Dynamical Scarring of an Interacting Floquet System: Resonance vs Emergent Conservation Laws

THERMALIZATION, MANY BODY LOCALIZATION AND HYDRODYNAMICS
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Ergodicity Expectations in Closed Many-body Quantum Systems

In the level of Eigenstates:

- **Eigenstate Thermalization Hypothesis (ETH):** (roughly) Expectation values of local operators on each individual eigenstate of an interacting generic Hamiltonian is equal to their microcanonical averages around the eigenstate. Or, in other words, expectation values of all local observables are slowly varying, smooth functions of energy.

In Dynamics:

- **Quench:** Quenching a narrow (in energy and local observables) wave-packet over the eigenstates of the quench Hamiltonian produces a ***thermal*** diagonal ensemble, if the quench Hamiltonian is generic and interacting.
  Only Constraint is Energy Conservation ⟹ Locally Gibb's Distribution

- **Periodic drive:**

No apparent Constraint/Conservation ⟹ Locally Infinite-T like scenario

Ergodicity Exceptions in Closed Many-body Quantum Systems

In the level of Eigenstates:

- **Many-body Localization (MBL) in Disordered Systems:** Localized (lowly entangled) eigenstates in the middle of the spectrum, violates ETH. **Random** local conserved quantities (*l*-bits).

- **Scars:** Measure zero eigenstates embedded all across the spectrum which does not satisfy ETH – have low (e.g., area law entanglement) but finite energy density.

In Dynamics:

- **Stability of MBL Under Periodic Drive:** Above a threshold disorder strength and drive frequency, an MBL state remains stable under periodic drive, and does not heat up to locally infinite-T like states.

- **Stability of Scars Under Periodic Drive:** Scar states can be stable under periodic drive under certain drive conditions (Arnab Sen, next Monday talk).

- **Our Scenario**: Emergence of constraints ***due*** to external drive in Floquet systems.

Floquet in a Nutshell

In general doesn't commute with any local operator

T-Periodic Hamiltonian $H(t+T) = H(t)$

Time-Evolution Operator: $\hat{U}(0,t)|\psi(0)\rangle = |\psi(t)\rangle$

Effective Hamiltonian (time-independent): $H_{eff} : \hat{U}(0,T) = e^{-iH_{eff}T}$

Observation Instants: $t = nT$

Stroboscopic Wave-function: $|\psi(nT)\rangle = e^{-iH_{eff}nT}|\psi(0)\rangle; \quad \left[\hat{U}(0,T)\right]^n = e^{-iH_{eff}nT}$

Evolution due to the time-independent Hamiltonian H_{eff} till time nT

- The eigenstates of H_{eff} (Floquet States) play the same role as that played by the Energy-Eigenstates of a time-independent drive Hamiltonian in a quench, provided we observe at stroboscopically.

Floquet-Diagonal Ensemble: The Quench Problem

- $\{|\mu_i\rangle\} \Rightarrow$ Floquet Eigenstates corresponding to $U(0,T)$
- Initial State $|\psi(0)\rangle = \sum_i c_i |\mu_i\rangle$
- Local Operator : $\hat{O}$

$$\begin{aligned}
\lim_{n\to\infty} \langle\psi(nT)|\hat{O}|\psi(nT)\rangle &= \lim_{n\to\infty} \sum_i c_i c_j^* e^{-i(\mu_i-\mu_j)nT} \langle\mu_j|\hat{O}|\mu_i\rangle \\
&\approx \sum_i |c_i|^2 \langle\mu_i|\hat{O}|\mu_i\rangle \\
&\equiv tr[\hat{\rho}_{Diag}\hat{O}], \text{ where } \hat{\rho}_{Diag} = \sum_i |c_i|^2 |\mu_i\rangle\langle\mu_i|
\end{aligned}$$

The Floquet Diagonal Ensemble (DEA) Average.

The Floquet Diagonal Ensemble (the infinite time limit)

(P Reimann, PRL **101**, 2008).

