

# Compressibility Effects in Turbulence: Revisited

Sanjiva K. Lele

STANFORD UNIVERSITY  
AERONAUTICS AND ASTRONAUTICS, AND  
MECHANICAL ENGINEERING

B. Olson, J. Larsson, I. Bermejo Moreno, J. Freund, P. Moin

Ack: AFOSR, ONR, NASA, SciDac

Happy 70th Sreeni !

Int. Symposium on *Turbulence from Angstroms to Light Years*,  
ICTS, Bangalore, Jan 21-25 2018

# Outline

## A Glimpse of compressibility effects on Turbulence

### Fundamental Picture

### More Recent Studies

Homogeneous Turbulence

Shock-Turbulence Interaction

Variable density/variable property effects

[Annual Reviews  
www.annualreviews.org/aronline](http://www.annualreviews.org/aronline)

*Annu. Rev. Fluid Mech. 1994, 26: 211-54  
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### Summary and Open Issues

Physics and Modeling

DNS/LES

## COMPRESSIBILITY EFFECTS ON TURBULENCE

*Sanjiva K. Lele*

Department of Mechanical Engineering and Department of Aeronautics  
and Astronautics, Stanford University, Stanford, California 94305-4035

KEY WORDS: shock-turbulence interaction, supersonic mixing, turbulence  
modeling

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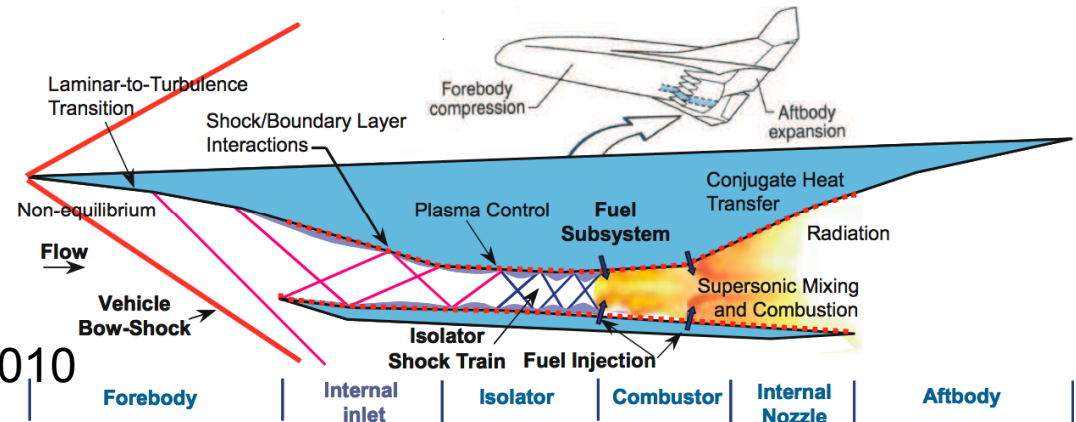
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## A Glimpse of compressibility effects on Turbulence

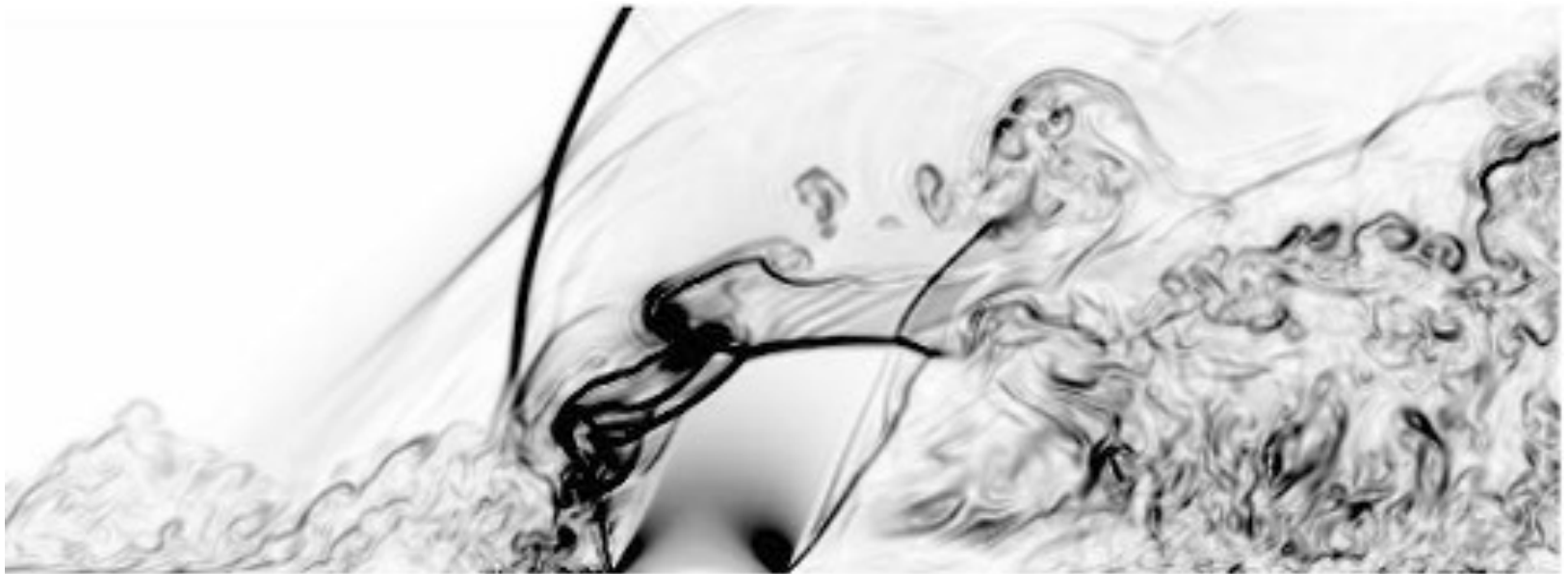
Supersonic mixing -- Scramjet

Supersonic/Hypersonic Turbulent Boundary layers – High speed Aerodynamics

Shock-Turbulence interaction -- High speed Aerodynamics



Kawai & Lele, AIAA J, vol 48, 2010



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# Shock-induced Separation

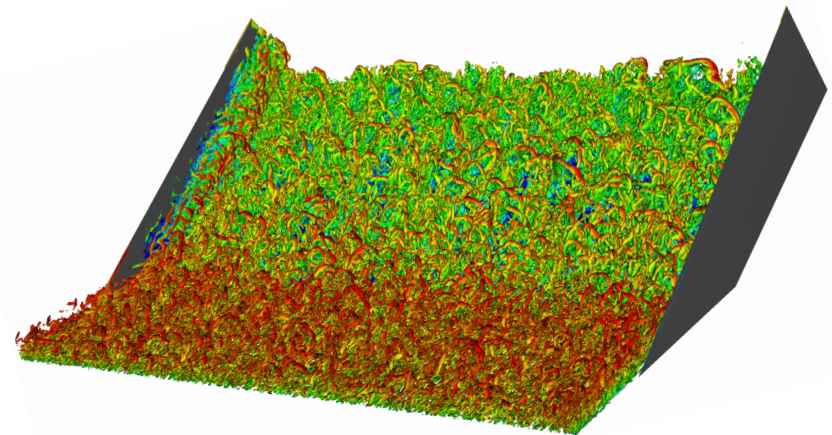
- ❖ Shock-induced separation
- ❖ Shock-induced unsteadiness

Olson & Lele AIAA 2011-3908

## Over-expanded Planar Nozzle



3D Shock-Turbulent Boundary Layer  
Interaction  
Compression ramp with fence



Dawson & Lele, AIAA 2015-1518

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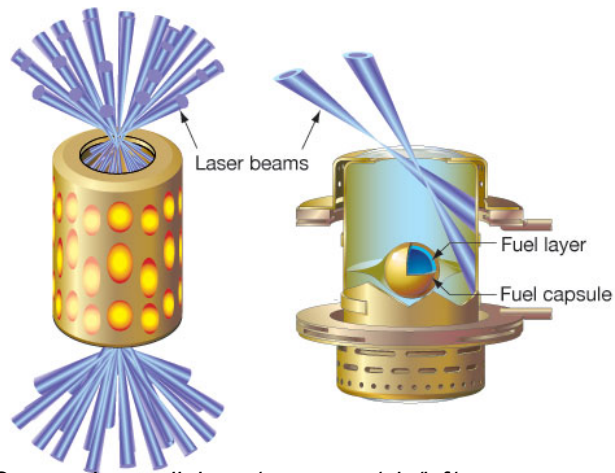
# A Glimpse of compressibility effects on Turbulence

