Settling-driven Instabilities in Mammatus Clouds

S. Ravichandran,
Eckart Meiburg and Rama Govindarajan

Turbulence from angstroms to light years

JNCASR UCSB
International Centre for Theoretical Sciences
tifr

24th January, 2018

Mammatus Clouds

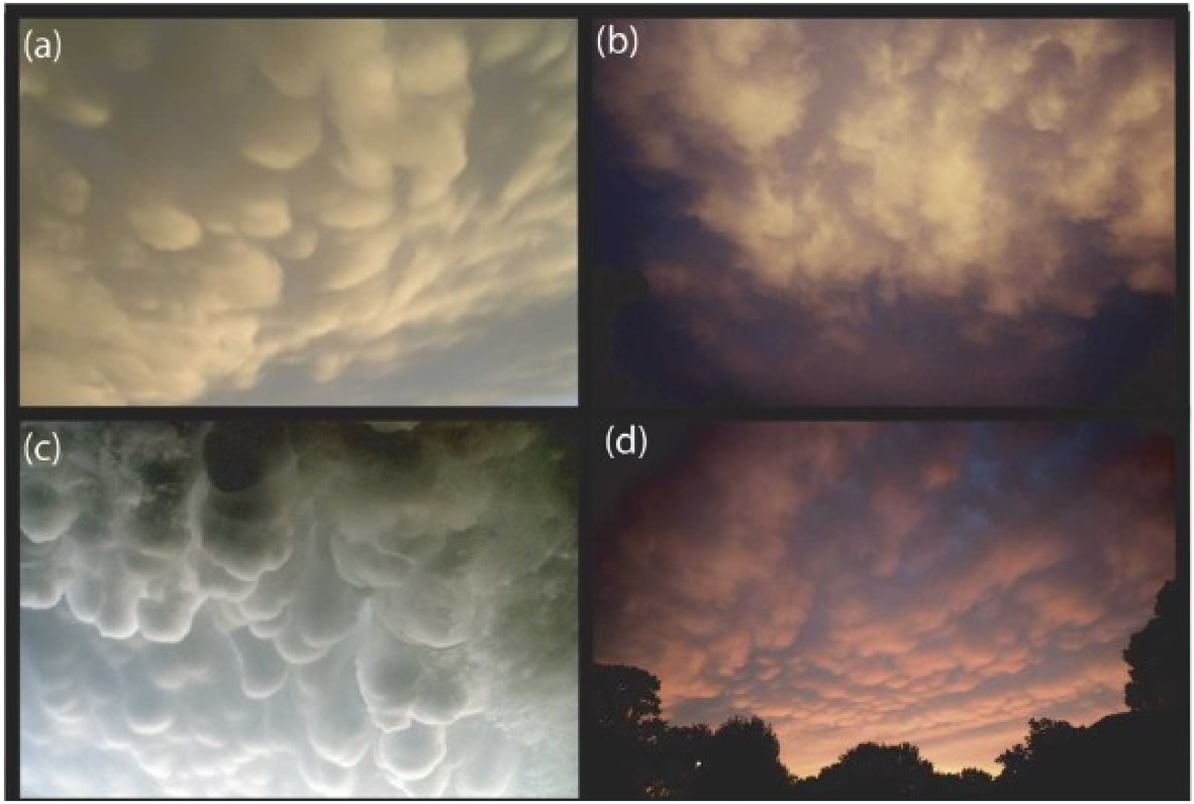
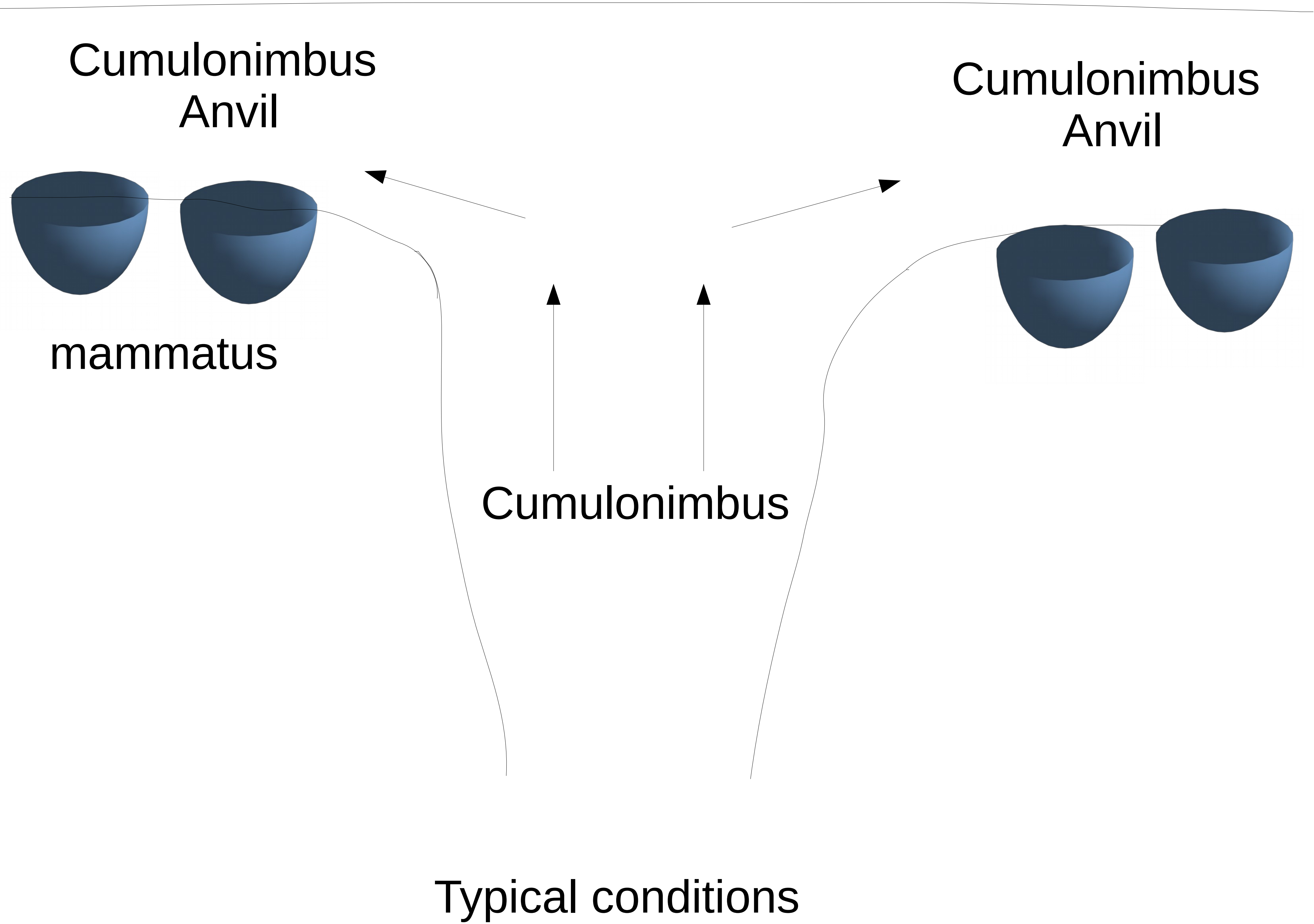


Fig. 2 from Schultz et al. 2006

Asperitas Clouds

In Cincinnati, Ohio by Ron Steele on August 3, 2015.

A dramatic asperitas cloud. (WMO International Cloud Atlas/Gary McArthur)


Cumulonimbus
Anvil
Cumulonimbus
Anvil
mammatus
Cumulonimbus
Typical conditions

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REVIEW

The Mysteries of Mammatus Clouds: Observations and Formation Mechanisms

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TIMOTHY J. GARRETT,[##] PETRA M. KLEIN,[#] AND DOUGLAS K. LILLY[@@]

Schultz *et al*., Journal of the Atmospheric Sciences **63** (2006)

Mammatus clouds: a primer

- Altitude: 2-8 km
- Horizontal extent: O(100m – 1km)
- Velocities: 1-3 m/s
- Lobe-like structures are smooth
- Lifetime: O(10 minutes)

- Typically found under cumulonimbus anvils

- Can also be found under
  - volcanic ash clouds
  - jet contrails

Mammatus clouds: a primer

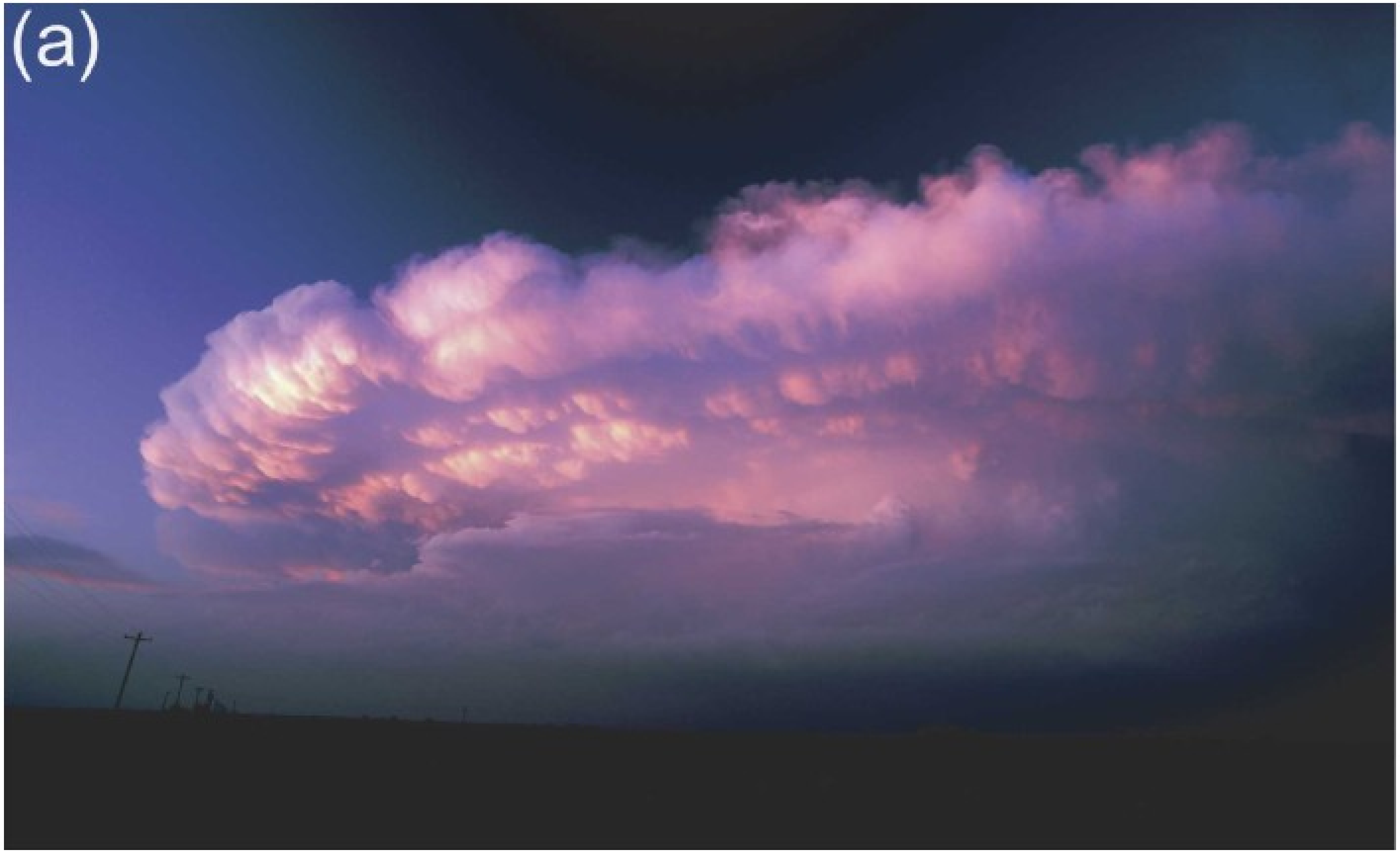
Several possible mechanisms suggested:

