# Prethermalization and Thermalization in Isolated Quantum Systems

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K. Mallayya, MR, and W. De Roeck, Phys. Rev. X **9**, 021027 (2019).
K. Mallayya and MR, arXiv:1907.04261.

# Outline

1. Introduction
   - Prethermalization (theory and experiments)
2. Prethermalization-thermalization: Universal two-step phenomena
   K. Mallayya, MR, and W. De Roeck, PRX **9**, 021027 (2019)
   - Setup and numerical experiments
   - General considerations and analytical results
   - Unperturbed strongly interacting quantum-chaotic model
   - Unperturbed strongly interacting integrable model
3. Prethermalization-thermalization in periodically driven systems
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   - Unperturbed strongly interacting quantum-chaotic model
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# Prethermalization & thermalization (theory)

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J. Berges, Sz. Borsányi, and C. Wetterich, PRL **93**, 142002 (2004).

**Sudden turn on of interactions in the Hubbard model**

M. Moeckel and S. Kehrein, PRL **100**, 175702 (2008).
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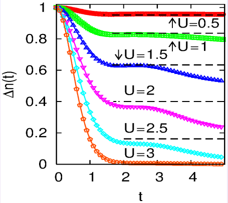
M. Kollar, F. A. Wolf, and M. Eckstein, PRB **84**, 054304 (2011).



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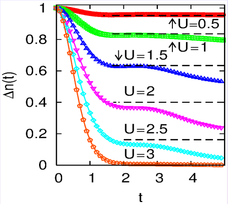
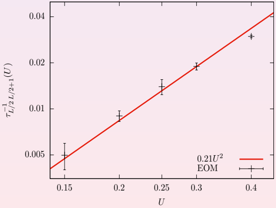



**Quenches in weakly interacting spinless fermions models (EOM)**

Essler, Kehrein, Manmana, and Robinson, PRB **89**, 165104 (2014).
Bertini, Essler, Groha, and Robinson,
PRL **115**, 180601 (2015); PRB **94**, 245117 (2016). $\Longrightarrow$

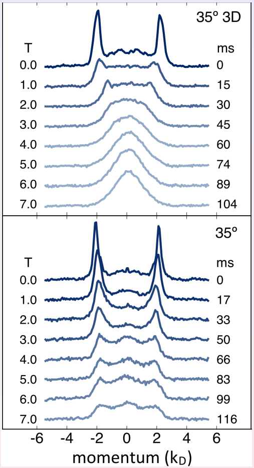
Rates $\propto U^2$




**Quenches in weakly interacting models (time-dependent GGEs)**

M. Stark and M. Kollar, arXiv:1308.1610.
D'Alessio, Kafri, Polkovnikov, and MR, Adv. Phys. **65**, 239 (2016).

# Prethermalization & thermalization (QNC Dysprosium)

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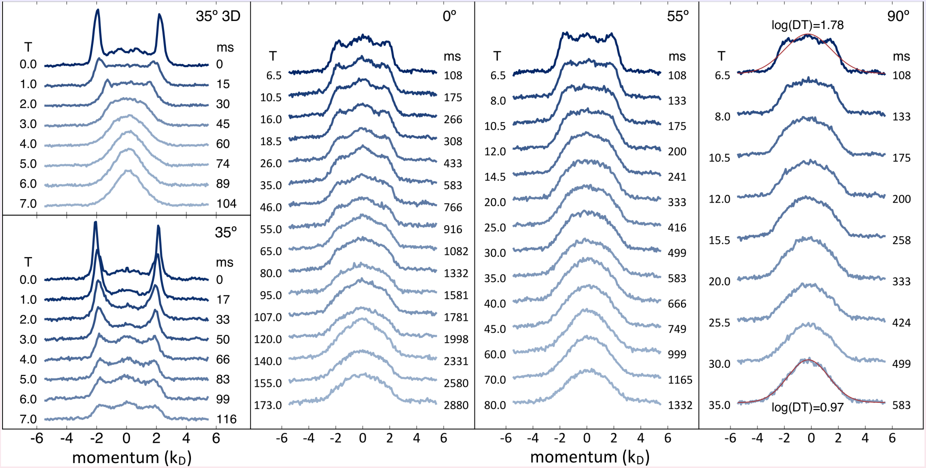


$$\text{DT} = \sqrt{\sum_k \left[n(k) - n_G(k)\right]^2}$$

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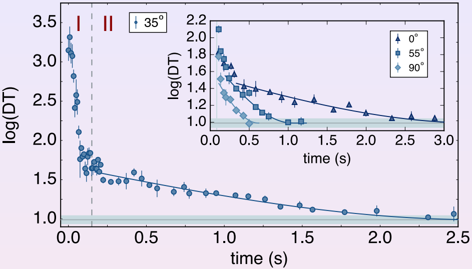

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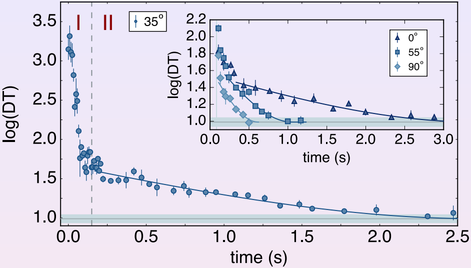
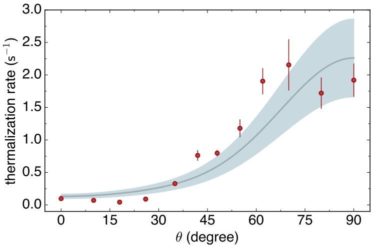



Consistent with FGR $\propto U_{\text{total}}^2(\theta)$



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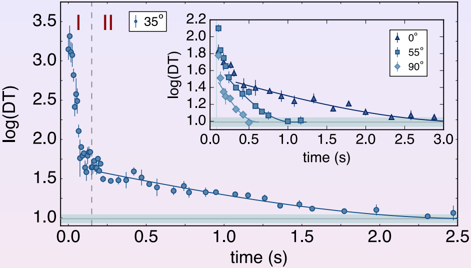


