Happy Birthday,
Sreeni!

“When I was about 12 or 13, my family priest taught me a prayer and said that I was to recite it 108 times a day: one hundred for myself and eight for the rest of humanity. If I did not find the time for 108 recitations, I should do 58, 50 for myself and eight for humanity. And if I couldn't do 58, I should do 33, 25 for myself and eight for humanity. The point is that no matter how much or how little one does for oneself, one should always contribute a constant amount for humanity. Coming to ICTP and furthering its causes may be my way of contributing to the rest of humanity.”

News from ICTP 103

Phenomenology of turbulent thermal convection

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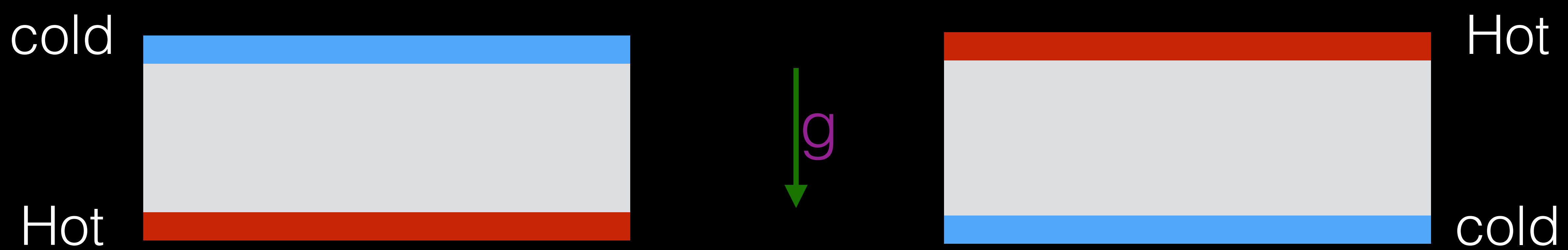
http://turbulencehub.org

Verma et al., New J. Phys. 2016

Acknowledgements
KAUST- computer time
DST/SERB for funding

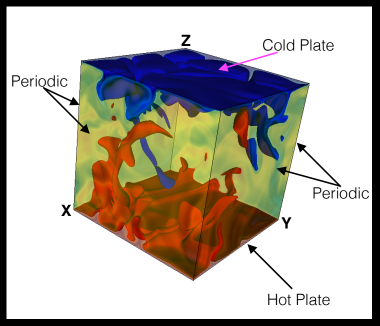
Verma, Physics of Buoyant Flows: From Instabilities to Turbulence , World Sci. 2018

Rayleigh Bénard Convection
&
Stably Stratified flow



RBC
Unstable

Stably-stratified flow
Stable



Equations

Velocity field

Pressure

Buoyancy

Ext. Force

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla\sigma + \alpha g\theta\hat{z} + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

$$\partial_t \theta + (\mathbf{u}.\nabla)\theta = -\frac{dT}{dz}u_z + \kappa\nabla^2\theta$$

Kinematic viscosity

Thermal fluctuations

Temperature stratification

Thermal diffusivity

Bossiness approximation

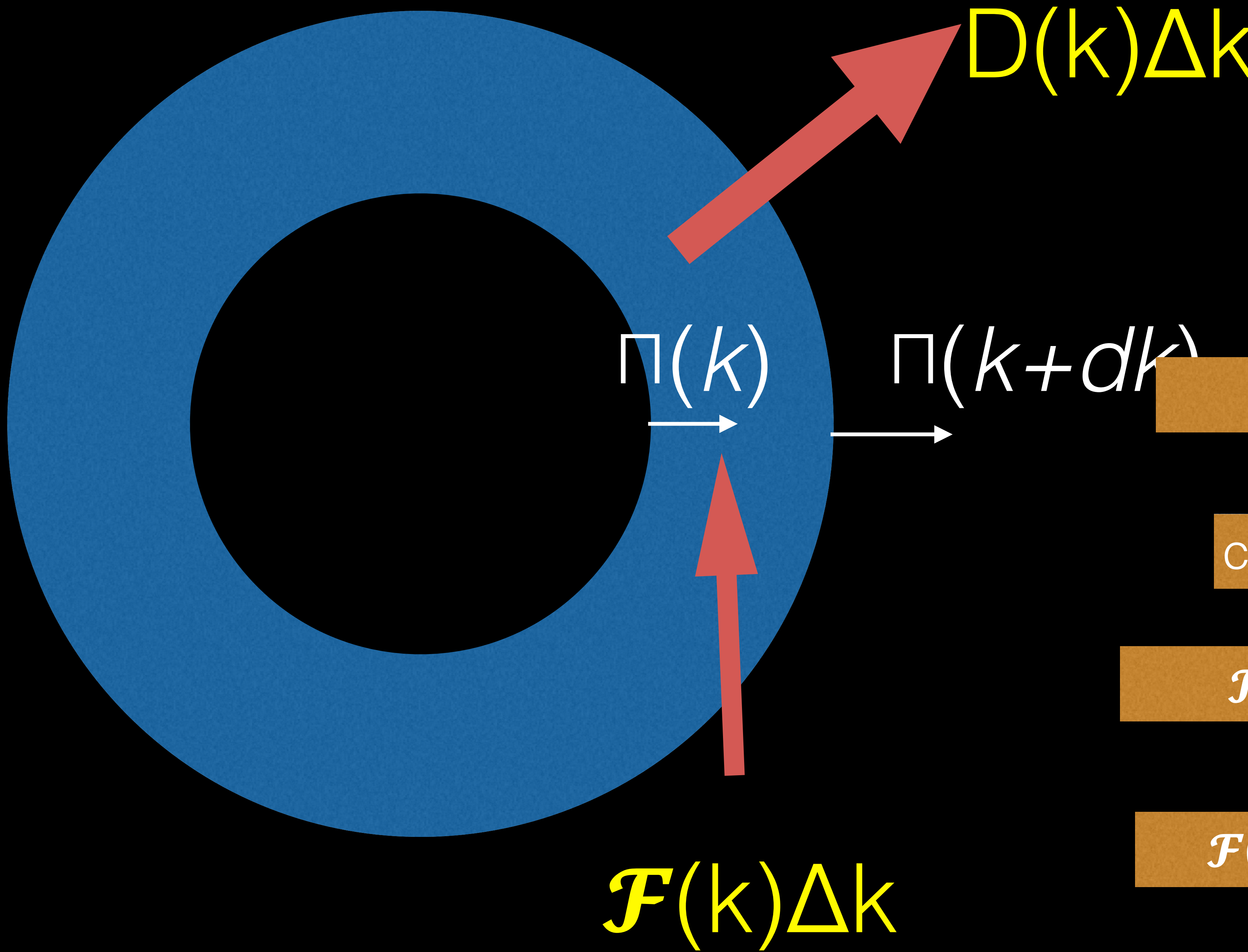
$T = T_C + \theta$

$$\nabla.\mathbf{u} = 0$$

Prandtl number Pr = ν/κ

Rayleigh number Ra = $\frac{\alpha g d^4}{\nu\kappa}\left|\frac{d\bar{T}}{dz}\right|$