- (re-)circulation associated with cumulonimbus convection
- Radiation
- Gravity waves
- Kelvin-Helmholtz
- Rayleigh-Taylor

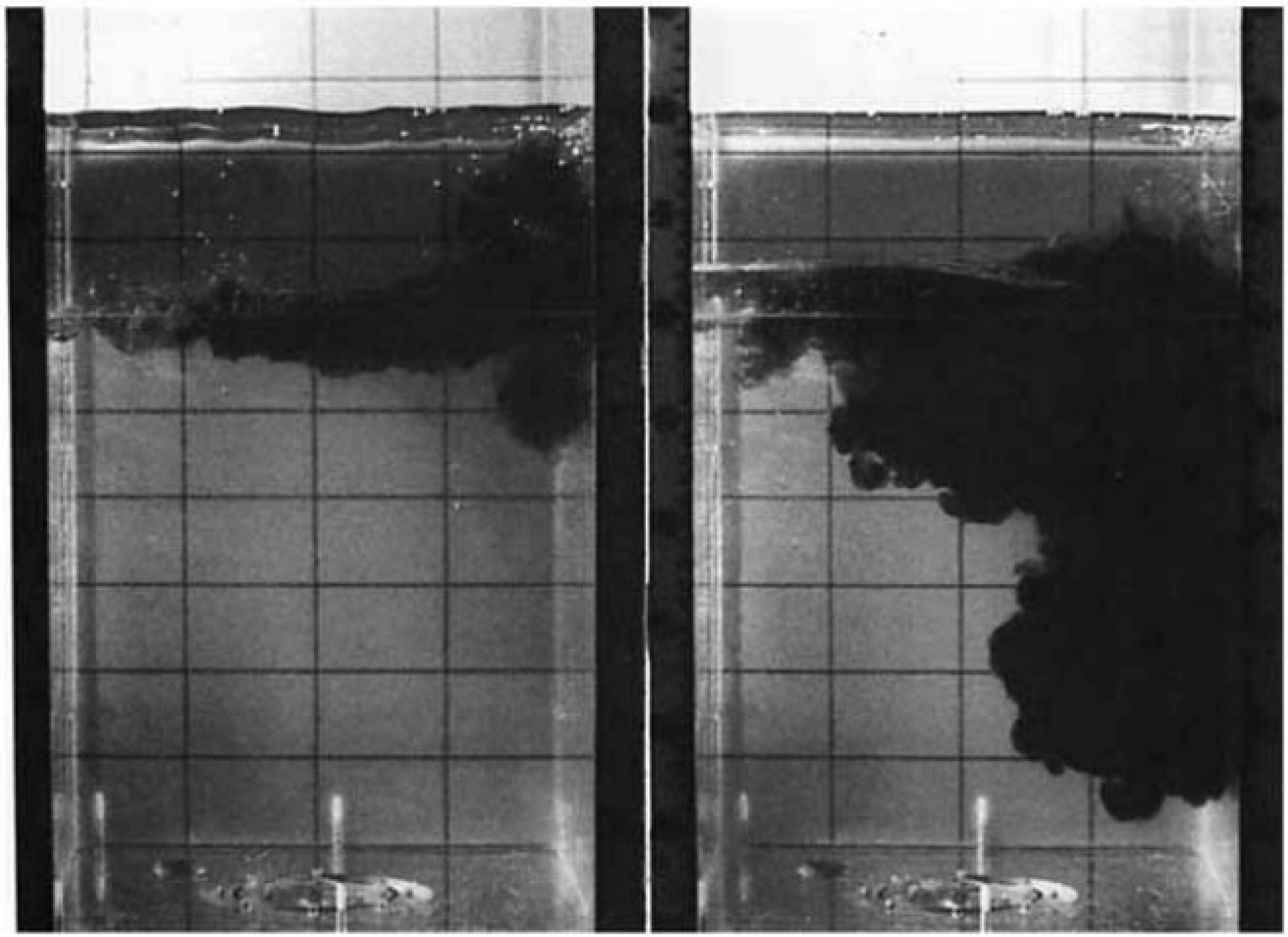
- **Detrainment at cloud-base**

- Settling- or precipitation-driven

Shear (=>mixing) => Mammatus Clouds



Mixing-driven stratocumulus cloud-top instabilities



(*a*) (*b*)

Shy and Breidenthal, J. Fluid Mech. **214** (1990)

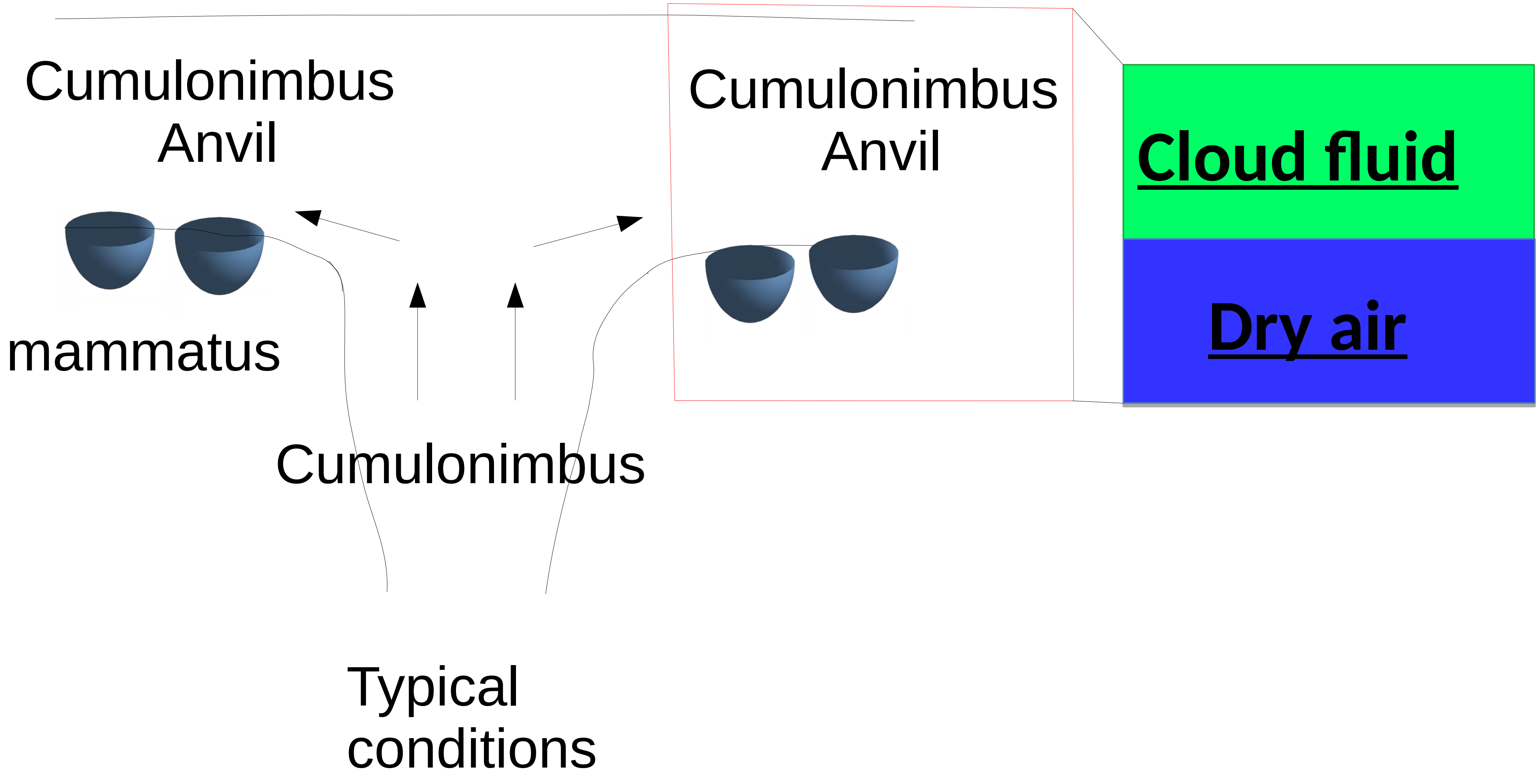
Mammatus clouds: a primer

Several possible mechanisms suggested:

- (re-)circulation associated with cumulonimbus convection
- Radiation
- Gravity waves
- Kelvin-Helmholtz
- Rayleigh-Taylor