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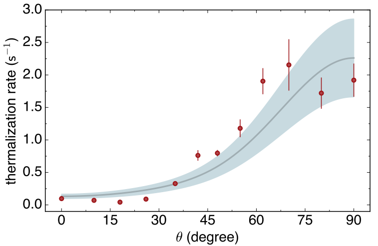
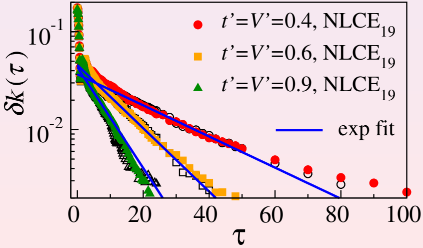



## Breaking integrability in the XXZ model (NLCEs)




K. Mallayya and MR, PRL **120**, 070603 (2018).

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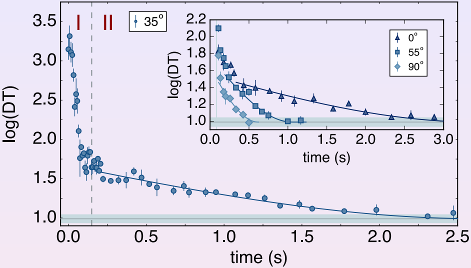


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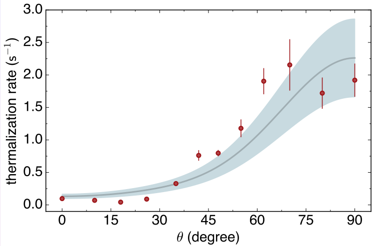


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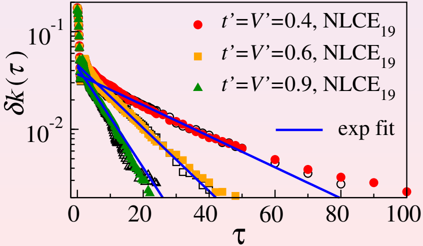


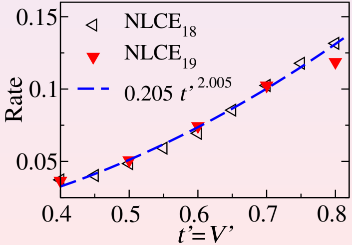


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# Outline

# General setup and specific model Hamiltonian

We have in mind Hamiltonians of the form: $\hat{H} = \hat{H}_0 + g\hat{U}$ with $g \ll 1$

- $\hat{H}_0$ (integrable or not) has at least one conserved quantity $\hat{Q}$, $[\hat{H}_0, \hat{Q}] = 0$
- $[\hat{U}, \hat{Q}] \neq 0 \Rightarrow [\hat{H}, \hat{Q}] \neq 0$

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Numerical experiments (NLCE): Quenches with hard-core bosons in 1D

$$\hat{H}_0 = \sum_i \Big[ -t \left( \hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.} \right) + V \left( \hat{n}_i - \frac{1}{2} \right)\left( \hat{n}_{i+1} - \frac{1}{2} \right) \\ -t' \left( \hat{b}_i^\dagger \hat{b}_{i+2} + \text{H.c.} \right) + V' \left( \hat{n}_i - \frac{1}{2} \right)\left( \hat{n}_{i+2} - \frac{1}{2} \right) \Big]$$

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Two perturbations: $g_\alpha \hat{U}_\alpha$, $\alpha = 1, 2$, with $[\hat{U}_\alpha, \hat{N}] \neq 0$

$$g_1 \hat{U}_1 = g_1 \sum_i \left[ \hat{b}_i + \frac{1}{2} \left( \hat{b}_i \hat{b}_{i+1} - \hat{b}_i^\dagger \hat{b}_{i+1} \right) + \text{H.c.} \right],$$
$$g_2 \hat{U}_2 = g_2 \sum_i \left( \hat{b}_i + \frac{1}{2} \hat{b}_i \hat{b}_{i+1} + \text{H.c.} \right).$$

# Outline

# ‘Mori-Zwanzig’ approach

- Analytical results obtained within ‘Mori-Zwanzig’ approach (describe systems with slow variables $(e_0, q)$ that can be separated from fast ones)
  - Liouville superoperator $\mathcal{L} = -i[\hat{H}, \cdot]$,
    split $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ where $\mathcal{L}_0 = -i[\hat{H}_0, \cdot]$ and $\mathcal{L}_1 = -ig[\hat{V}, \cdot]$
  - Projection $\mathcal{P}$: $\hat{\rho} \to \hat{\rho}_{e_0,q}$
  - Rewrite $\mathcal{P}$-projected Liouville equation $\partial_\tau \mathcal{P}\hat{\rho}(\tau) = \mathcal{P}\mathcal{L}\hat{\rho}(\tau)$
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- A similar formulation was used for open quantum systems in:
  - Z. Lenarčič, F. Lange, and A. Rosch, "Perturbative approach to weakly driven many-particle systems in the presence of approximate conservation laws", PRB **97**, 024302 (2018).
  - F. Lange, Z. Lenarčič, and A. Rosch, "Time-dependent generalized Gibbs ensembles in open quantum systems", PRB **97**, 165138 (2018).

# Assumptions and analytical results

- Let $\tau^*$ be the (generalized) thermalization time of the unperturbed dynamics, namely, at times $\tau \gtrsim \tau^*$ observables are described by the thermal density matrix $\hat{\rho}_{e_0,q}$, with $(e_0, q) = (\langle \hat{h}_0 \rangle_{\hat{\rho}_I}, \langle \hat{q} \rangle_{\hat{\rho}_I})$, or by a GGE.

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- Main results, thermalization under perturbed ($\hat{H}$) dynamics
  - For $\tau \gg \tau^*$ observables are well described by intermediate equilibrium states of $\hat{H}_0$, $\text{Tr}[\hat{\rho}(\tau)\hat{O}] \approx \langle \hat{O} \rangle_{e_0,q(\tau)}$, where $\partial_\tau q(\tau) = d[e_0, q(\tau)]$ and $d[e_0, q(\tau)]$ is given by Fermi's golden rule. Corrections from $\langle \hat{O} \rangle_{e_0,q(\tau)}$ are generally described by first order perturbation theory.