Turbulence energetics



Fluid: $\mathcal{F}(k)=0$, $D(k)\to 0$

const Π(k) in the inertial range

$\mathcal{F}(k) < 0 \quad \Rightarrow$ Π(k) decreases

$\mathcal{F}(k) > 0 \quad \Rightarrow$ Π(k) increases

$$\Pi_u(k+\Delta k) = \Pi_u(k) + \left[\mathcal{F}(k) - D(k)\right]\Delta k$$

$$\frac{d\Pi_u(k)}{dk} = \mathcal{F}(k) - D(k)$$

$$\frac{d\Pi_u(k)}{dk} = \mathcal{F}(k) - D(k)$$

Energetics arguments: General

Independent of isotropy assumption

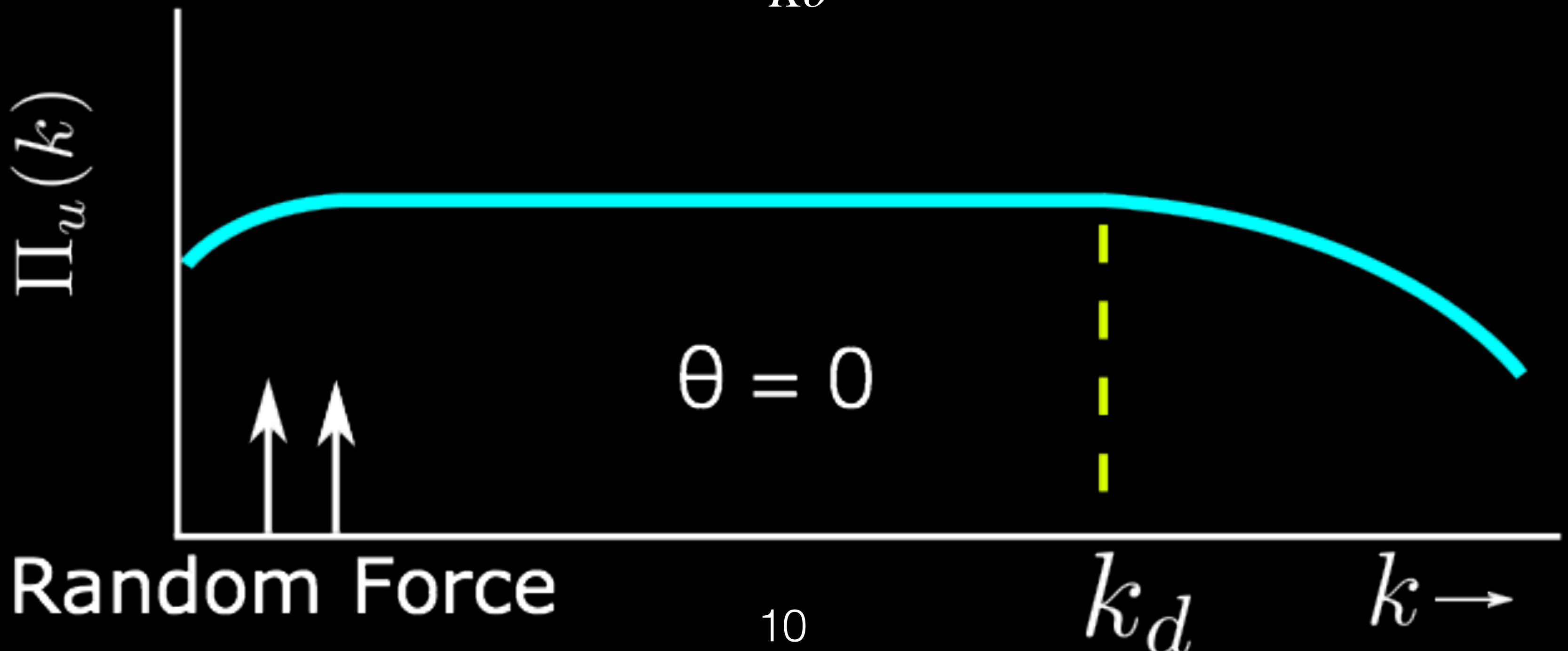
Kolmogrov's theory of turbulence

Energy supplied at large scale

Energy cascades … scale by scale

Constant flux Π(k)

$$E(k) = K_{Ko}\Pi^{2/3}k^{-5/3}$$



BO Phenomenology

Bolgiano, 1959
Obukhov, 1959

$k<k_B$

$\Pi_u(k) \sim k^{-4/5}$

$E_u(k) \sim k^{-11/5}$

$E_\theta(k) \sim k^{-7/5}$

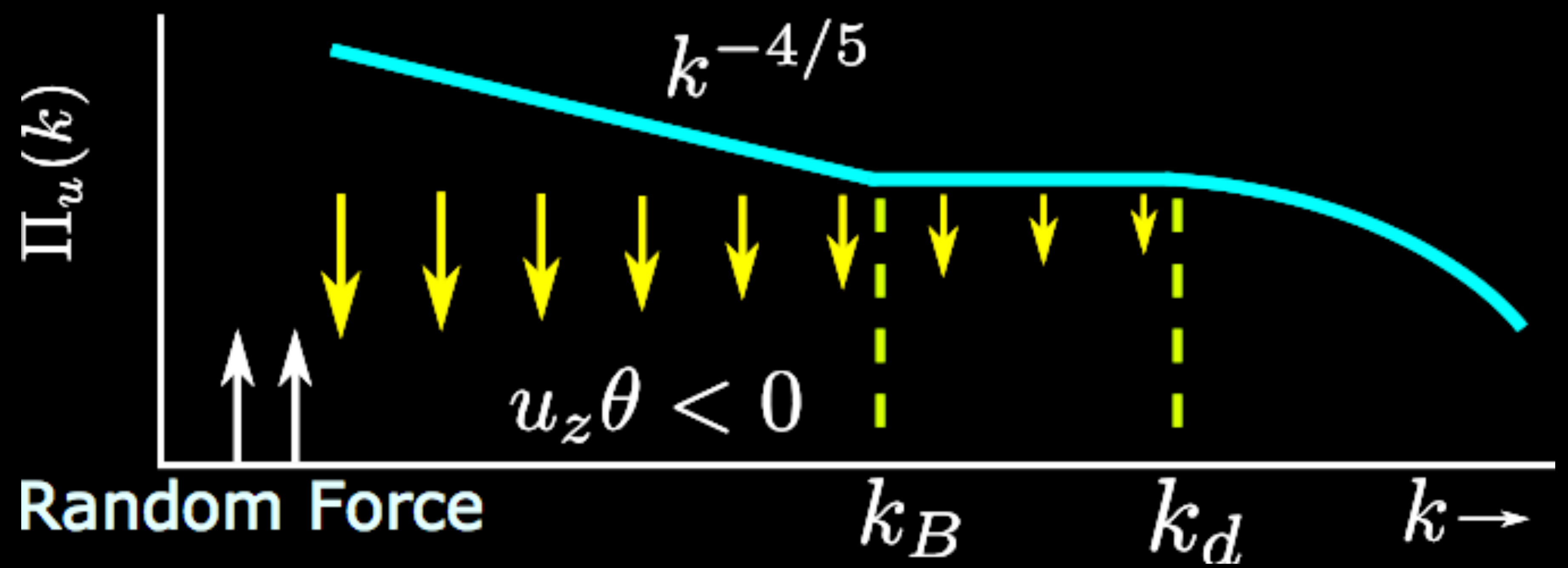
$k>k_B$

$\Pi_u(k)$ = const.

$\Pi_\theta(k)$ = const.