Solar convection

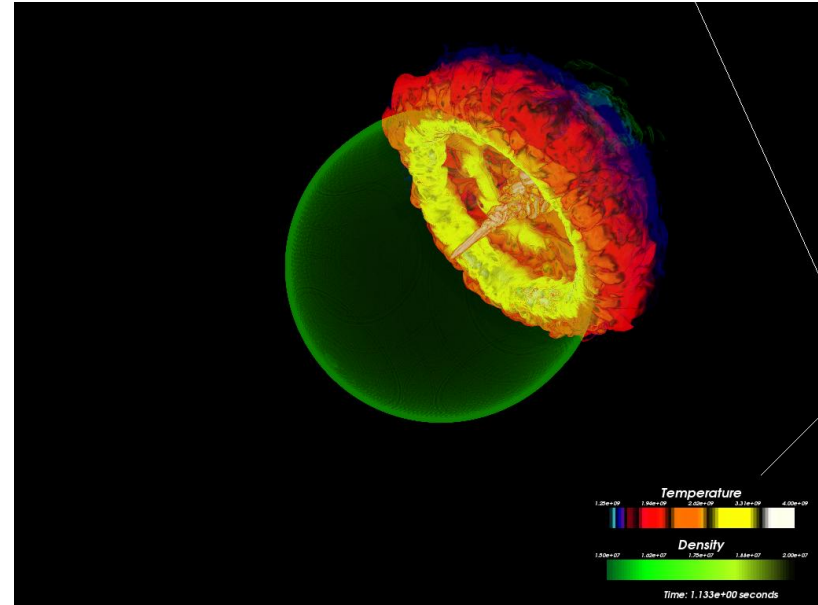
Supernova collapse

Interstellar density fluctuations

Inertial confinement fusion



Source: [lasers.llnl.gov/programs/nic/icf/](http://lasers.llnl.gov/programs/nic/icf/)



Source: [flash.uchicago.edu](http://flash.uchicago.edu)

Supernova explosion

Inertial confinement fusion by shock-induced implosion of a deuterium/tritium capsule

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# Compressibility Effects on Turbulence - Fundamentals

Three Modes of fluctuations      Yaglom (1948), Moyal (1952), Kovasznay (1953)

- ❖ **Vortical mode** or Gust Mode -- Solenoidal velocity, no pressure or density variations
- ❖ **Acoustic mode** – No vorticity, irrotational velocity, pressure, density fluctuation-isentropic
- ❖ **Entropic mode** – No velocity or pressure, density and temperature fluctuations

Linearized modes of uniform state

Evolve independently at first order in amplitude

mode coupling: due to mean gradients and nonlinearity

**Incompressible turbulence – Nonlinearly interacting vortical modes**

Not just gust modes; (incompressible) Pressure effects (non-locality) important

**Variable density turbulence** (low  $M$  reacting flow, low  $M$  mixing)

**Compressible turbulence** – density of fluid parcel changes in response to pressure

Low  $M_t$  -- Pseudo sound (Ristorcelli, 1997) Enslaved dilatational motions

**Linearized modes of non-uniform state ?** Linear stability analysis; non-linear interactions

Primary Instability → Secondary Instability → Transition to turbulence

Useful framework but not well suited for broadband turbulence dynamics

global modes/DMD/POD useful for modeling large-scale dynamics

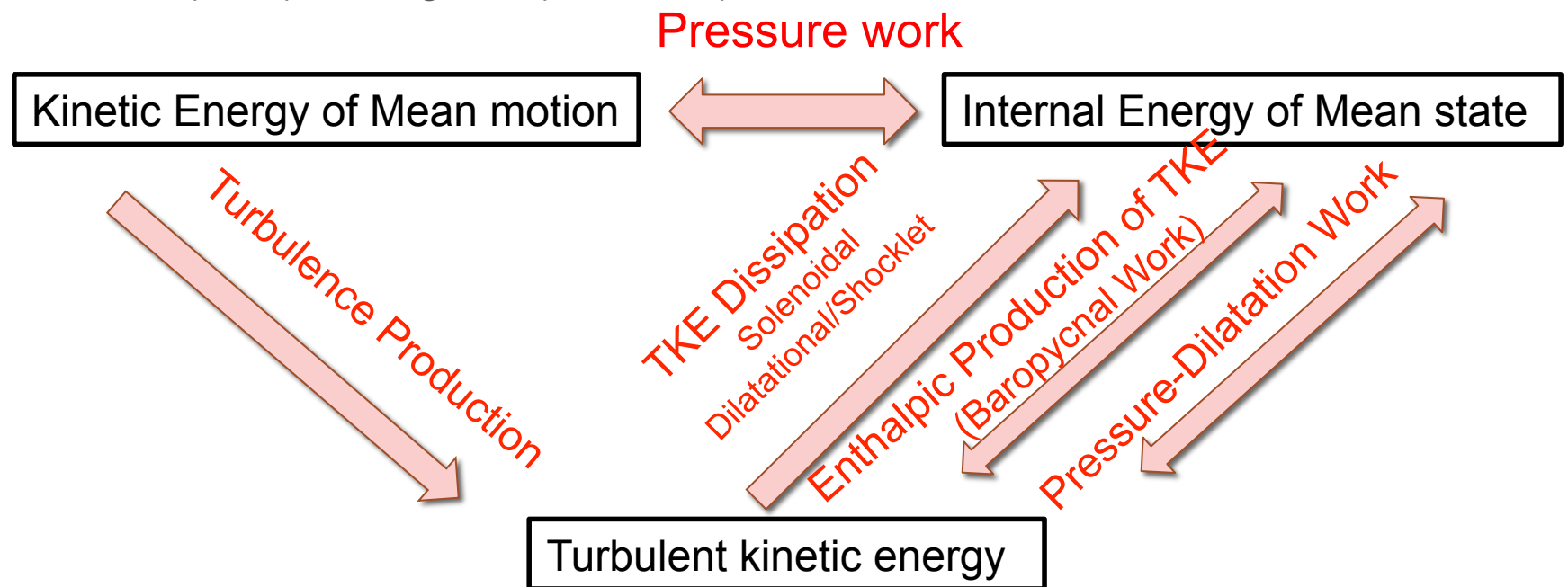
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# Compressibility Effects on Turbulence - Fundamentals

Decomposition of variables: **Favre decomposition** (mass weighted velocity)  
(define)

## Energy Transfer pathways (coupling of momentum with thermodynamics)

Favre (1965), Huang et al (1995,JFM), ...



# Compressibility Effects on Turbulence - Fundamentals

## Dimensionless parameters -- summary

Relative/Convective Mach Number  $M_c, M_r = (U_1 - U_2)/(C_1 + C_2)$

Turbulence Mach Number  $M_t = q/C$

Gradient/Deformation Mach Number  $M_g = SL/C = (L/C) / (1/S) = L / (C/S)$

Friction Mach Number  $M_T = u_T / C$

Mixing Layer  $u_T / \Delta U \sim 0.2$ ; Boundary Layer  $u_T / U_\infty \sim 1/25$

So for TBL require  $M_\infty$  which is at least 5 times larger than mixing layers for significant compressibility effects

In practice, even at  $M_\infty = 10$  intrinsic compressibility effects seem absent! Why ?

Intense aerodynamic heating in supersonic/hypersonic TBL →

Very low density and high viscosity near the wall

# Compressibility Effects on Turbulence - Fundamentals

## Flow Physics of Supersonic Mixing -- Observations I

### Suppression of mechanical mixing

Brown & Roshko (1974), Bogdonoff (1983), Papamoschou & Roshko (1986)