# Outline

# Dynamics of the particle filling (slow variable)

Quenches: $\beta_I = 0.1$, $\mu_I = 2$, $t_I = 0.5$, $V_I = 1.5$, $t' = V' = 0.7$, $g_1 = 0$
$\Longrightarrow t = V = 1.0$, $t' = V' = 0.7$, $g_1 \in [0.03, 0.12]$

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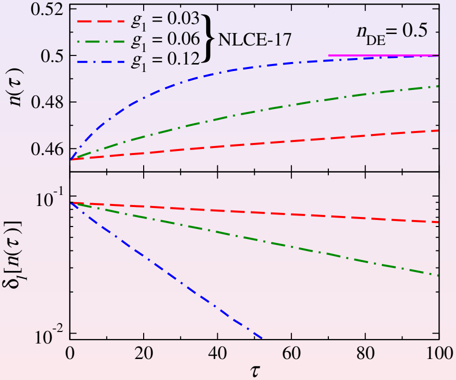


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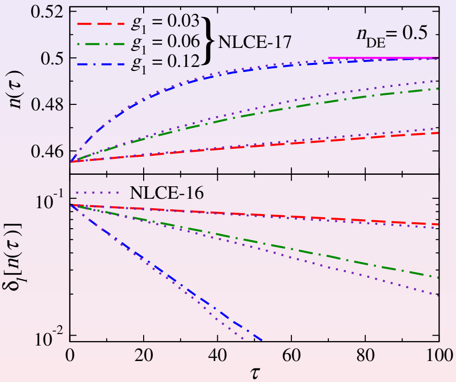


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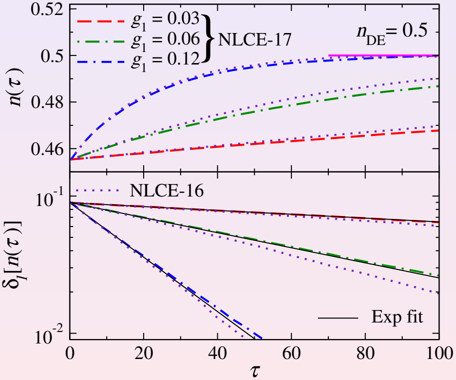


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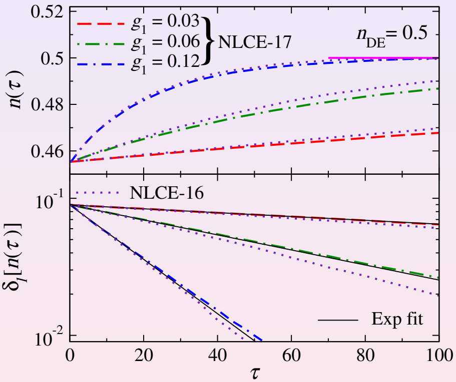


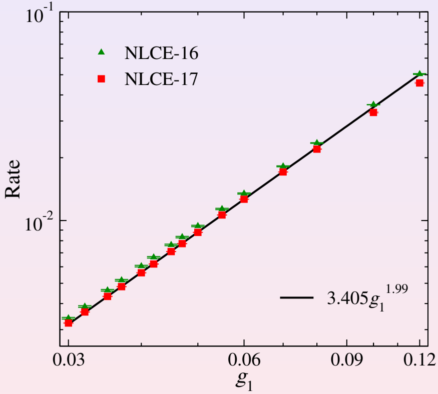


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Fermi's golden rule (exact diag.):

$$\dot{n}(\tau) = \frac{2\pi g_1^2}{L} \sum_{i,j} \delta(E_j^0 - E_i^0)\,(N_j - N_i)\,P_i^0(\tau) \times \left|\langle E_j^0|\hat{U}_1|E_i^0\rangle\right|^2,$$

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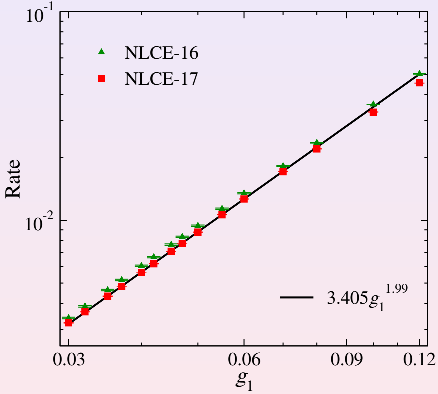
Thermalization rate

$$\Gamma^{\text{Fermi}}(g_1) = -\frac{\dot{n}(\tau)}{n(\tau) - 0.5}$$



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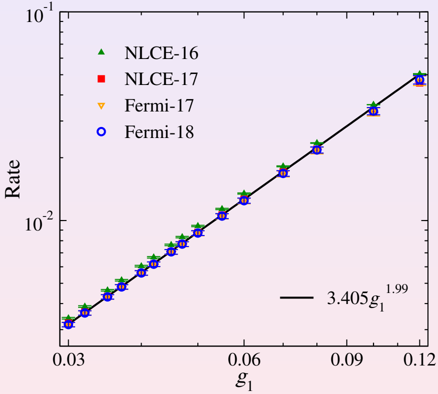
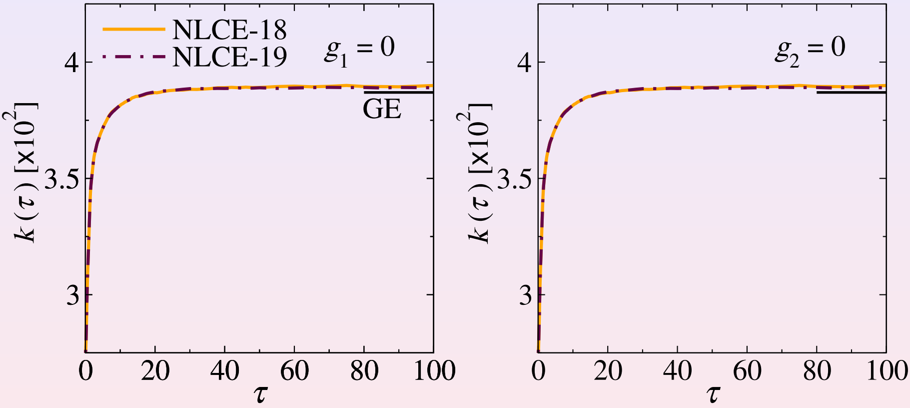


