Soft-wall turbulence.

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Dedicated to Prof. K. R. Sreenivasan on his 70^{th} birthday.

Acknowledgments

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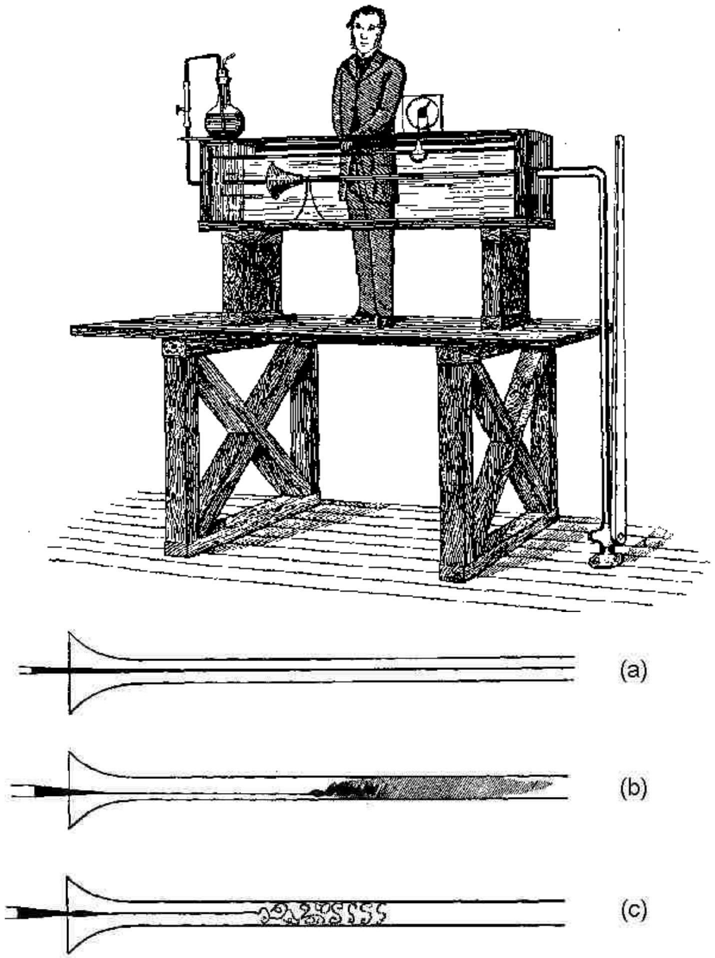
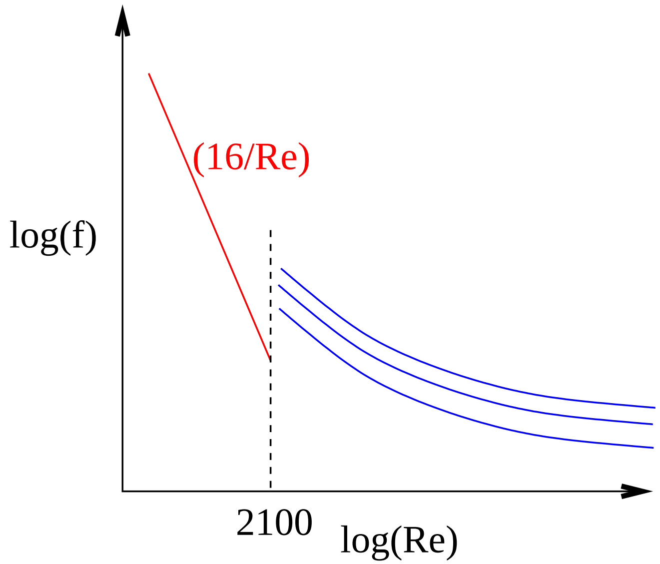
- Mr. S. S. Srinivas (TRDDC).
- Department of Science and Technology, Government of India.
- J. R. D. Tata Trust.

Transition from laminar to turbulent flow:



*N. Rott, 1990 Ann. Rev. of Fluid Mech.

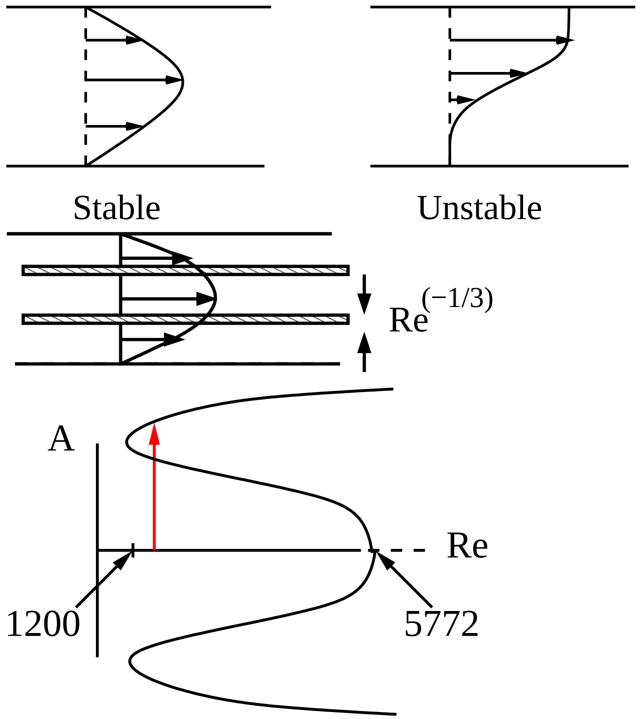
- Pipe flow $Re \sim 2100$, channel flow $Re \sim 1200$.
- Discontinuous transition in drag coefficient $f = (\nabla p/(\rho V^2/2D))$.



Flow in rigid channels

- Experiments — transition $Re \sim 1200$.
- Inviscid: *Parabolic flow always stable*.
- Include viscous effects — flow unstable at $Re = 5772$.
- Viscous stresses important in 'internal critical layer' of thickness $Re^{-1/3}$.
- Discrepancy between experiments and theory: instability highly sub-critical.

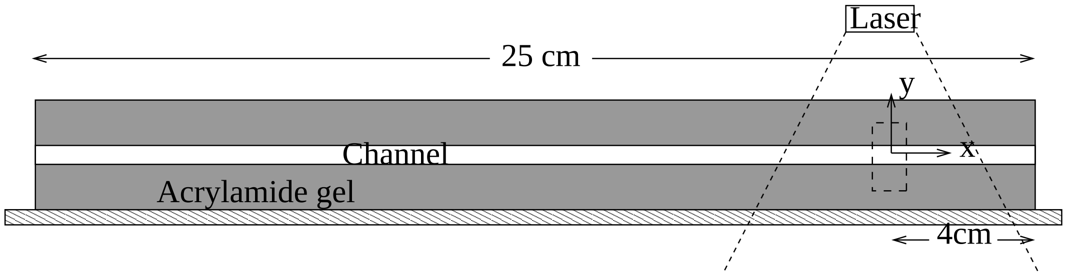
Rayleigh criterion:



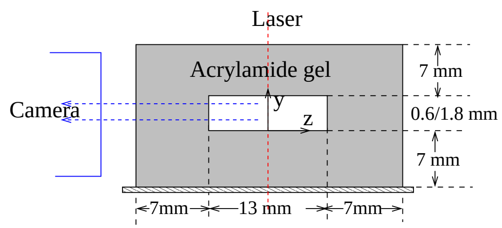
Algebraic growth — non-normal Orr-Sommerfeld opeartor (Schmid ARFM 39, 129, 2007).

Traveling wave solutions for pipe flows (Kerswell, Nonlinearity, 2005).

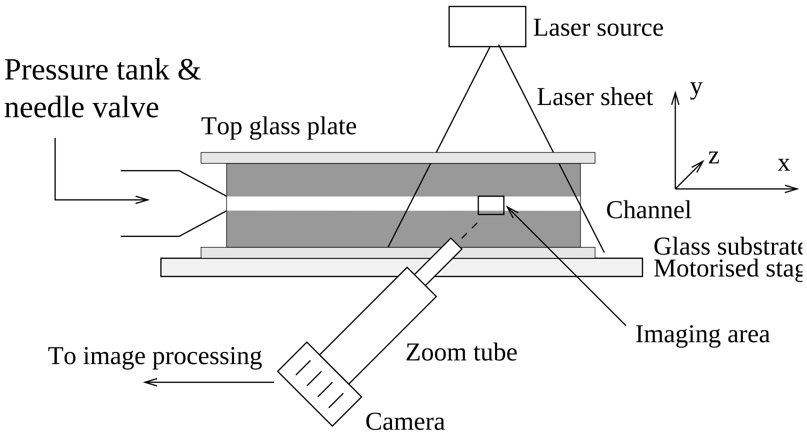
Turbulence in a rigid channel:



Side view

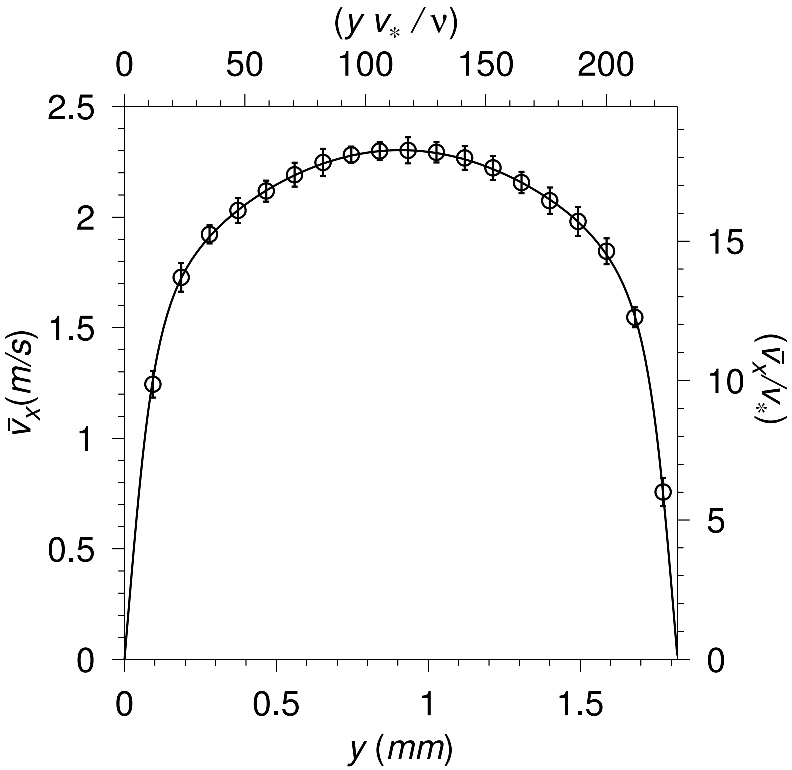


Cross-section

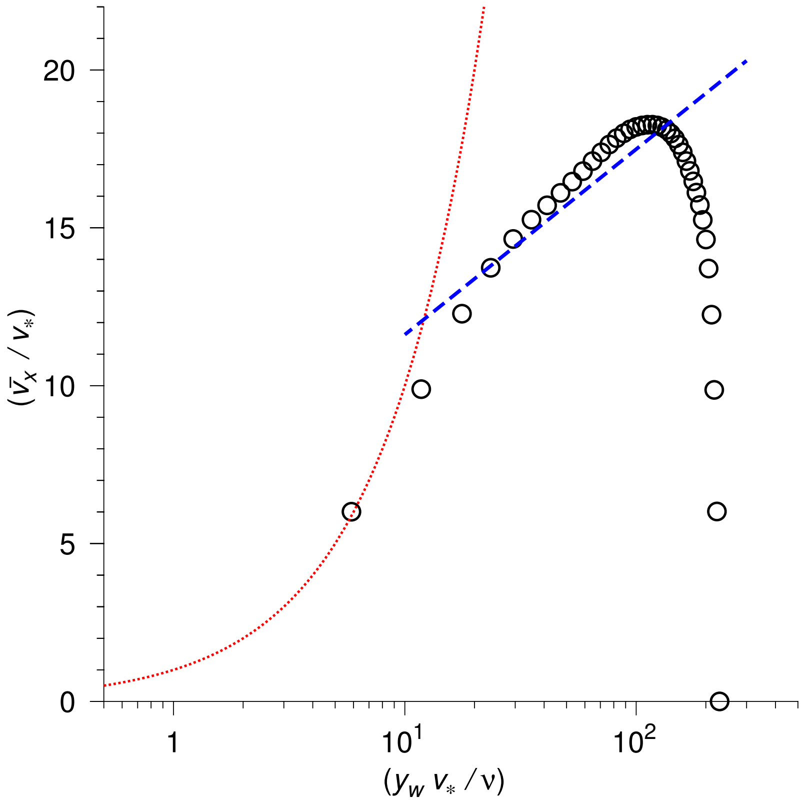


Turbulence in a rigid channel: Re=3500

Mean velocity:



Log law:

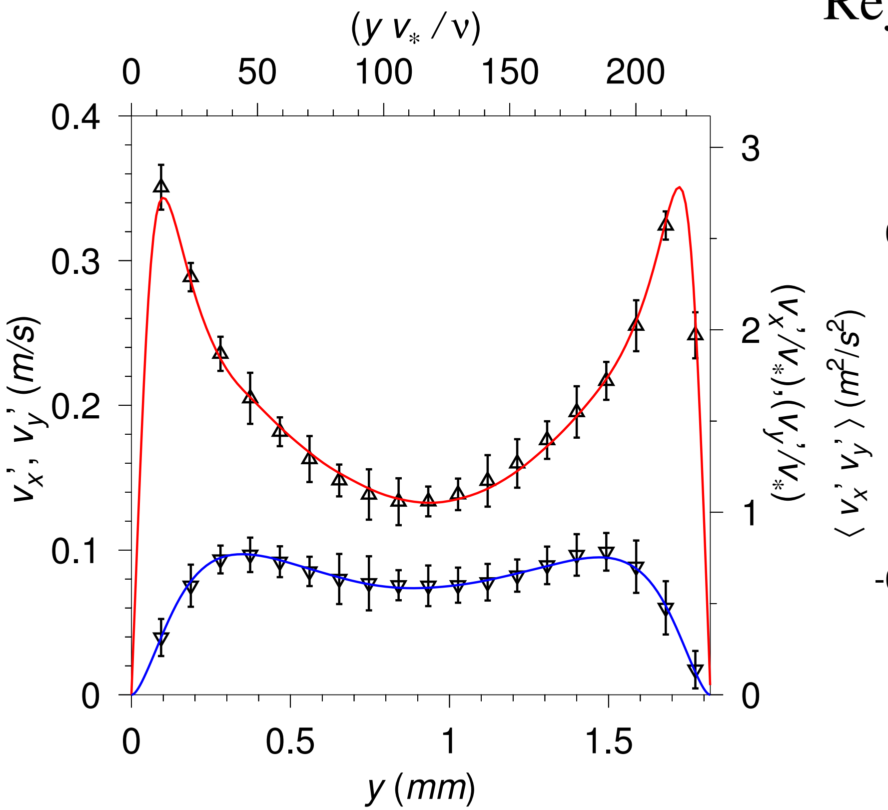


$$v_* = \sqrt{\tau_w/\rho}$$

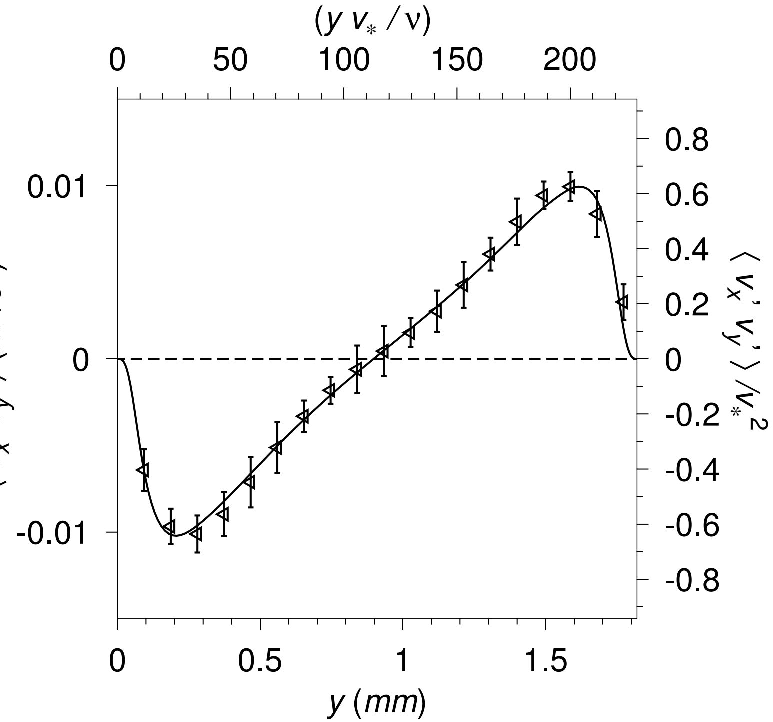
$\cdots$ $(v_x/v_*) = (yv_*/\nu)$; $- - -$ $(v_x/v_*) = 2.44 \log{(yv_*/\nu)} + 5.5$

Turbulence in a rigid channel: Re=3500

RMS fluctuating velocity v'_x, v'_y



Reynolds stress $\langle v'_x v'_y \rangle$



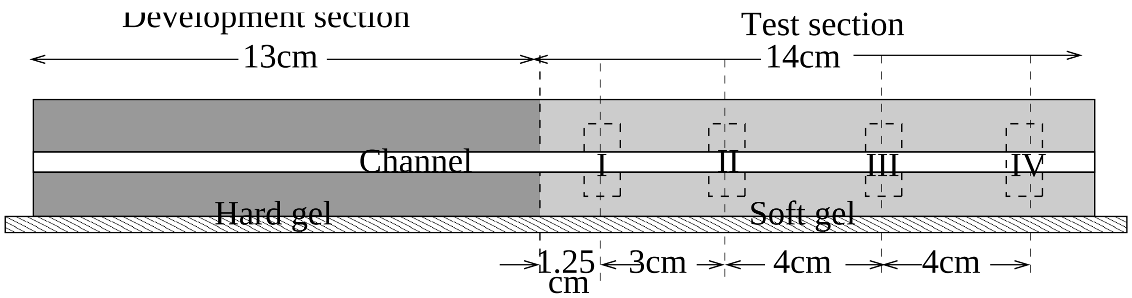
Soft-wall turbulence.

- Fluid — $\rho(\mathcal{M}\mathcal{L}^{-3}), \eta(\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1})$.
- Length scale — $R(\mathcal{L})$.
- Reynolds number $(\rho U R/\mu)$.
- Elasticity of wall material — $G(\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2})$.
- Dimensionless parameter $\Sigma = (\rho G R^2/\eta^2)$.
- Transition a function of two dimensionless parameters Re & Σ.
- Soft interfaces — $G \sim 10^4 - 10^5 Pa \sim 10^{-6} G(\text{steel})$

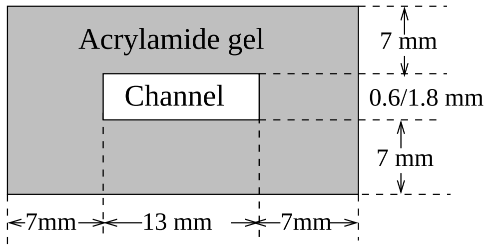
Outline:

- Experiments on transitions in channel with soft walls.
- Stability analysis.
- Theoretical predictions.
- Open questions.
- Mixing.

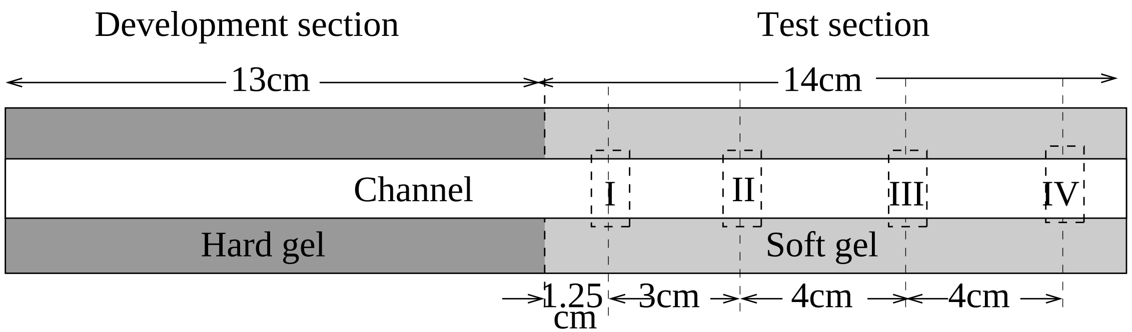
Soft-walled channel: Polyacrylamide gal $G = 0.75 kPa$



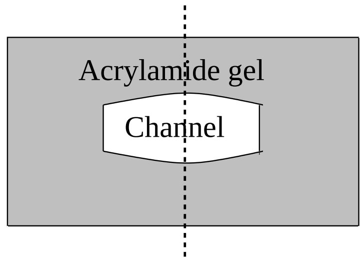
Side view



Cross-section

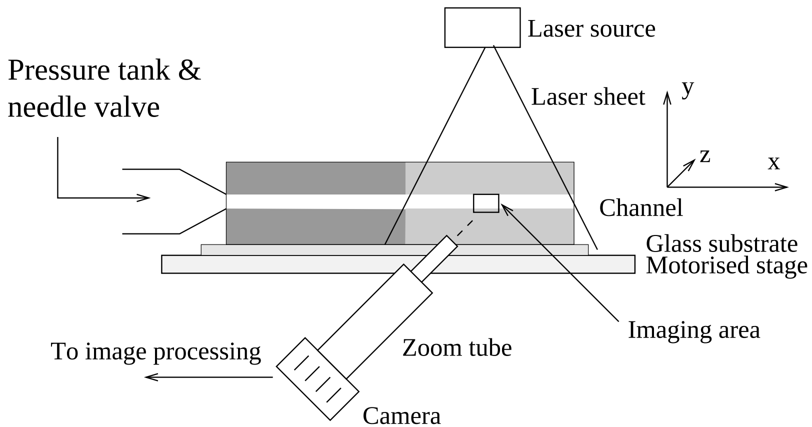


Top view

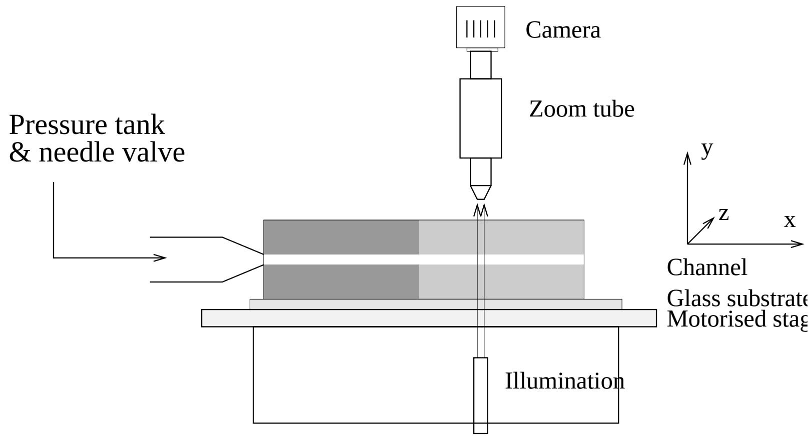


Deformed

Soft-walled channel: Experimental configurations.

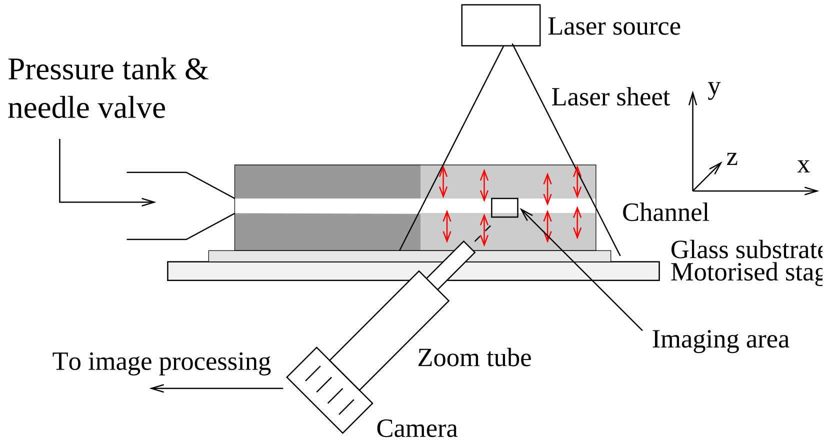


Fluid velocity (PIV), Wall displacement.



Wall displacement

Soft-walled channel: Experimental configurations.



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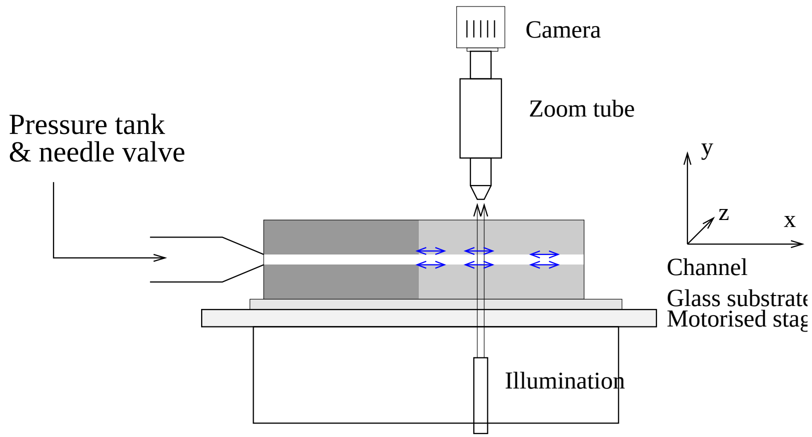


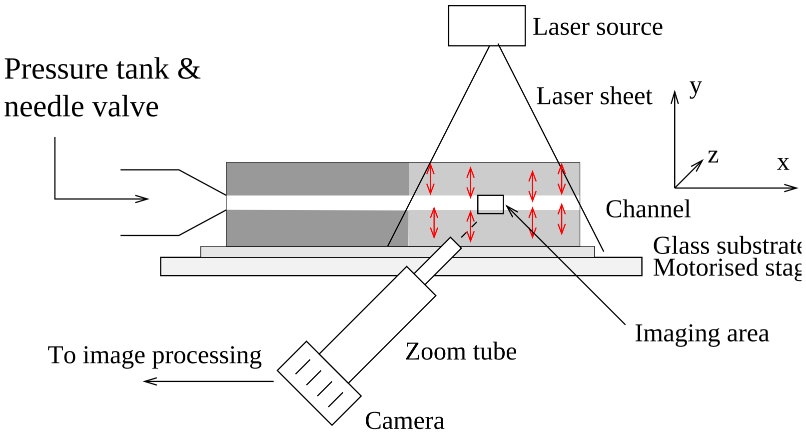
Wall displacement

Soft-walled channel:

Channel height ≈ 0.6 mm, 1.8 mm.

Fluid — water at 20^o. Wall — Polyacrylamide gel
$G = 0.75 kPa, 2.19 kPa.$

Dimensionless parameters:
$Re = (\rho Q / W \eta) = (\rho U h / \eta)$
$\Sigma = (\rho G h^2 / \eta^2)$
Measurements:



Mean velocity $\bar{v}_x(y)$, fluctuations $v'_x(y), v'_y(y), \langle v'_x v'_y \rangle (y)$.

Wall displacement fluctuations u'_y

Wall displacement fluctuations u'_x, u'_z.