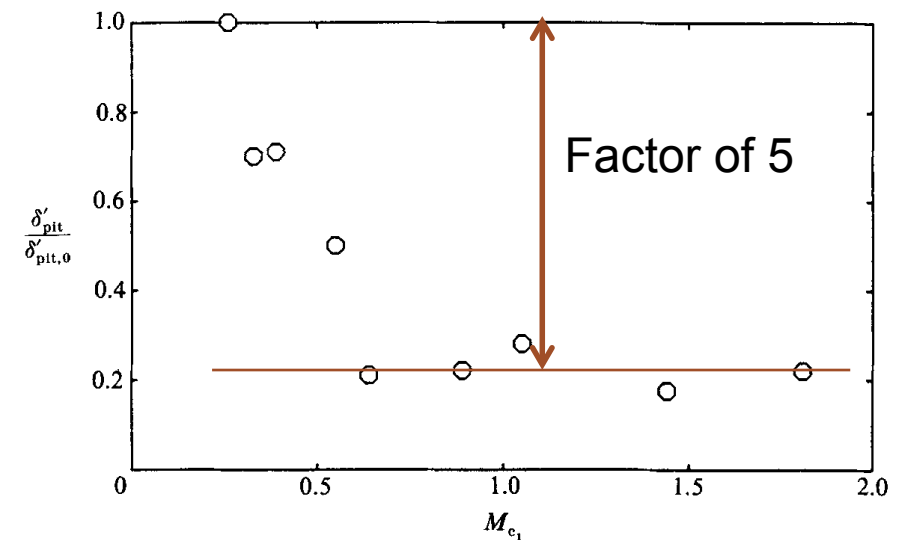
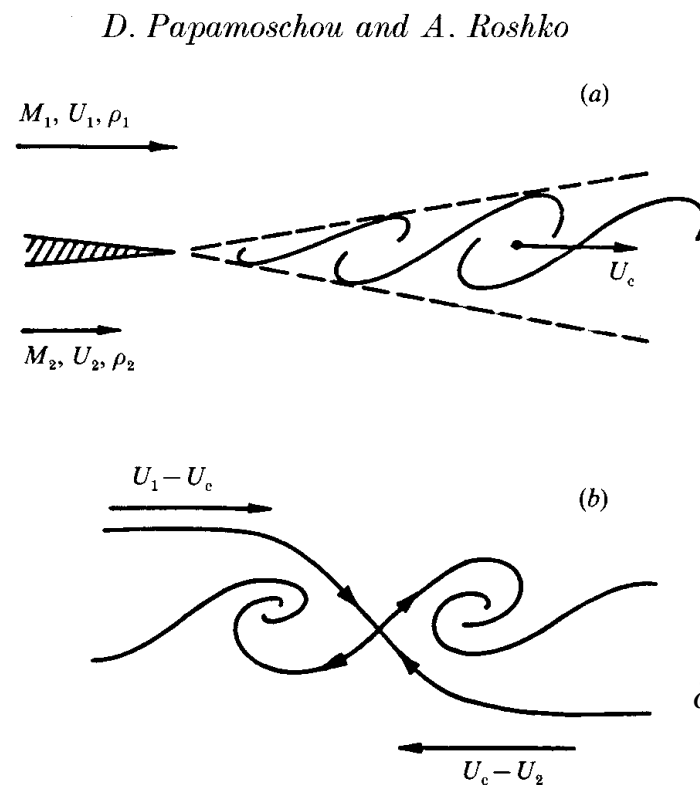


FIGURE 16. Normalized Pitot-thickness growth versus  $M_{c1}$ .

$$\delta'_0(x; r, s) \simeq C_\delta \frac{(1-r)(1+s^{1/2})}{2(1+s^{1/2}r)} \left\{ 1 - \frac{(1-s^{1/2})/(1+s^{1/2})}{1+2.9(1+r)/(1-r)} \right\},$$

$$M_c = (U_1 - U_2)/(C_1 + C_2)$$

# Compressibility Effects on Turbulence - Fundamentals

## Flow Physics of Supersonic Mixing -- Observations II

### Explanations/Hypotheses

Dilatational/Shocklet dissipation Zeman (1990), Sarkar et al (1991)

Damping via Acoustic Radiation

Suppression of linear instability

Sandham & Reynolds (1990), Ragab & Wu (1989), Grosch & Jackson (1989), Lu & Lele (1993)

Turbulence structure change Vreman et al (1996), Pantano & Sarkar (2002)

Sonic Eddy Hypothesis Briedenthal (1992) AIAA J.

# Compressibility Effects on Turbulence - Fundamentals

## Flow Physics of Supersonic Mixing -- Observations III

Freund, Lele & Moin, JFM (2003)

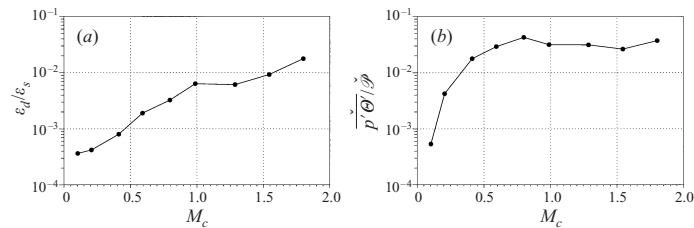
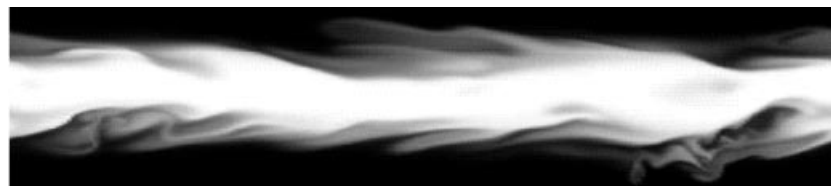
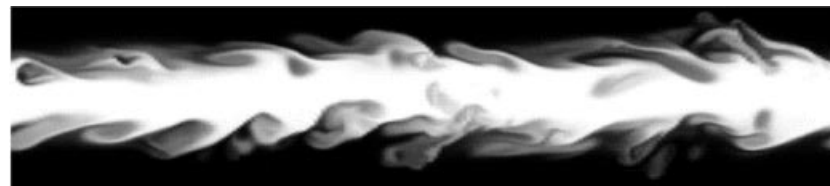
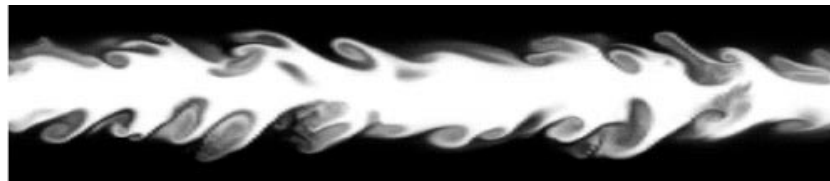
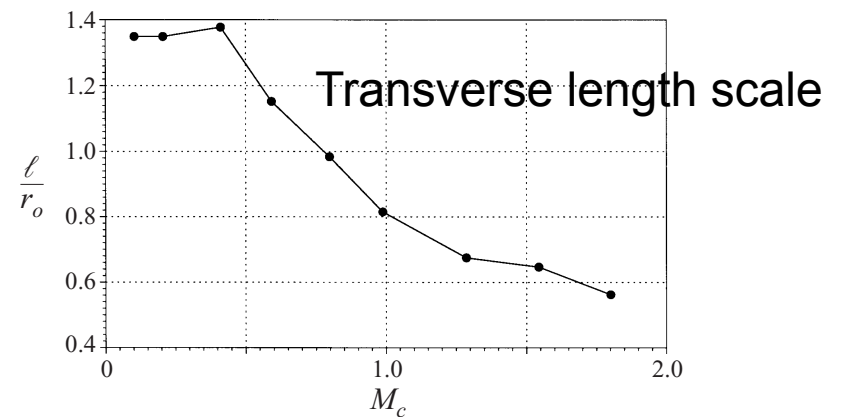


FIGURE 21. (a) Ratio of dilatational to solenoidal dissipation and (b) ratio of integrated pressure dilatation to the integrated production.



Passive tracer cuts



$M_c = 0.2$

FIGURE 6. Transverse large-eddy lengthscale.

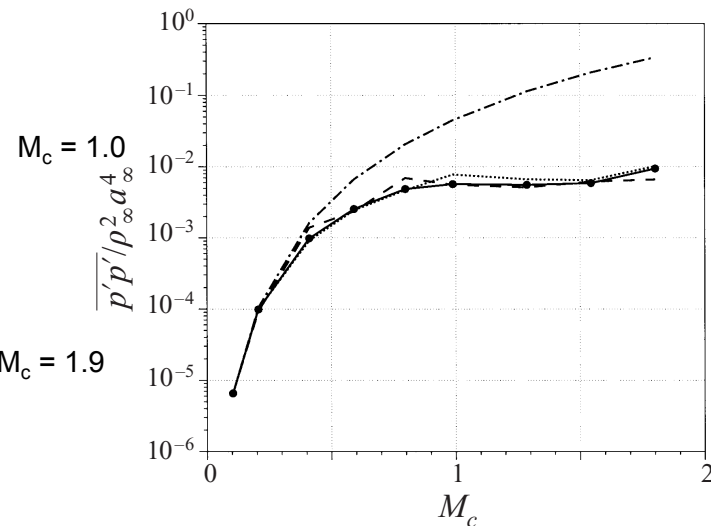


FIGURE 30. 'Un-normalized' pressure fluctuations at  $r = r_o$ : —•— from the simulation; ---,  $\sim M_g^4$  from (6.4); ..... ,  $\sim M_{tr}^4$  from (6.5); —•—,  $\sim U_j^4$  (see text).