$E_u(k) \sim k^{-5/3}$

$E_\theta(k) \sim k^{-5/3}$



KE → PE → dissipation

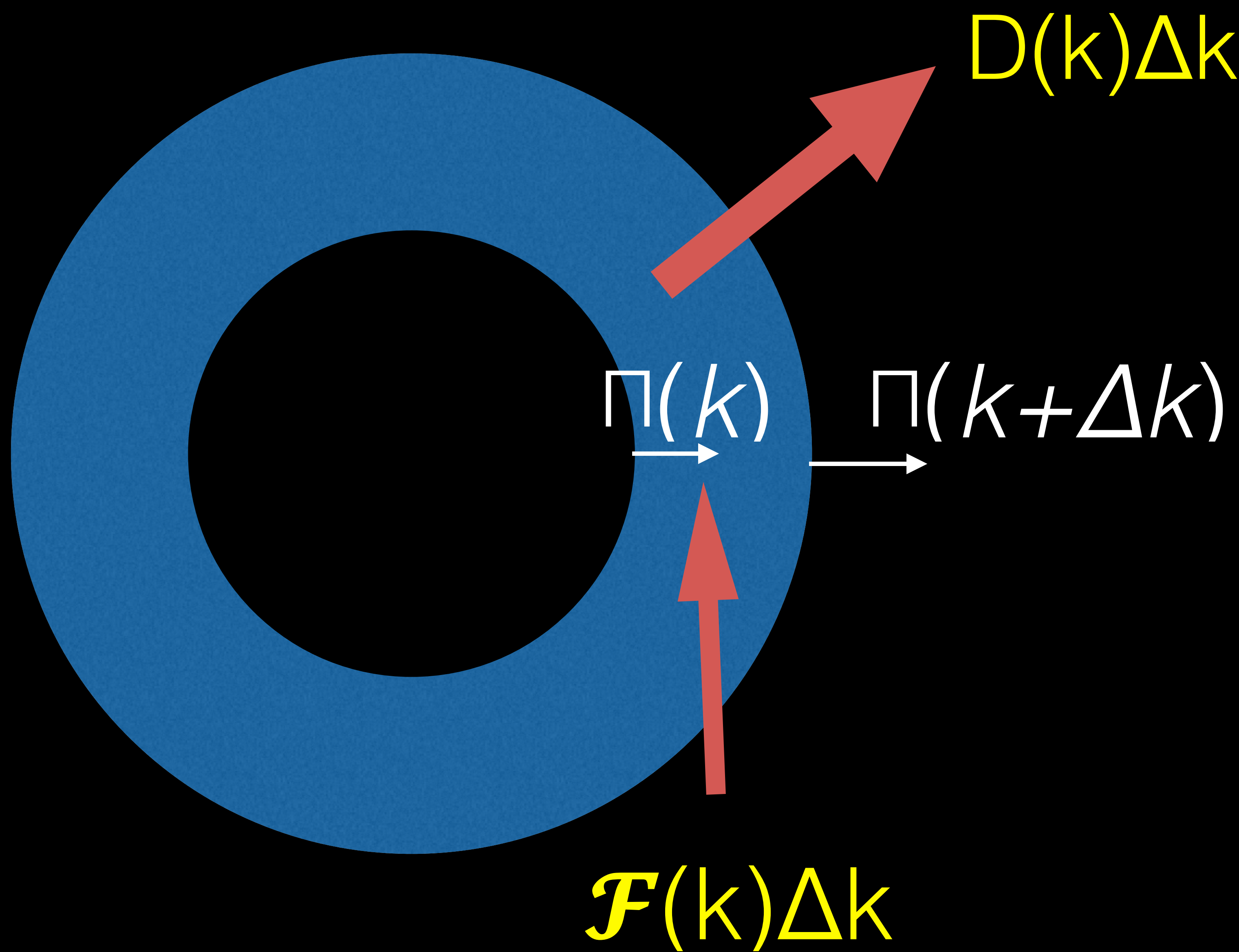
Flux decreases with k

Ri ≈ 1

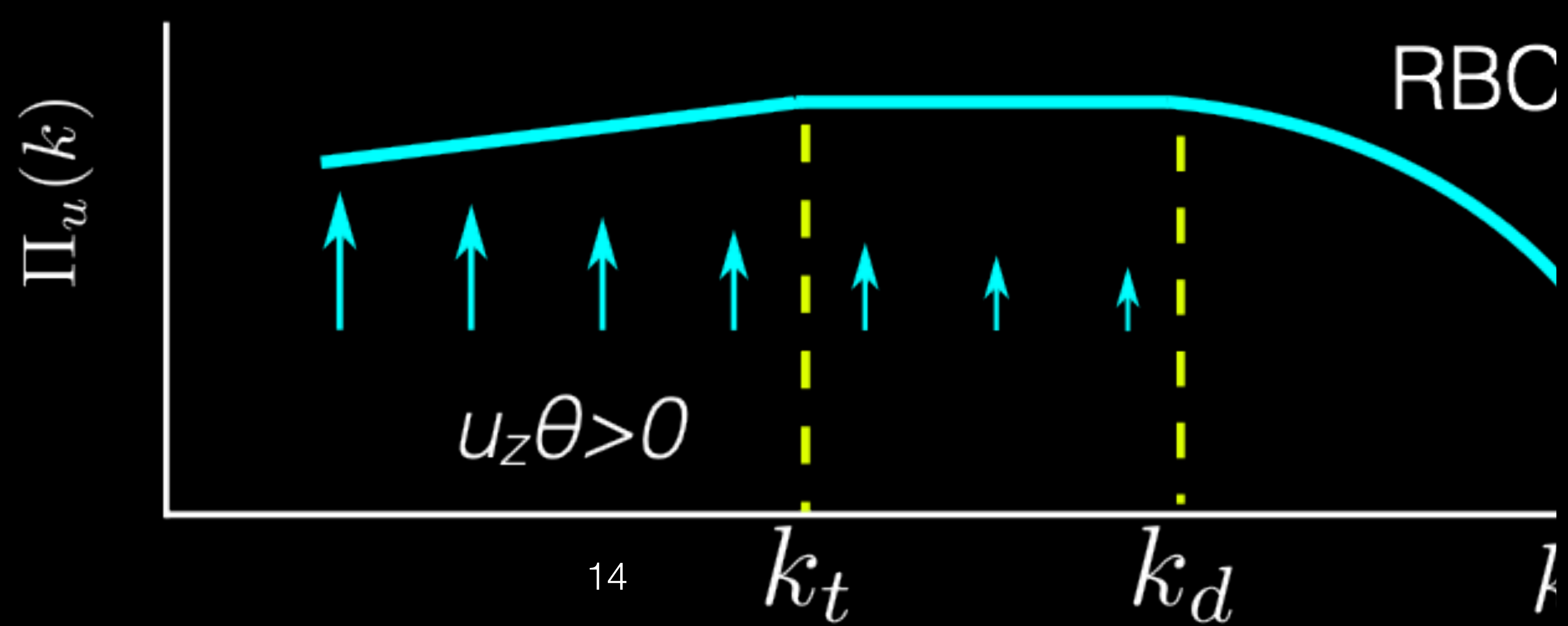
Kumar et al. 2014

Spectrum & Fluxes for RBC

BO phenomenology extended to RBC
using field-theoretic argumetns:
Procaccia & Zaitak, 1989;
Lvov & Falkovich, 1991, 1992
Rubinstein, 1994