# Dynamics of n.n. one-body $\hat{K} = \sum_i (\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i)$

One-body nearest-neighbor correlations have dynamics even when $g_\alpha = 0$

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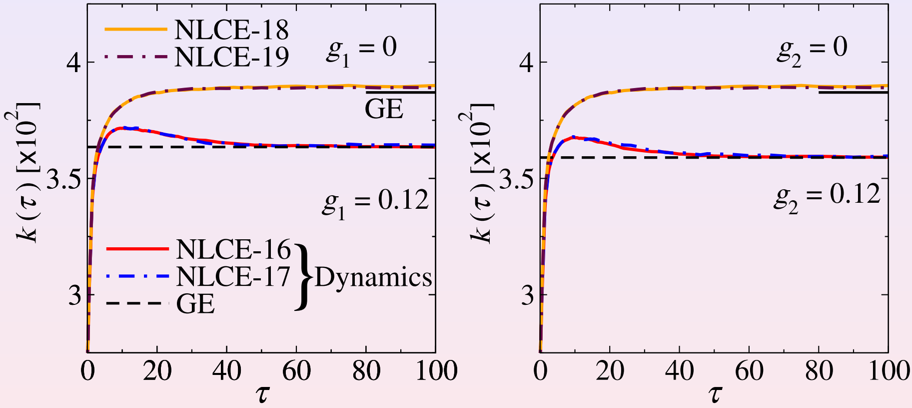


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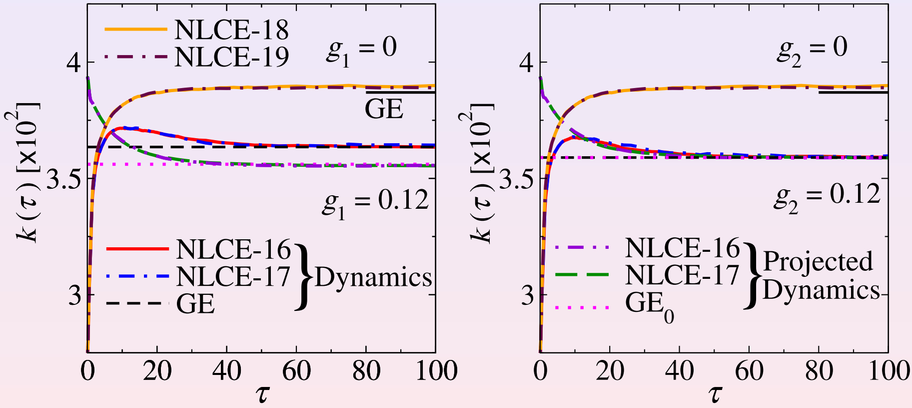


$$g_1\hat{U}_1 = g_1 \sum_i \left[\hat{b}_i + \frac{1}{2}\left(\hat{b}_i\hat{b}_{i+1} - \hat{b}_i^\dagger\hat{b}_{i+1}\right) + \text{H.c.}\right] \qquad g_2\hat{U}_2 = g_2 \sum_i \left(\hat{b}_i + \frac{1}{2}\hat{b}_i\hat{b}_{i+1} + \text{H.c.}\right)$$

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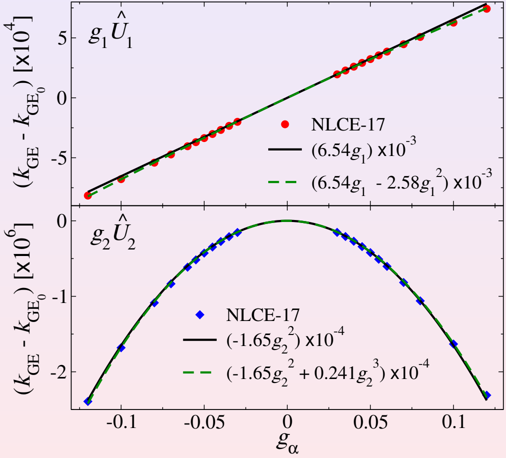
Correction as $\tau \to \infty$ vs $g_\alpha$



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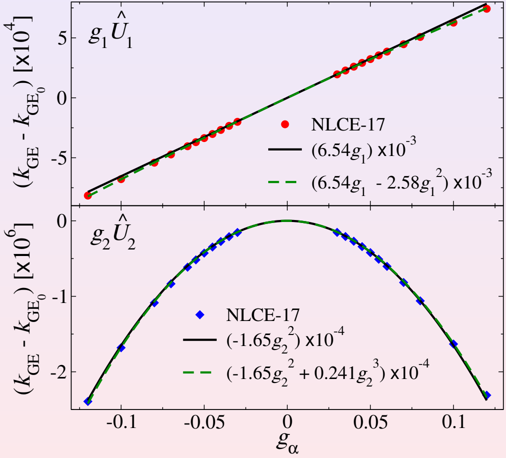



The first order correction is:

$$i g_\alpha \int_0^\infty \mathrm{d}s\, \mathrm{Tr}\left(\left[\hat{U}_\alpha(-s), \hat{K}\right] \hat{\rho}_0(\tau)\right)$$

where

$$\hat{U}_\alpha(-s) = e^{-is\hat{H}_0}\hat{U}_\alpha e^{is\hat{H}_0}$$

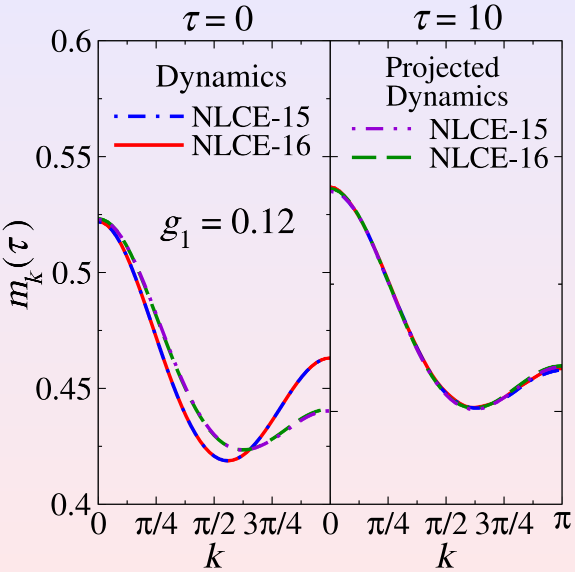
$\hat{\rho}_0(\tau)$ is the projected $\hat{\rho}(\tau)$

$\hat{K}$ and $\hat{\rho}_0(\tau)$ are block diagonal in the particle number basis

$\hat{b}_i^\dagger \hat{b}_{i+1} \implies O(g_1) \neq 0$

lack thereof $\implies O(g_2) = 0$.