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# Compressibility Effects on Turbulence - Fundamentals

Absence of significant effects in Supersonic/Hypersonic TBLs

Morkovin (1962), Bradshaw (1977), Smits & Dussauge (1986)

DNS Duan & Martin, JFM (2011)

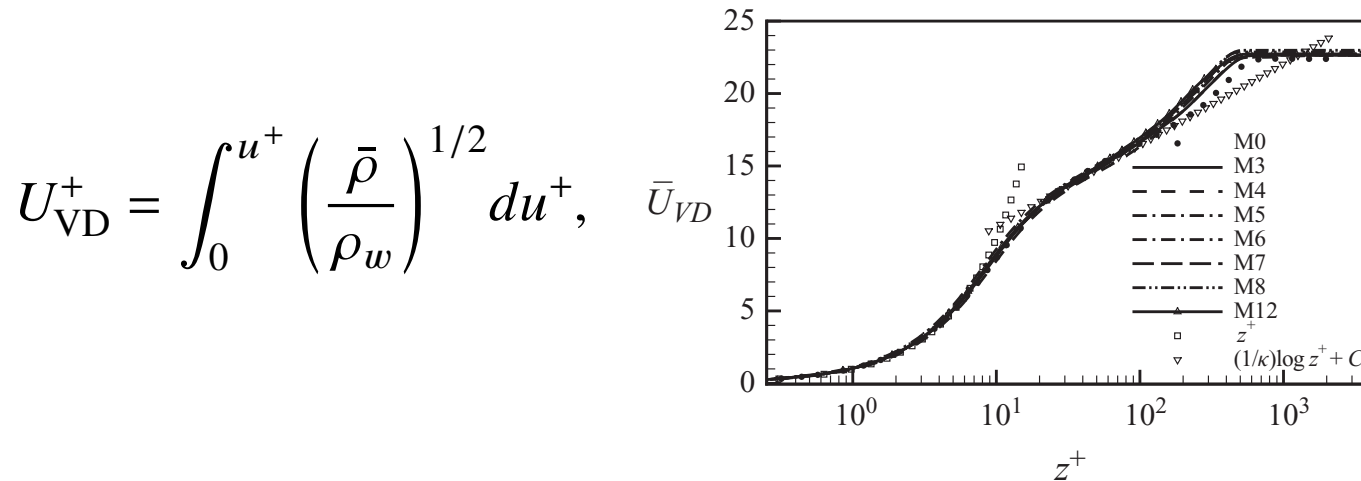


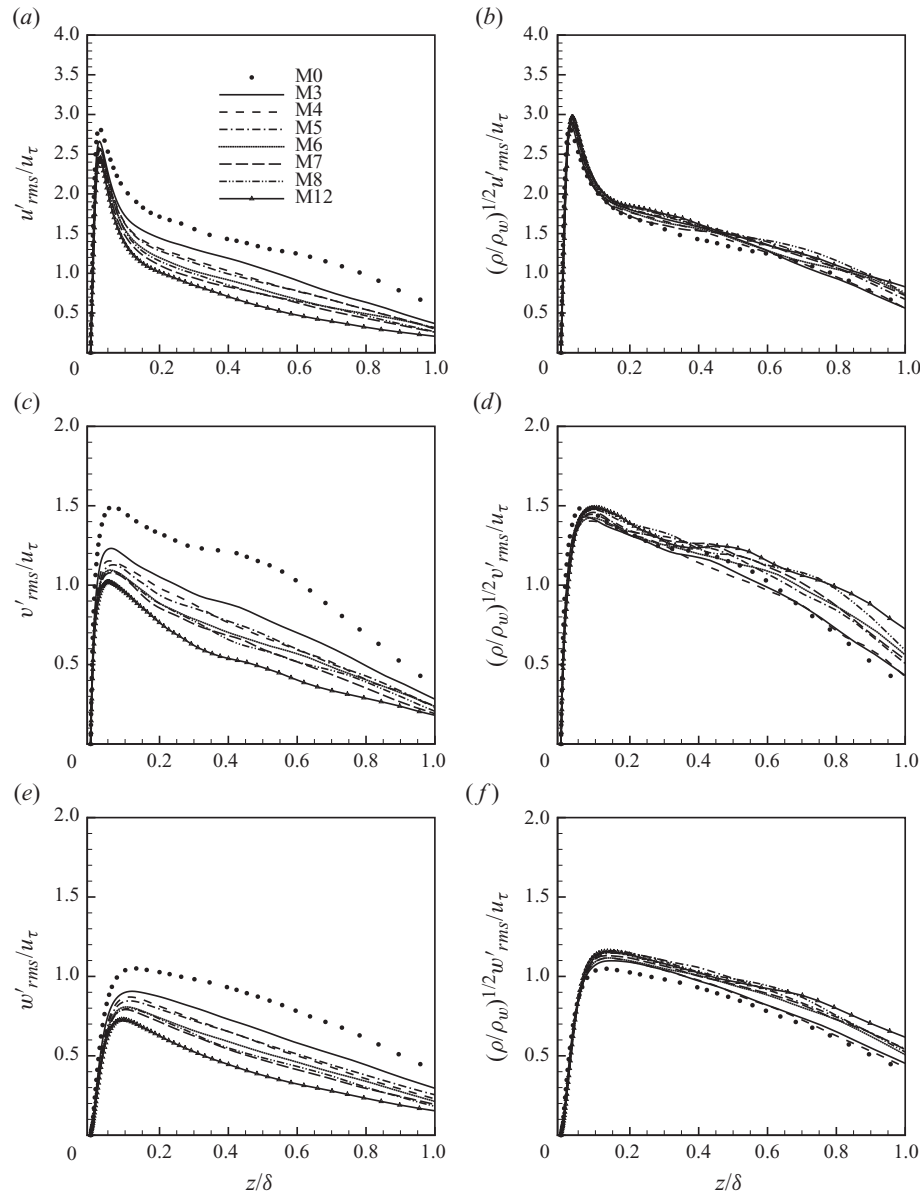
FIGURE 4. Van Driest transformed velocity for different free-stream Mach number cases.

*Direct numerical simulation of hypersonic turbulent boundary layers* 247

Case	$M_\delta$	$\rho_\delta$ (kg m <sup>-3</sup> )	$T_\delta$ (K)	$T_w/T_\delta$	$Re_\theta$	$Re_\tau$	$Re_{\delta_2}$	$\theta$ (mm)	$H$	$\delta$ (mm)
M0	0.30	0.0900	220.0	1.00	1514.7.6	569.9	1515.2	2.76	1.41	23.0
M3	2.97	0.0910	219.9	2.51	3028.6	486.9	1586.7	0.619	5.09	8.85
M4	3.98	0.0902	219.2	3.83	4093.7	438.2	1587.4	0.658	8.01	12.0
M5	4.90	0.0962	224.4	5.31	4931.7	416.5	1578.0	0.682	11.29	15.1
M6	5.81	0.0990	230.7	7.02	5775.1	412.8	1582.2	0.730	15.4	19.7
M7	6.89	0.0929	224.2	9.49	7207.3	391.7	1586.4	0.838	20.2	28.1
M8	7.70	0.0990	232.8	11.2	7508.3	397.5	1577.5	0.861	24.2	31.8
M12	11.93	0.0906	228.0	27.6	11356.4	376.8	1577.5	1.33	46.1	84.7

TABLE 1. Dimensional boundary-layer edge and wall parameters for the DNS database.

# Compressibility Effects on Turbulence - Fundamentals



Local density scaling

DNS  
Duan & Martin, JFM (2011)

FIGURE 6. Turbulence intensities and density-weighted turbulence intensities of the (a,b) streamwise, (c,d) spanwise and (e,f) wall-normal fluctuating velocity components for different free-stream Mach number cases.