Kumar et al., PRE 2014
Verma et al., NJP 2017



Rayleigh-Bénard convection

$\mathrm{Pr} = 1$

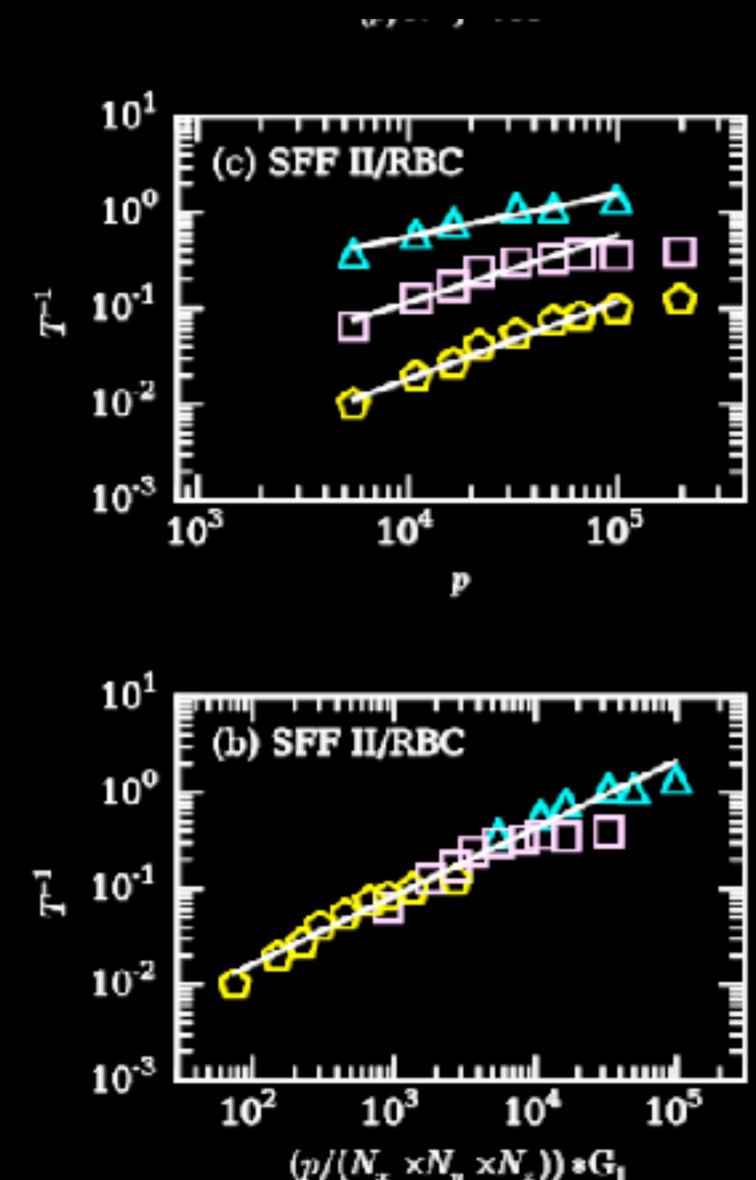
Grid: 4096^3

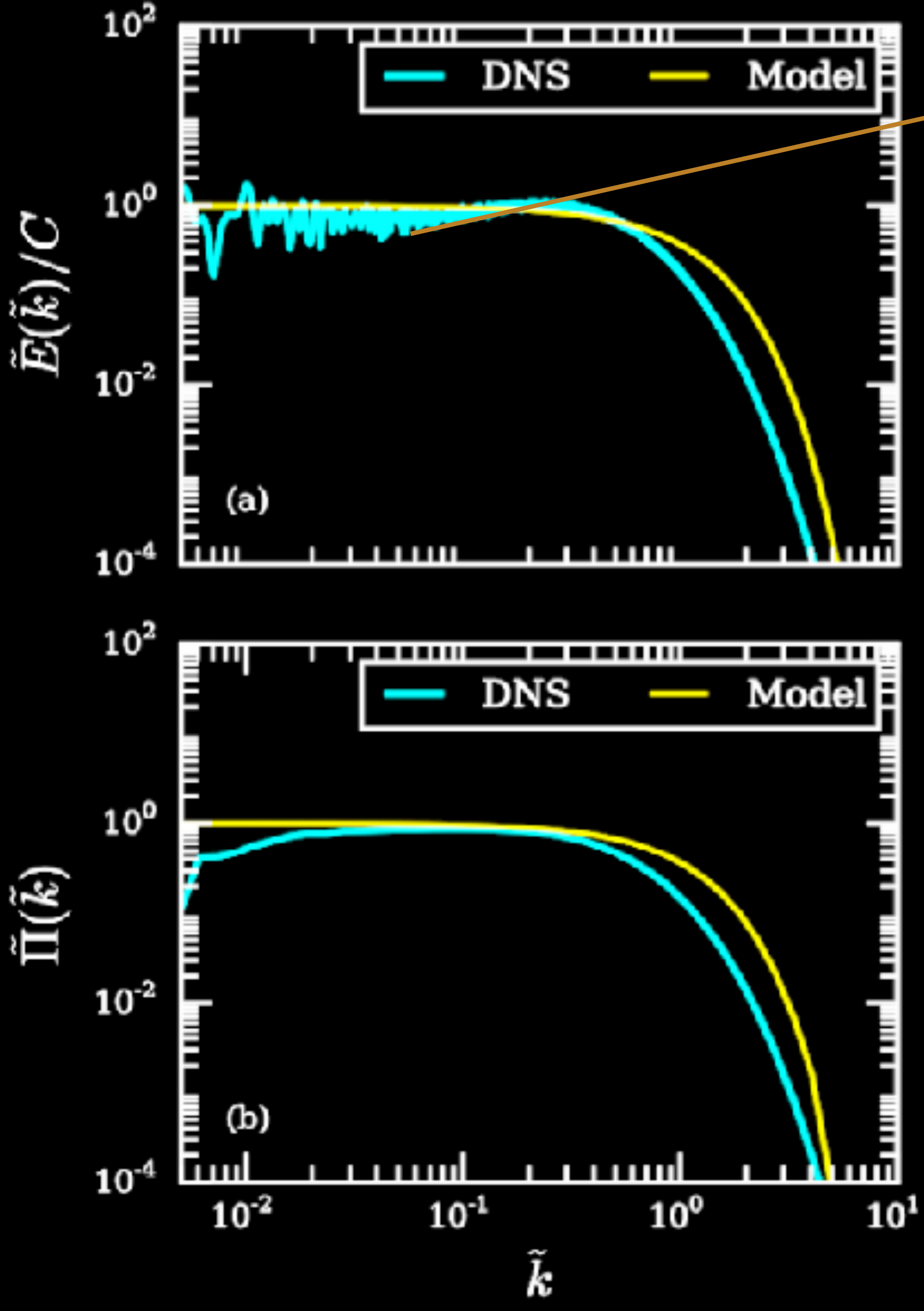
$\mathrm{Ra} = 1.1\times10^{11}$

$\mathrm{Re} = 4.5\times10^{4}$

Highest achieved so far

on 196608 processors
of Shaheen of KAUST





$E_u(k)k^{5/3}$

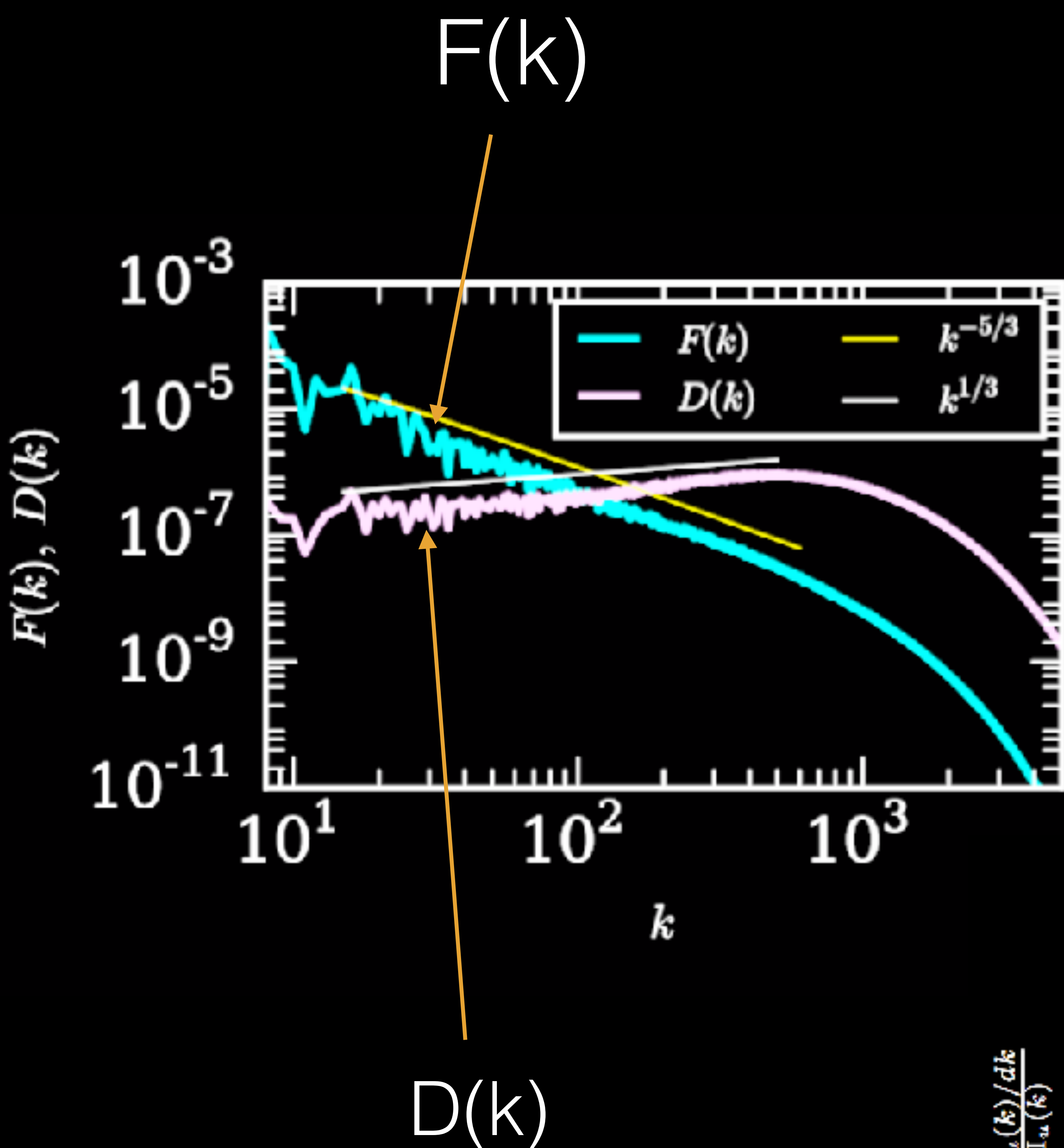
