_{j\sigma} + \mathrm{H.c.}) + U \sum_j n_{j\uparrow} n_{j\downarrow} + \Delta \sum_j \cos(2\pi\beta j)(n_{j\uparrow} + n_{j\downarrow})$$



Charge imbalance decay:

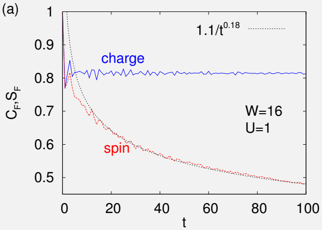
- Spinless $U = 0$: Aubry-Andre-Harper model, **mathematics** – The Ten Martini Problem (Fields 2014)

- Experiment, $U = 0$: Inguscio et al., Nature 2008.

What about the spin $S^z_j = n_{j\uparrow} - n_{j\downarrow}$?

Do charge and spin behave the same?

NO! (Prelovšek, Barišić, Žnidarič, PRB '16)



Symmetry:

$S(t \to \infty) \propto \sum_\alpha |S^z_{\alpha,\alpha}|^2$

S^z odd under $n_{j\uparrow} \leftrightarrow n_{j\downarrow}$, H even,

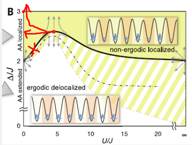
$\Rightarrow S^z_{\alpha,\alpha} = -S^z_{\alpha,\alpha} \equiv 0$

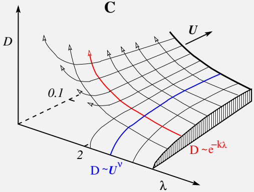
[caveat: an incorrect order of limits!]

- $SU(2)$ incompatible with localization? (Chandran, Khemani, Laumann & Sondhi PRB '14); (Potter & Vasseur PRB '16); (Protopopov, Ho & Abanin, PRB '17)
- Ensuing discussion: Parameswaran & Gopalakrishnan '17; Zhao et. al. '17; Bonča & Mierzejewski '17, Zakrzewski & Delande '18; Kozarevski et al. '18; Yo et al. '18, Protopopov & Abanin '18,...

Ultimate fate of charge/spin?

Spinless fermions: even then such phase diagram would be wrong





(MZ & ML PNAS '18)

quasiperiodic is different than random!

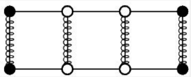
Also q.-p. Fibonacci model

(V.K.Varma & MZ PRB '19)

2a) Special eigenstates (Tom Iadecola & MZ, PRL '19), idea from MZ PRL '13

$$h^{||}_{k,k+1} = \sigma^{\rm x}_k\sigma^{\rm x}_{k+1} + \sigma^{\rm y}_k\sigma^{\rm y}_{k+1} + \tau^{\rm x}_k\tau^{\rm x}_{k+1} + \tau^{\rm y}_k\tau^{\rm y}_{k+1},$$

$$h^{\perp}_k = J(\sigma^{\rm x}_k\tau^{\rm x}_k + \sigma^{\rm y}_k\tau^{\rm y}_k) + \Delta_k\,\sigma^{\rm z}_k\tau^{\rm z}_k + h_k(\sigma^{\rm z}_k + \tau^{\rm z}_k).$$

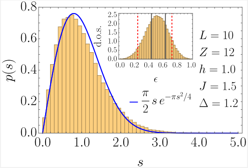


Using Jordan-Wigner, it represents spinfull fermions:

- Upper leg $|\uparrow\rangle$, lower leg $|\downarrow\rangle$. Charge density $d_k = (\sigma^{\rm z}_k + \tau^{\rm z}_k)/2$, spin density $s_k = (\sigma^{\rm z}_k - \tau^{\rm z}_k)/2$.
- $\langle d_k \rangle = \pm 1$ is doublon/holon.
- hopping is 1, interaction is $\sim \Delta_k(2n_{k\uparrow} - 1)(2n_{k\downarrow} - 1)$.
- on-site potential $\sim h_k(n_{k\uparrow} + n_{k\downarrow})$.
- rung hopping is $\sim J(c_{k\downarrow}c^{\dagger}_{k\uparrow} + \mathrm{H.c.})$.

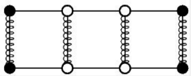
Important cases:

- Can be clean (MZ, PRL '13), or on-site, or/and interaction disorder.
- Integrable Hubbard ($J = 0$).
- In general chaotic model (e.g. $J \neq 0$):
  diffusion (for small disorder), RMT LSD



Symmetries:

$$h^{||}_{k,k+1} = \sigma^x_k\sigma^x_{k+1} + \sigma^y_k\sigma^y_{k+1} + \tau^x_k\tau^x_{k+1} + \tau^y_k\tau^y_{k+1},$$
$$h^{\perp}_k = J(\sigma^x_k\tau^x_k + \sigma^y_k\tau^y_k) + \Delta_k\, \sigma^z_k\tau^z_k + h_k(\sigma^z_k + \tau^z_k).$$



Always:

- U(1): conserved $Z = \sum_k \sigma^z_k + \tau^z_k$.
- $\mathbb{Z}_2$: $\sigma^\alpha_k \leftrightarrow \tau^\alpha_k$.

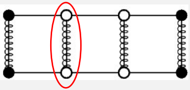
Hubbard model ($J \equiv 0$):

- one SU(2) for $J = 0$: spin-rotation symmetry.
  (the other SU(2) η-symm. broken by $h_k \neq 0$)
  - if $J \neq 0$ there are no SU(2) symmetries.
- In a ladder, $h_k \equiv 0$ and $J \neq 0$: "almost" η SU(2) symmetry (Kitazawa et al. JPA '03). Is broken by $h_k \neq 0$.