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- Shock-Turbulence Interaction

- Variable density/variable property effects

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- Physics and Modeling

- DNS/LES

# Compressibility Effects : Homogeneous Turbulence

## Spectral behavior

HIT Decaying and solenoidal forcing

Aluie (2011), Kritsuk et al (2013), Donzis & Jagannathan JFM (2013), Others  
**Wang, Gotoh & Watanabe PRF (2017)**

Helmholtz decomposition

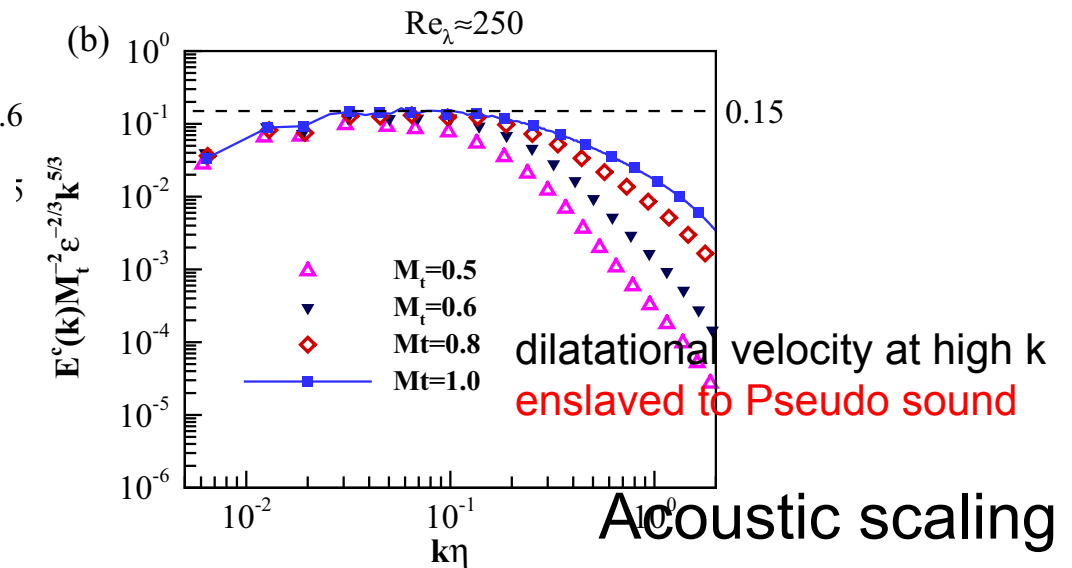
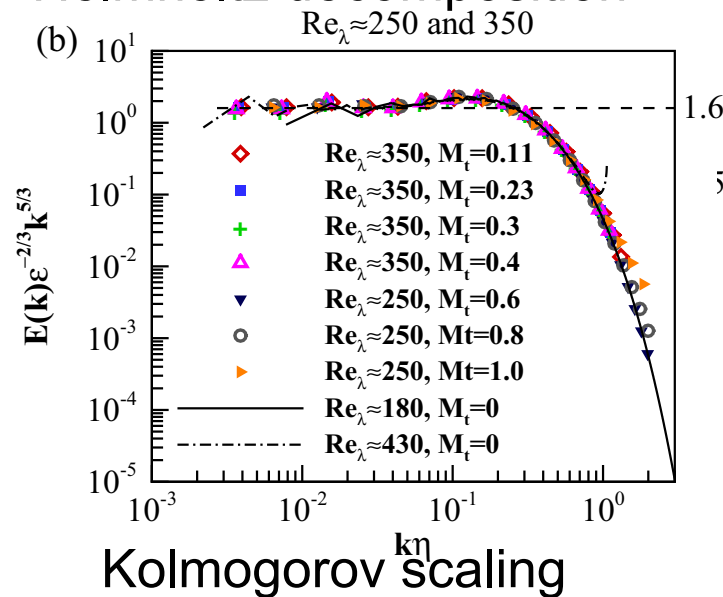
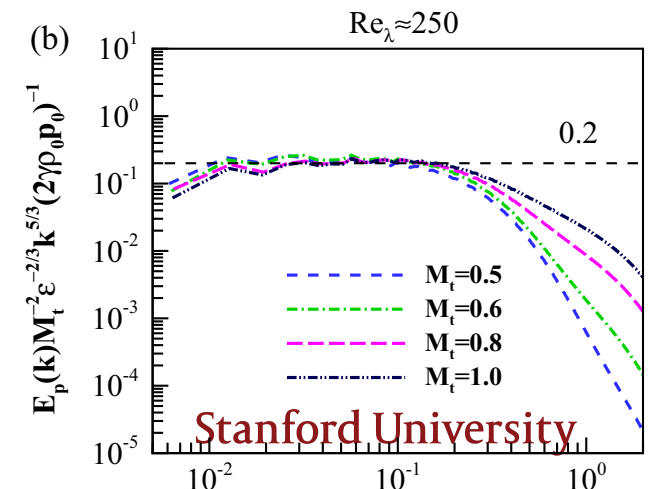


TABLE III. Simulation parameters and resulting flow statistics for  $1024^3$  grid resolution.

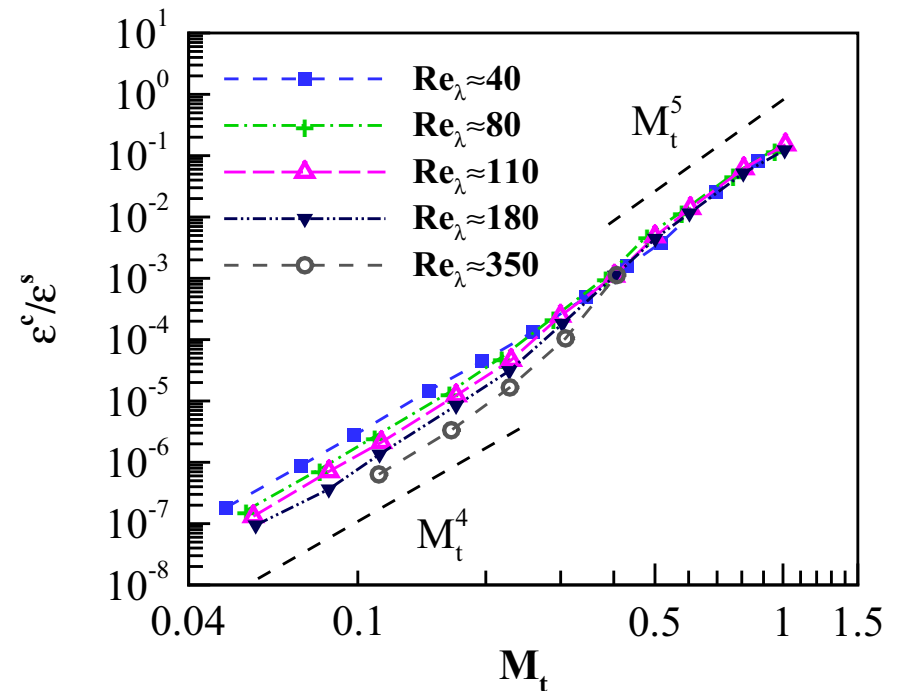
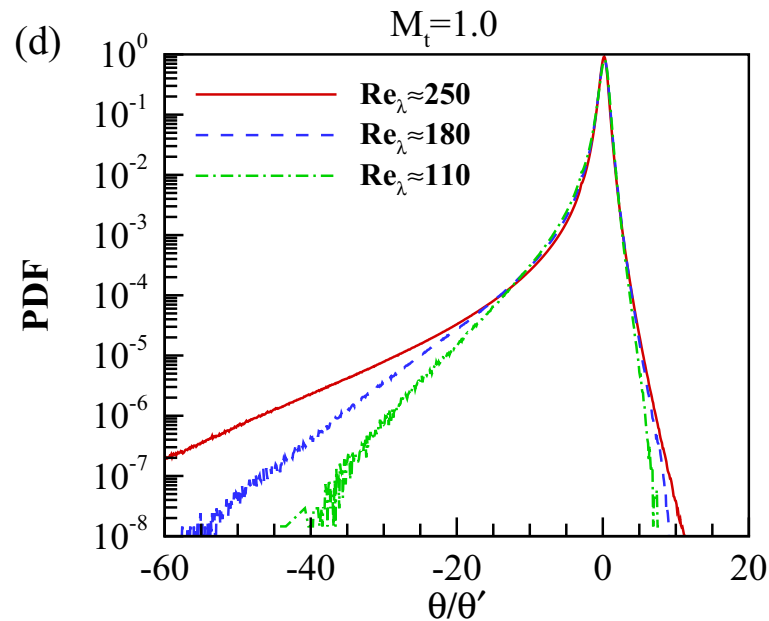
Resolution	$Re_\lambda$	$M_t$	$\eta/\Delta x$	$L_1/\eta$	$\lambda/\eta$	$S_3$	$D = \epsilon L_1/(u'/\sqrt{3})^3$
$1024^3$	350	0.11	0.64	371	36.5	-0.55	0.41
$1024^3$	355	0.17	0.64	374	36.8	-0.55	0.43
$1024^3$	369	0.23	0.62	382	37.7	-0.56	0.41
$1024^3$	361	0.30	0.58	395	37.4	-0.55	0.44
$1024^3$	365	0.40	0.59	390	37.6	-0.54	0.43
$1024^3$	253	0.51	1.00	233	31.2	-0.53	0.44
$1024^3$	262	0.60	1.02	231	31.6	-0.53	0.42
$1024^3$	261	0.79	1.05	229	30.9	-0.83	0.41
$1024^3$	250	1.02	1.04	226	29.1	-1.95	0.42