Central Finding:
Dynamical Scarring and Emergence of a Local Conserved Quantity

Ergodic System + 

(Strong Periodic Drive)

$$H(t);\;\; H(t+T) = H(t)$$

⇒ Emergence of New Constraints not present in the undriven system,
at certain points/regimes in the Drive Parameter Space
(Scar points/regimes).

- The ergodic system fails to thermalize to infinite-T like state under periodic drive at the scar points/regimes
- We show this happens because of emergence of a local conserved quantity.

A Concrete Example:
STRONG Integrable Drive + Non-Integrability Static Part

(Asmi Haldar., R. Moessner, A.D., PRB 2018)

$$H(t) = H_0(t) + V, \text{where}$$

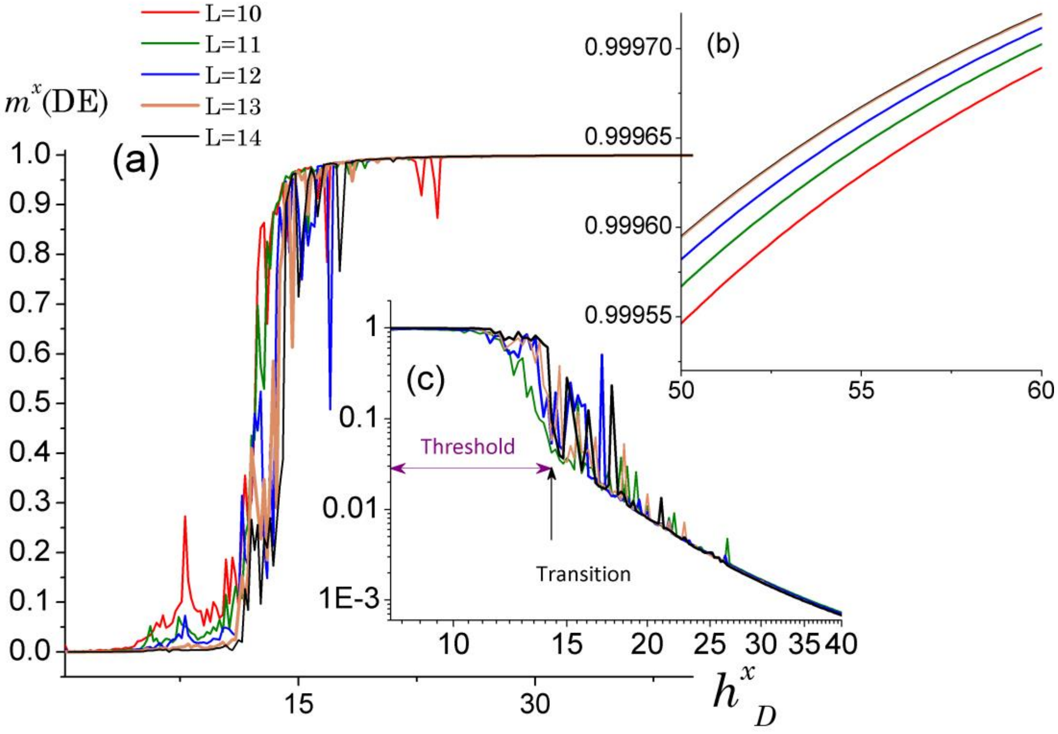
$$H_0(t) = H_0^x + \text{Sgn}(\sin(\omega t))H_D, \text{with}$$

$$H_0^x = -\sum_{n=1}^{L} J\sigma_n^x\sigma_{n+1}^x + \sum_{n=1}^{L} \kappa\sigma_n^x\sigma_{n+2}^x - h_0^x\sum_{n=1}^{L}\sigma_n^x,$$

$$H_D = h_D^x\sum_{n=1}^{L}\sigma_n^x, \text{and}$$

$$V = h^z\sum_{n=1}^{L}\sigma_n^z,$$

The Threshold (Reminiscence of KAM)