In short: in general no SU(2) symmetries.

Rung eigenbasis:

$$h_k^{\perp} = J(\sigma_k^x \tau_k^x + \sigma_k^y \tau_k^y) + \Delta_k\, \sigma_k^z \tau_k^z + h_k(\sigma_k^z + \tau_k^z)$$



Eigenstate	Notation	$\langle d_k \rangle$	Eigenenergy
singlet	$\lvert S\rangle := \frac{1}{\sqrt{2}}\left(\lvert\begin{smallmatrix}0\\1\end{smallmatrix}\rangle - \lvert\begin{smallmatrix}1\\0\end{smallmatrix}\rangle\right)$	0	$E_k^S = -2J - \Delta_k$
triplet	$\lvert T\rangle := \frac{1}{\sqrt{2}}\left(\lvert\begin{smallmatrix}0\\1\end{smallmatrix}\rangle + \lvert\begin{smallmatrix}1\\0\end{smallmatrix}\rangle\right)$	0	$E_k^T = 2J - \Delta_k$
doublon	$\lvert D\rangle := \lvert\begin{smallmatrix}0\\0\end{smallmatrix}\rangle$	$+1$	$E_k^D = \Delta_k + 2h_k$
holon	$\lvert H\rangle := \lvert\begin{smallmatrix}1\\1\end{smallmatrix}\rangle$	-1	$E_k^H = \Delta_k - 2h_k$

products of rung eigenstates are eigenstates of $H^{\perp}$

Important relations:

$$h_{k,k+1}^{||} = \sigma_k^x \sigma_{k+1}^x + \sigma_k^y \sigma_{k+1}^y + \tau_k^x \tau_{k+1}^x + \tau_k^y \tau_{k+1}^y$$

$$h_{k,k+1}^{||}\lvert TD\rangle \sim h_{k,k+1}^{||}\left(\lvert\begin{smallmatrix}0\\1\end{smallmatrix}\rangle\lvert\begin{smallmatrix}0\\0\end{smallmatrix}\rangle + \lvert\begin{smallmatrix}1\\0\end{smallmatrix}\rangle\lvert\begin{smallmatrix}0\\0\end{smallmatrix}\rangle\right) = \left(\lvert\begin{smallmatrix}0\\0\end{smallmatrix}\rangle\lvert\begin{smallmatrix}0\\1\end{smallmatrix}\rangle + \lvert\begin{smallmatrix}0\\0\end{smallmatrix}\rangle\lvert\begin{smallmatrix}1\\0\end{smallmatrix}\rangle\right) = \lvert DT\rangle$$

$$\lvert\{S,T\}\ \{D,H\}\rangle \xleftrightarrow{h_{k,k+1}^{||}} \lvert\{D,H\}\ \{S,T\}\rangle \qquad h_{k,k+1}^{||}\lvert\{ST, TS, HH, DD\}\rangle = 0$$

An innert background – a vacuum:

$$H^{||}\lvert STSTST\cdots\rangle = 0, \qquad H^{||}\lvert TSTSTS\cdots\rangle = 0, \qquad E = -\sum_k \Delta_k$$

Invariant subspaces:

- $|ST\cdots ST\mathbf{D}_k ST\cdots\rangle$: one D in a background
  $H^{||}$ moves D_k around
- $|ST\cdots \mathbf{D}_k ST\cdots \mathbf{D}_l ST\cdots\rangle$
- $\vdots$
- A fixed number **r** of **only doublons**, or **only holons**, in a background. Dimension $\binom{L}{r}$, total $4\cdot 2^L$.

Free subspace!

Possible eigenstates:

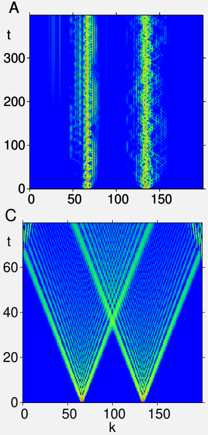
- $\Delta_j = \Delta$ and random h_j: Anderson localized doublon and holon subspaces for any h_j.

  Localization irrespective of disorder strength h, with SU(2), or without SU(2)!
- random $\Delta_k = h_k$: doublons localized, holons ballistic, irrspective of disorder strength h!
- quasiperiodic disorder: ballistic, diffusive, localized, depending on the disorder strength h.

- Exponentially large free subspaces, dim 2^L *vs.* 4^L.
- Everywhere in energy.
- **Are they relevant?** yes, they are easy to prepare

Is it stable to breaking $\mathbb{Z}_2$? simulation for 4% breaking of $\mathbb{Z}_2$, $J = 0$, $\Delta_k = h_k \in [-1, 1]$

shown is $\log_{10} |\langle d_k \rangle|$



- Two **doublons**.
- charge density localized on longer than $1/\epsilon$.

- Two **holons**.
- charge density ballistic.

(Iadecola & MZ arXiv:1811.07903)

Large invariant subspaces in chaotic and integrable models

- Hubbard-like system (ladder), $\mathbb{Z}_2$ symmetry and hopping along legs.
- $\mathbb{Z}_2$ can "stabilize" localization despite $SU(2)$.

  any possibile "proof" of no-MBL in SU(2) must take that into account
- In $\mathbb{Z}_2$ quasiperiodic Hubbard (experiment) there are exact localized eigenstates at any disorder h!
- Can engineer any other transport in the invariant subspaces.
- "Chaotic" model: ETH does not hold for all eigenstates.

Postdoc position starting 2020:

topic is Many-body transport engineering

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