# Dynamics of the momentum distribution function



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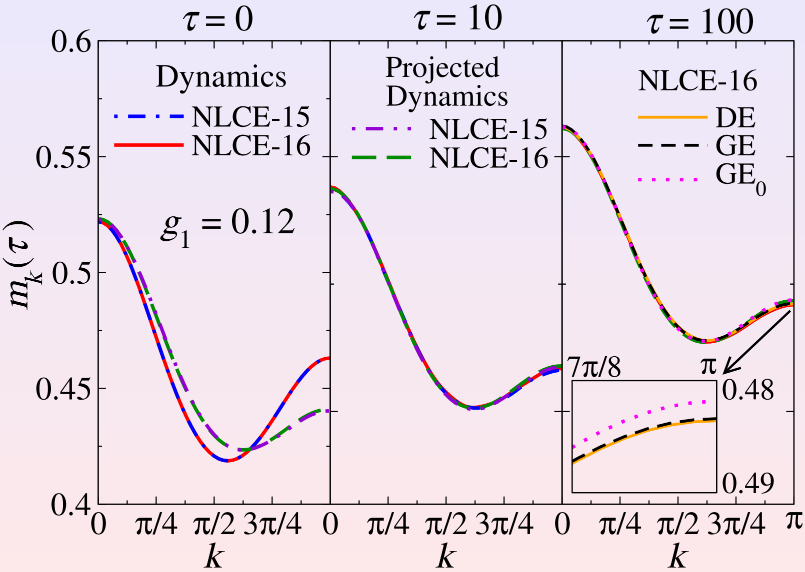

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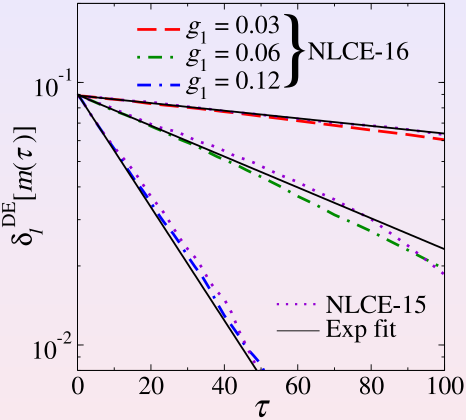


$$\delta_l^{\mathsf{DE}}\left[m(\tau)\right] = \frac{\sum_k \left| m_k^l(\tau) - m_k^{l,\mathsf{DE}} \right|}{\sum_k m_k^{l,\mathsf{DE}}}$$

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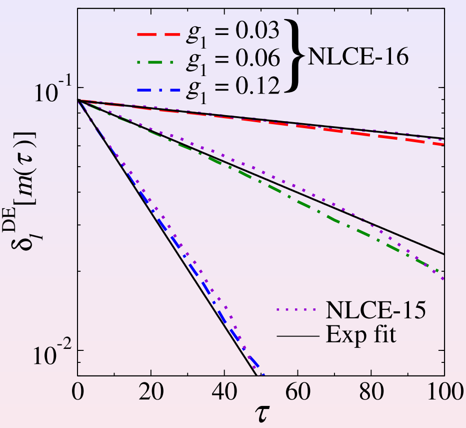


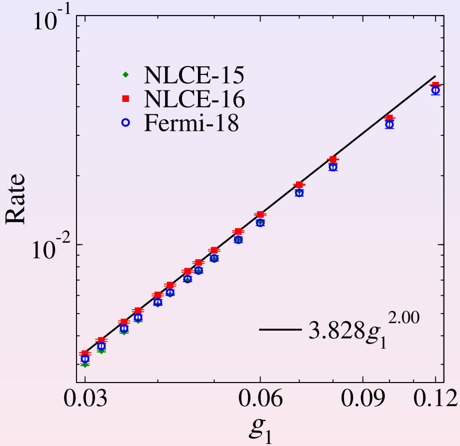


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# Outline

# Dynamics of the particle filling (slow variable)

Quenches: $\beta_I = 0.1$, $\mu_I = 2$, $t_I = 0.5$, $V_I = 1.5$, $t' = V' = 0$, $g_2 = 0$
$\Longrightarrow t = V = 1.0$, $t' = V' = 0$, $g_2 \in [0.03, 0.12]$

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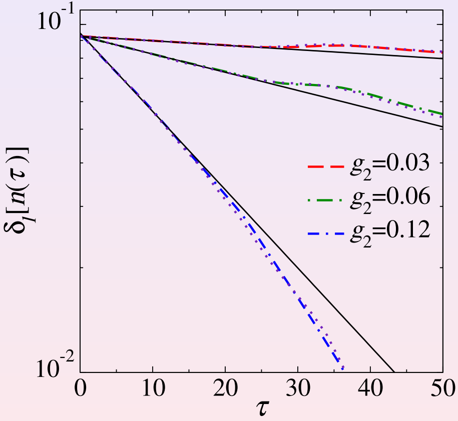


$$\delta_l[n(\tau)] = \left| \frac{n_l(\tau) - 0.5}{0.5} \right|$$

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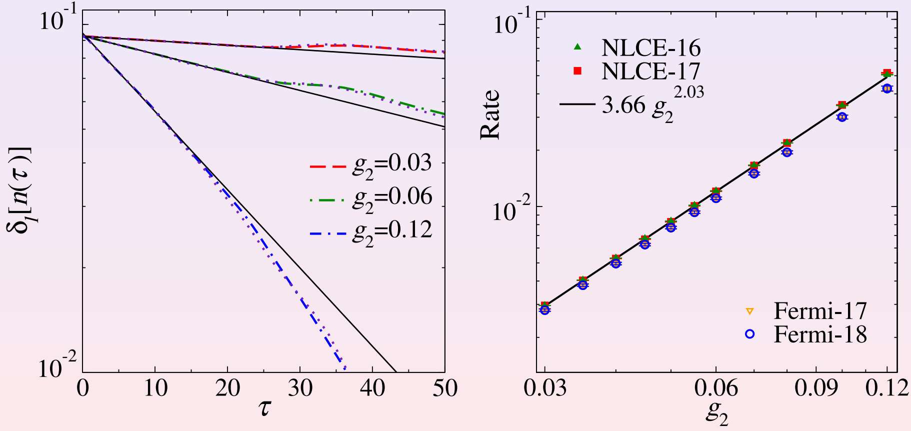