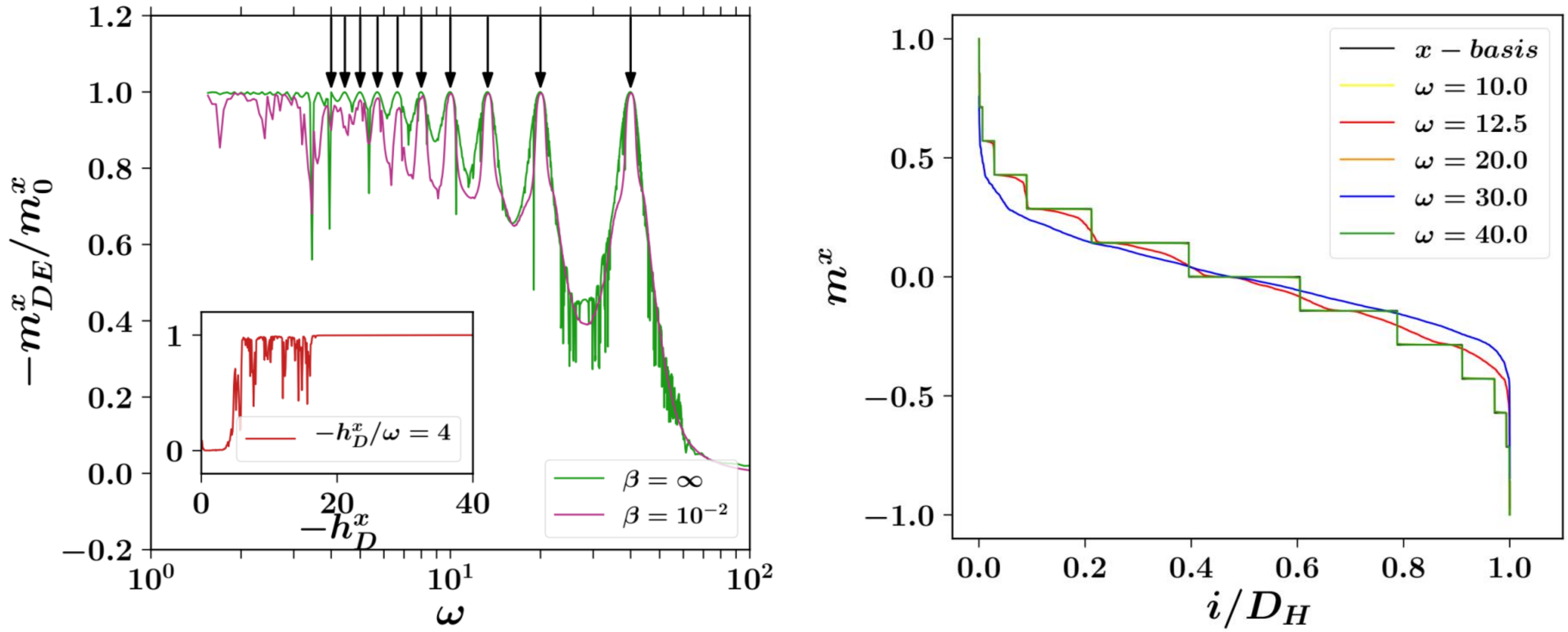
$$J = 1, \kappa = 0.7, \omega = 0.1, h_0^x = 0.01, h^z = 1.2$$

Initial State = the Ground State of *H(t=0)*

- The threshold doesn't move with the system-size.
- Finer resolution shows, m^x is more strongly frozen for larger L above the threshold.