- Detrainment at cloud-base

- **Settling- or precipitation-driven**

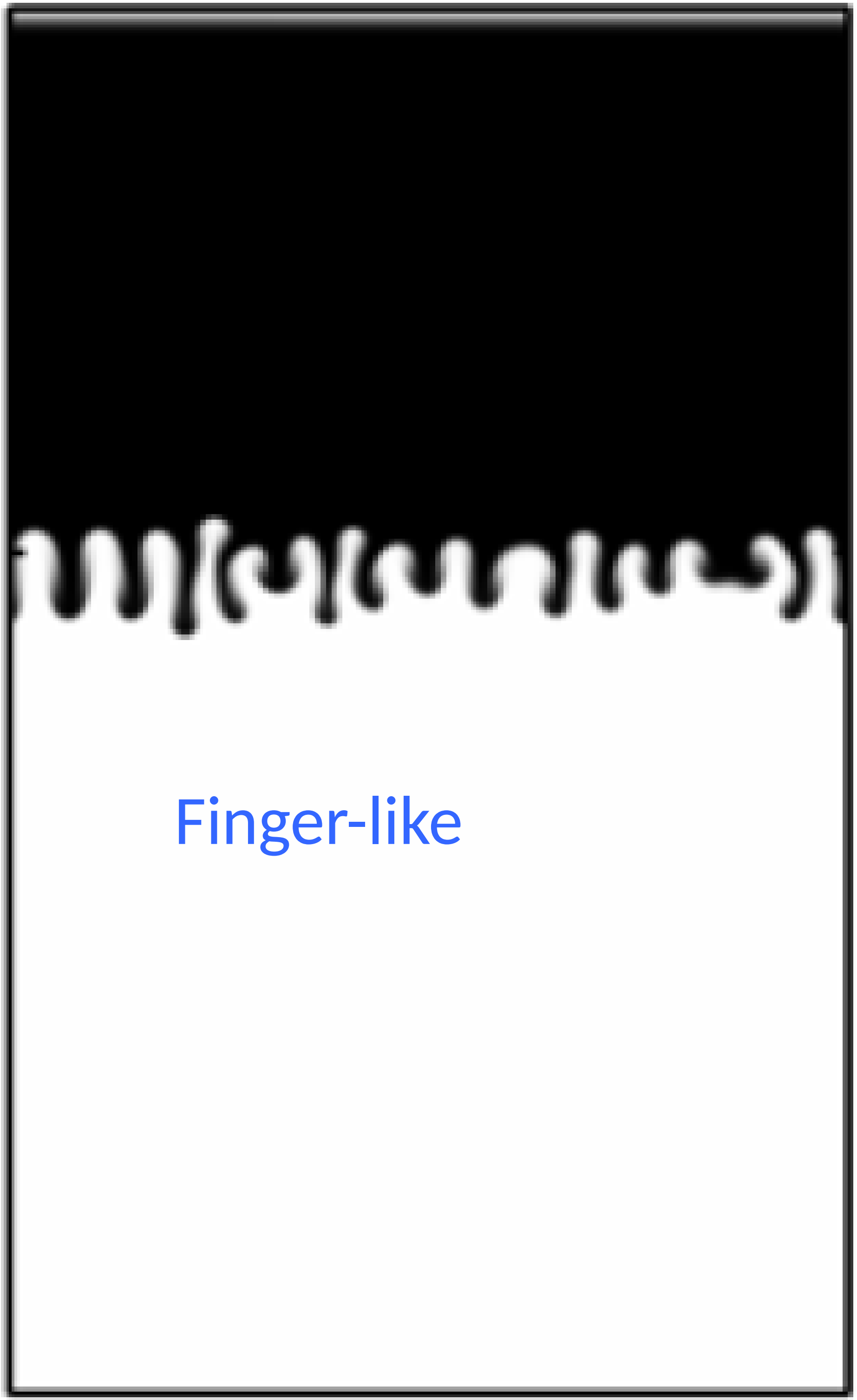


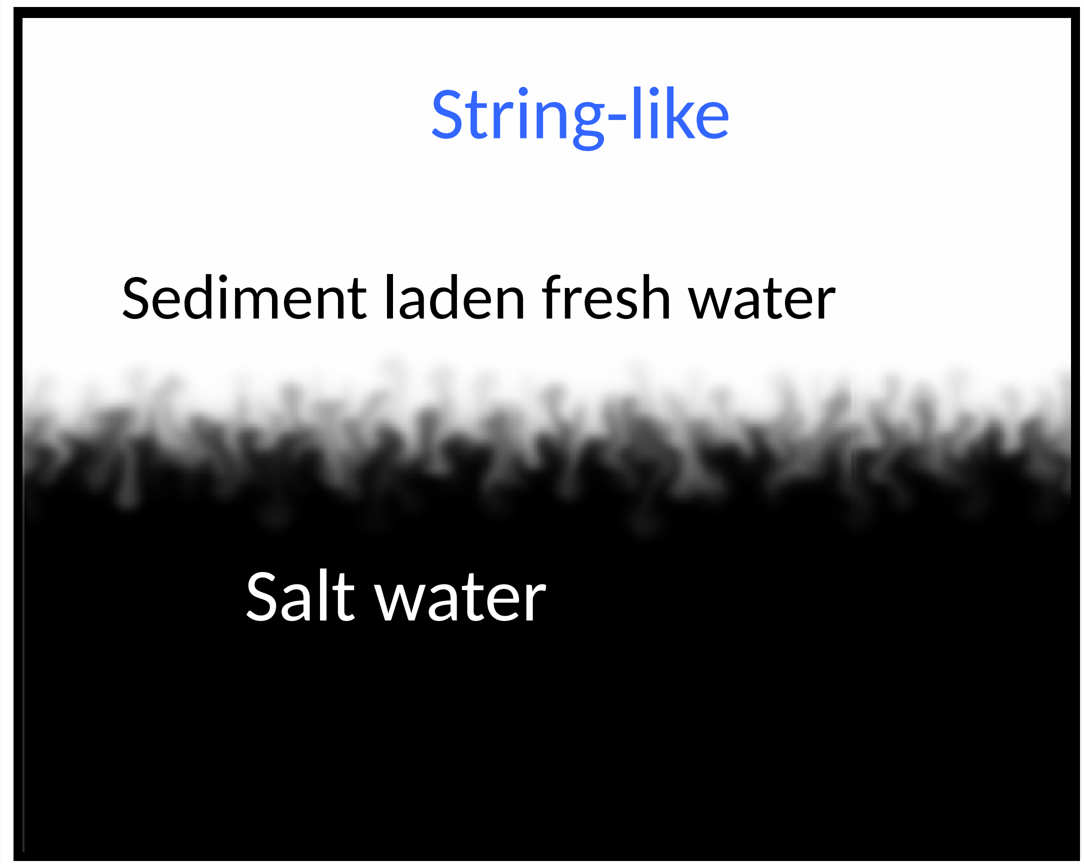
Cumulonimbus
Anvil
mammatus
Cumulonimbus
Anvil
Cloud fluid
Dry air
Cumulonimbus
Typical
conditions

Burns & Meiburg

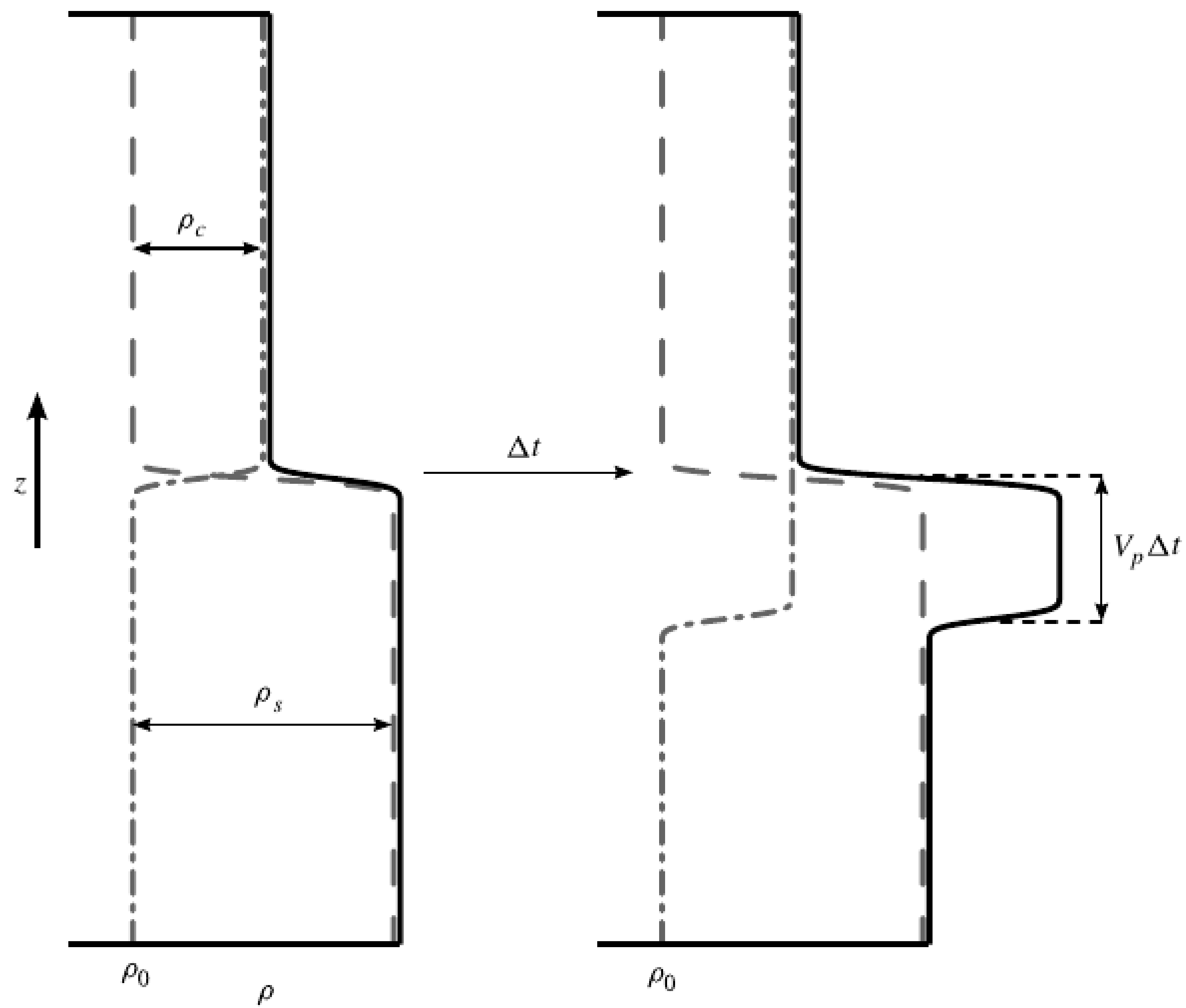
JFM 2012

JFM 2015





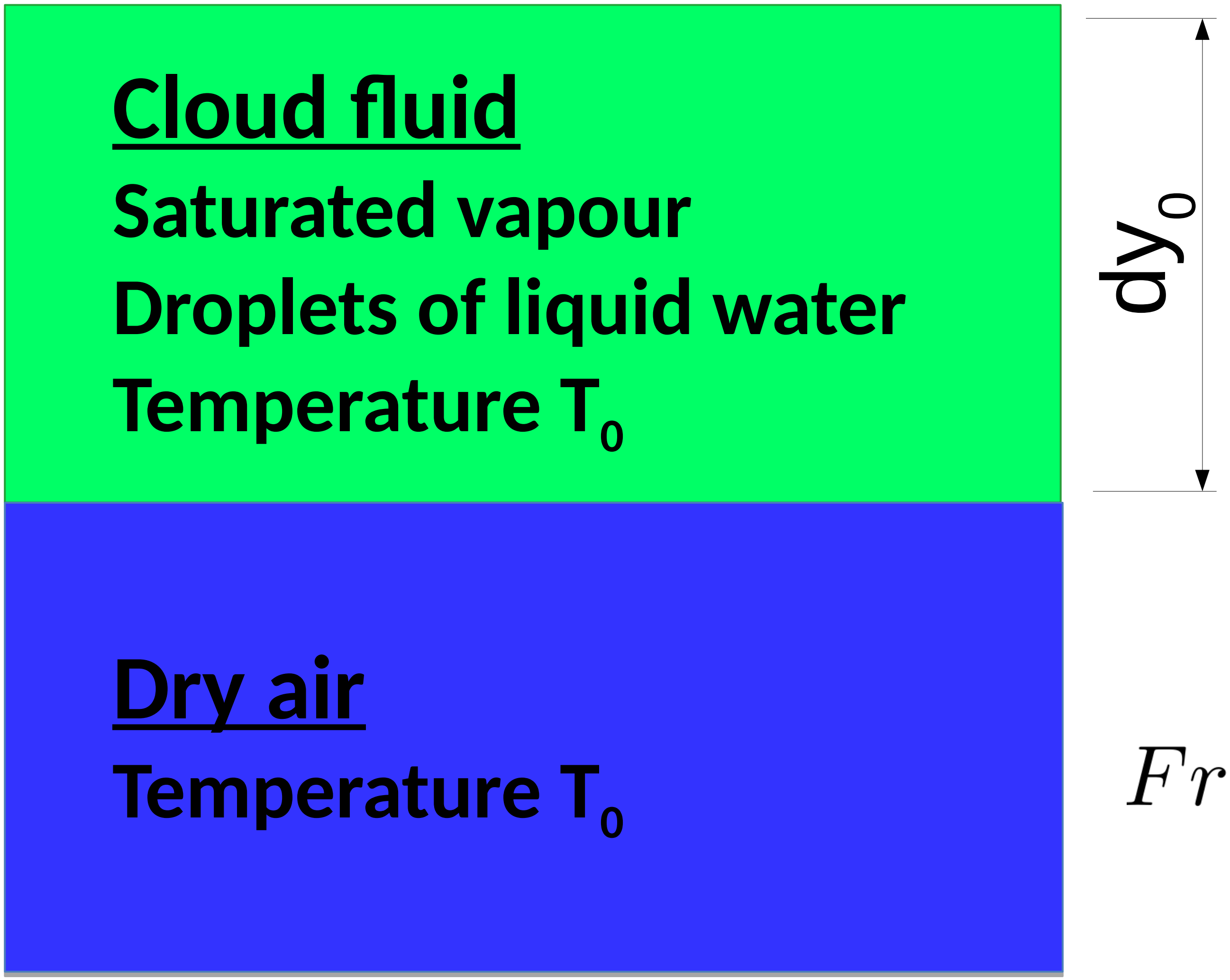
Density overhang



Silt Vs Salt

Burns and Meiburg, 2012

Parameter space:



$T_0 \quad \Delta T$

$$r_s^0 = r_s^0(T_0)$$

n, a_0 → $\tau_s \; r_l$

v_p

$$Fr^2 = \frac{U^2}{g\mathcal{L}\Delta T/T_0} \equiv 1$$

Pr_i i = 1,2,3

$$L_1 = \frac{L_v r_s^0}{C_p \Delta T} \qquad L_2 = \frac{L_v \Delta T}{R_v T_0^2}$$

Re

Model Initial Conditions

<u>Cloud fluid</u>

Saturated vapour

Droplets of liquid water

Temperature T_0

<u>Dry air</u>

Temperature T_0

$$\theta = \left(T - T_0\right) / \Delta T = 0$$

$$\omega = 0$$

$$r_v = \begin{cases} 1 & y > 0 \\ 0 & y < 0 \end{cases}$$

$$r_l = \begin{cases} r_l^0 & \delta y^0 > y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Flow parameters $Fr^2 \equiv 1$ Re Pr_i

Thermodynamic parameters

$$\theta = (T - T_0) / \Delta T$$

$$r_s(T) \approx \exp(L_2 \theta)$$

$$L_2 = \frac{L_v \Delta T}{R_v T_0^2}$$

$$L_1 = \frac{L_v r_s^0}{C_p \Delta T}$$

T_0 $r_s^0 = r_s^0(T_0)$ ΔT

Mechanical parameters

$$r_l^0 \sim n a_0^3$$

$$v_p \sim a_0^2$$

$$\tau_s \sim (n a_0)^{-1}$$

(n, a_0)

Nondimensional Equations

$$\frac{D\omega}{Dt} = \frac{1}{Fr^2}\frac{\partial}{\partial x}\theta + \frac{1}{Re}\nabla^2\omega$$

$$\frac{D\theta}{Dt} = \frac{1}{Re \cdot Pr_1}\nabla^2\theta - L_1 E$$

$$\frac{Dr_v}{Dt} = \frac{1}{Re \cdot Pr_2}\nabla^2 r_v + E$$

$$\frac{Dr_l}{Dt} = v_p\frac{\partial r_l}{\partial y} + \frac{1}{Re \cdot Pr_3}\nabla^2 r_l - E$$

$$E = -\mathcal{H}\frac{1}{\tau_s}\left(\frac{r_v}{r_s} - 1\right)$$

$$\mathcal{H} = \begin{cases} 1 & \text{saturated, or} \\ 0 & \text{unsaturated.} \end{cases}$$

Clausius-Clapeyron

$$r_s\left(T\right) = r_s^0 exp\left(\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right) \approx \exp\left(L_2\theta\right)$$

Linear Stability, 1D Nonlinear evolution

- Standard approach: linearise the equations, including the thermodynamics
- Problem: frozen-flow approximation is wrong. Instability time-scale ~ phase-change time-scale
- Our approach: evolve the (~)equations in 1D, get the density profile, do the stability on the net density profile

1D Nonlinear evolution

- Allows for parametric study of $\left(r_l^0, \delta y^0, \tau_s, v_p\right)$

$$\frac{\delta y}{\delta y^0} = \delta y\left(\frac{\delta y^0}{v_p \tau_s}, r_l^0\right)$$

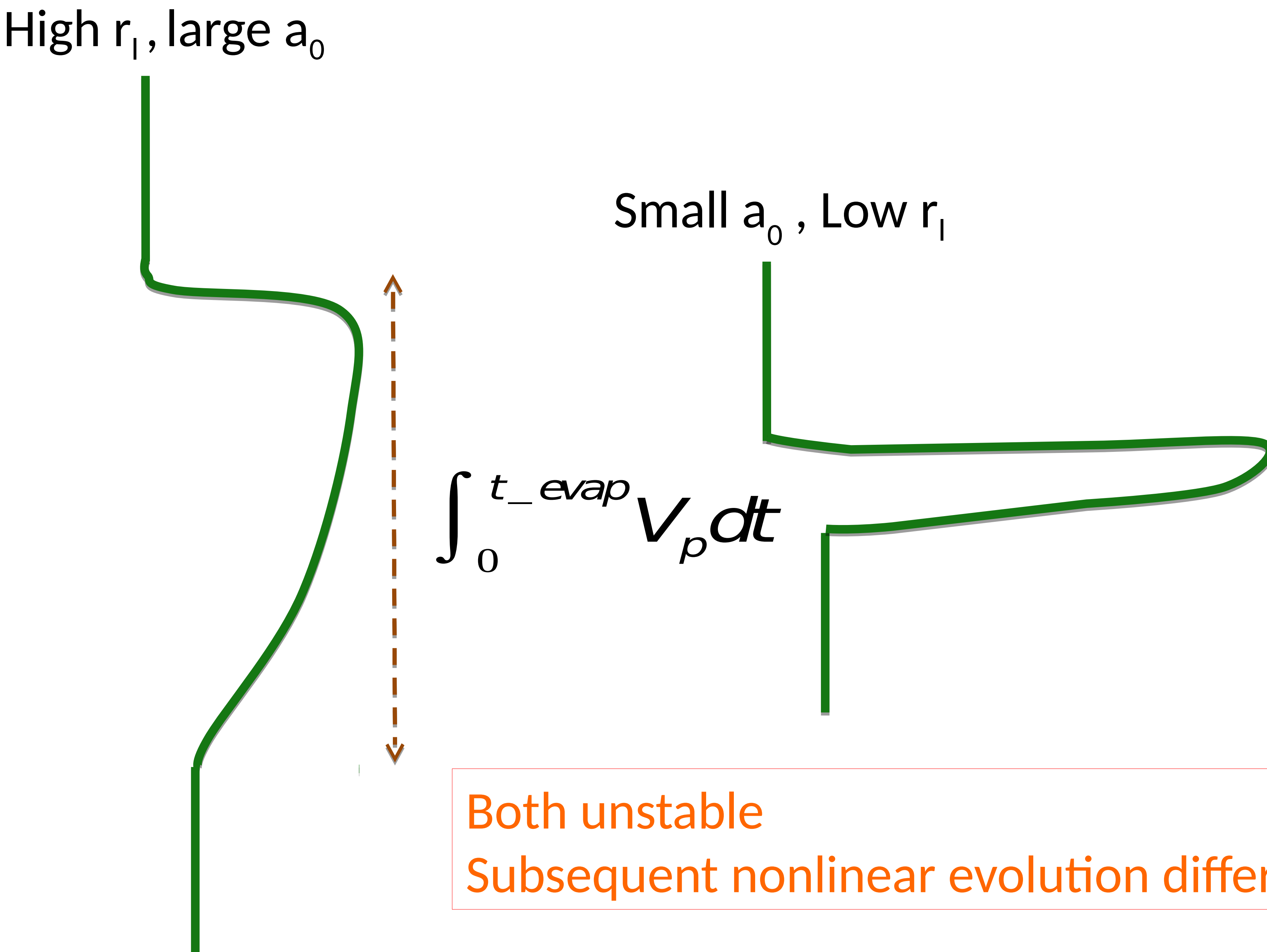
$$\theta_{max} = \theta_{max}\left(\frac{\delta y^0}{v_p \tau_s}, r_l^0\right)$$