Soft-walled channel:

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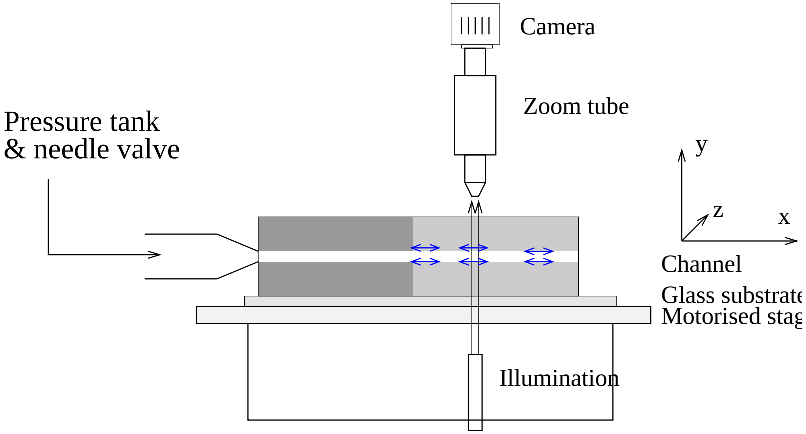


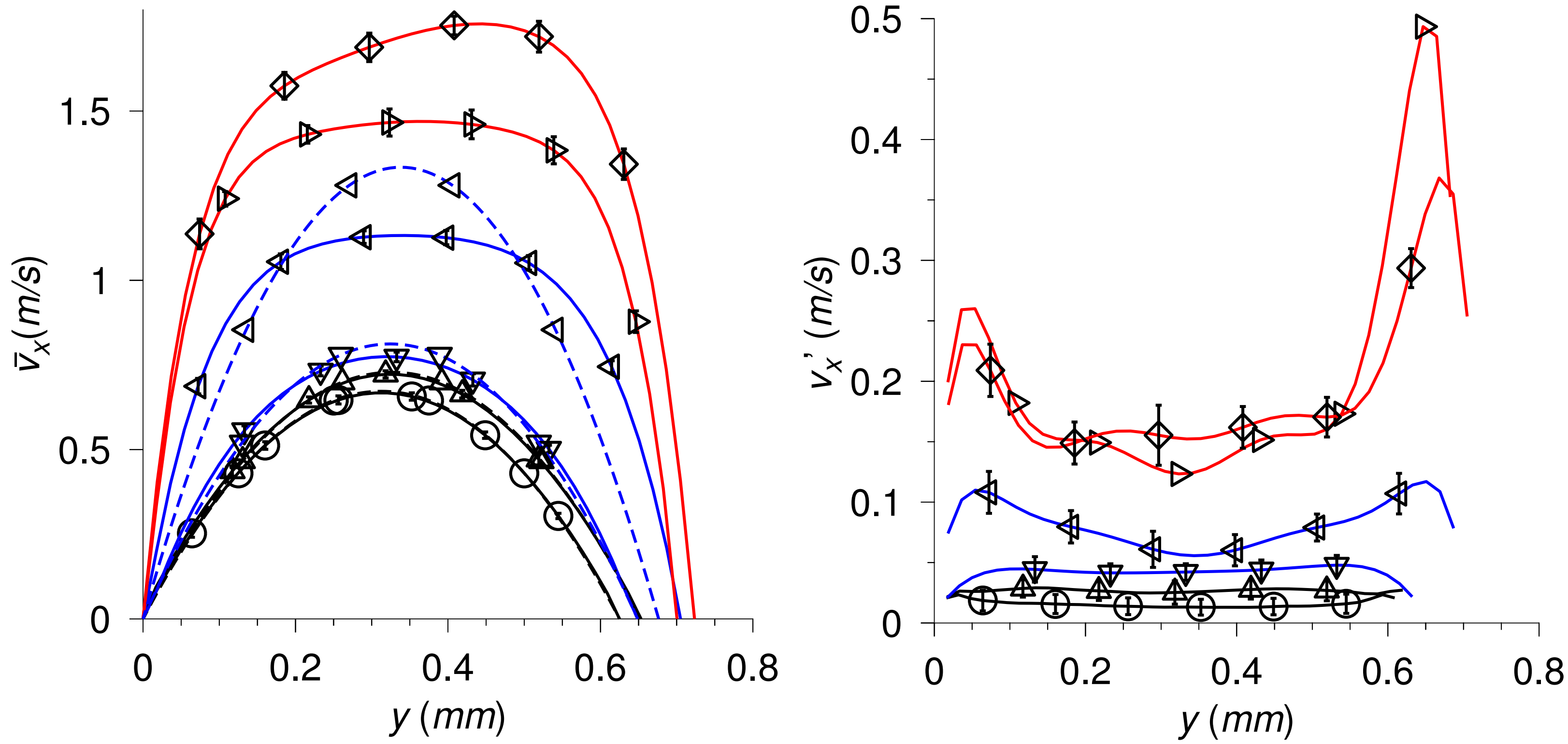
Mean velocity $\bar{v}_(y)$, fluctuations $v'_x(y), v'_y(y), \langle v'_x v'_y \rangle (y)$.

Wall displacement fluctuations u'_y

Wall displacement fluctuations u'_x, u'_z.

Soft-walled channel: Fluid velocity profiles Location III.

h=0.6mm

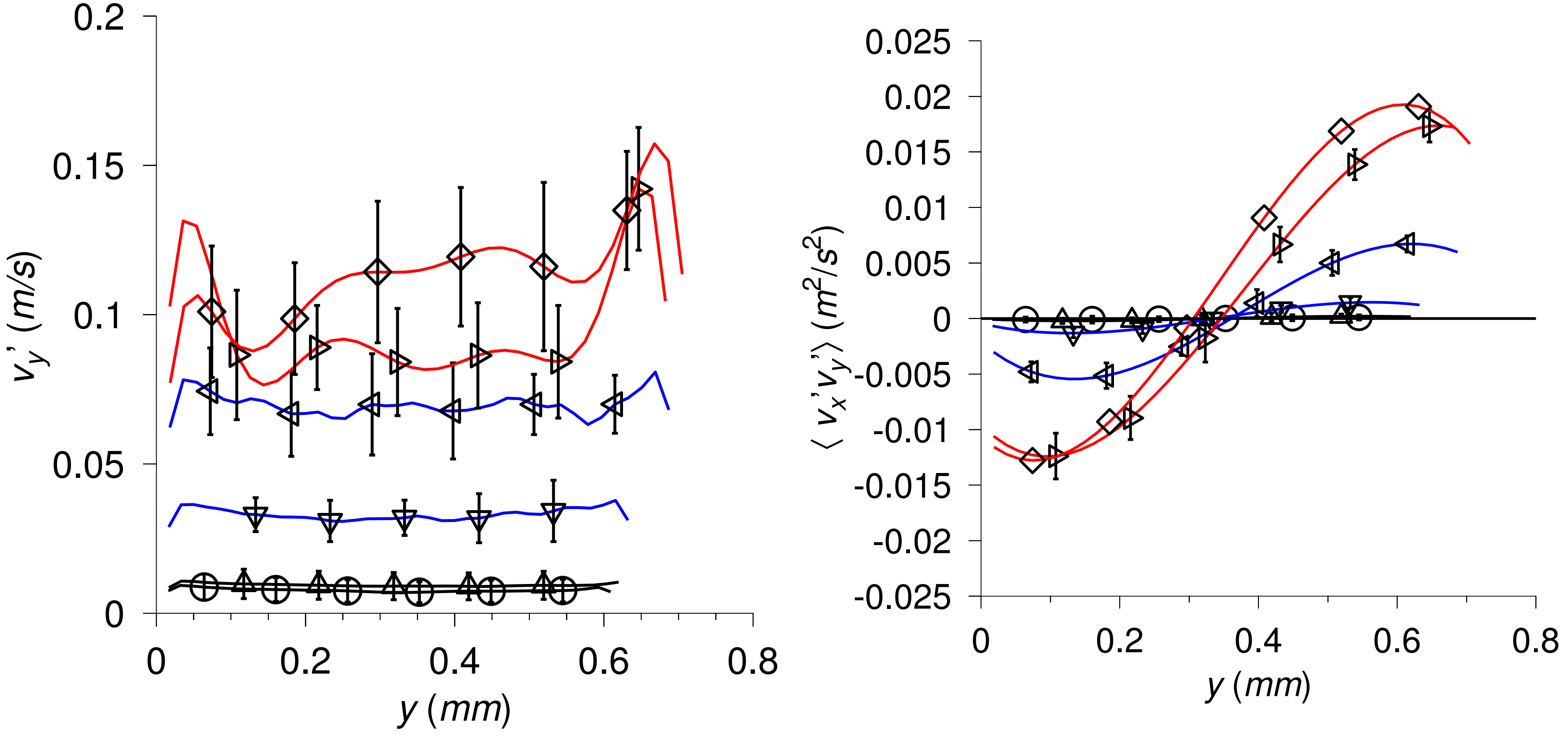


Mean velocity $\bar{v}_x$ Streamwise RMS v_x'

○ Re=278, △ Re=301, ▽ Re=335, ◁ Re=545, ▷ Re=741, ◇ Re=860.

Soft-walled channel: Fluid velocity profiles Location III.

h=0.6mm



Cross-stream RMS v'_y

Correlation $\langle v'_x v'_y \rangle$

○ Re=278, △ Re=301, ▽ Re=335, ◁ Re=545, ▷ Re=741, ◇ Re=860.

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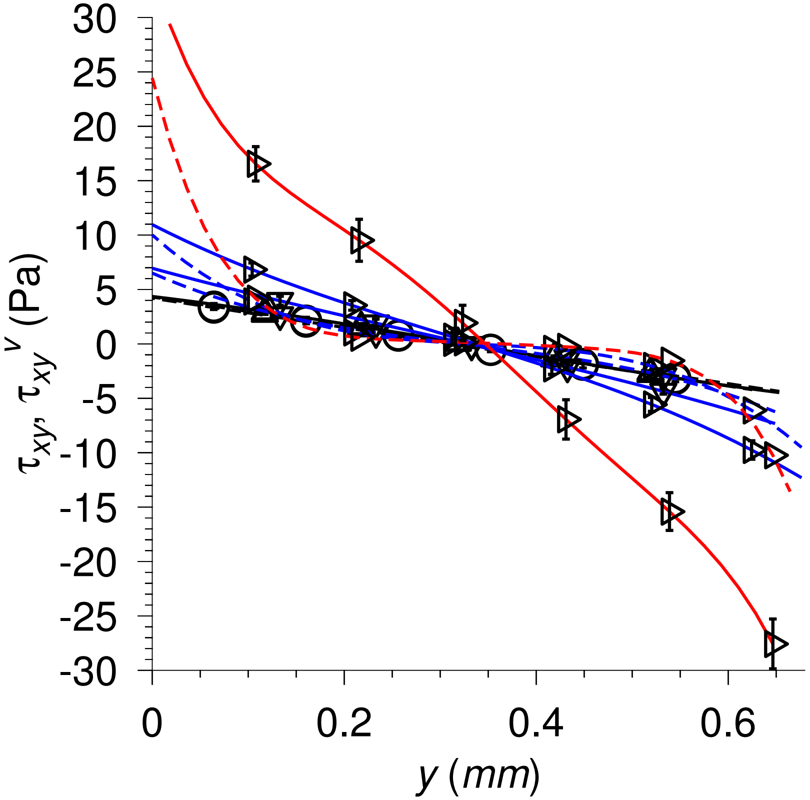
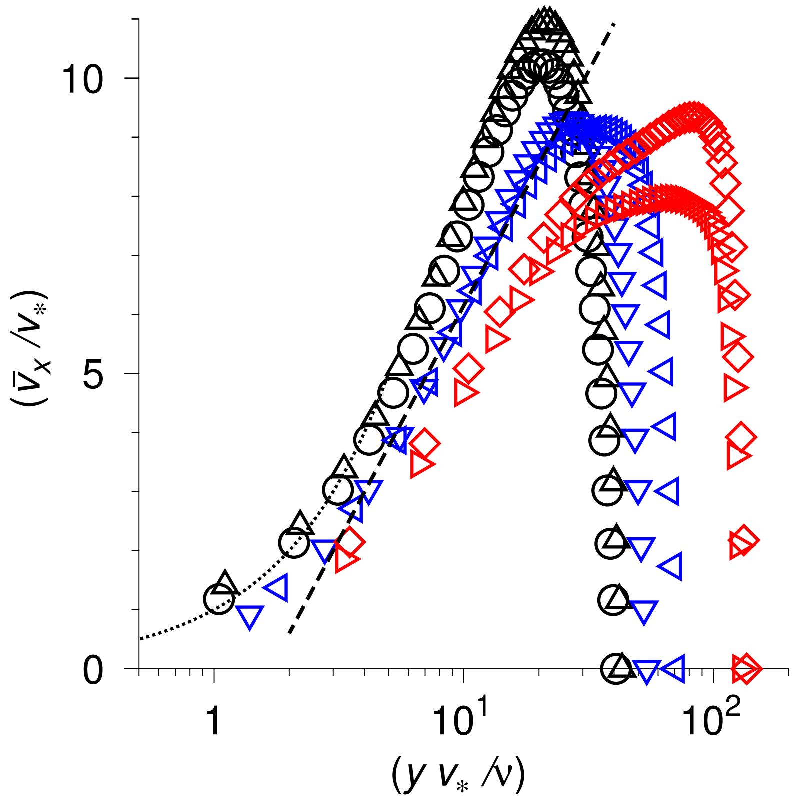
Soft-walled channel: Fluid velocity profiles Location III.

h=0.6mm

Stress



Logarithmic profile

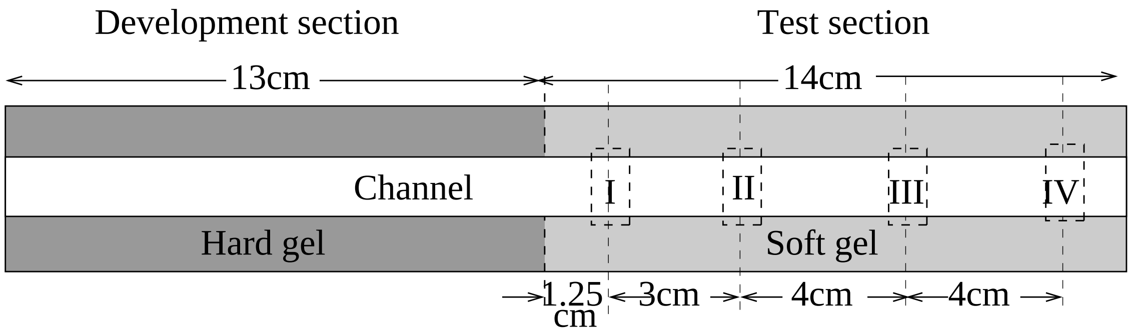


— τ_{xy}, – – – $\eta(d\bar{v}_x/dy)$

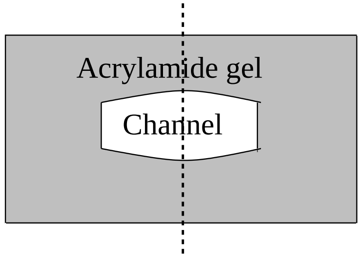
Line $(\bar{v}_x/v_*) = 3.45(yv_*/\nu) - 1.8$

○ Re=278, △ Re=301, ▽ Re=335, ◁ Re=545, ▷ Re=741, ◇ Re=860.