The Scar Phenomenology: Freezing and Quasi-Conservation

$$J = 1, \kappa = 0.7\pi/3, h_0^x = e/10, h_D^x = 40, L = 14$$



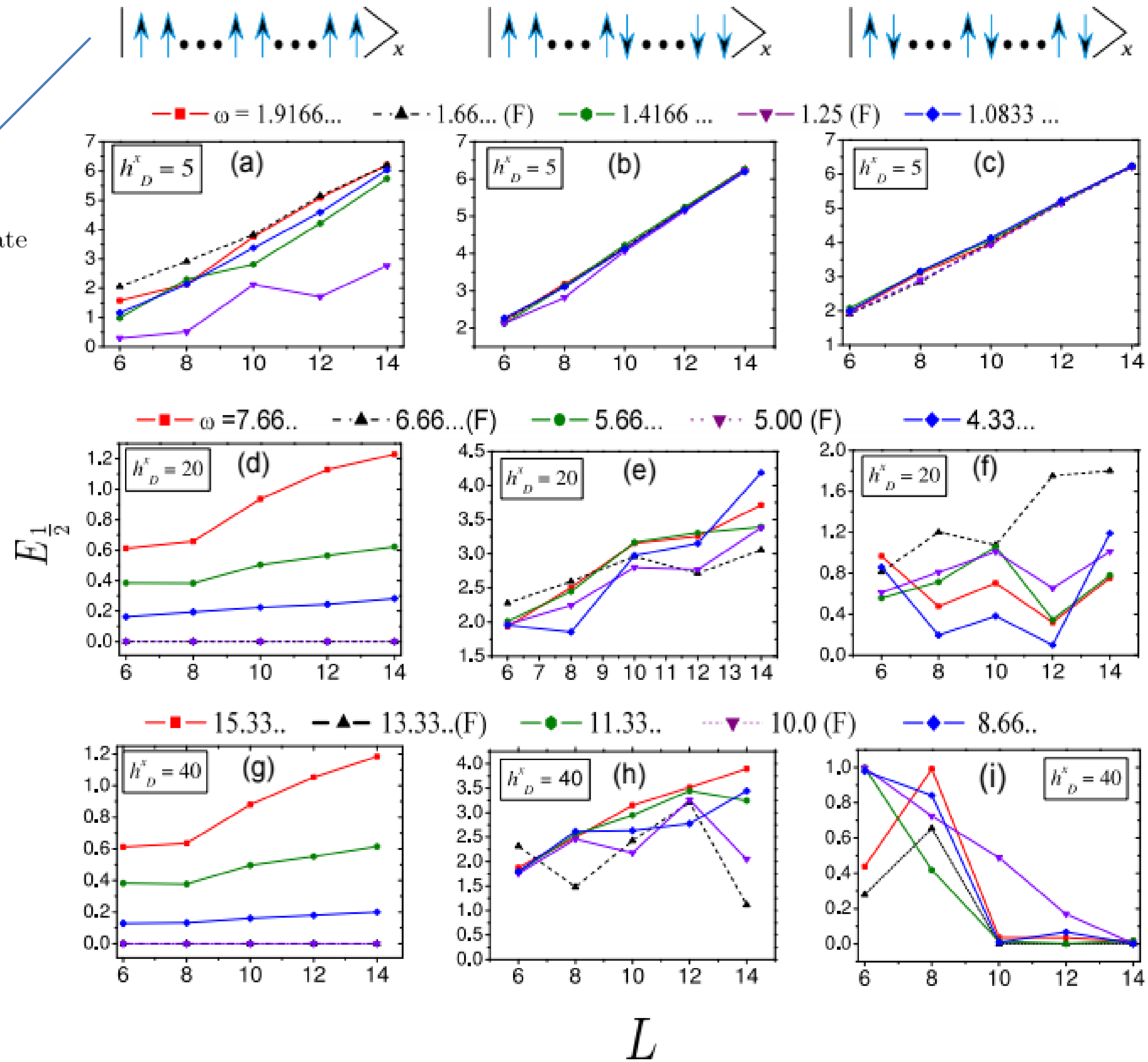
Longitudinal magnetization emerges as a quasi-conserved quantity under the Drive condition:

$$\boxed{h_D^x = k\omega}$$

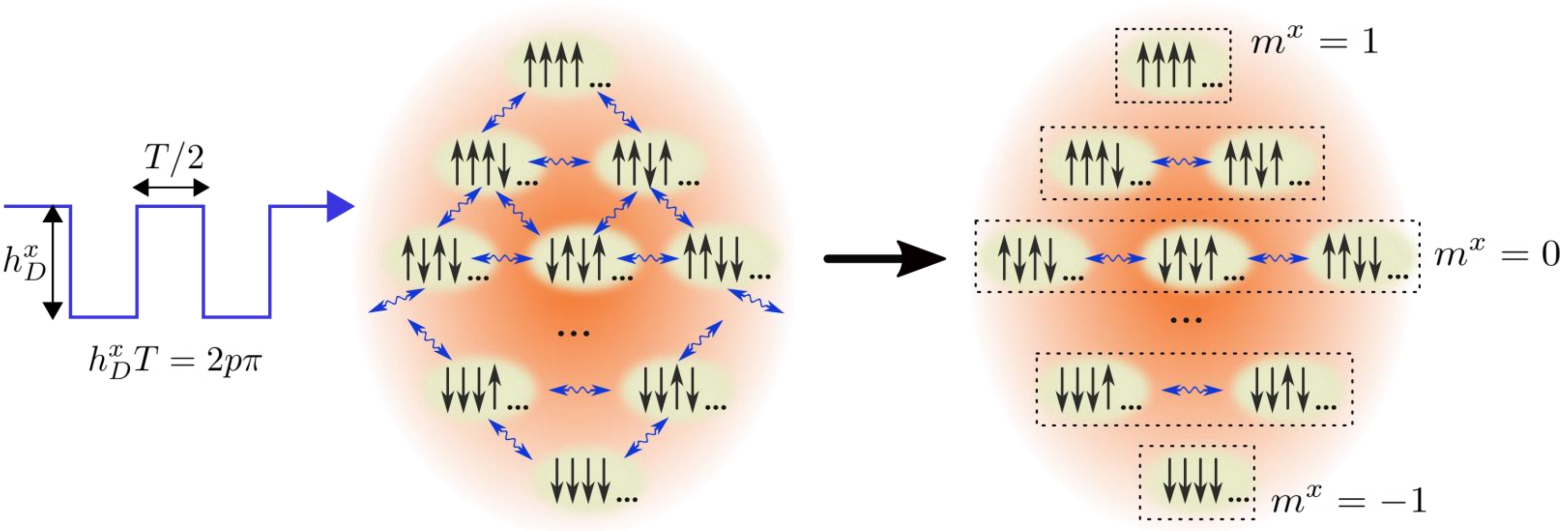
This happens for a very broad range of ω

Dynamics of the Unentangled Eigenstates of mx: Growth of Entanglement Entropy

$x-$ basis state
Simultaneous Eigenstate
of all σ_i^x



Interpretation of the Emergent Conserved Quantity



$$[m^x, H_{eff}] \approx 0$$

But

$$H_{eff} \not\approx H_D$$

Note that $m^x = -\frac{h_D^x}{L} H_D$

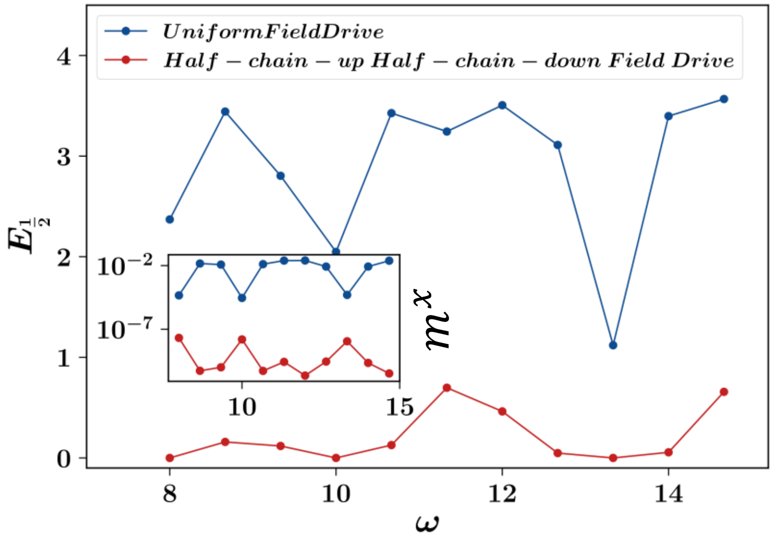
Conserved Quantity

Recipe for Freezing an Arbitrary Spin Pattern

- Choose the drive term such that the spin pattern state lies in a non-degenerate sector of H_D