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# Outline

# General setup and specific model Hamiltonian

We have in mind time-periodic Hamiltonians (period $T = 2\pi/\Omega$) of the form:

$\hat{H}(\tau) = \hat{H}_0 + g(\tau)\hat{K},$ with $g(\tau) = g(\tau + T) \ll 1$ and $\overline{g(\tau)} = 0$

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Numerical experiments: Hard-core bosons in 1D lattices

$$\hat{H}_0 = \sum_i \Bigg[\left(-t\,\hat{b}_i^\dagger \hat{b}_{i+1} - t'\,\hat{b}_i^\dagger \hat{b}_{i+2} + h\,\hat{b}_i^\dagger\right) + \text{H.c.}$$

$$+V\left(\hat{n}_i - \frac{1}{2}\right)\left(\hat{n}_{i+1} - \frac{1}{2}\right) + V'\left(\hat{n}_i - \frac{1}{2}\right)\left(\hat{n}_{i+2} - \frac{1}{2}\right)\Bigg]$$

$$\hat{K} = -\sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}\right)$$

# Dynamics of the energy [defined using $\hat{H}_0 = \overline{\hat{H}(\tau)}$]

Quench + drive: $t_I = 0.5$, $V_I = 2.0$, $t' = V' = 0.8$, $h = 1.0$

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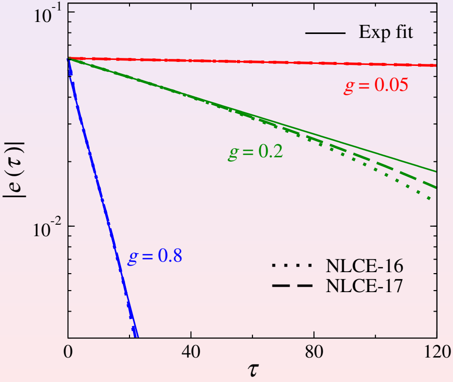

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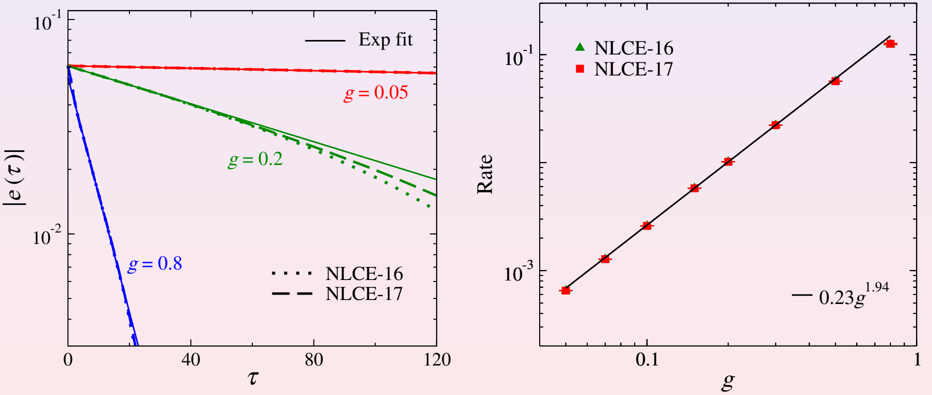

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$$\dot{E}(\tau) = \sum_{m>0} \dot{E}_m(\tau)\,, \quad \text{where}$$

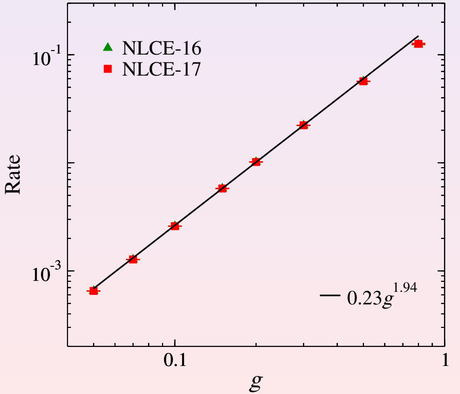
$$\dot{E}_m(\tau) = 2\pi g_m^2 \sum_{i,f} \delta(E_f^0 - E_i^0 \pm m\Omega)$$

$$\times (E_f^0 - E_i^0) P_i^0(\tau)\, |\langle E_f^0|\hat{K}|E_i^0\rangle|^2$$

$$P_i^0(\tau) = \langle E_i^0|\hat{\rho}(\tau)|E_i^0\rangle \quad \text{(DE)}$$

Heating rates: $\Gamma(\tau) = \sum_{m>0} \Gamma_m(\tau)$

$$\Gamma_m(g) = -\frac{\dot{E}_m(\tau)}{E_\infty - E(\tau)}$$



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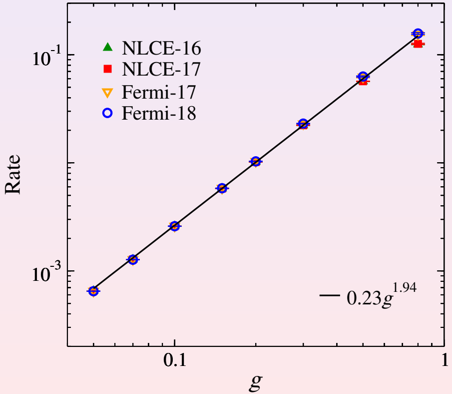