Soft-walled channel: Transition measures: h=0.6mm



Top view



Cross section

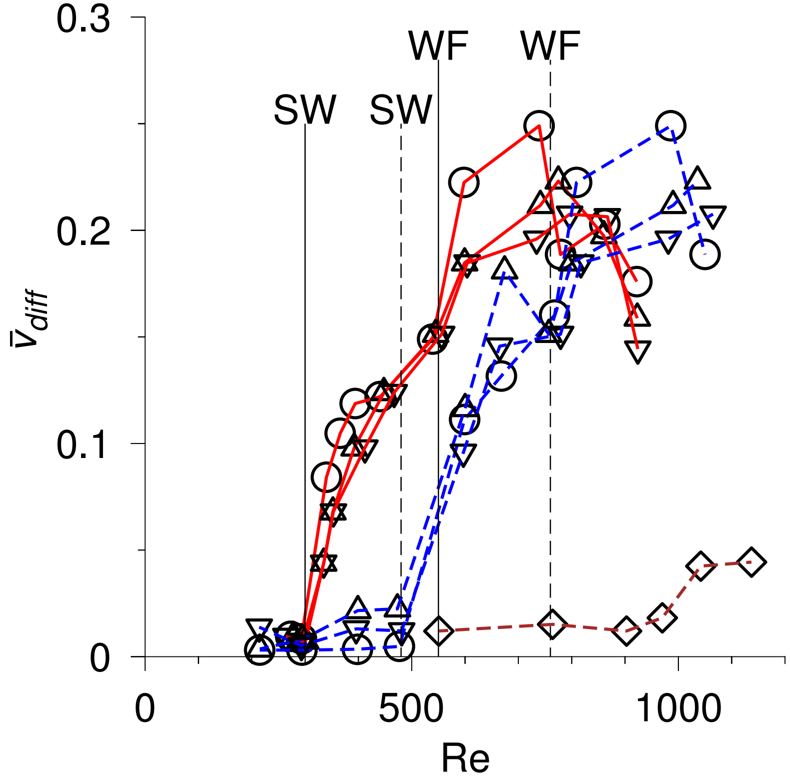
$\bar{v}$ Cross-section averaged velocity, $\bar{v}_x^l$ laminar velocity profile.

$$\bar{v}_{diff} = \sqrt{\frac{1}{h(\bar{v})^2} \int_0^h dy (\bar{v}_x(y) - \bar{v}_x^l(y))^2}$$

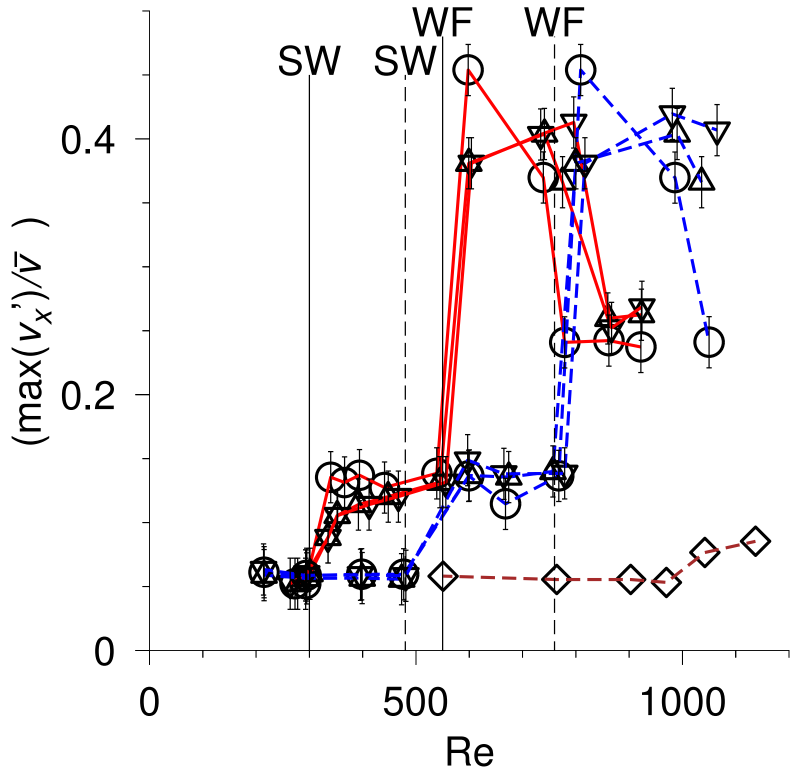
Maximum of $(v_x')/\bar{v}$, $(v_y')/\bar{v}$, $(\langle v_x' v_y' \rangle)/\bar{v}^2$ across the channel.

SW — Soft Wall trans.; WF — Wall Flutter; HW — Hard Wall trans.

Soft-walled channel: Transition measures: h=0.6mm



$\bar{v}_{diff}$



v'_x

G = 0.75 kPa; G = 2.19 kPa; Hard wall

○ Location II; △ Location III; ▽ Location IV.

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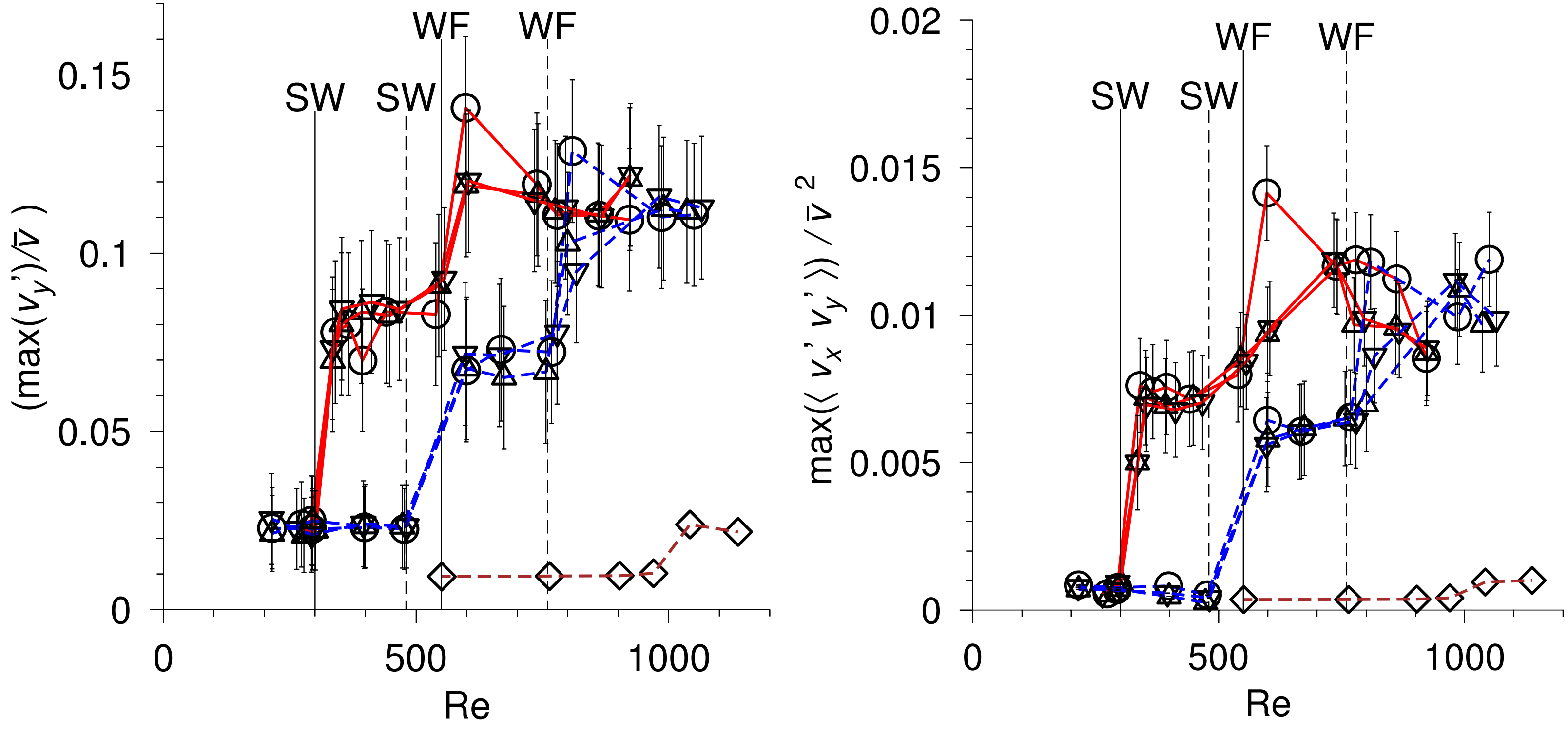
Soft-walled channel: Transition measures: h=0.6mm



v_y' $\langle v_x' v_y' \rangle$

G = 0.75 kPa; G = 2.19 kPa; Hard wall

○ Location II; △ Location III; ∇ Location IV.

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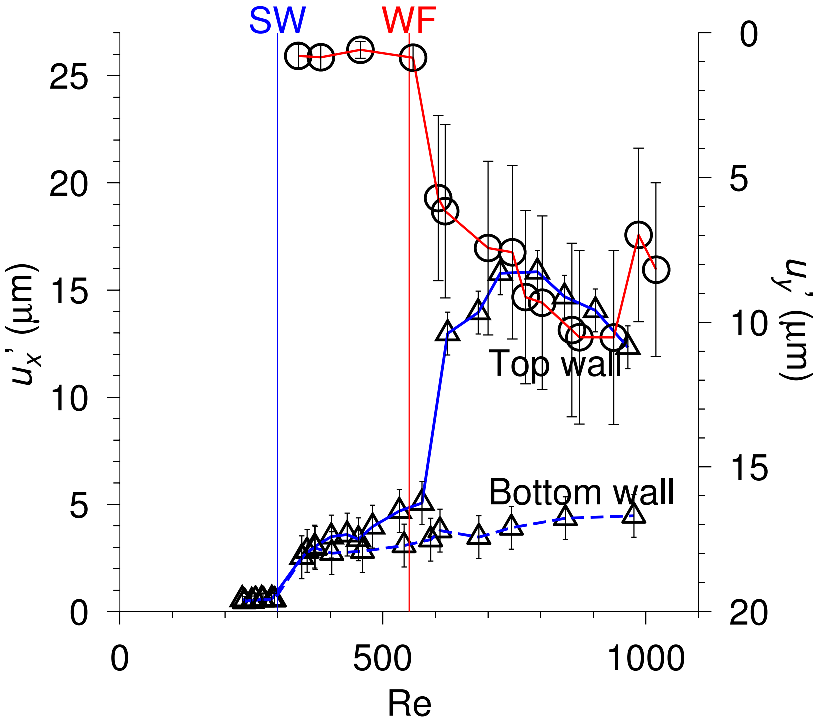
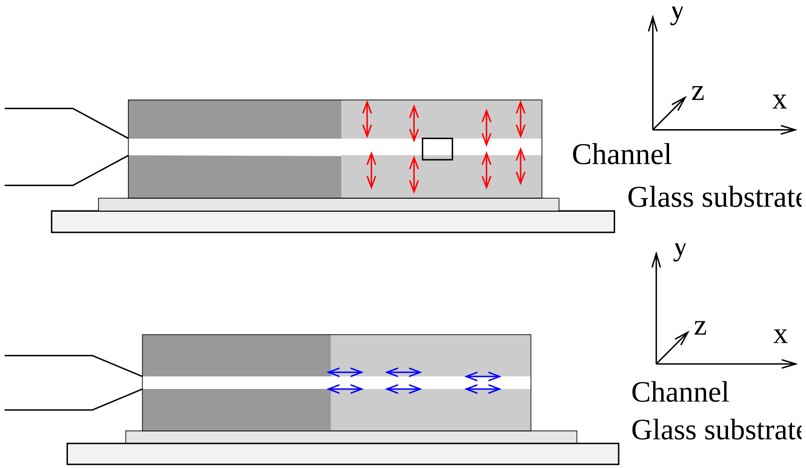
Soft-walled channel: Wall displacement Location III.

h=0.6mm





u'_x, u'_y

$G = 0.75kPa$; $\circ$ Location II; $\triangle$ Location III; ∇ Location IV.

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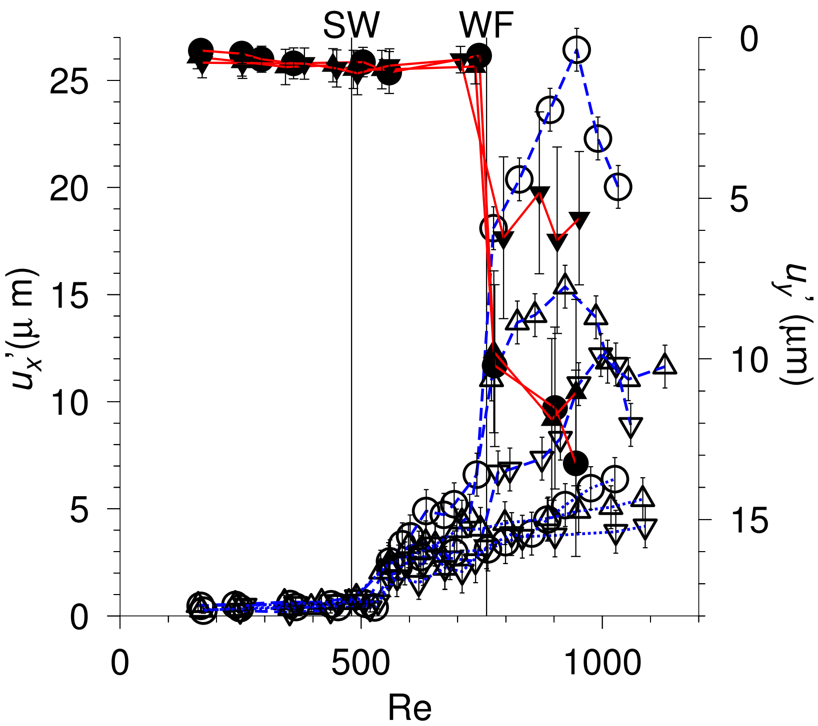
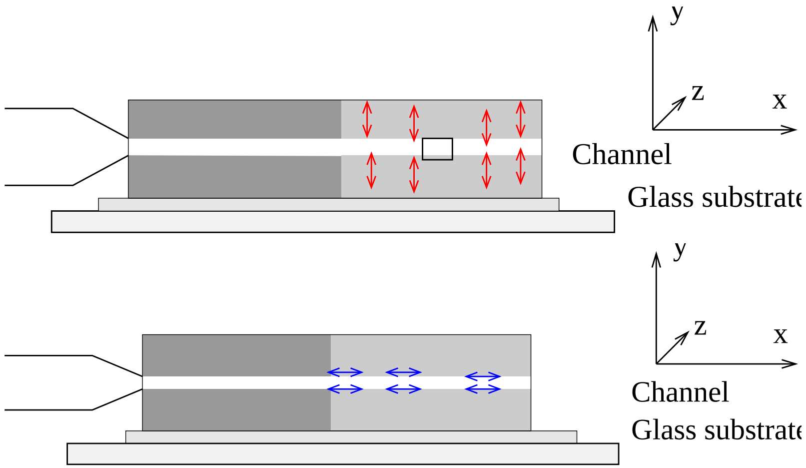
Soft-walled channel: Wall displacement Location III.

h=0.6mm





u'_x, u'_y

$G = 2.19kPa$; ○ Location II; △ Location III; ▽ Location IV.