$$E(k) = K_{Ko}\epsilon^{2/3}k^{-5/3}$$

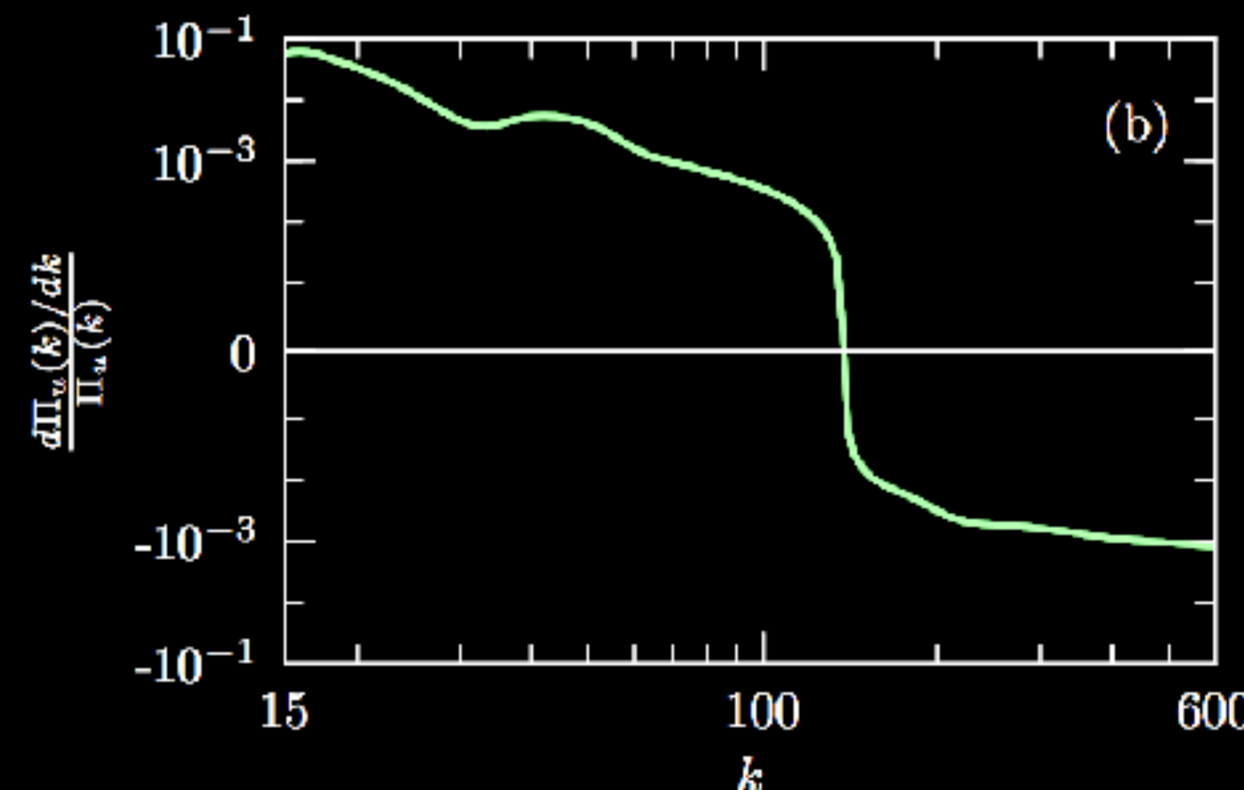
$$\Pi(k) = \epsilon$$

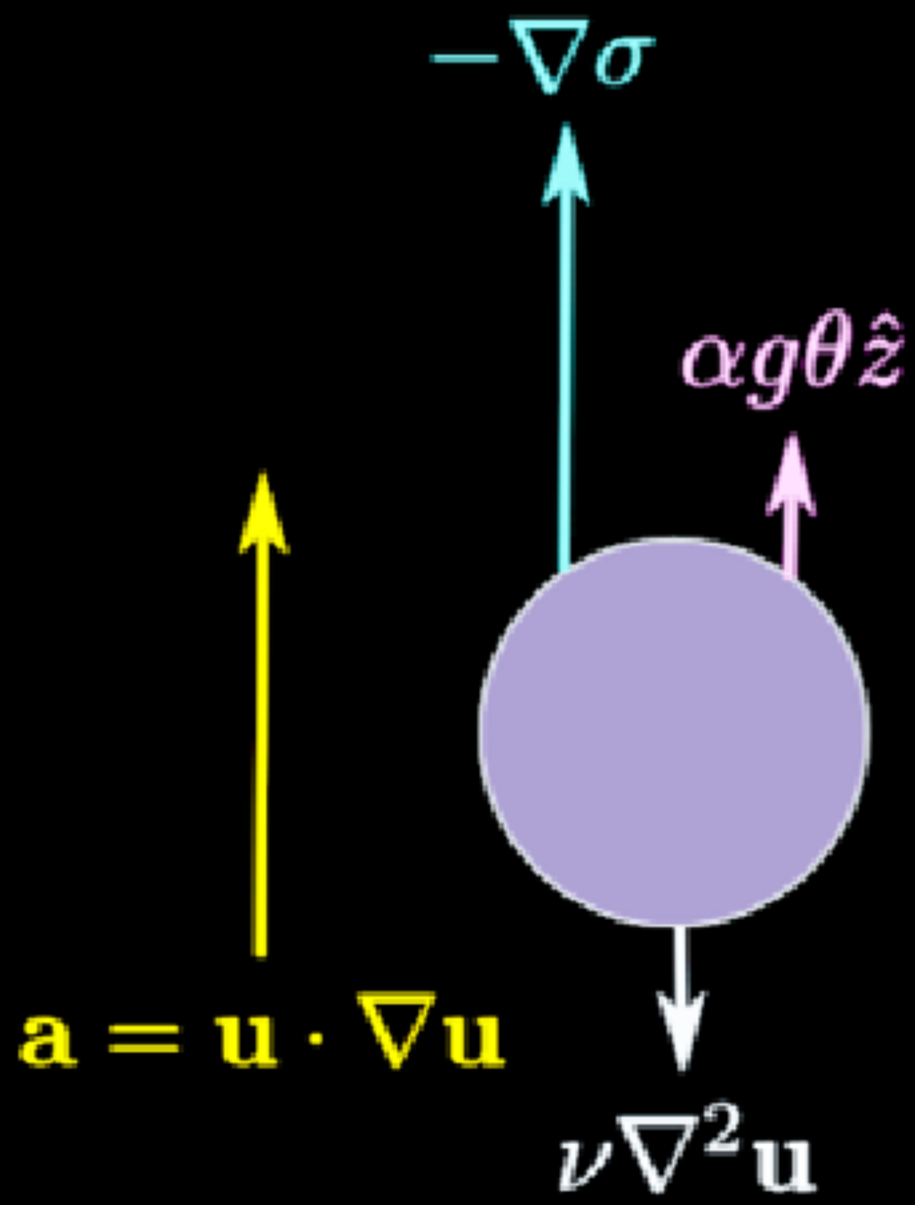
$$\tilde{k} = k / k_d$$

Verma et al., 2016

$\varepsilon = 1.6 \times 10^{-3}$


F(k)
D(k)
F(k), D(k)
k
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{1}
10^{2}
10^{3}
F(k)
D(k)
$k^{-5/3}$
$k^{1/3}$