# Compressibility Effects : Homogeneous Turbulence

## Shocklets & Dissipation

Lee et al (1991), Samtany et al. (2001), **Wang et al PRF (2017)**

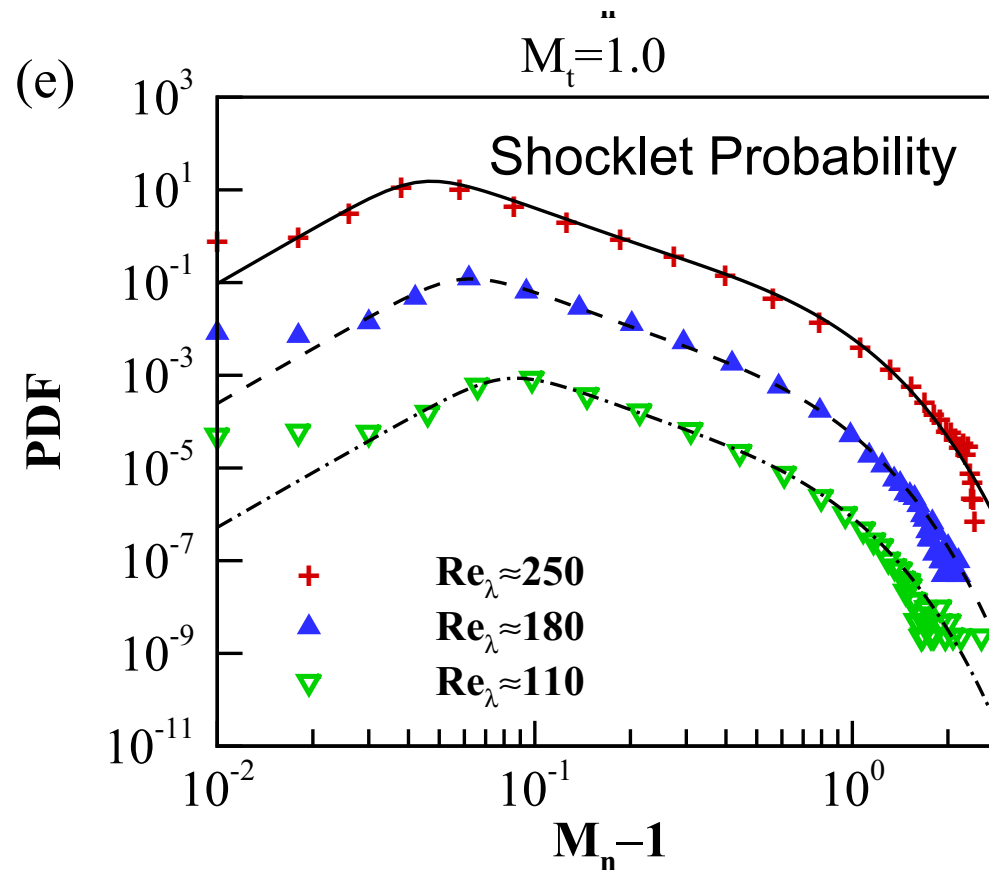


$$\epsilon^c/\epsilon^s \sim M_t^4 \text{Re}_L^{-1} \log(\text{Re}_L),$$

# Compressibility Effects : Homogeneous Turbulence

## Shocklets & Dissipation

Lee et al (1991), Samtany et al. (2001), **Wang et al PRF (2017)**



$$P(M_n - 1) = C_1 \exp \left[ -\frac{M_n - 1}{\beta_2} \right] \left[ 1 + \left( \frac{M_n - 1}{M_n^{\text{exp}} - 1} \right)^{-\beta_1} \right] \left[ 1 + \left( \frac{M_n - 1}{\beta_3} \right)^{-\beta_1 - 4} \right]^{-1},$$



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# Canonical shock/turbulence interaction

Isotropic turbulence passing through a normal shock in a perfect gas  
Isolates the core interaction between turbulence and shock

Parameters:

$M_1$ ,  $M_t$ ,  $Re_t$

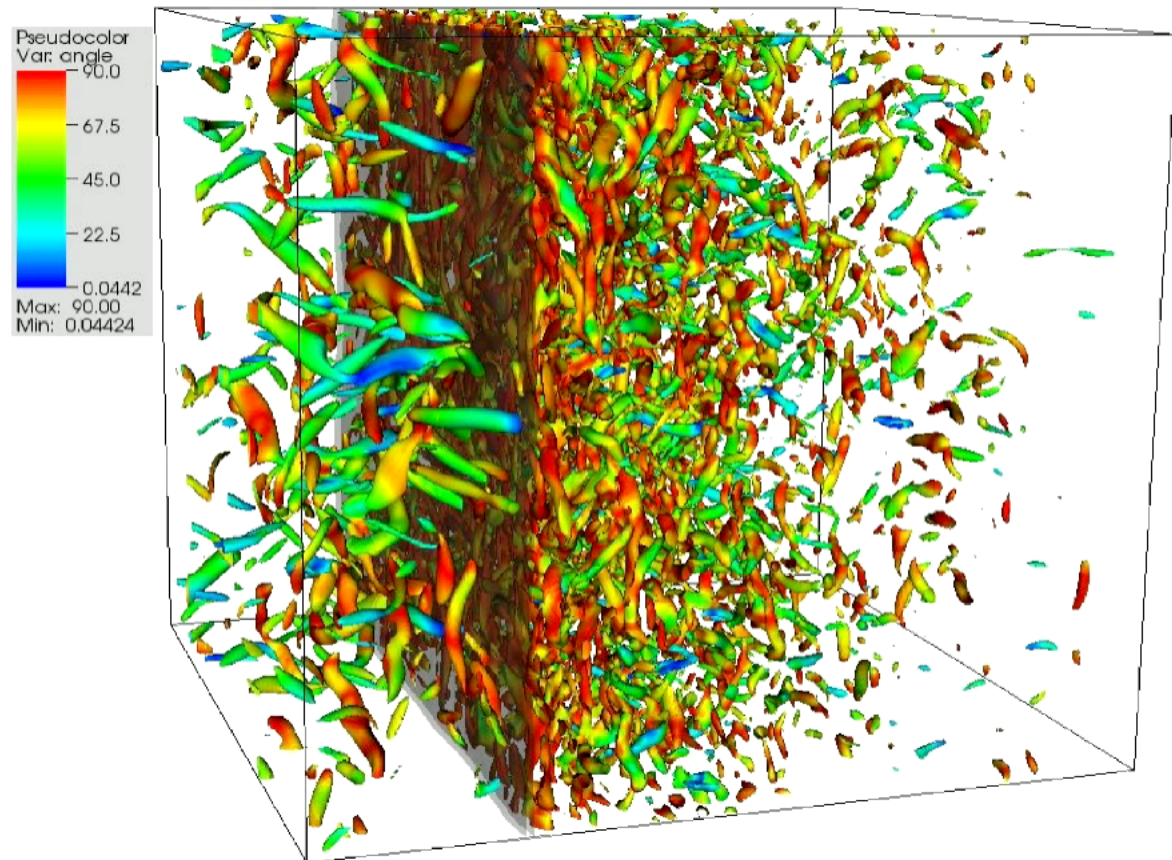
Entropy fluctuations

Acoustic fluctuations

$\delta_{\text{shock}}/L$

Real-gas effects

Eddies visualized by Q-criterion, colored by angle vorticity from  $x_1$ . Shock visualized by dilatation contour.



From Larsson & Lele, Phys. Fluids, 2009

**Larsson, Bermejo-Moreno & Lele, JFM, 2013**

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## Difference compared to previous studies