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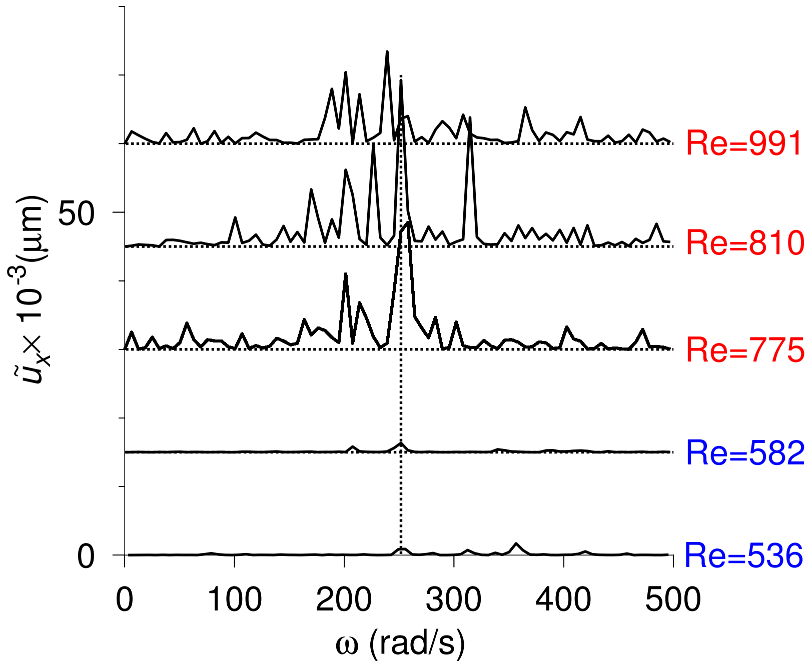
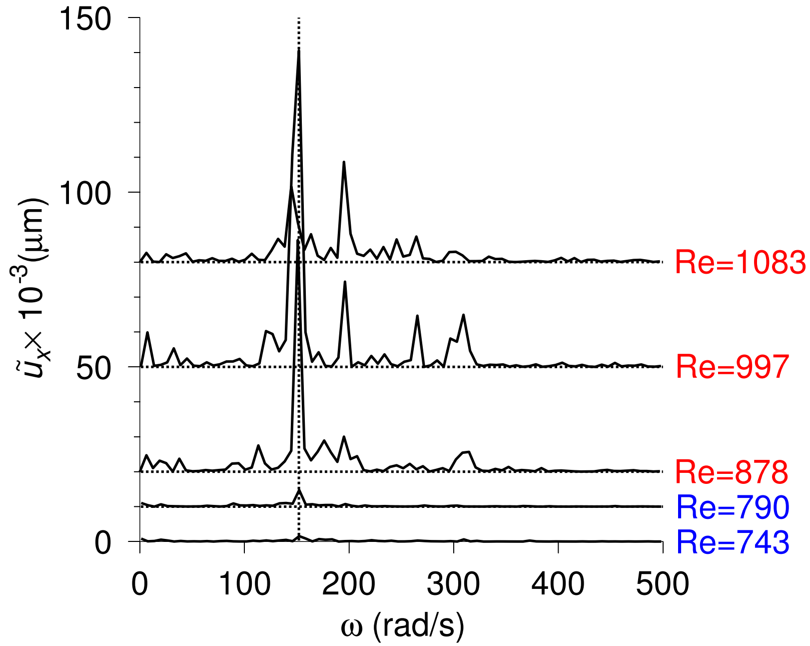
Soft-walled channel: Wall displacement spectrum. h=0.6mm





$G = 0.75kPa$

$G = 2.19kPa$

$\omega \sim \sqrt{G/\rho h^2} \sim 200 - 500Hz$

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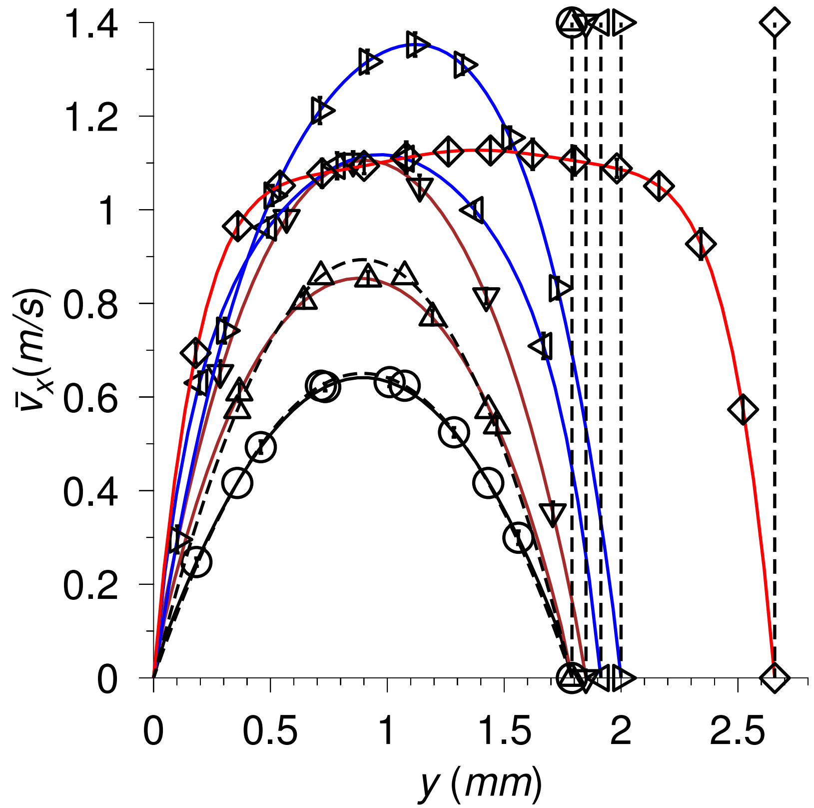
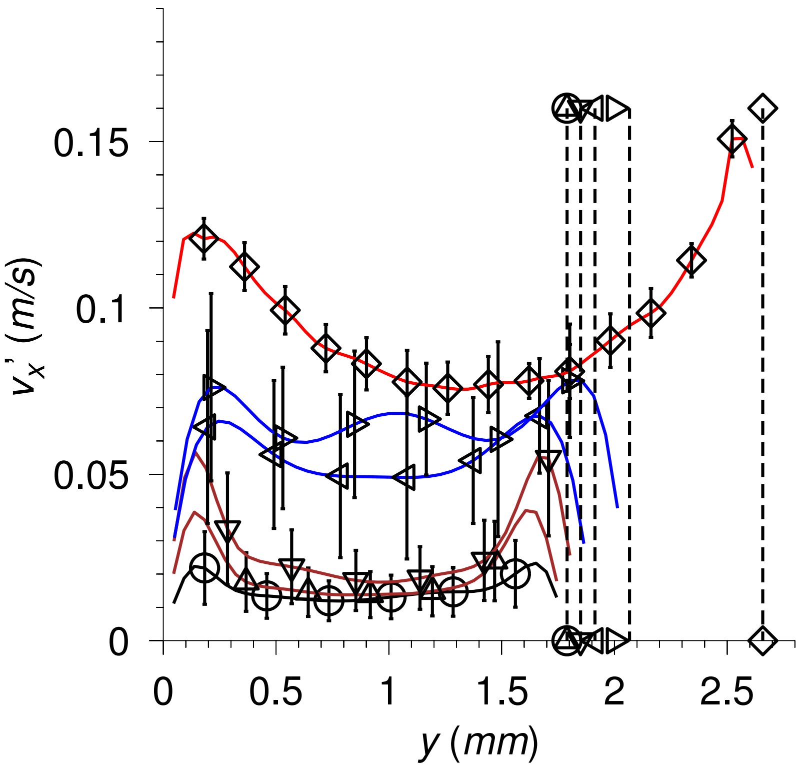
Soft-walled channel: Fluid velocity profiles Location III.

h=1.8mm



Mean velocity $\bar{v}_x$

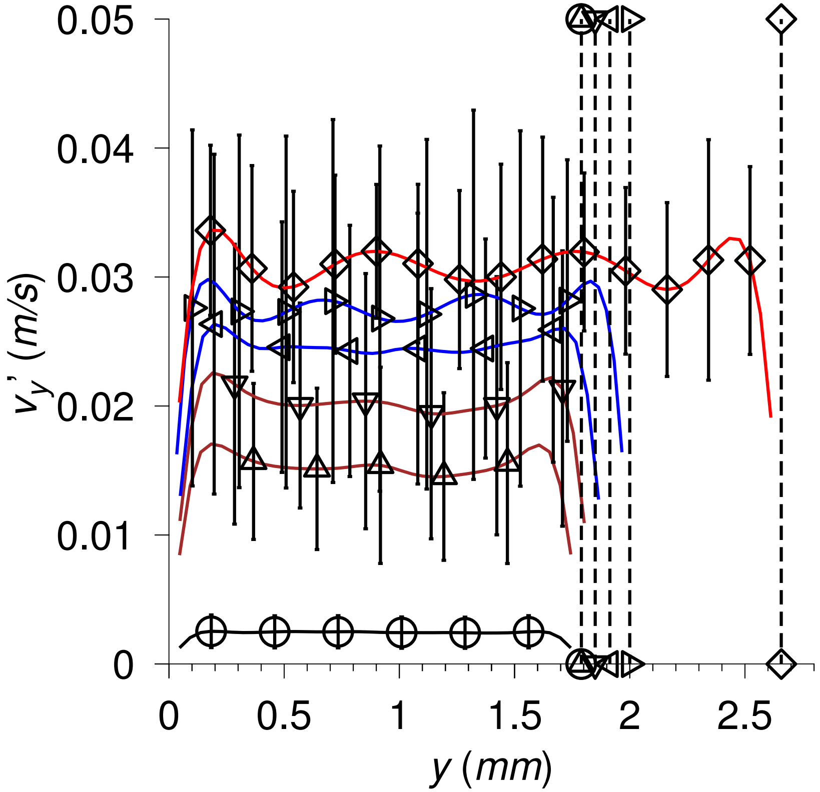


Streamwise fluctuation v_x'

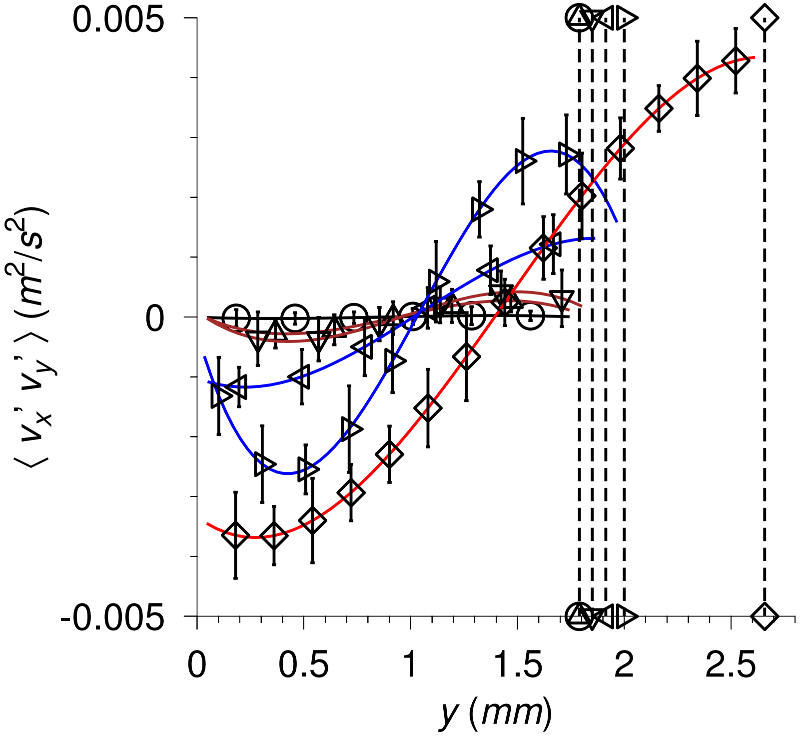
○ Re=768, △ Re=1071, ▽ Re=1332, ◁ Re=1515, ▷ Re=1734, ◇ Re=1973.