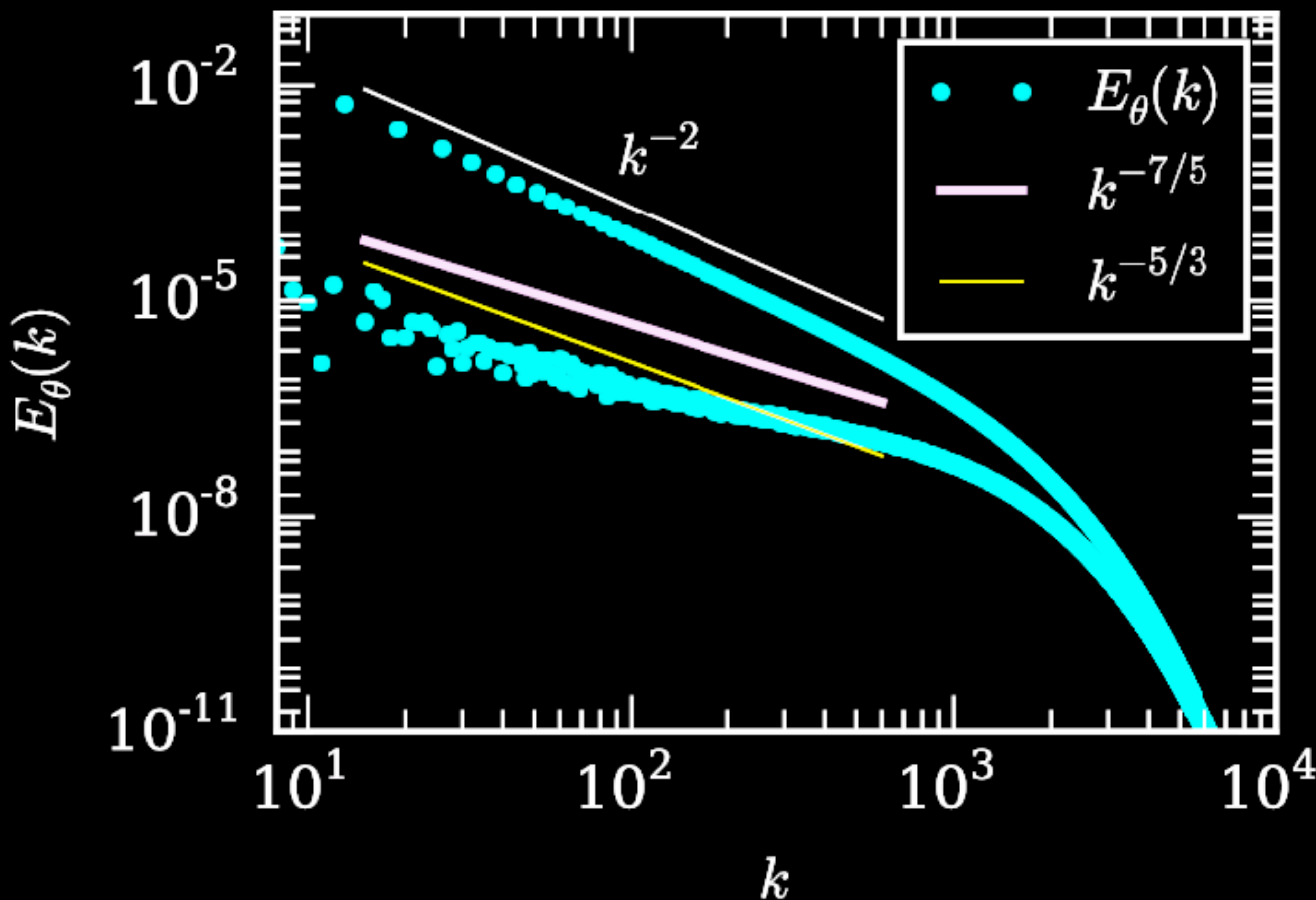

(b)
$\frac{d\Pi_u(k)/dk}{\Pi_u(k)}$
k
10^{-1}
10^{-3}
0
-10^{-3}
-10^{-1}
15
100
600



-∇p >> buoyancy

Pandey & Verma, PoF 2016

Temperature spectrum

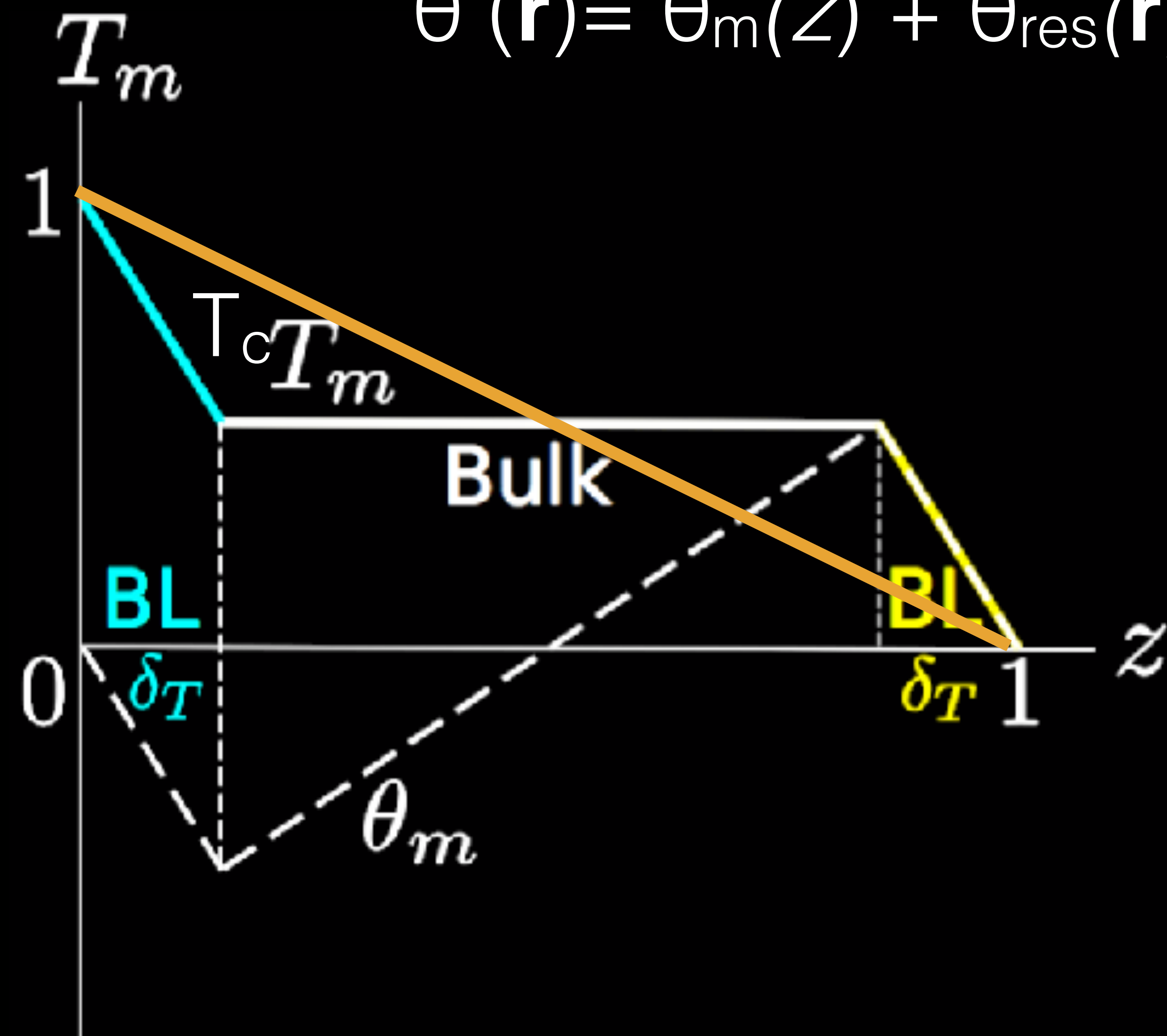


$$E_\theta(k) \sim \frac{1}{k^2}$$

Bispectrum

$$T(\mathbf{r}) = T_c(z) + \theta(\mathbf{r})$$

$$\theta(\mathbf{r}) = \theta_m(z) + \theta_{res}(\mathbf{r})$$



$$\hat{\theta}(0,0,2n) = -\frac{1}{2n\pi}$$

Anisotropy in RBC

Anisotropy

$A = E_\perp/(2E_\parallel)$

Table I. Simulation parameters in grid size of 512 × 512 × 512.