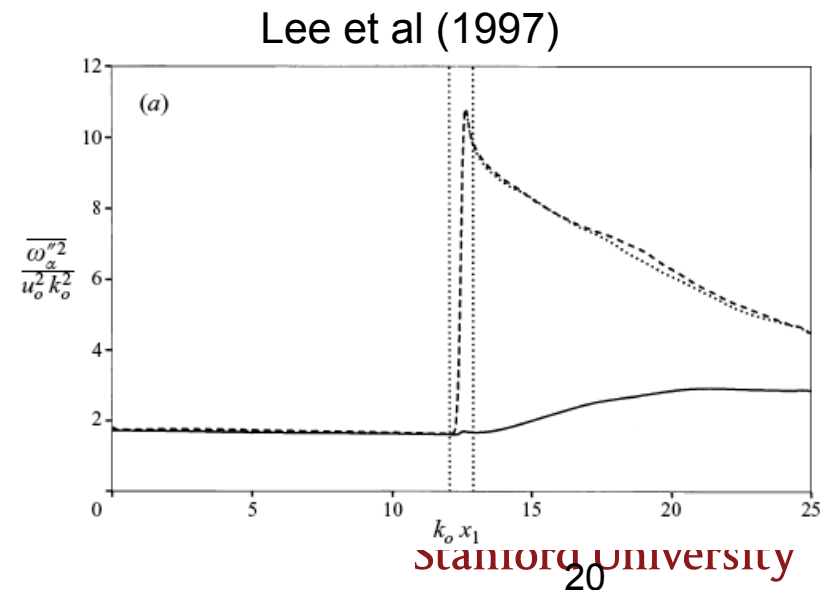
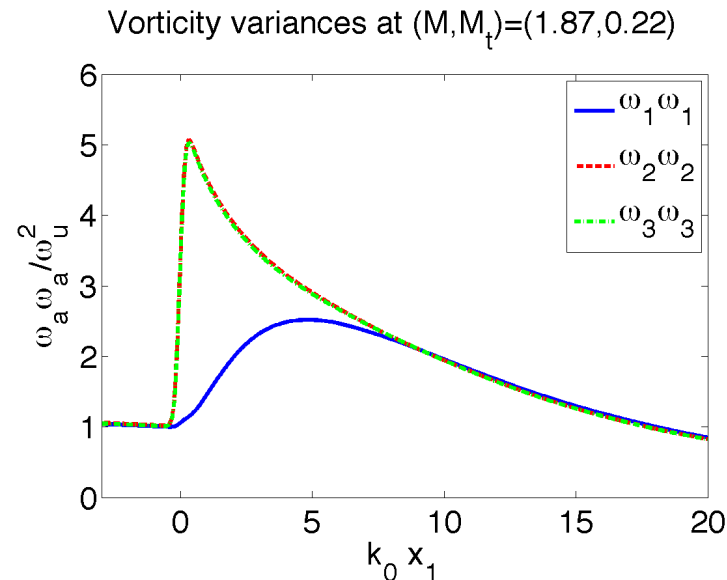
Past studies:  $Re_\lambda = 20$  on  $129 \times 64^2$  grids (Lee et al, 1993, 1997, Mahesh et al, 1997)

Present:  $Re_\lambda = 40$  on  $1040 \times 384^2$  grid;  $Re_\lambda = 60$  on  $1675 \times 512^2$  grid;  $Re_\lambda = 74$  on  $2234 \times 1024^2$  grids

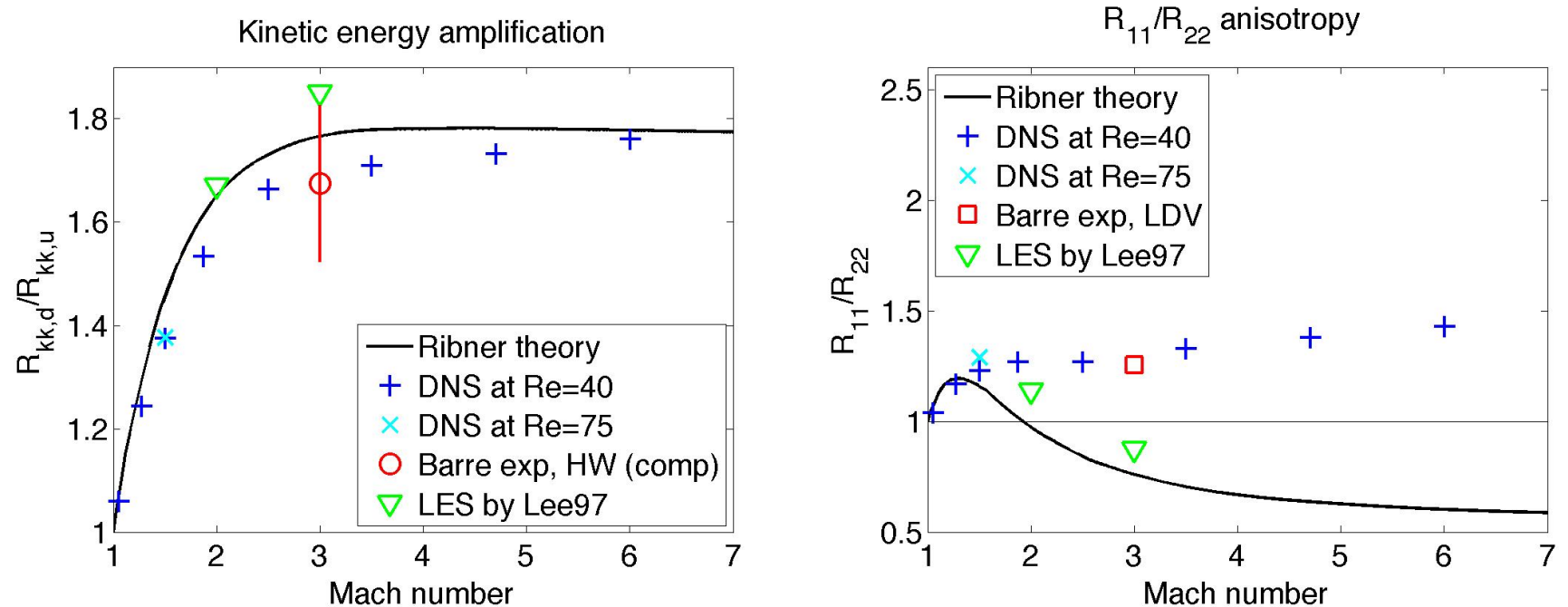
Return to local (small-scale) isotropy in present DNS, but not in past work

Vorticity redistribution is a nonlinear process

Past work did not fully capture these processes due to lack of grid resolution



# Reynolds stresses: amplification across shock



Linear theory predicts TKE amplification correctly, but misses the return to isotropy of small-scale turbulence and (post-shock) Reynolds stress anisotropy

Also see, Donzis (2012) Phys. Fluids and Ryu & Livescu (2014) JFM

Also K. Sinha and collaborators

## Shock motion and approach flow at $(M, M_t) = (1.50, 0.38)$

Contours of streamwise velocity  
scaled by upstream speed  
of sound (colors)

Contours of dilatation at shock  
(gray scale)

$u' > 0$ :

- “pushed back” shock
- strong shock (high compression)

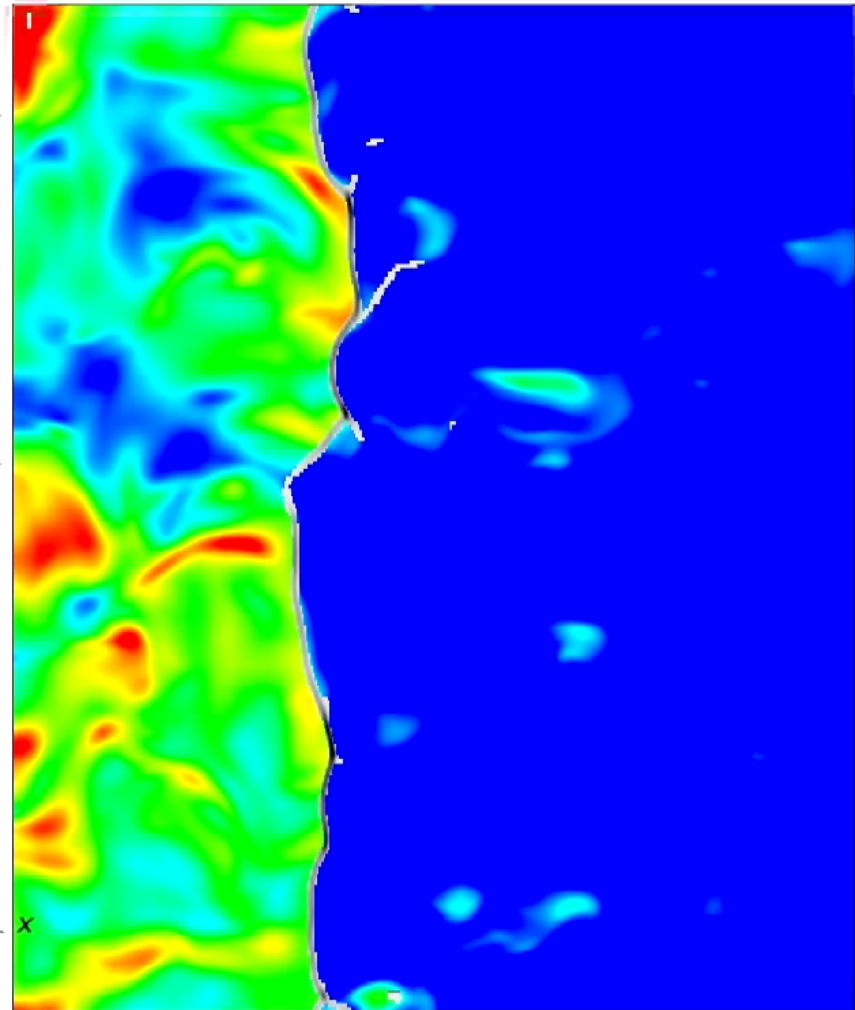
$u' < 0$ :

- “pushed forward” shock
- weak shock (low compression)