Soft-walled channel: Fluid velocity profiles Location III.

h=1.8mm



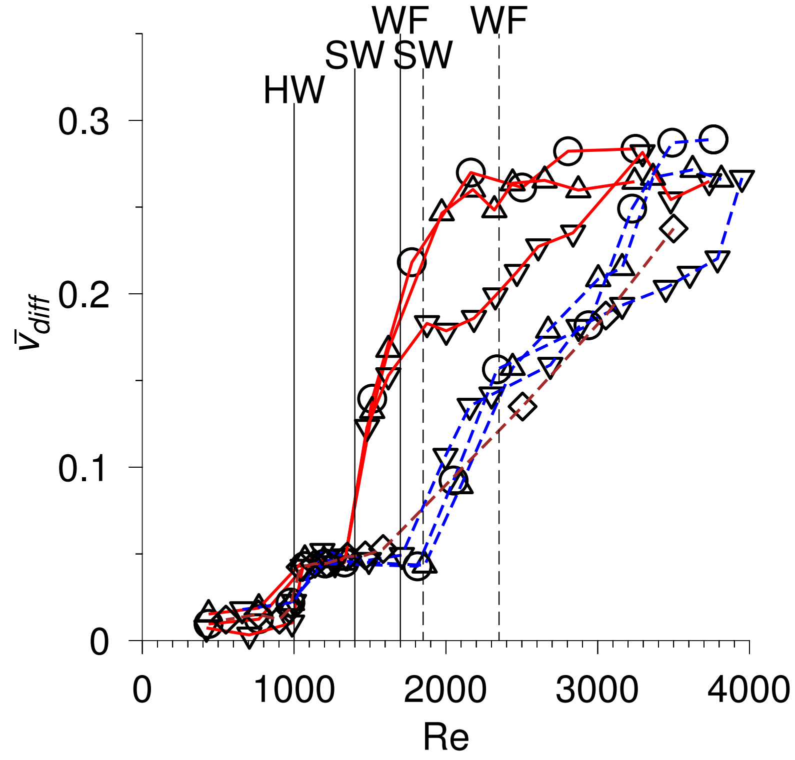
Mean velocity v_y'



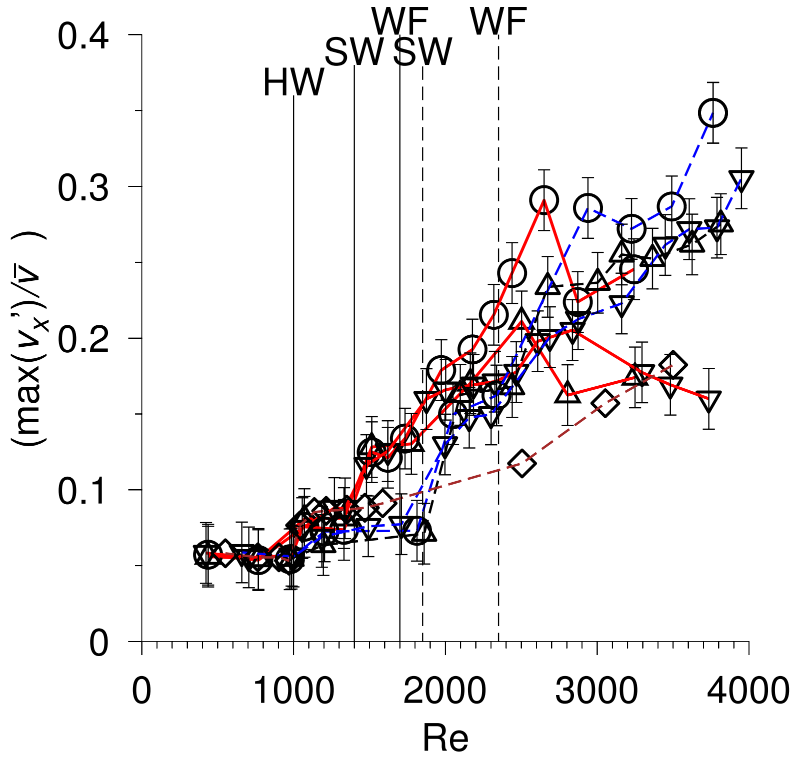
Streamwise fluctuation $\langle v_x' v_y' \rangle$

○ Re=768, △ Re=1071, ▽ Re=1332, ◁ Re=1515, ▷ Re=1734, ◇ Re=1973.

Soft-walled channel: Transition measures: h=1.8mm



$\bar{v}_{diff}$

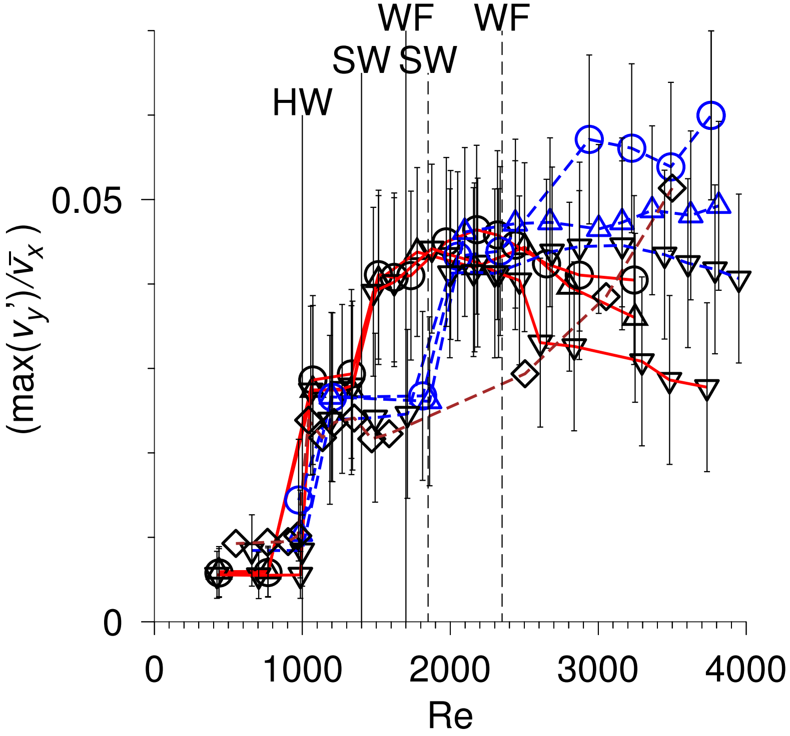


v'_x

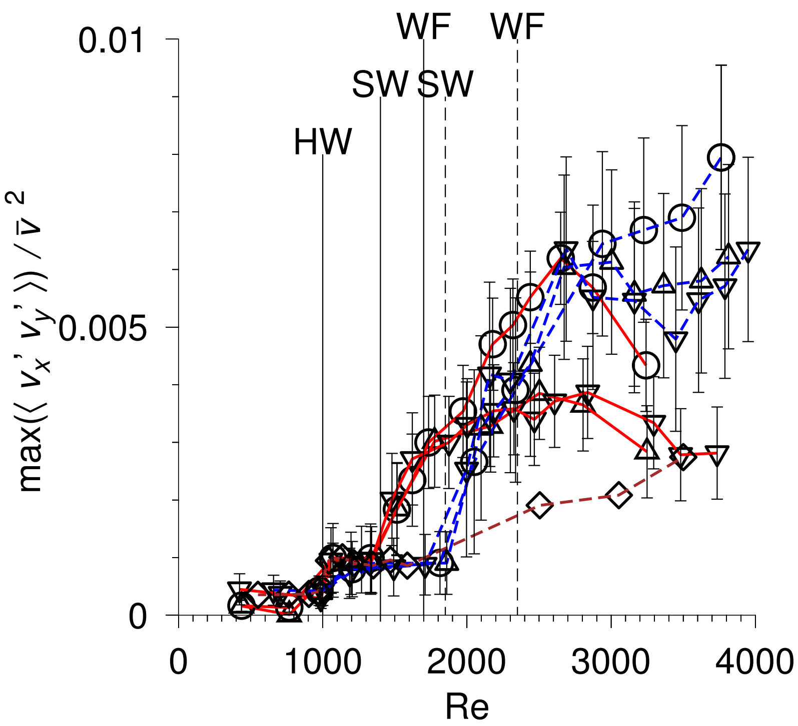
G = 0.75 kPa; G = 2.19 kPa; Hard wall

○ Location II; △ Location III; ∇ Location IV.

Soft-walled channel: Transition measures: h=1.8mm



v'_y



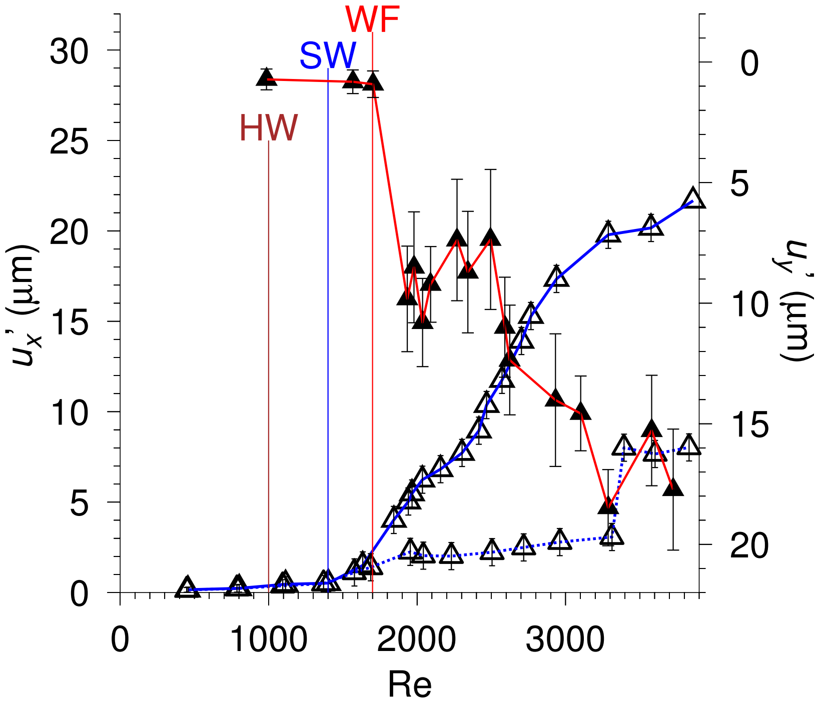
$\langle v'_x v'_y \rangle$

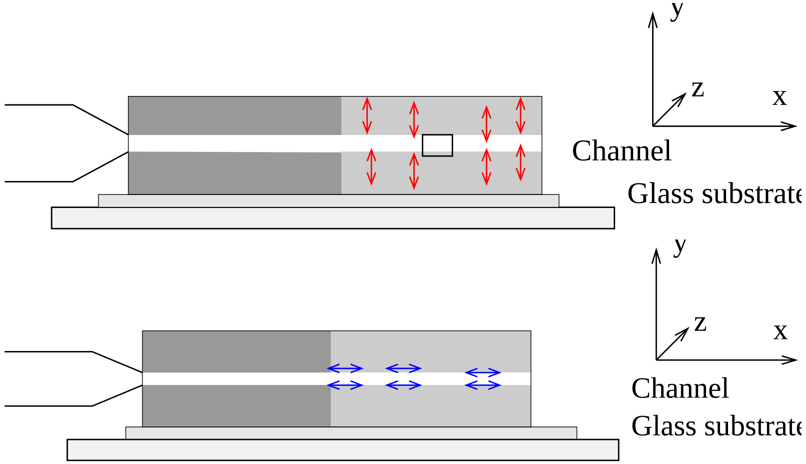
G = 0.75 kPa; G = 2.19 kPa; Hard wall

○ Location II; △ Location III; ∇ Location IV.

Soft-walled channel: Wall displacement Location III.

h=1.8mm





u'_x, u'_y

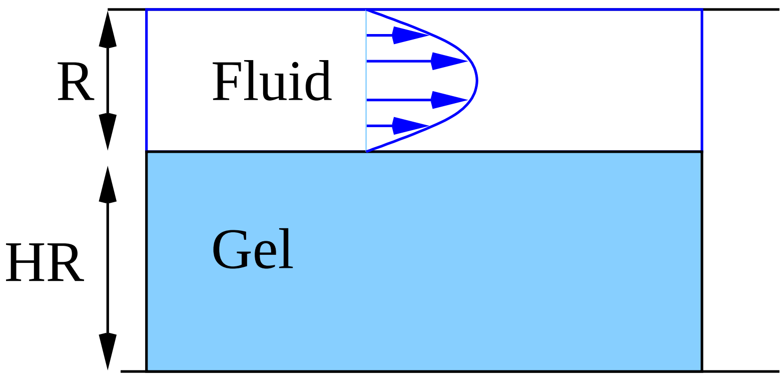
Summary: Two distinct transitions:

- Soft-wall transition:
  - Fixed & unrestrained walls.
  - Fluctuating velocity profiles symmetric.
  - Mean velocity flatter at the center and steeper at the wall.
  - Characteristic peak in the stream-wise fluctuating velocity close to the wall.
  - Fluctutating velocity, Reynolds stress appear to be non-zero at the wall.
  - Tangential motion of the wall, no visible normal wall motion.
  - No visible viscous sub-layer.
  - Logarithmic layer, but log law depends on wall elasticity.
  - Characteristic frequency?

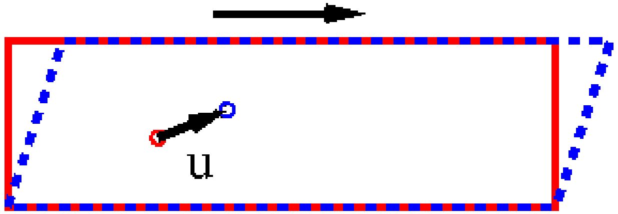
Summary: Two distinct transitions:

- Wall flutter transition:
  - Only at unrestrained top wall.
  - Normal wall velocity only at unrestrained top wall.
  - Wall displacement profiles asymmetric.
  - Velocity profile asymmetric.
  - Fluctuating velocity, Reynolds stress appear non-zero at the wall.
  - No visible viscous sub-layer, log layer.
  - Characteristic frequency from shear wave speed in solid.

Wall — viscoelastic continuum



- Displacement field $\mathbf{u}$ — displacement of material points from steady state positions.

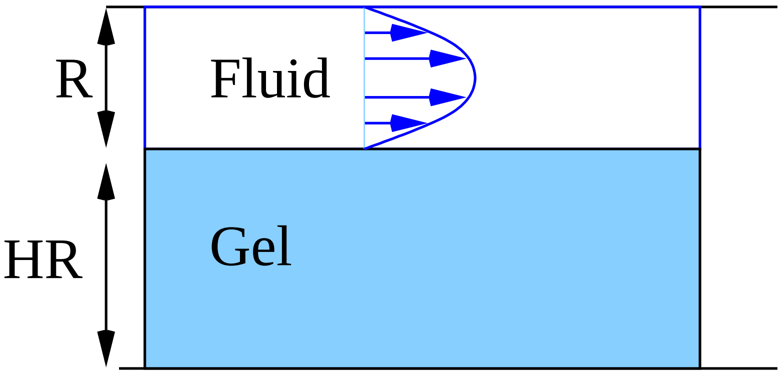


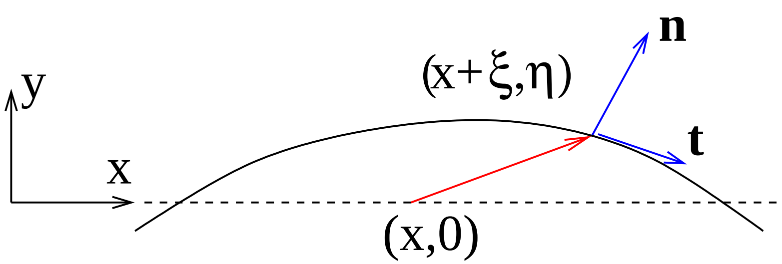
Gel stress $\sigma = -p\mathbf{I} + G\mathbf{e} + \eta_g \dot{\mathbf{e}}$ Deformation tensor: Neo-Hookean: $\mathbf{e} = (1/2)(\nabla\mathbf{u} + (\nabla\mathbf{u})^T - (\nabla\mathbf{u}).(\nabla\mathbf{u}^T))$	Dimensional parameters. Fluid density ρ, viscosity η. Wall elasticity G, viscosity η_g. Length scale R, HR.