$$t_{evap} \sim \tau_s \delta y / v_p$$

$$\left(r_l^0, \tau_s, v_p\right)$$

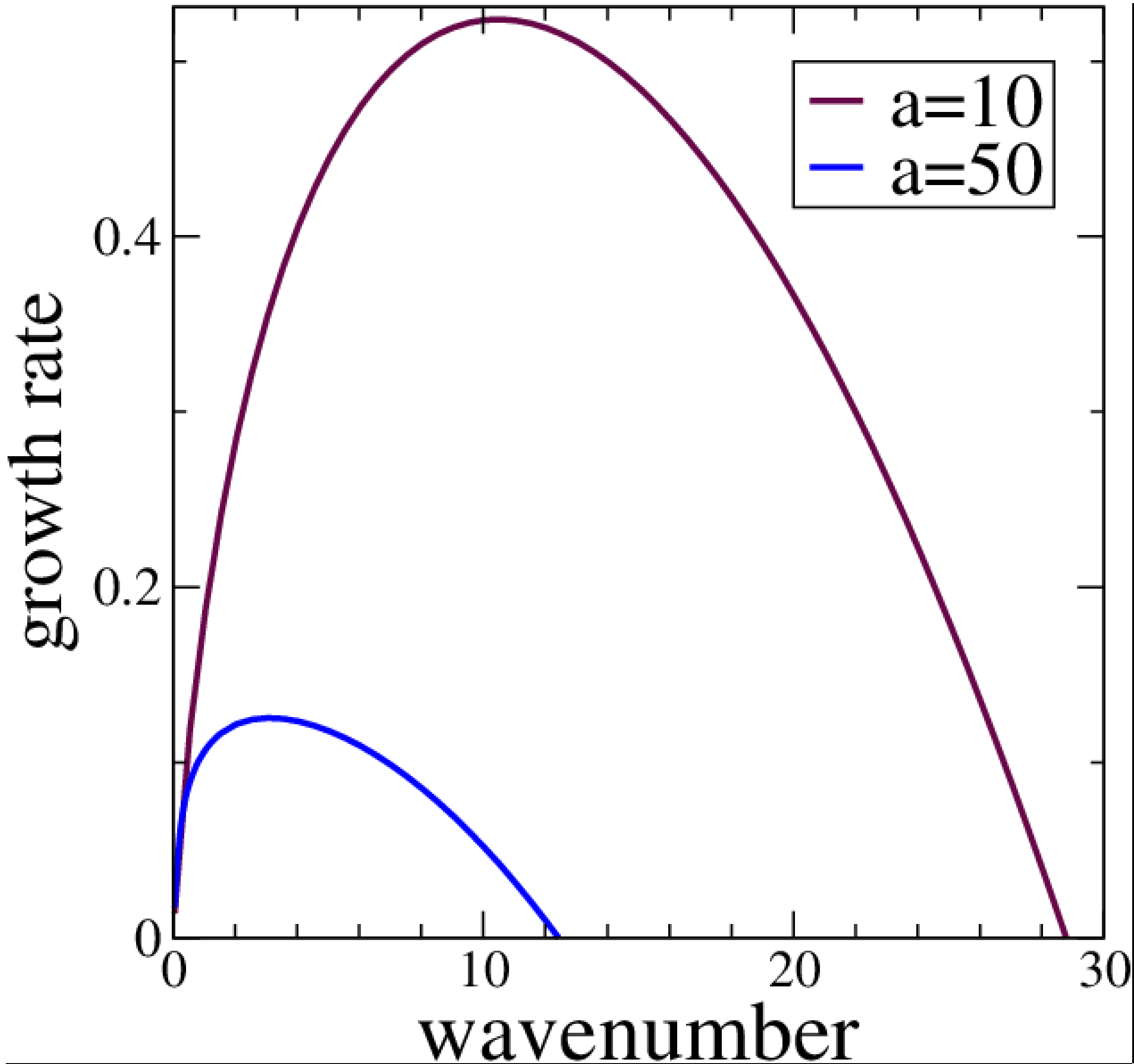
$$\delta y^0$$

Evolving density contours

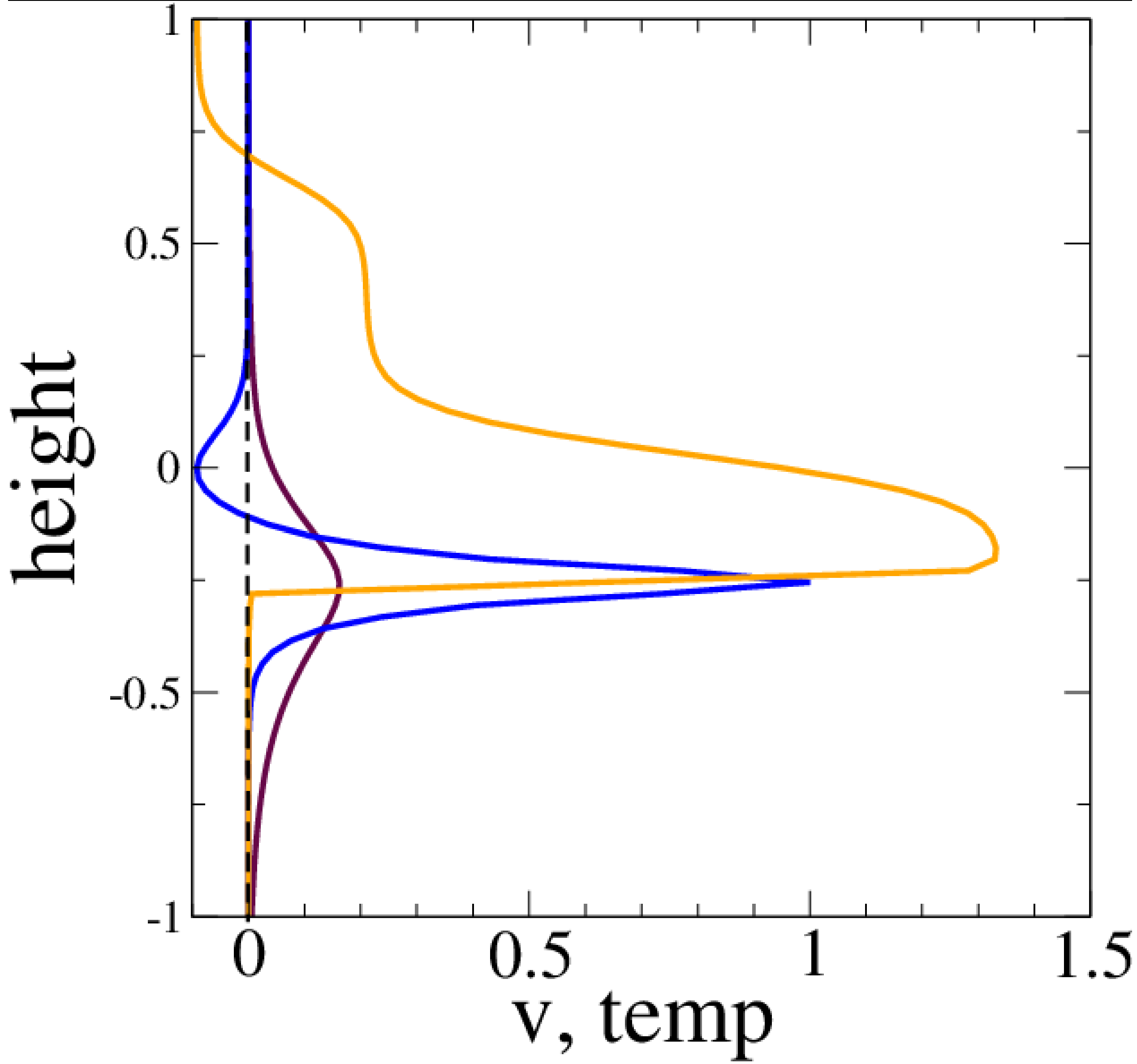


Both unstable
Subsequent nonlinear evolution different

Stability results



Stability results



2D Numerical Simulations

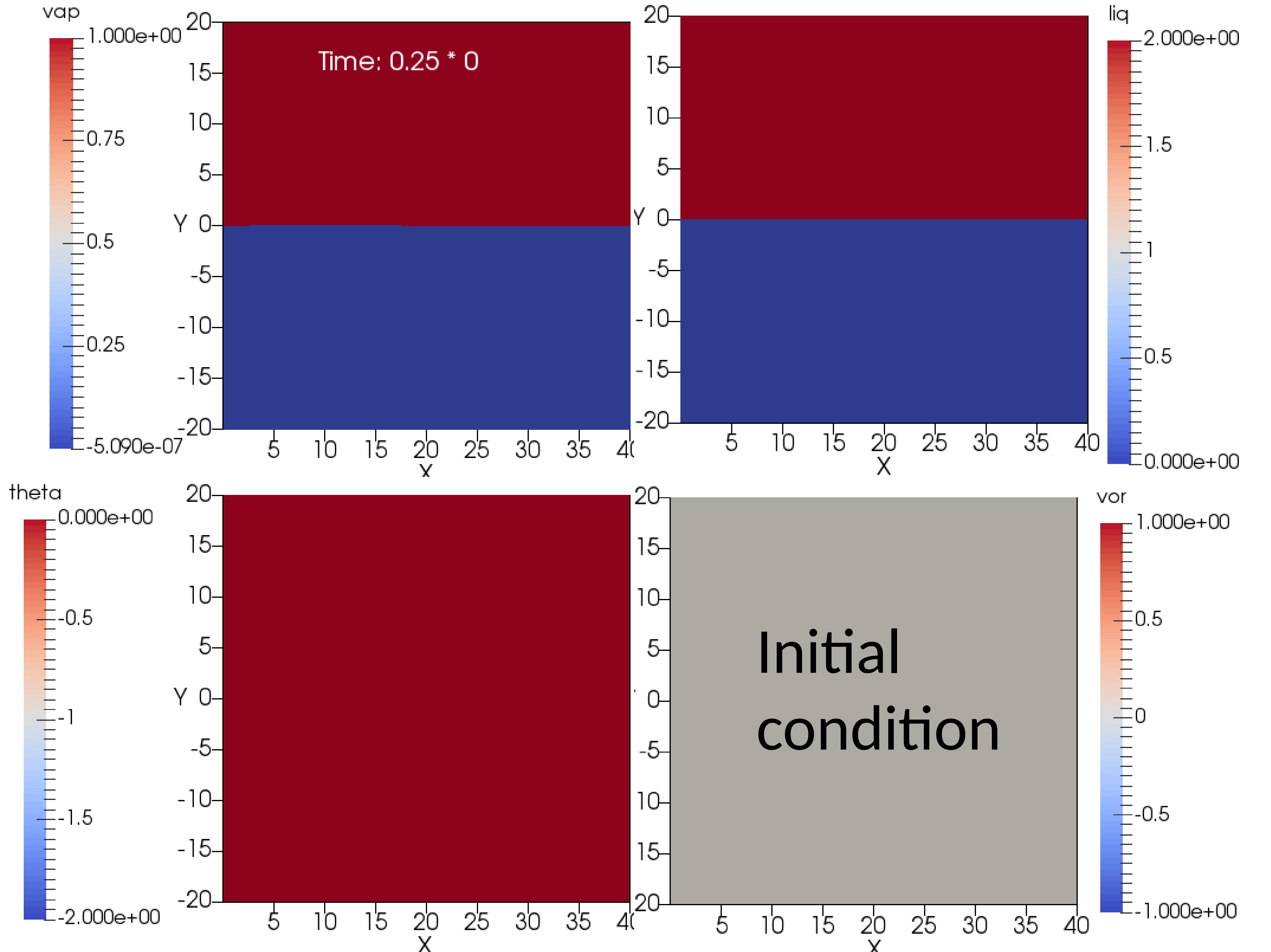
Message:

Settling velocity matters more than liquid density

$$v_p \sim a_0^2$$

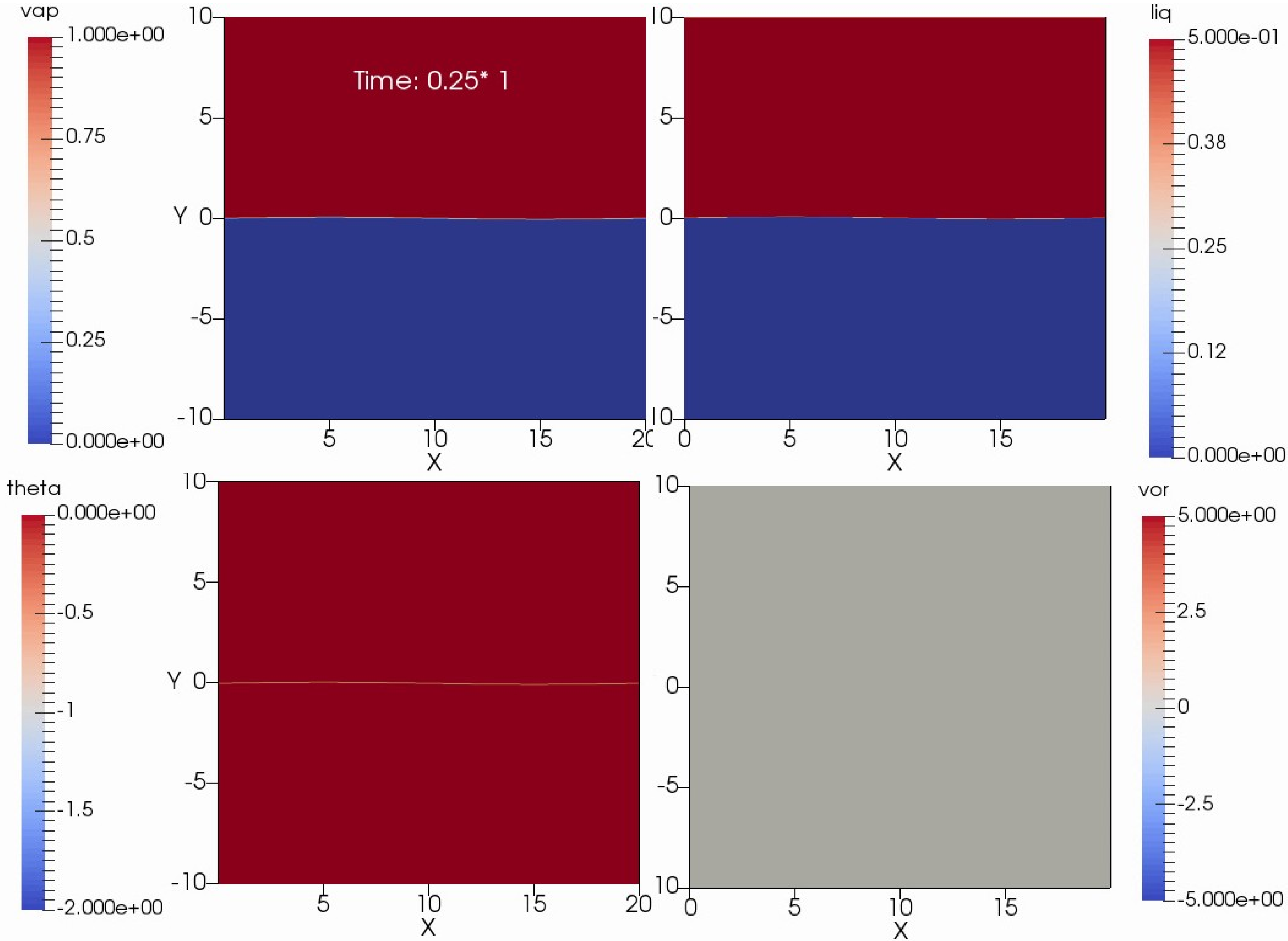
$$r_l^0 \sim n a_0^3$$

Simulations: small/large r_l^0, small/large a_0

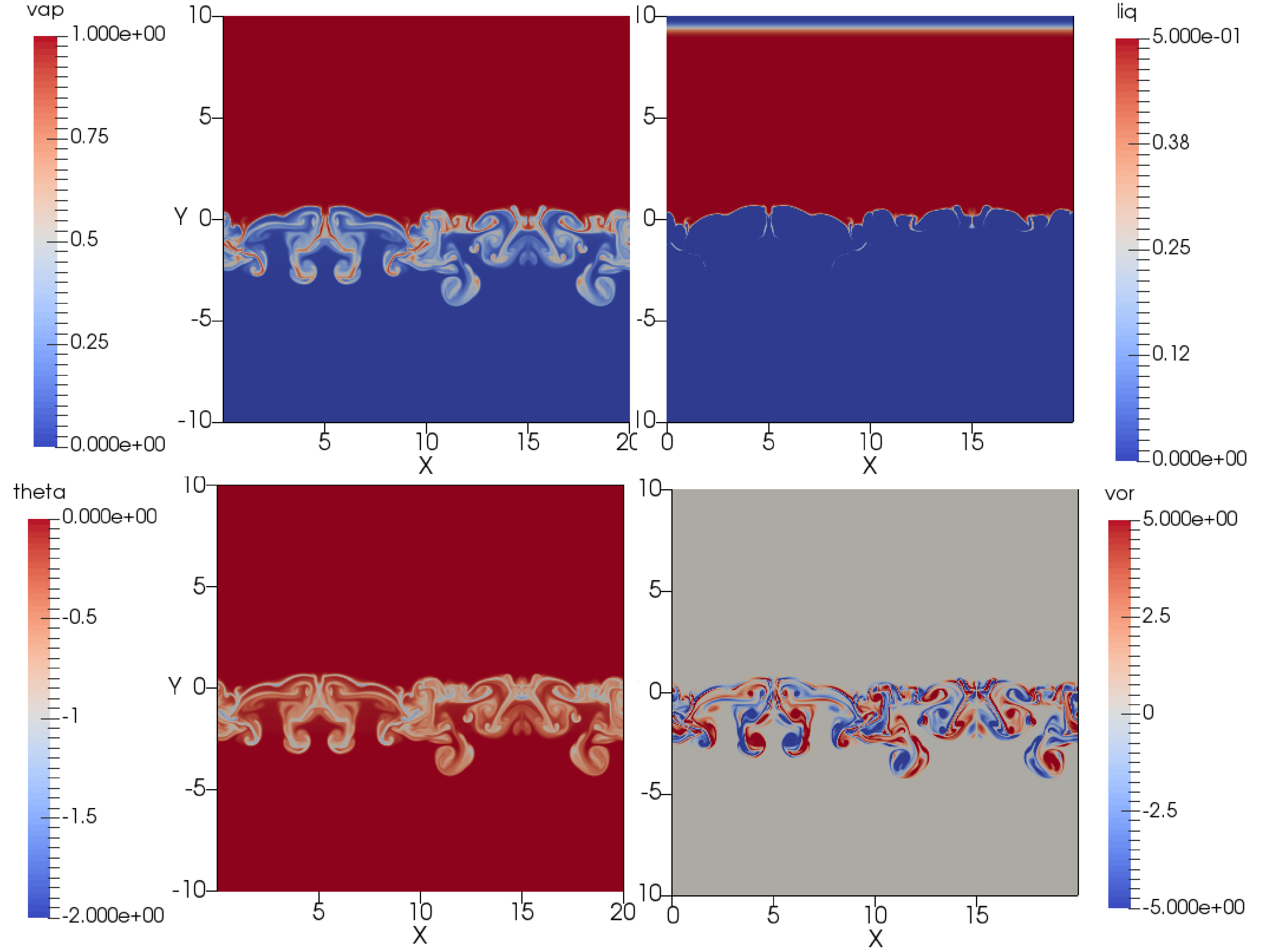

vap
1.000e+00
0.75
0.5
0.25
-5.090e-07
Time: 0.25 * 0
liq
2.000e+00
1.5
1
0.5
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
1.000e+00
0.5
0
-0.5
-1.000e+00
X
Y
Initial condition

Small r_l^0 , small a_0

$r_l^0 = 0.5$, $a_0 = 10\ \mu$

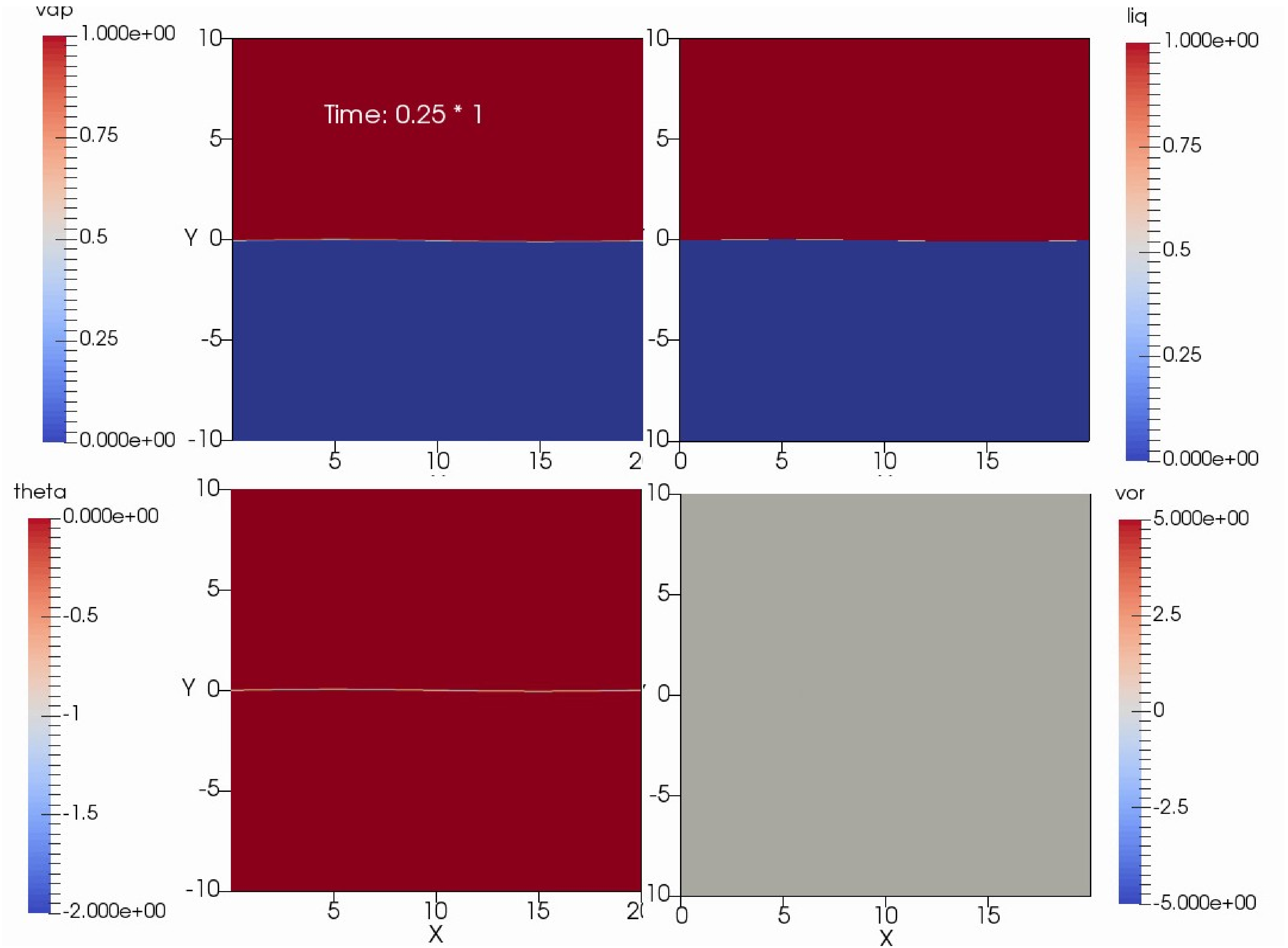


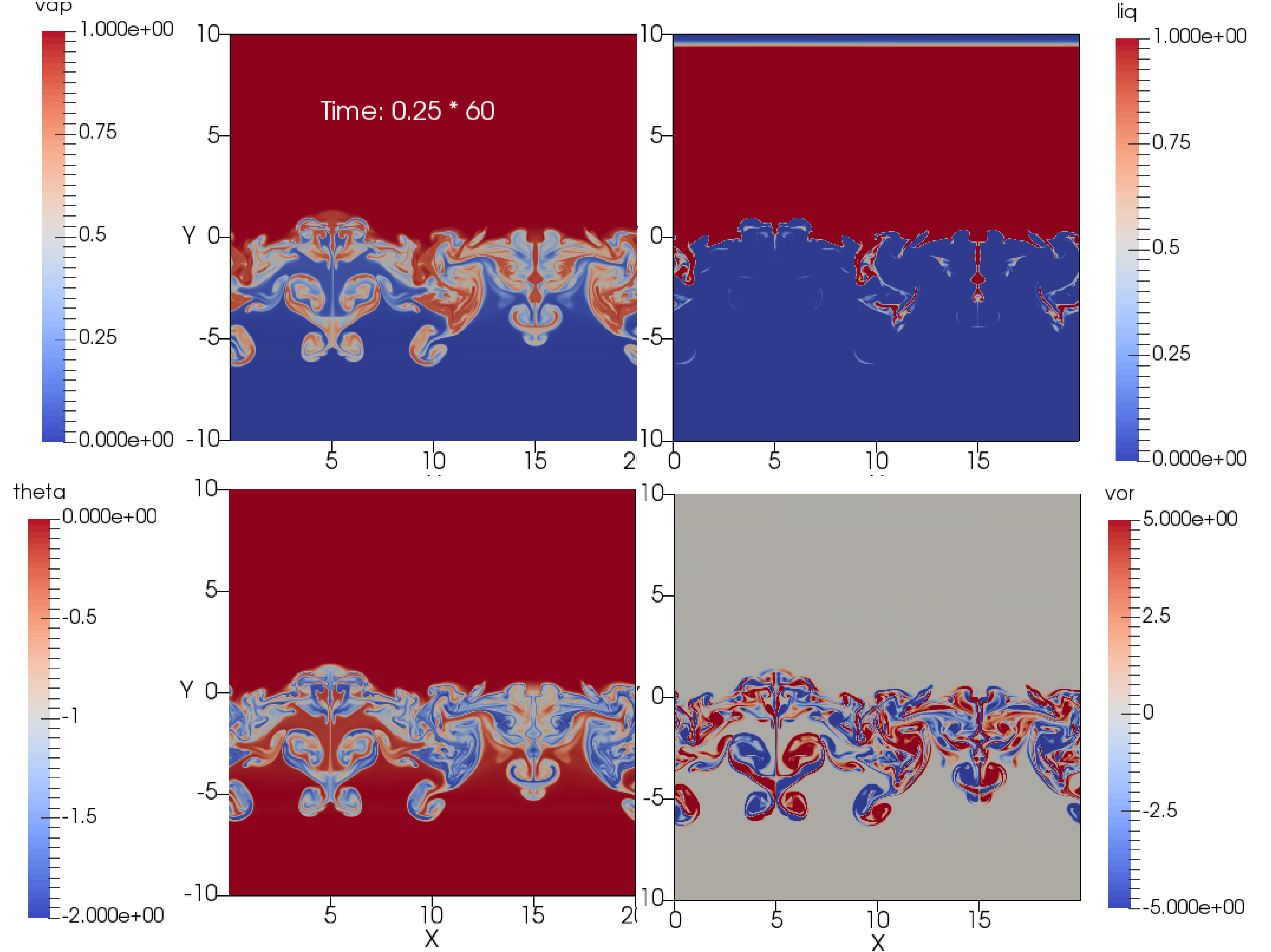
vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25* 1
liq
5.000e-01
0.38
0.25
0.12
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
X
Y