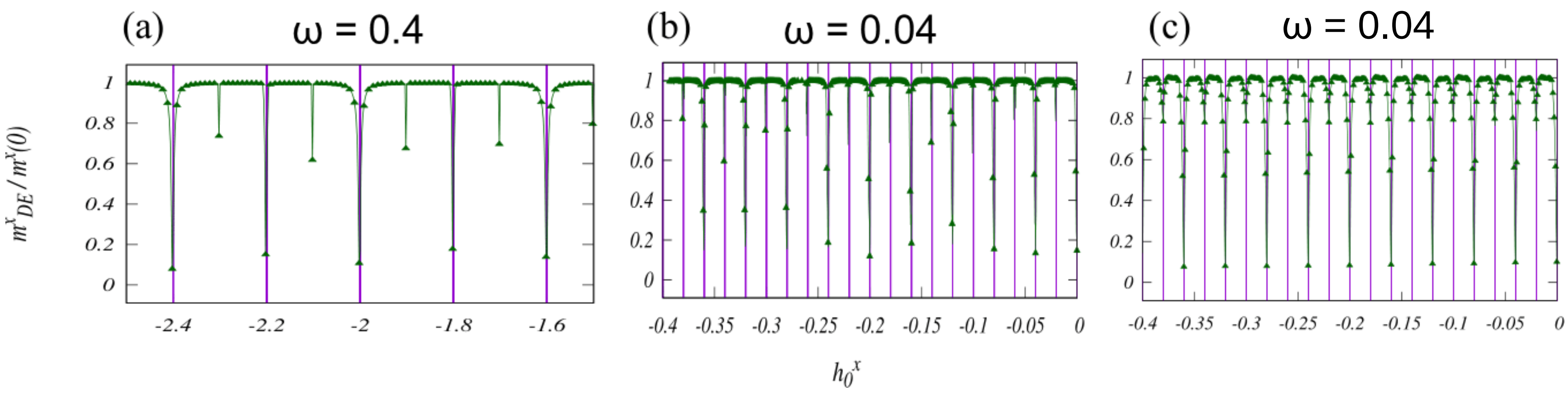
Suppose we want to freeze the x-basis state with consecutive L/2 spins up (in x-direction) and next consecutive L/2 spins down.
Then the choice of our drive Hamiltonian will be:

$$H_D = -h_D^x \sum_{i=1}^{L/2} \sigma_i^x + h_D^x \sum_{i=L/2+1}^{L} \sigma_i^x$$



Resonances

Resonances are sufficiently Isolated!



T = 0 | T = 0 | β = 0.01

- ➢ Resonances are mostly 1st order at low ω, and those are isolated!

⇩

- ➢ Resonance-free parameter regimes: No Heating!

Analytical Approaches

- Magnus Expansion in a Moving Frame
(Expected to explain the high ω regime)

- A Floquet Perturbation Theory
(Expected to explain the Resonances)

Strong-Drive Magnus Expansion in A Rotating Frame

Standard Magnus Expansion

1st order is qualitatively wrong at scar points!

$$H_{eff} = \sum_{n=0}^{\infty} H_F^{(n)} \text{where}$$

$$H_F^{(0)} = \frac{1}{T}\int_0^T dt\ H(t),$$

$$H_F^{(1)} = \frac{1}{2!(i)T}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)]$$