Transition Re a function of $\Sigma = (\rho G R^2/\eta^2)$, $\eta_r = (\eta_g/\eta)$, length ratio.

Soft interfaces — $G \sim 10^4 - 10^5 Pa \sim 10^{-6} G(\text{steel})$

Interface





- Deformed interface:

  Velocity continuity

  $\mathbf{v}.\mathbf{n} = D_t(\mathbf{u}).\mathbf{n},\ \mathbf{v}^t = (D_t\mathbf{u})^t$

  Stress continuity

  $\tau_{nn} = \sigma_{nn}, \tau_{tn} = \sigma_{tn}.$

- Top rigid surface:

  $v_x = 0; v_y = 0.$

- Bottom rigid surface:

  $u_x = u_y = 0$

Theoretical analysis:

- Modification of rigid wall instabilities $Re \propto \Sigma^{1/2}$.
- Viscous instability $Re \propto \Sigma$
- Low Reynolds number inertial instability $Re \propto \Sigma^{1/2}$
- High Reynolds number wall layer instability $Re \propto \Sigma^{3/4}$

High Reynolds number:

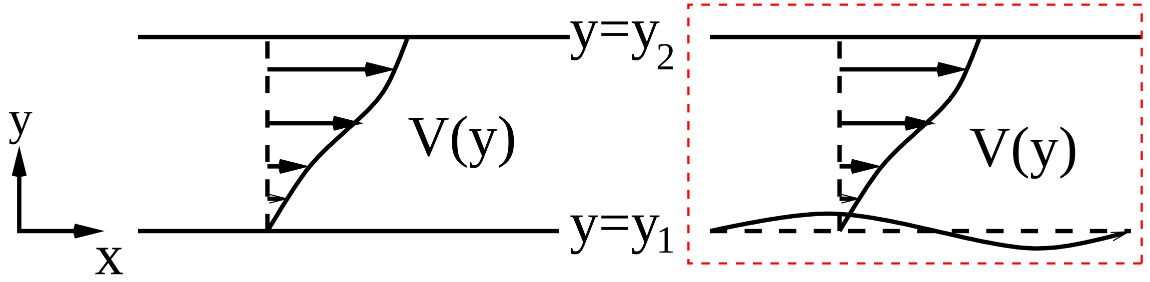
$$G \sim \rho V^2$$

$$Re \propto \Sigma^{1/2}$$

Kumaran JFM **294***, 259, (1995); JFM* **320***, 1, (1996); Shankar and Kumaran JFM* **395***, 211, (1999); JFM* **407***, 291, (2000).*

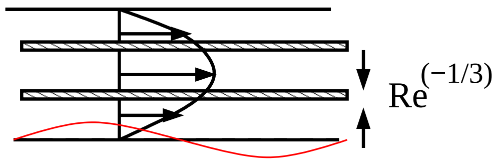
High Reynolds number:

Rayleigh theorem assumes zero normal velocity at walls ×



Rayleigh theorem — parabolic flow could be unstable.

Fjortoft theorem — wave speed between maximum and minimum of fluid velocity.



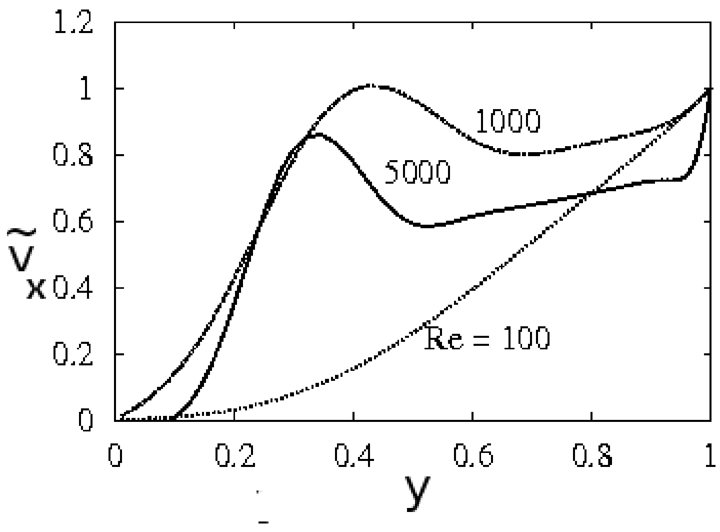
Internal critical layers.

Linear stability analysis:

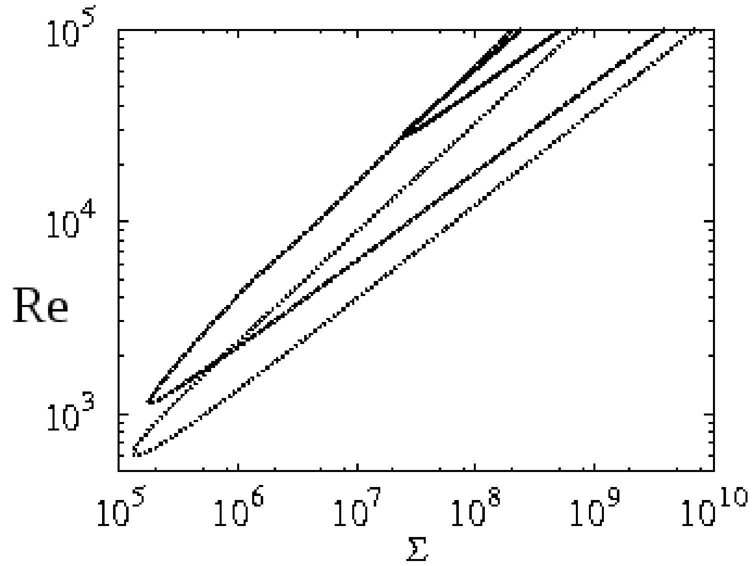
Orr-Sommerfeld equation for fluid velocity, coupled to wall displacement.

High Reynolds number instability:

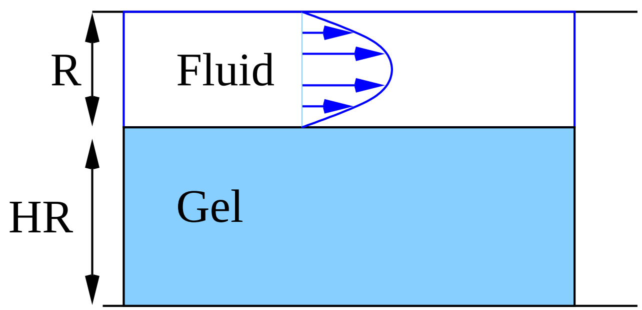
Sharp gradients in critical layer



Multiple solutions



H = 5, k = 1, η_r = 1



Viscous instability $Re \ll 1; Re \propto \Sigma$

Kumaran, Fredrickson & Pincus, J. Phys. France II **4***, 893, (1994).*

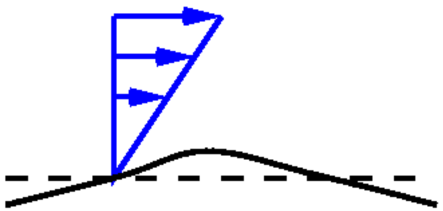
Kumaran, JFM **294***, 259, (1995).*

Shankar & Kumaran, JFM **434***, 337, 2001.*

Chokshi & Kumaran, PRE, **77***, 056303, 2008.*

Viscous instability

- Low Reynolds number — neglect inertia.
- Balance between viscous stresses in the fluid and elastic stresses in the wall material $\Gamma = (V\eta/GR) \sim 1; Re \propto \Sigma$.
- Equations *linear* — no coupling between mean flow and fluctuations.
- Coupling — tangential velocity boundary condition due to variation of mean velocity.

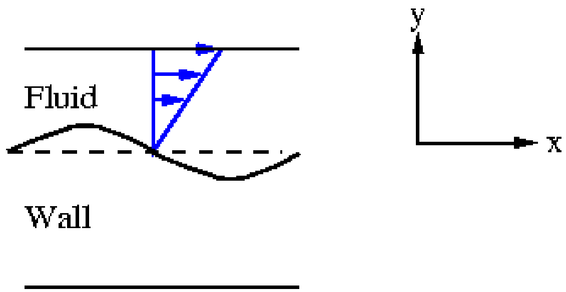


$$\tilde{v}_x|_{u_y} = \tilde{v}_x|_0 + \Gamma \tilde{u}_y|_0$$

Kumaran, Fredrickson & Pincus, J. Phys. France II **4***, 893, (1994);*
Kumaran, JFM **294***, 259, (1995).*

Results of stability analysis

- Flow becomes unstable for $\Gamma > \Gamma_c(k, H, \eta_r)$
- Mechanism of instability — transfer of energy from mean flow to fluctuations due to shear work done at the interface.

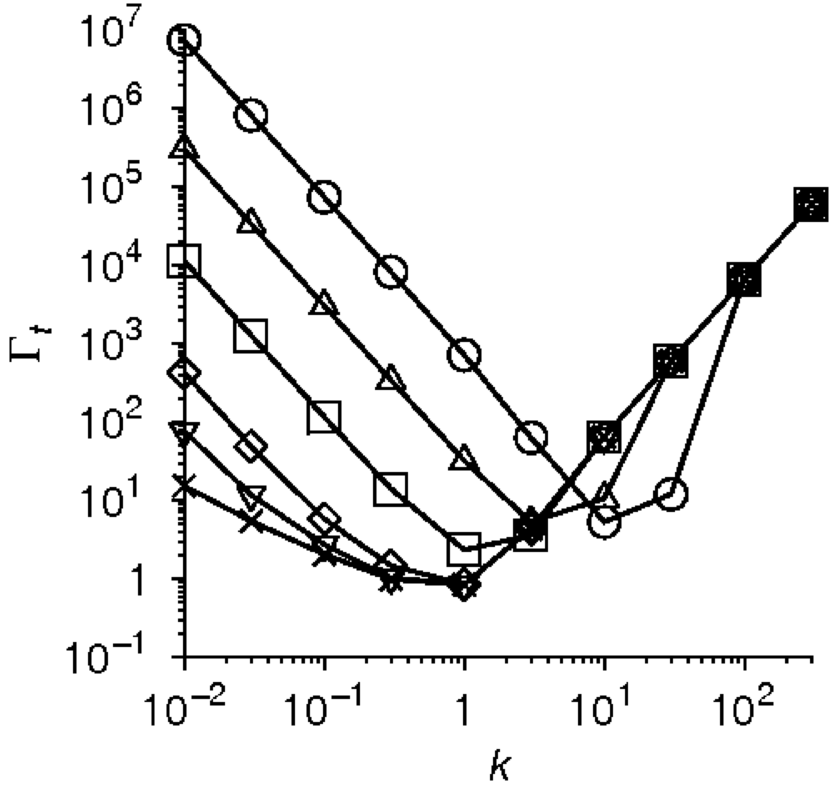


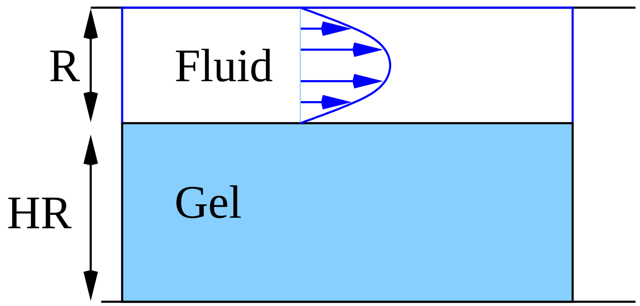
- Tangential motion of interface necessary for inducing instability.

Kumaran, Fredrickson & Pincus, J. Phys. France II **4***, 893, (1994);*
Kumaran, JFM **294***, 259, (1995).*

Neutral stability curves (Γ_t vs. k; $\eta_r = 0$)

$\circ$ $H = 1.1$; $\triangle$ $H = 1.3$; $H = 2$; $\diamond$ $H = 5$, ∇ $H = 10$; $\times$ $H = 100$.





Kumaran, Fredrickson & Pincus, J. Phys. France II **4***, 893, (1994);*
Kumaran, JFM **294***, 259, (1995).*

High Reynolds number wall layer instability

$Re \gg 1; Re \propto \Sigma^{3/4}$

Kumaran, EPJB **4**, *519, 1998.*

Shankar & Kumaran EPJB, **19**, *607, 2001.*

Shankar & Kumaran Phys. Fluids, **14**, *2324, 2002.*

Chokshi & Kumaran Phys. Fluids, **20**, *094109, 2008.*

High Reynolds number wall layer: $Re \sim \Sigma^{3/4}$

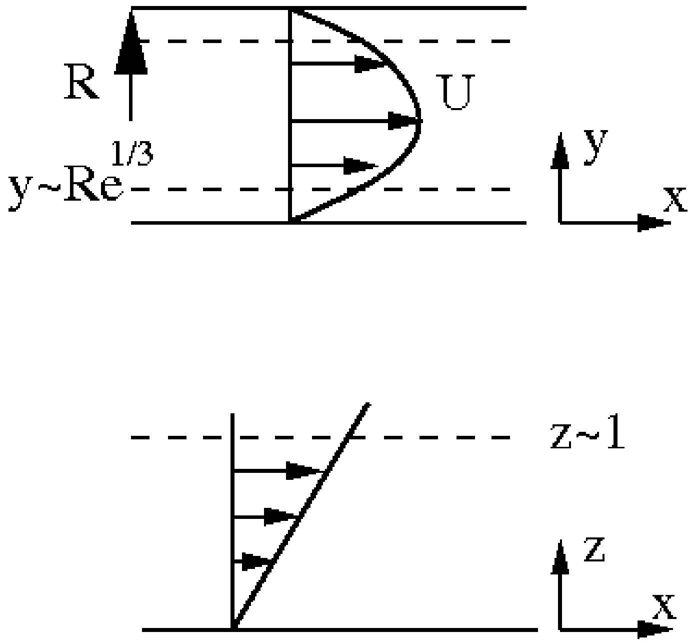
- Variation in y direction rapid compared to x direction.
- Equation for vorticity (strain rate)

$$\frac{\partial \tilde{\omega}}{\partial t} + V \frac{\partial \tilde{\omega}}{\partial x} = \frac{1}{Re} \left(\frac{\partial^2 \tilde{\omega}}{\partial y^2} + \frac{\partial^2 \tilde{\omega}}{\partial x^2} \right)$$

$$\frac{\partial \tilde{\omega}}{\partial t} + y \frac{\partial \tilde{\omega}}{\partial x} = \frac{1}{Re} \left(\frac{\partial^2 \tilde{\omega}}{\partial y^2} + \frac{\partial^2 \tilde{\omega}}{\partial x^2} \right)$$

- Rescale $z = (y/\delta)$, $\delta = Re^{-1/3}$, $t^* = (t/\delta)$.