Pr	Ra	Re	$k_{max}\eta$	$0.5E_\perp$	$E_\parallel$	A	D
0.02	2×10^6	7.05×10^3	3.4	0.118	0.187	0.63	1.02×10^{-4}
1	10^8	3.11×10^3	5.9	0.013	0.018	0.73	1.02×10^{-4}
6.8	10^8	9.08×10^2	3.2	0.010	0.017	0.59	2.65×10^{-4}
100	10^8	1.25×10^2	1.6	0.002	0.004	0.49	1.02×10^{-3}
∞	2×10^8	0	4.2	0.221	0.725	0.30	7.21×10^{-5}

Nath et al., Phys. Rev. Fluids 2016

Rayleigh Taylor turbulence

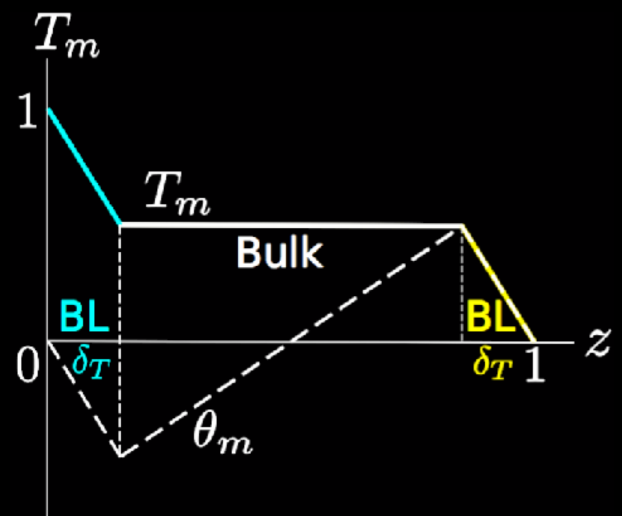
Bubbly turbulence

Unstable stratification
(salt above water): Arakeri et al.

Cahn-Hilliard turbulence
(Pandit et al.)

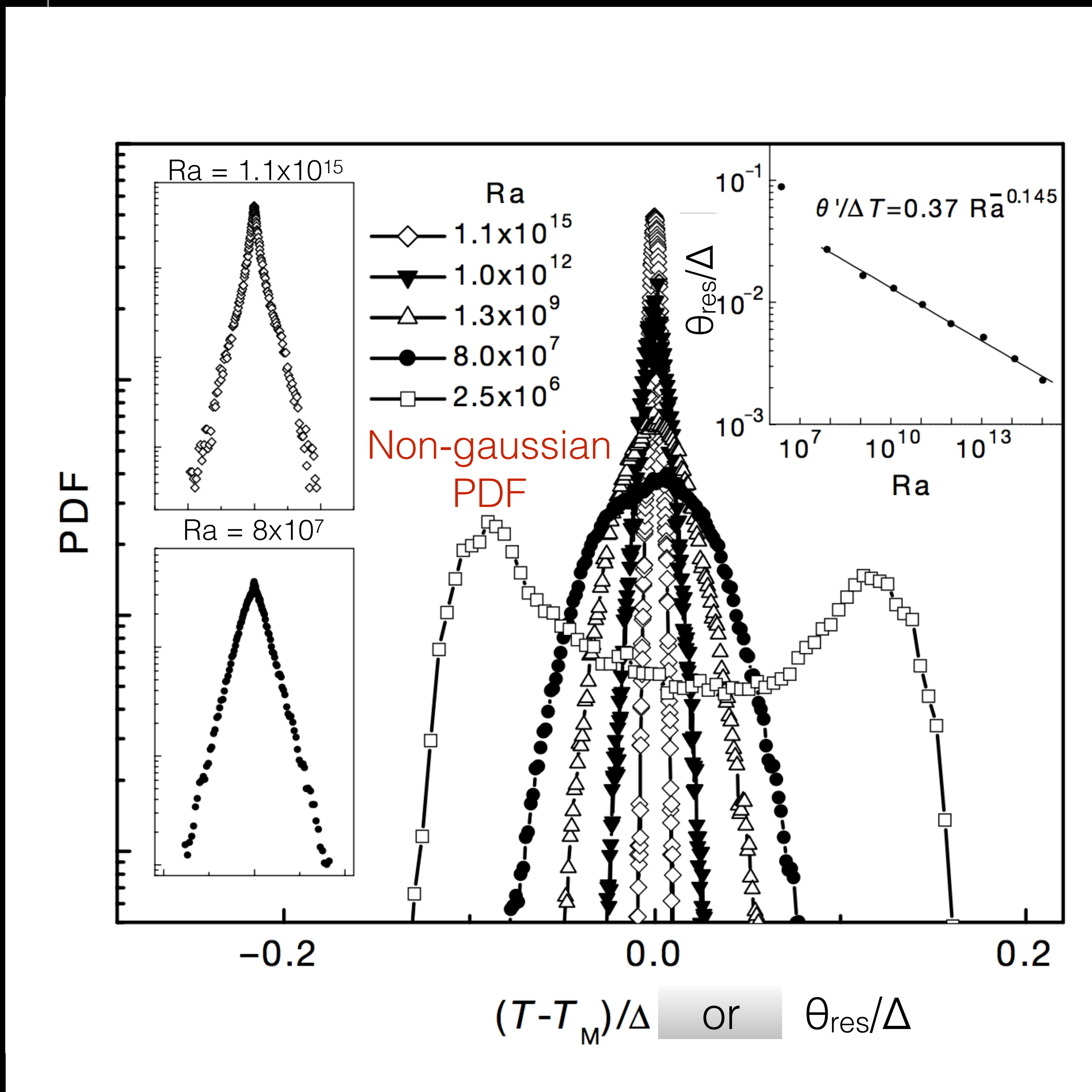
Fluctuations in RBC

Walls matter!