Contains large terms in our case

Switching to a Rotating Frame

$$|\psi_{mov}(t)\rangle = W(t)^\dagger |\psi(t)\rangle,$$

$$\hat{O}_{mov} = W(t)^\dagger \hat{O} W(t)$$

$$W(t) = \exp\left[-i\int_0^t dt' H_D \times r(t')\right]$$

This is chosen to cancel out the large term exactly

$$H_{mov} = W(t)^\dagger H(t) W(t) - iW(t)^\dagger \partial_t W$$

$$H_{mov} = H_0^x - h^z \sum_i \left[\cos(2\theta)\sigma_i^z + \sin(2\theta)\sigma_i^y\right],$$

$$\theta(t) = -h_D^x \int_0^t dt' \mathrm{sgn}(\sin \omega t')$$

ME →

The large number goes into the phase

The Effective Hamiltonian in the Moving Frame

$$H_F^{(0)} = H_0^x - \frac{h^z}{2h_D^x T}\left[\sin\left(2h_D^x T\right)\sum_i \sigma_i^z + \left(1+\cos\left(2h_D^x T\right) - 2\cos\left(h_D^x T\right)\right)\sum_i \sigma_i^y\right]$$

$$\underbrace{\qquad}_{\Sigma_0}$$

$$H_F^{(1)} = \frac{1}{i\ (2T)}\ (\Sigma_1\ +\ \Sigma_2\ +\ \Sigma_3)$$

$$\Sigma_1 = -\ h^z\ [H_0^x,\ \mathcal{S}_z]\left\{\frac{1}{4\ (h_D^x)^2}\left(2\cos\left(2h_D^x T\right) - \cos\left(h_D^x T\right) - 1\right) + \frac{3\ T}{4\ h_D^x}\sin\left(h_D^x T\right)\ +\ \frac{T}{4\ h_D^x}\sin\left(h_D^x T\right)\right\}.$$

$$\Sigma_2 = -\ h^z\ [H_0^x,\ \mathcal{S}_y]\left\{\left[\frac{1}{2\ (h_D^x)^2}\ +\ \frac{T^2}{4}\right]\sin\left(h_D^x\ T\right)\ +\ \frac{T}{4\ h_D^x}\left[1-\cos h_D^x T\right]\right\}.$$

$$\Sigma_3 = 0.$$

$$h_D^x = k\omega \Rightarrow \Sigma_0 = \Sigma_1 = \Sigma_2 = \Sigma_3 = 0;\quad H_{eff} \approx H_0^x$$

Any H_0^x

Resonances: A Floquet Perturbation Theory

$$H(t) = H_0(t) + V; \quad [H_0(t), H_0(t')] = 0 \ \ \forall t, t'$$ and $$\langle n|V|n\rangle = 0$$

$$H_0(t)|n\rangle = E_n(t)|n\rangle; \ \langle m|n\rangle = \delta_{mn}$$

- Here V is the perturbation (small) and for V = 0, |n> are the Floquet states.
- Goal = Finding the Floquet State for finite V expanding perturbatively around |n>.

TDSE:

$$i\frac{\partial|\psi_n\rangle}{\partial t} = H(t)|\psi_n(t)\rangle$$

Expansion:

$$|\psi_n(t)\rangle = \sum_m c_m(t)\ e^{-i\int_0^t dt' E_m(t')}\ |m\rangle$$

(to 1st order in V)

Coefficients:

$$c_m(0) = -i\ \langle m|V|n\rangle \frac{\int_0^T dt\ e^{i\int_0^t dt'[E_m(t')-E_n(t')]}}{e^{i\int_0^T dt[E_m(t)-E_n(t)]} - 1}$$

The resonance condition:

$$e^{i\int_0^T dt[E_m(t)-E_n(t)]} = 1$$

In our Case:

$H(t) = H_0(t) + V$, where

$H_0(t) = H_0^x + \mathrm{Sgn}(\sin(\omega t))H_D$, with

$$H_0^x = -\sum_{n=1}^{L} J\sigma_n^x\sigma_{n+1}^x + \sum_{n=1}^{L} \kappa\sigma_n^x\sigma_{n+2}^x - h_0^x\sum_{n=1}^{L}\sigma_n^x,$$

$$H_D = h_D^x\sum_{n=1}^{L}\sigma_n^x, \text{ and}$$

→ f (σ^x)

$$V = h^z\sum_{n=1}^{L}\sigma_n^z,$$

→ Single Spin-flip Perturbation

1st order Resonance Condition (isolated resonances)

$$h_0^x\sigma_0 + J\sigma_0(\sigma_{-1} + \sigma_1) - \kappa\sigma_0(\sigma_{-2} + \sigma_2) = \frac{p\omega}{2}$$

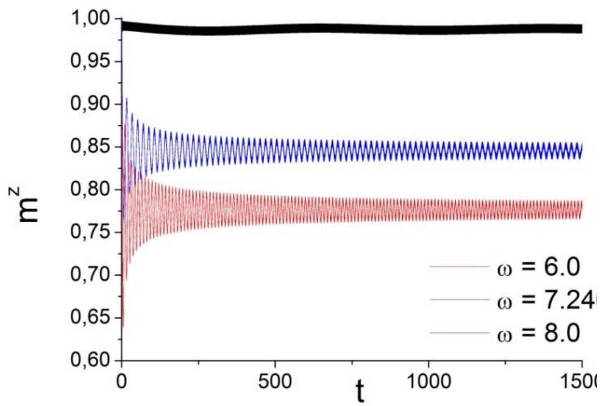
↓

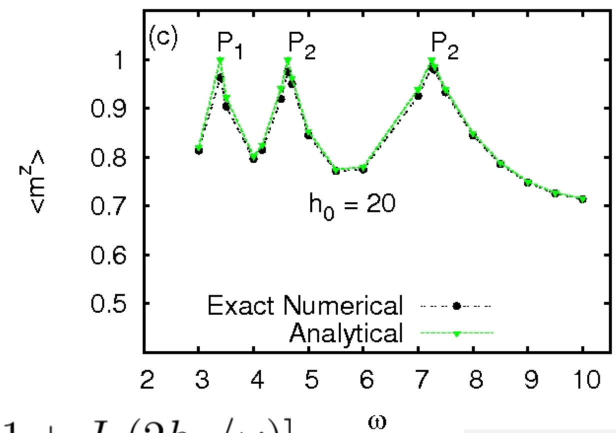
The violet vertical lines matching the dips

Dynamical Scarring in Non-interacting Systems (in retrospect)

$$H = -\frac{J}{2}\left[\sum_{i=1}^{L}\sigma_i^x\sigma_{i+1}^x + h_0\cos(\omega t)\sum_i \sigma_i^z\right]$$

AD, PRB (2010)





$$Q = \langle m^z\rangle = 1/[1 + J_0(2h_0/\omega)]$$

Periodic Gibbs" Ensemble

$$\hat{H}_{eff} = \sum_{p=1}^{L}\omega_p \tilde{a}_p^\dagger \tilde{a}_p \text{ with } \mathcal{I}_p = \tilde{a}_p^\dagger \tilde{a}_p$$

Exact (periodic) Conserved Quantities

m^z

Is **NOT** among them!

$$\hat{\rho}_{PGE}(t) = \mathcal{Z}^{-1}\exp\left[-\sum_p \lambda_p \mathcal{I}_p(t)\right];\ \mathcal{Z}(t) = tr[\hat{\rho}_{PGE}(t)]$$

A. Lazarides, AD, R. Moessner, PRL (2014)

Conclusion Outlook

- Strong evidence in favour of existence of stable non-thermal Floquet states in interacting systems are presented.

- Two types of series expansions and their comparison with numerical results indicate they are asymptotic in nature, and first order indicates there are only isolated resonances: Possibility of stable non-thermal states in the thermodynamic limit

- This opens up a new possibility – that of stable Floquet engineering in interacting systems. Stable Floquet time-crystals without disorder or dissipation cannot be ruled out in view of this.