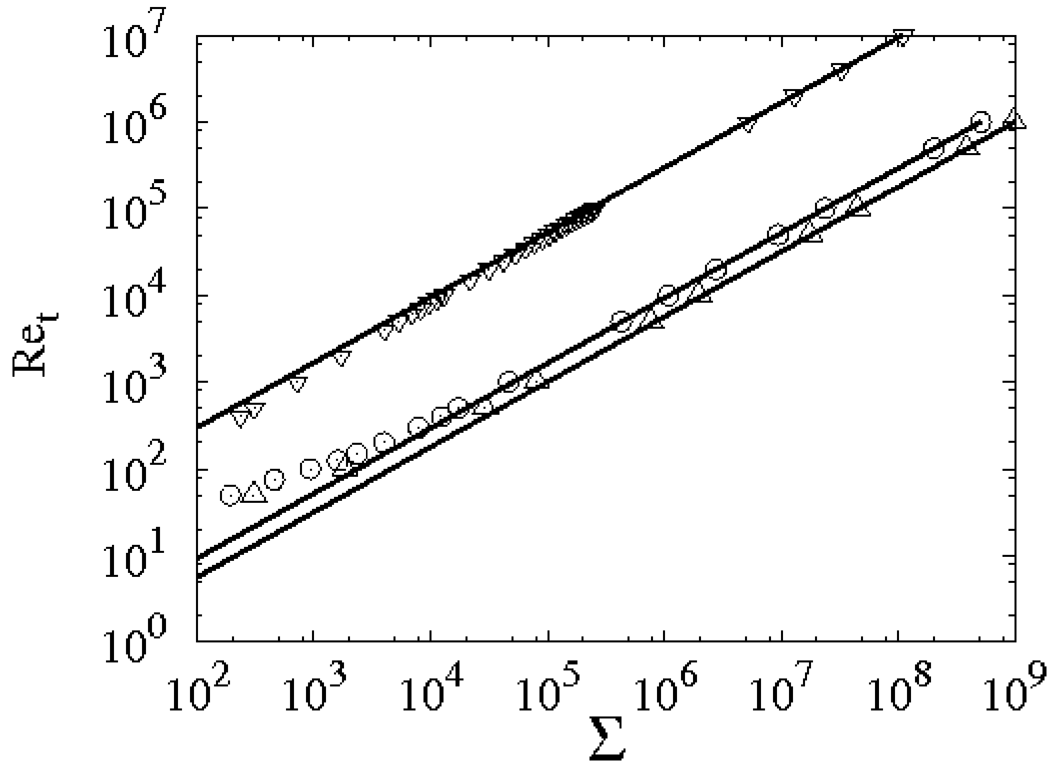
$$\frac{\partial \tilde{v}_x}{\partial t^*} + z \frac{\partial \tilde{v}_x}{\partial x} = \frac{\partial^2 \tilde{v}_x}{\partial z^2}$$

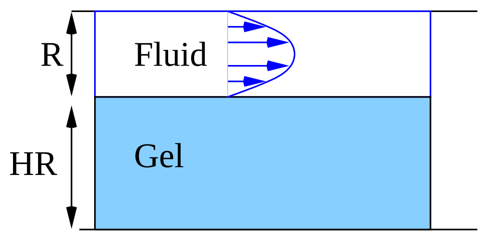


Kumaran, EPJB **4***, 519, 1998; Shankar & Kumaran EPJB,* **19***, 607, 2001.*

Wall mode instability

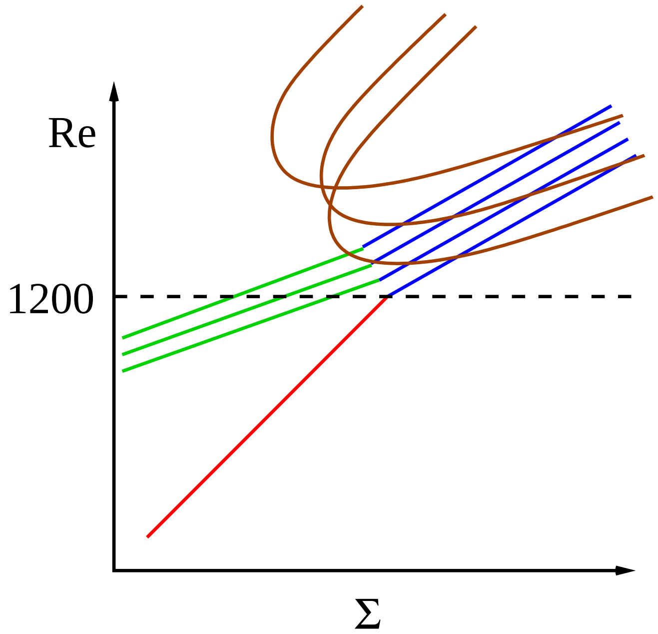
Neutral stability curves





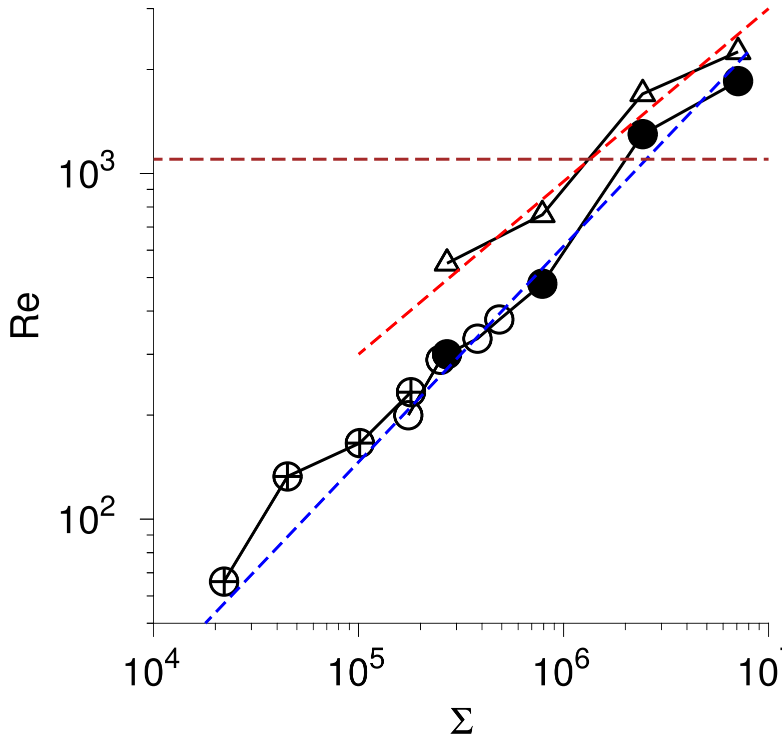
$H = 5, k = 1, \eta_r = 0$

Shankar & Kumaran Phys. Fluids, **14**, *2324, 2002.*

Summary: 

Regime	Mechanism	Flow structure	Non-linear
Viscous $Re \propto \Sigma \ll 1$	Shear work at interface		Sub-critical
Low Re inertial $Re \propto \Sigma^{1/2} \ll 1$	Reynolds stress		
Internal $Re \propto \Sigma^{1/2} \gg 1$	Reynolds stress	Internal viscous layer $Re^{-1/3}$	
Wall mode $Re \propto \Sigma^{3/4} \gg 1$	Shear work at interface	Wall viscous layer $Re^{-1/3}$	Super-critical

Conclusions: Theory & experiment.



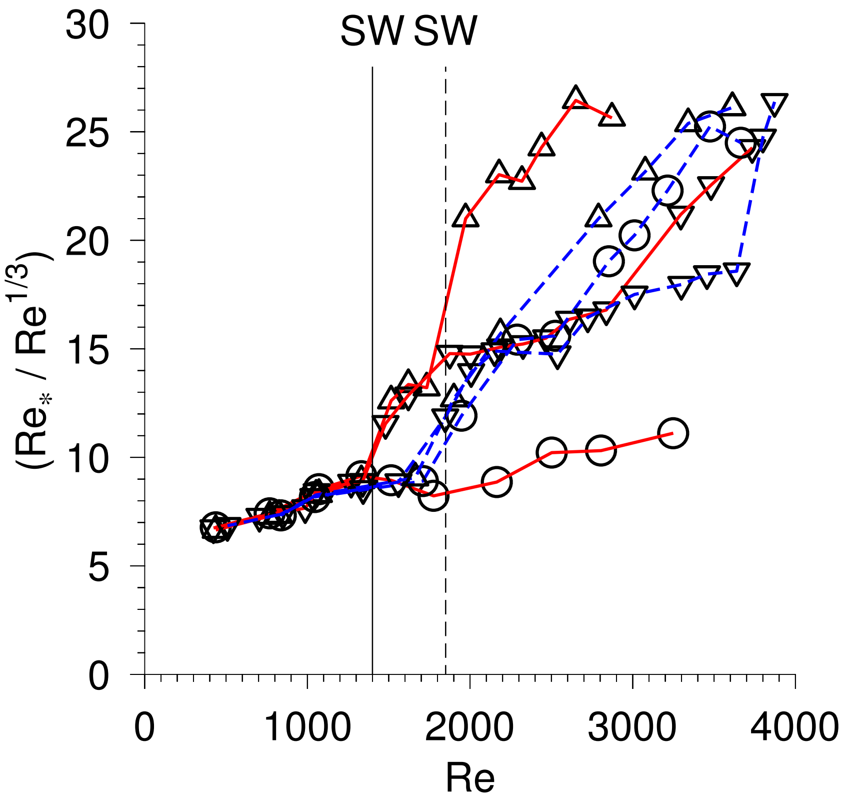
Hard-wall, Soft-wall, Wall flutter

⊕ Kumaran & Bandaru *Chem. Eng. Sci.* 2016, ∘ Verma & Kumaran *JFM* 2015, •, △ Srinivas & Kumaran *JFM* 2017.

1. Soft-wall transition — wall mode.
   (a) Need to include channel deformation, and consequent modification of velocity field and pressure gradient.
   (b) Wall deformation — Re $\propto \Sigma^{5/8}$; quantitatively accurate prediction of transition Reynolds number.
2. Wall flutter
   (a) Inviscid instability.
   (b) Transition Re $\propto \Sigma^{1/2}$.

Conclusions: Transitions from a turbulent flow.

- Wall-mode instability applicable if viscous sub-layer comparable or larger than wall layer in stability calculation.
- Viscous sub-layer $(30\nu/v_*) = (30h/\mathrm{Re}_*)$.
- Wall layer in stability analysis $(\delta_w/h) \sim \mathrm{Re}^{-1/3}$.
- Stability analysis applicable if $(\mathrm{Re}_*/\mathrm{Re}^{1/3}) < 30$.

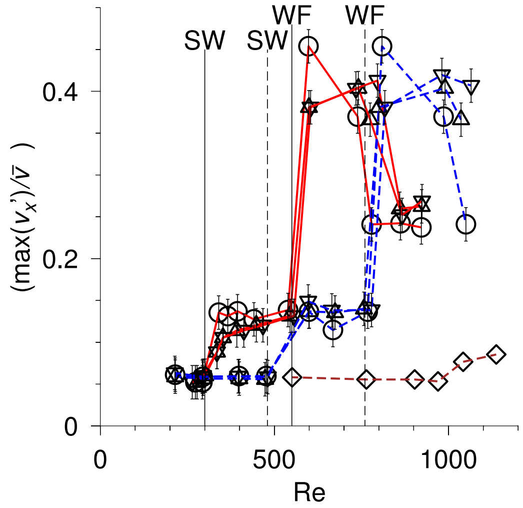


G = 0.75 kPa; G = 2.19 kPa; ○ Location II; △ Location III; ▽ Location IV.
Srinivas & Kumaran JFM 2017.

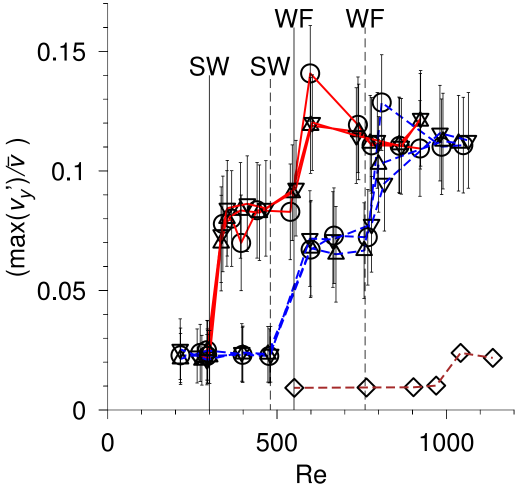
Open questions:

- Very large turbulent fluctuations at low Reynolds number.
- Ultra-fast mixing.
- Velocity fluctuations at the wall.
- Viscous sub-layer and logarithmic layer.
- Wall frequency and dynamics.

$(v'_x|_{max}/\bar{v}_x)$



$(v'_y|_{max}/\bar{v}_x)$



$(\langle v'_x v'_y \rangle|_{max} / (\bar{v}_x)^2)$