$$T(\mathbf{r}) = T_c\,(z) + \theta(\mathbf{r})$$

$$\theta\,(\mathbf{r}) = \theta_m(z) + \theta_{res}(\mathbf{r})$$



$$\Theta/\Delta = Ra^{-0.145}$$

Niemela et al., 2000

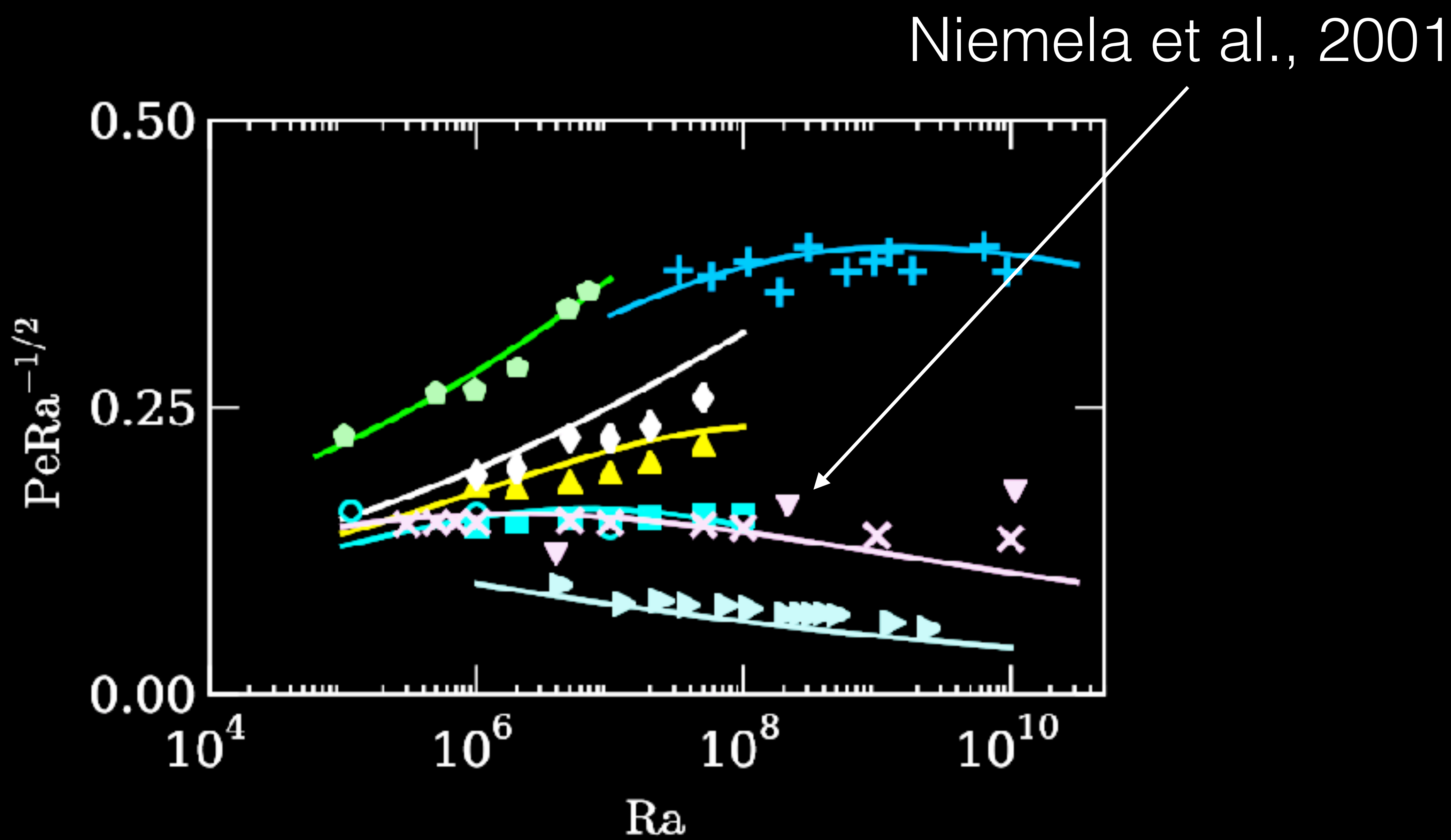
Reynolds number scaling

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla \sigma_{\mathrm{res}} + \alpha g \theta_{\mathrm{res}} \hat{z} + \nu \nabla^2 \mathbf{u}$$

$$c_1 \frac{U^2}{d} = c_2 \frac{U^2}{d} + c_3 \alpha g \Theta_{\mathrm{res}} - c_4 \nu \frac{U}{d^2}$$

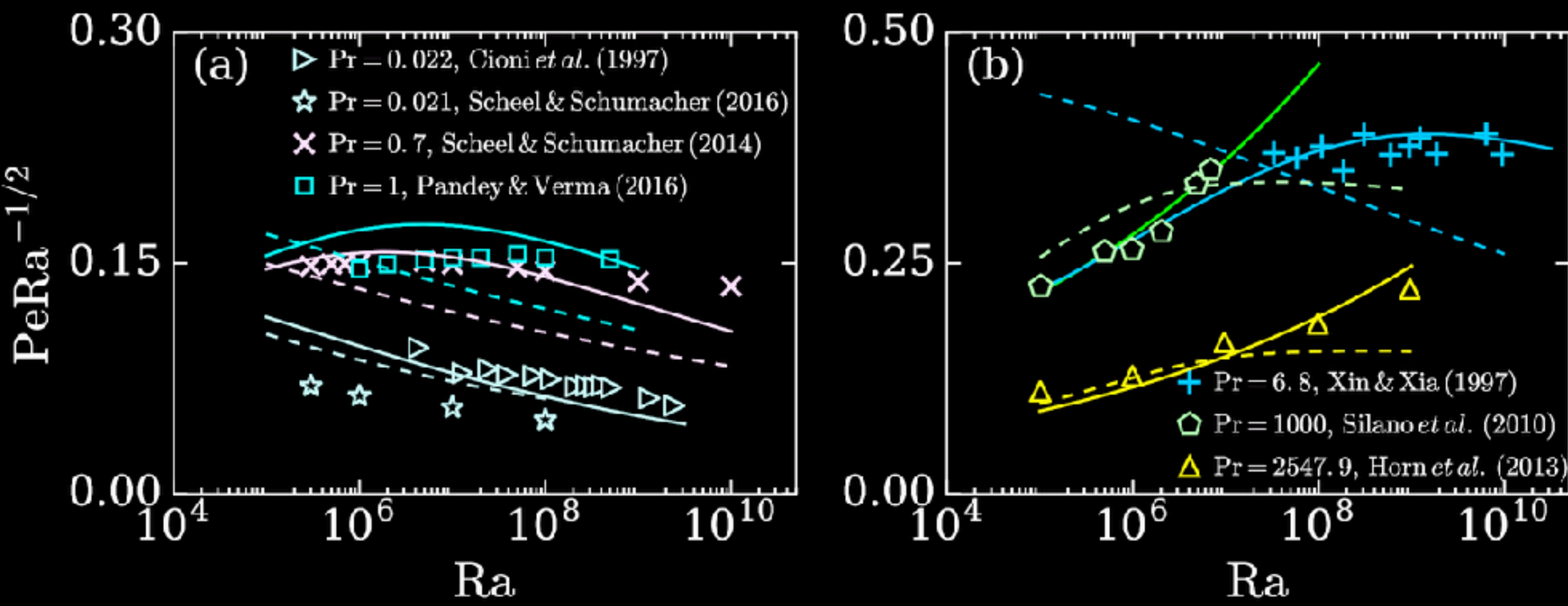
$$c_1 = \frac{\langle | \mathbf{u} \cdot \nabla \mathbf{u} | \rangle}{U^2 / d}; \quad c_2 = \frac{\langle | \nabla \sigma |_{\mathrm{res}} \rangle / \rho_0}{U^2 / d}; \quad c_3 = \Theta_{\mathrm{res}} / \Delta; \quad c_4 = \frac{\langle | \nabla^2 \mathbf{u} | \rangle}{U / d^2}$$

$$\mathrm{Pe} = \frac{-c_4 \mathrm{Pr} + \sqrt{c_4^2 \mathrm{Pr}^2 + 4(c_1 - c_2) c_3 \mathrm{Ra} \mathrm{Pr}}}{2(c_1 - c_2)}$$



Niemela et al., 2001
0.50
0.25
0.00
10^4
10^6
10^8
10^{10}
$PeRa^{-1/2}$
Ra

Comparison between GL and our model



Dissipation rate

Hydrodynamic $\varepsilon_u \sim (U^3/d)$

RBC $\varepsilon_u \sim (U^3/d)Ra^{-0.2}$

Conclusions

- Kolm-like spectrum: $k^{-5/3}$
- $-\nabla p$ >> buoyancy; nearly isotropic.
- Walls matter!

Verma, Physics of Buoyant Flows: From Instabilities to Turbulence , World Sci. 2018

Thank you!

Acknowledgements
KAUST- computer time
DST/SERB for funding