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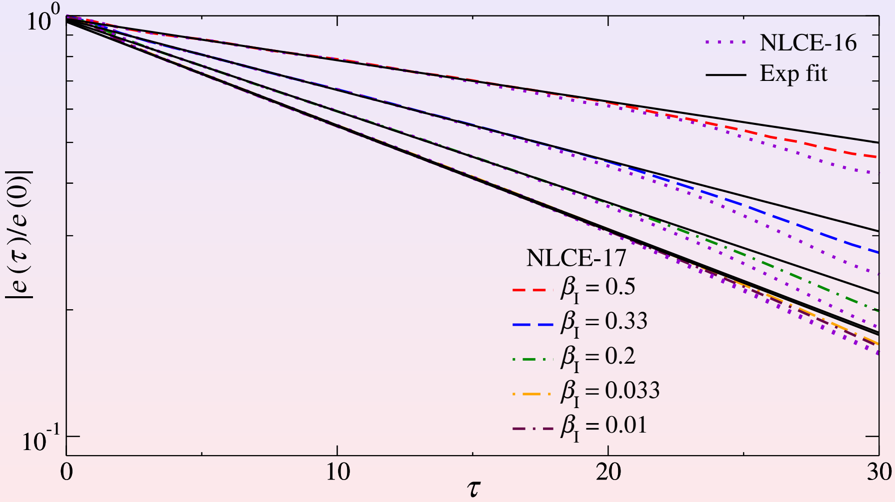

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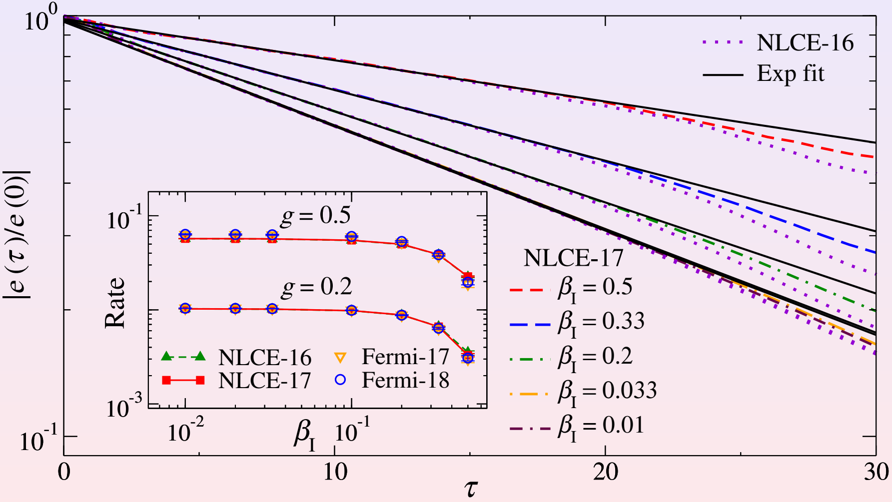

# Heating rates and eigenstate thermalization

Heating rates: $\Gamma_m = -\frac{\dot{E}_m(\tau)}{E_\infty - E(\tau)}$, where

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Eigenstate thermalization hypothesis

M. Srednicki, J. Phys. A **32**, 1163 (1999); L. D'Alessio *et al.*, Adv. Phys. **65**, 239 (2016).

$$O_{\alpha\beta} = O(E)\delta_{\alpha\beta} + [D(E)]^{-1/2} f_O(E,\omega) R_{\alpha\beta}$$

where $E \equiv (E_\alpha + E_\beta)/2$, $\omega \equiv E_\alpha - E_\beta$, $D(E)$ is the density of states at energy $E$, and $R_{\alpha\beta}$ is a random number with zero mean and unit variance.

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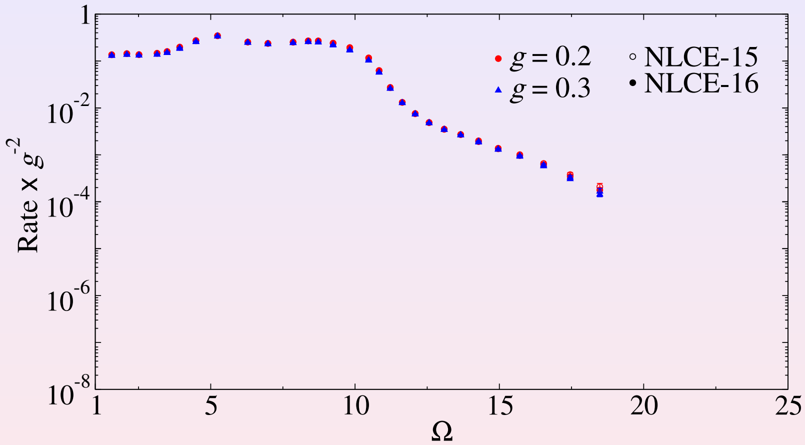
At high temperatures [$\beta(\tau) \ll 1$], one obtains:

$$\Gamma_m = \frac{2\pi(m\Omega g_m)^2}{\mathrm{Tr}(\hat{H}_0^2)} \int_{E_{\mathrm{min}}+m\Omega/2}^{E_{\mathrm{max}}-m\Omega/2} dE\,|f_K(E,m\Omega)|^2 \frac{D(E+m\Omega/2)D(E-m\Omega/2)}{D(E)}$$

# Heating rates and eigenstate thermalization

Results for $\beta_I = 0.033$, $\mu_I = 0$, and different values of $g$.