vap
1.000e+00
0.75
0.5
0.25
0.000e+00
liq
5.000e-01
0.38
0.25
0.12
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
X
Y

large r_l^0 , small a_0

$r_l^0 = 2.0$, $a_0 = 10\ \mu$


vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 * 1
liq
1.000e+00
0.75
0.5
0.25
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
Y
X



vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 * 60
liq
1.000e+00
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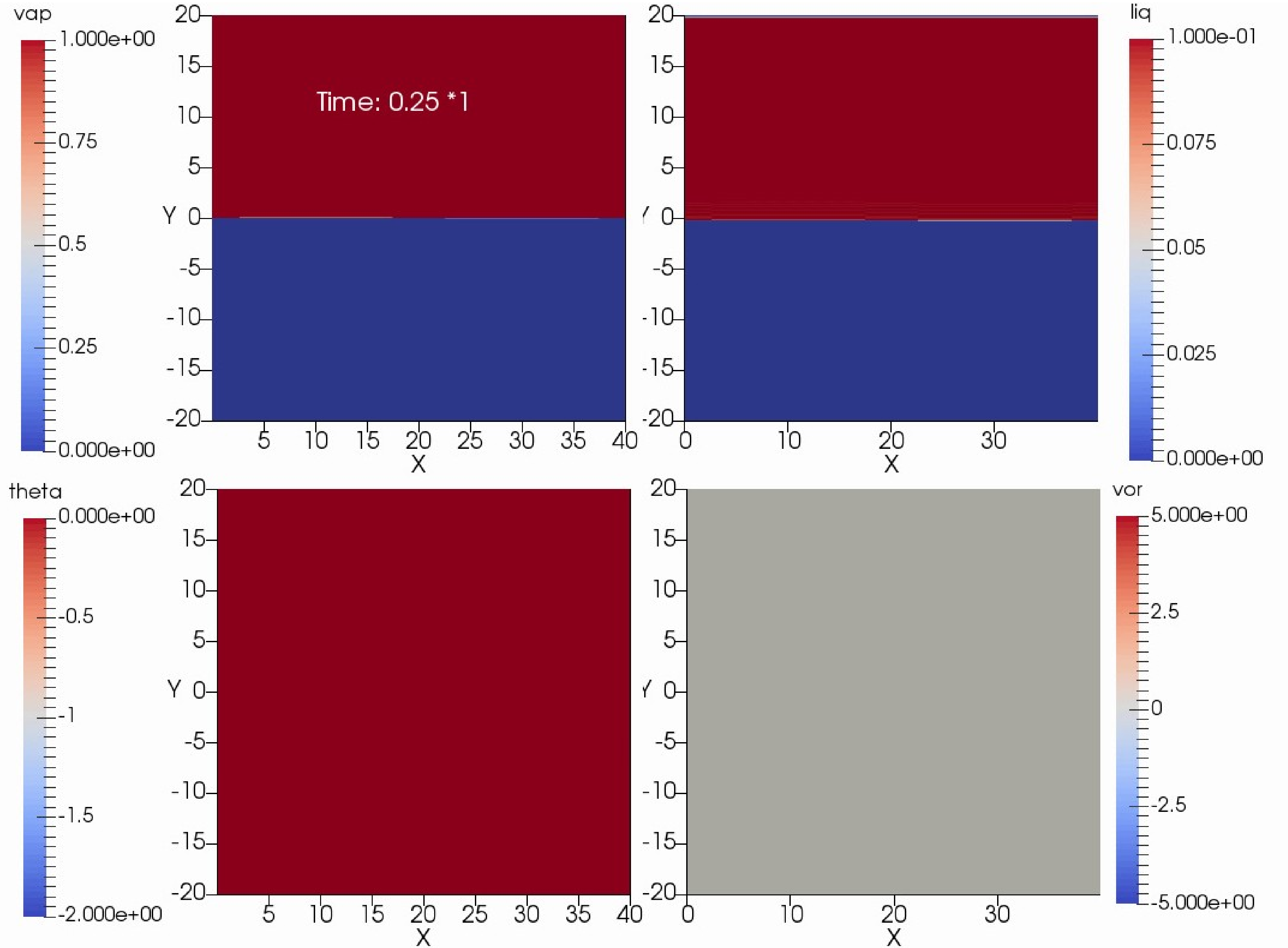
0.5
0.25
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
X
Y

Very Small r_l^0, large a_0

$r_l^0 = 0.1$, $a_0 = 50\ \mu$



vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 *1
liq
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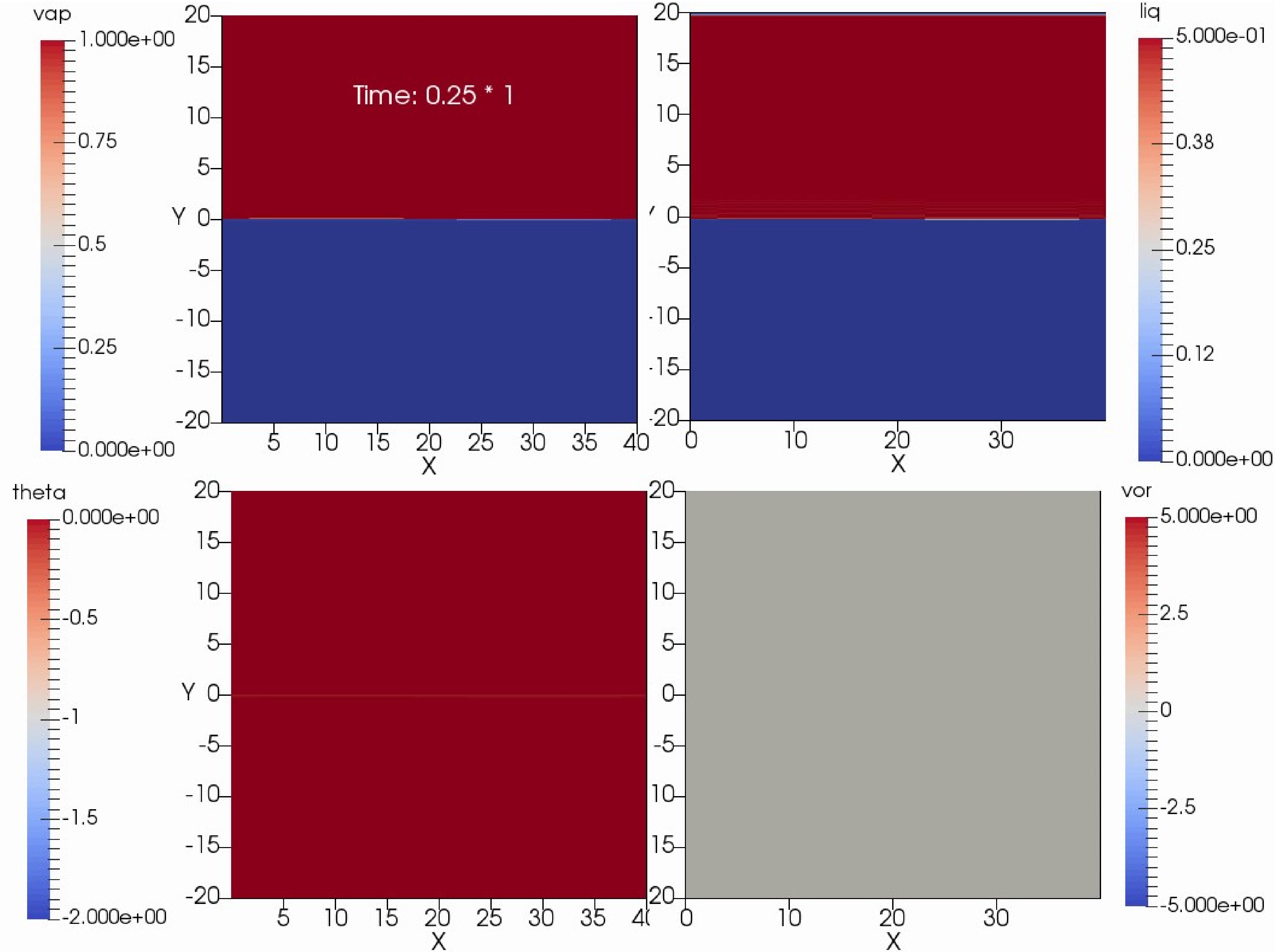
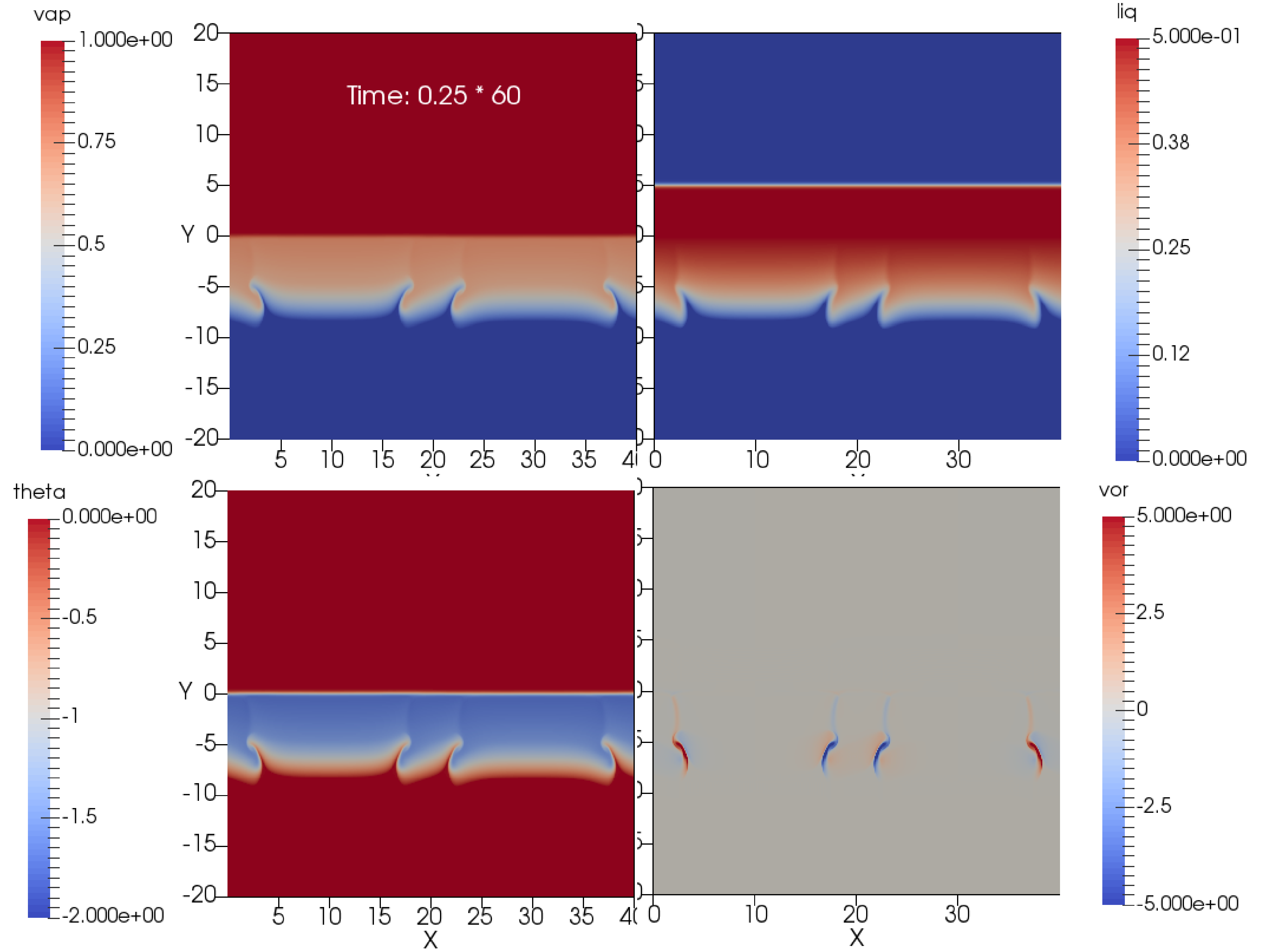
0.075
0.05
0.025
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
X
Y

Small r_l^0, large a_0

$r_l^0 = 0.5$, $a_0 = 50\ \mu$



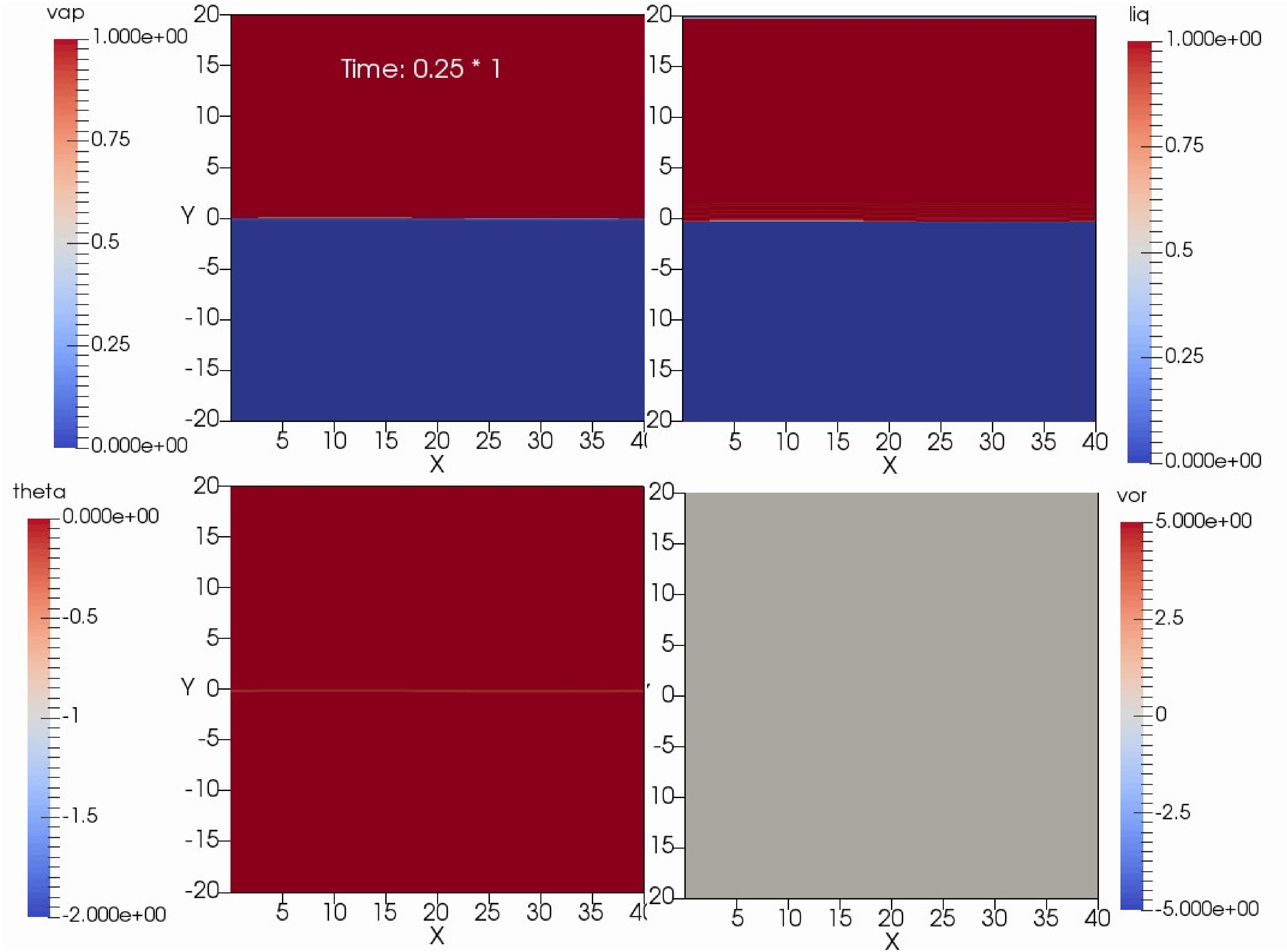
vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 * 1
liq
5.000e-01
0.38
0.25
0.12
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
X
Y



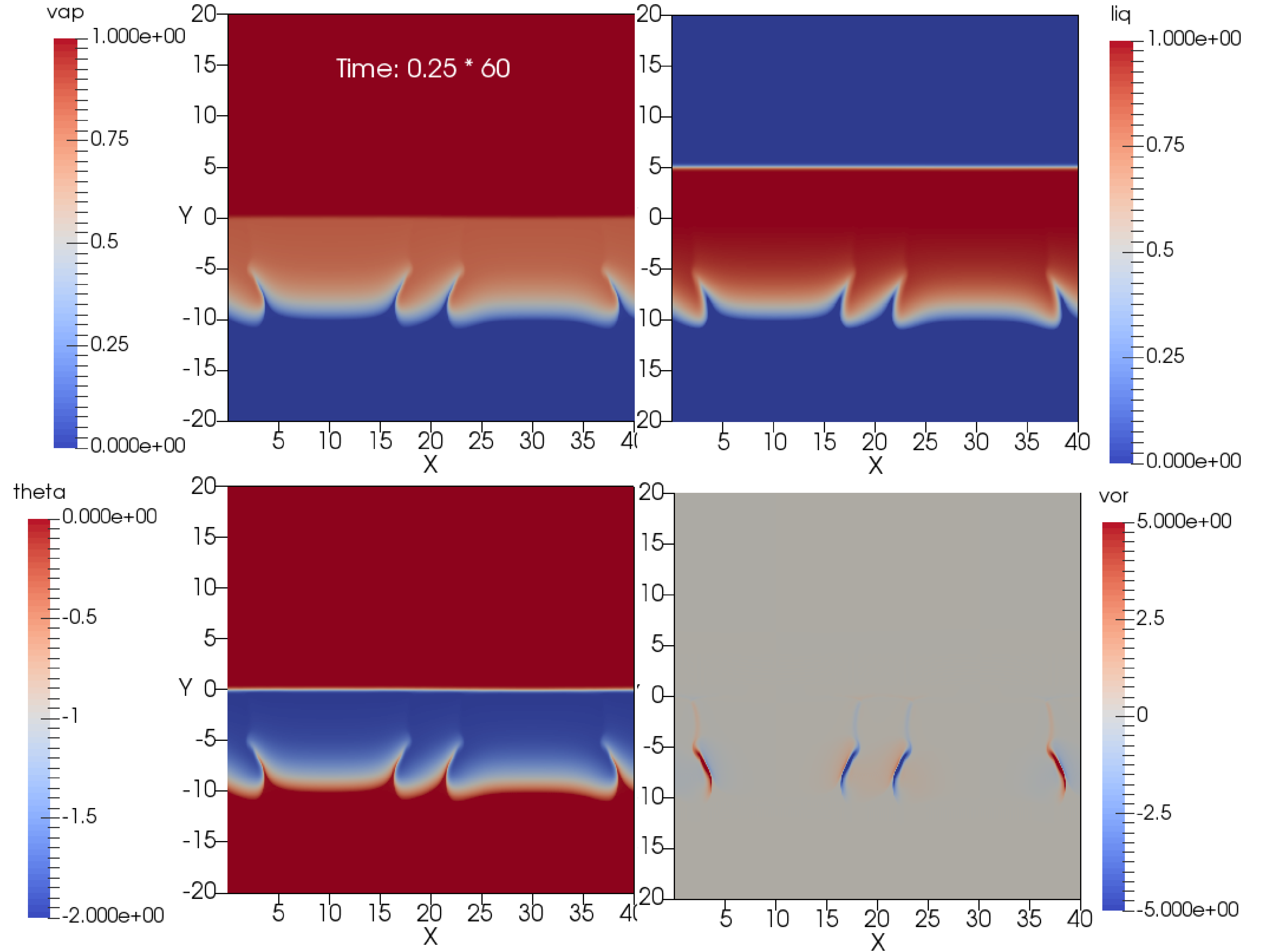
vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 * 60
liq
5.000e-01
0.38
0.25
0.12
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00
Y
X

Large r_l^0, large a_0

$r_l^0 = 1.0$, $a_0 = 50\ \mu$



vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 * 1
Y
X
liq
1.000e+00
0.75
0.5
0.25
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00



vap
1.000e+00
0.75
0.5
0.25
0.000e+00
Time: 0.25 * 60
Y
X
liq
1.000e+00
0.75
0.5
0.25
0.000e+00
theta
0.000e+00
-0.5
-1
-1.5
-2.000e+00
vor
5.000e+00
2.5
0
-2.5
-5.000e+00

Message:

Settling velocity matters more than liquid density

Summary

- A minimal model for mammatus clouds

- More to be done:
  - Shear/Mixing ("detrainment instability"): Asperitas?
  - 3D simulations
  - Compare predictions with observations