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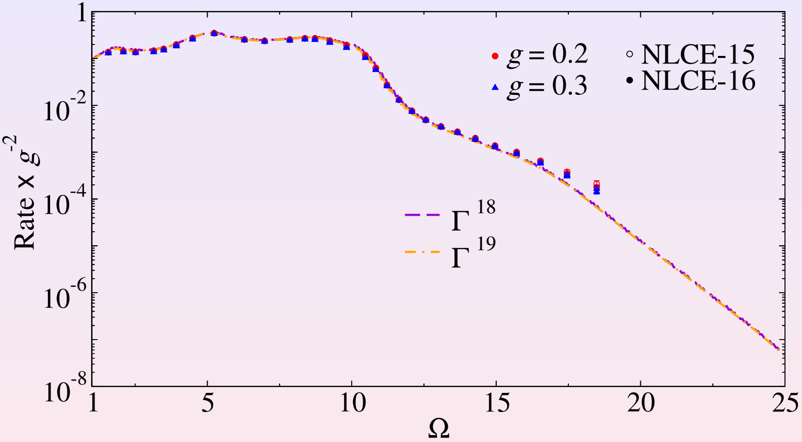

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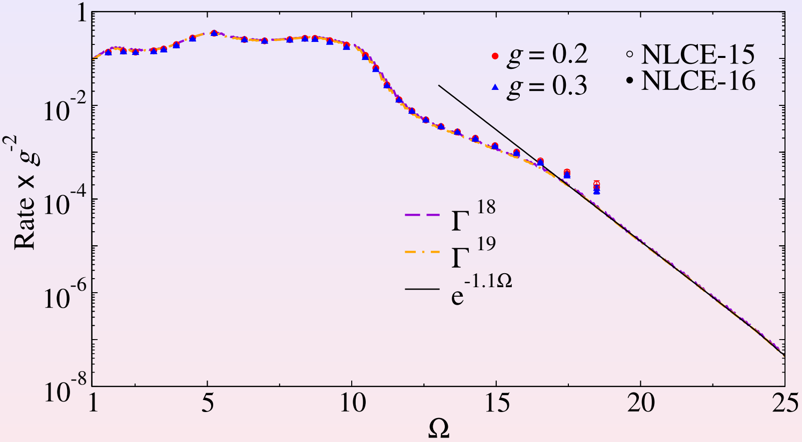

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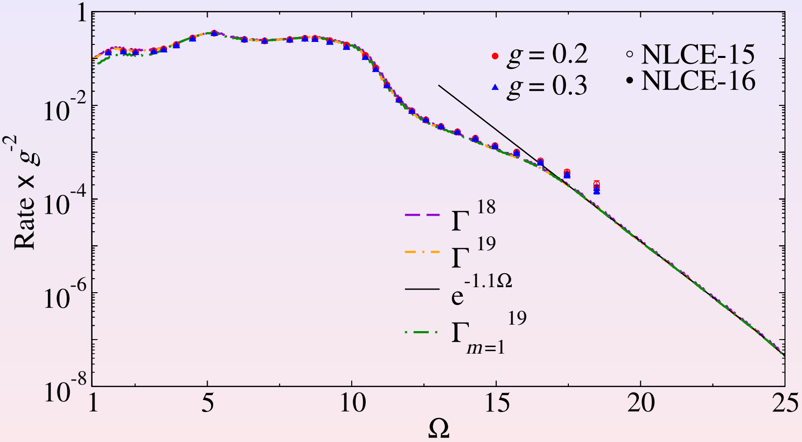

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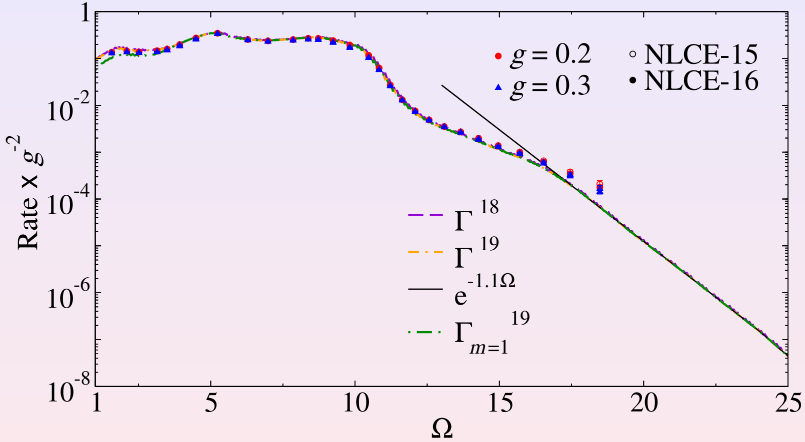


In the thermodynamic limit, since $E$ is extensive but $\Omega$ is not, one obtains:

$$\Gamma^{\infty}_{m=1} = \frac{2\pi(\Omega g_1)^2}{\text{Tr}(\hat{H}_0^2)} |f_K(E_\infty, \Omega)|^2 Z(\beta = 0)$$

# Outline

# Heating rates and $f_O(E, \omega)$

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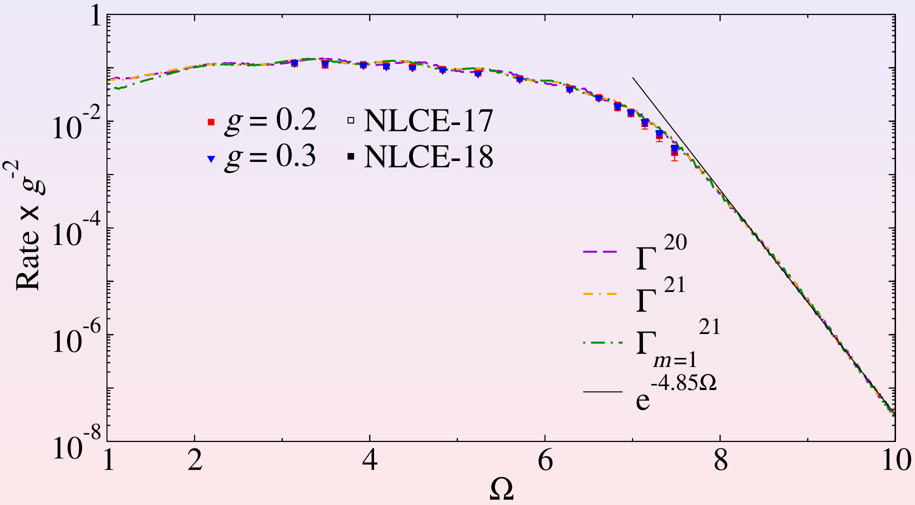

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## Collaborators

- Wojciech De Roeck (KULeuven)
- Sarang Gopalakrishnan (CUNY)
- Ben Lev & group (Stanford)
- *Krishna Mallayya (PSU)*

## Support