Pseudocolor  
Var:  $U$   
2.00  
1.75  
1.50  
1.25  
1.00  
Max: 2.487  
Min: 0.1445

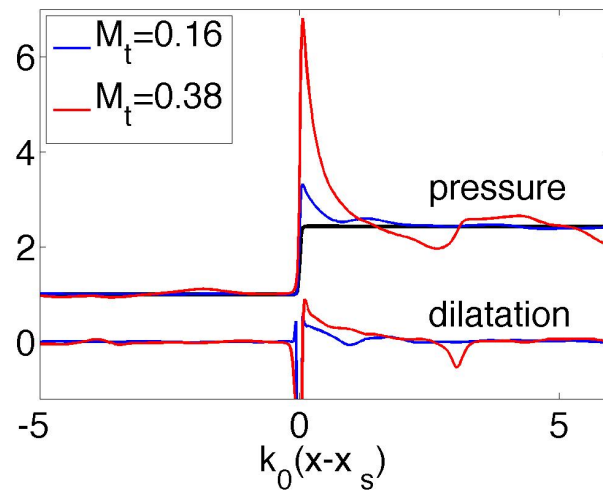
Pseudocolor  
Var:  $dilat$   
-5.00  
-21.2  
-47.5  
-73.8  
-100.  
Max: 19.48  
Min: -130.7

y  
z  
x

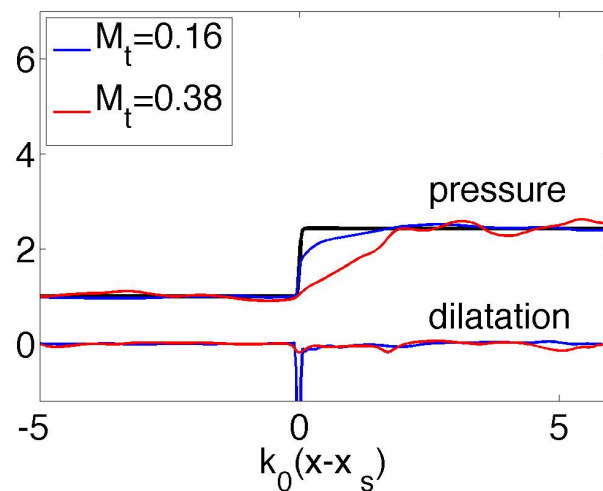


# Instantaneous profiles through shock

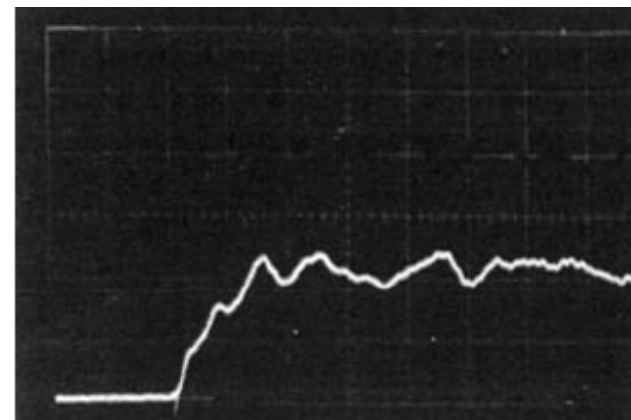
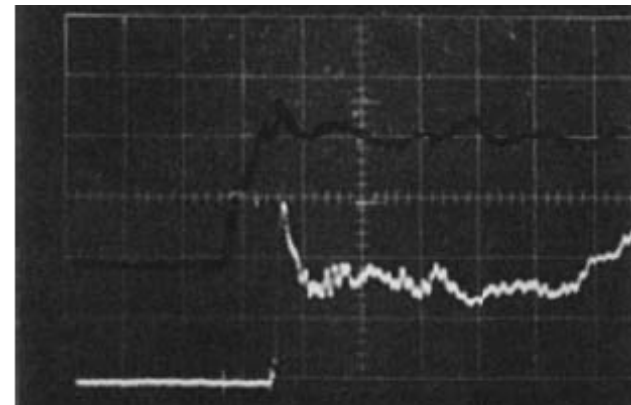
Peaked profiles at  $M = 1.50$



Rounded profiles at  $M = 1.50$



Hesselink & Sturtevant (JFM 1988): “Peaked” and “rounded” pressure profiles for shock propagating through random medium at  $M \leq 1.1$



Source: Hesselink & Sturtevant, JFM 1988

## Interaction regimes

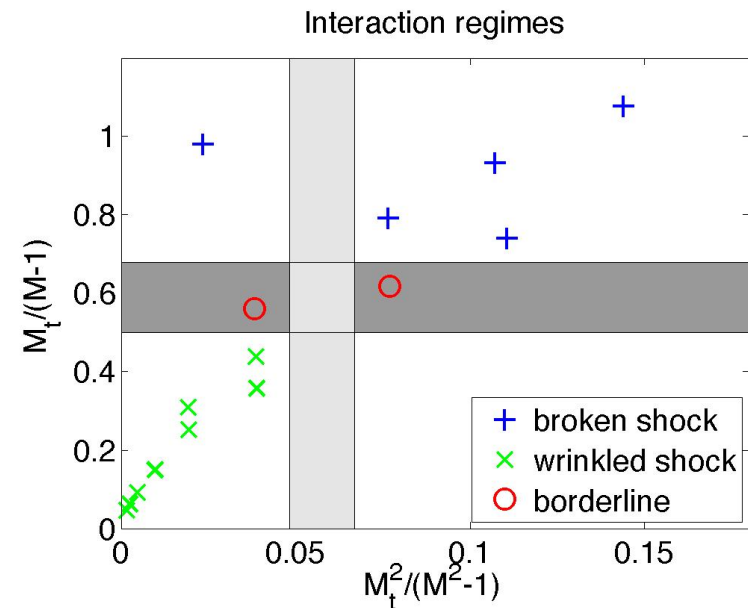
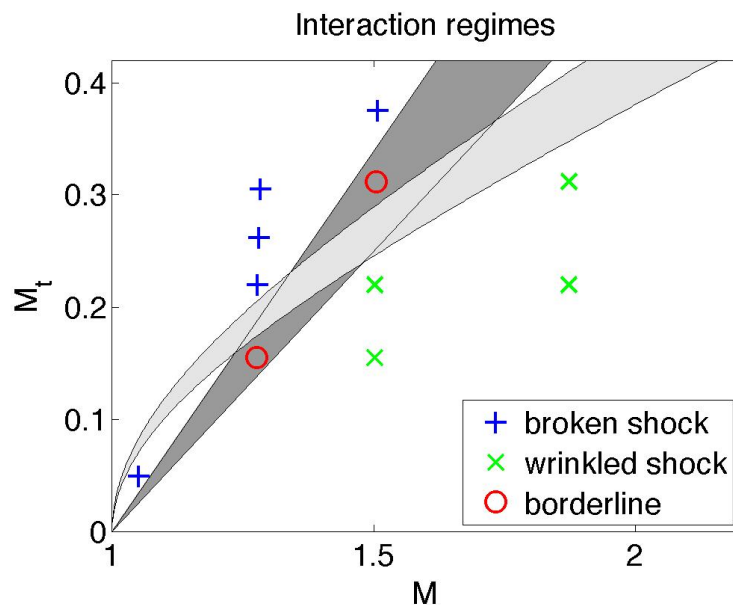
“Broken shock” regime -- existence of smooth (isentropic) profiles through shock

“Wrinkled shock” regime -- every local interaction has a clear shock

Under what conditions can “broken shock” regime be expected?

- Lee et al (1993):  $\frac{p'_{\text{turb}}}{\Delta p_{\text{shock}}} \sim \frac{M_t^2}{M^2 - 1} \gtrsim \text{const}$

- Present: finite prob. of  $u < c \Rightarrow \frac{M_t}{M - 1} \gtrsim \text{const}$  Consistent with Donzis 2013



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DNS/LES



# Variable Property Effects: density and viscosity change

**Semi-local scaling** -- includes scaling of transport eqn (recall Laminar BL theory)

Coleman et al (1995), Patel et al (2016), **Pecnik & Patel JFM (2017)**

$$U_{VD}^+ = \int_0^{u^+} \left( \frac{\bar{\rho}}{\rho_w} \right)^{1/2} du^+,$$

$$U^+ = \int_0^{u^+} \left( \frac{\bar{\rho}}{\rho_w} \right)^{1/2} \left[ 1 + \frac{1}{2} \frac{1}{\bar{\rho}} \frac{d\bar{\rho}}{dy} y - \frac{1}{\bar{\mu}} \frac{d\bar{\mu}}{dy} y \right] du^+,$$

Van Driest Scaling

Trettel & Larsson, PoF 2016

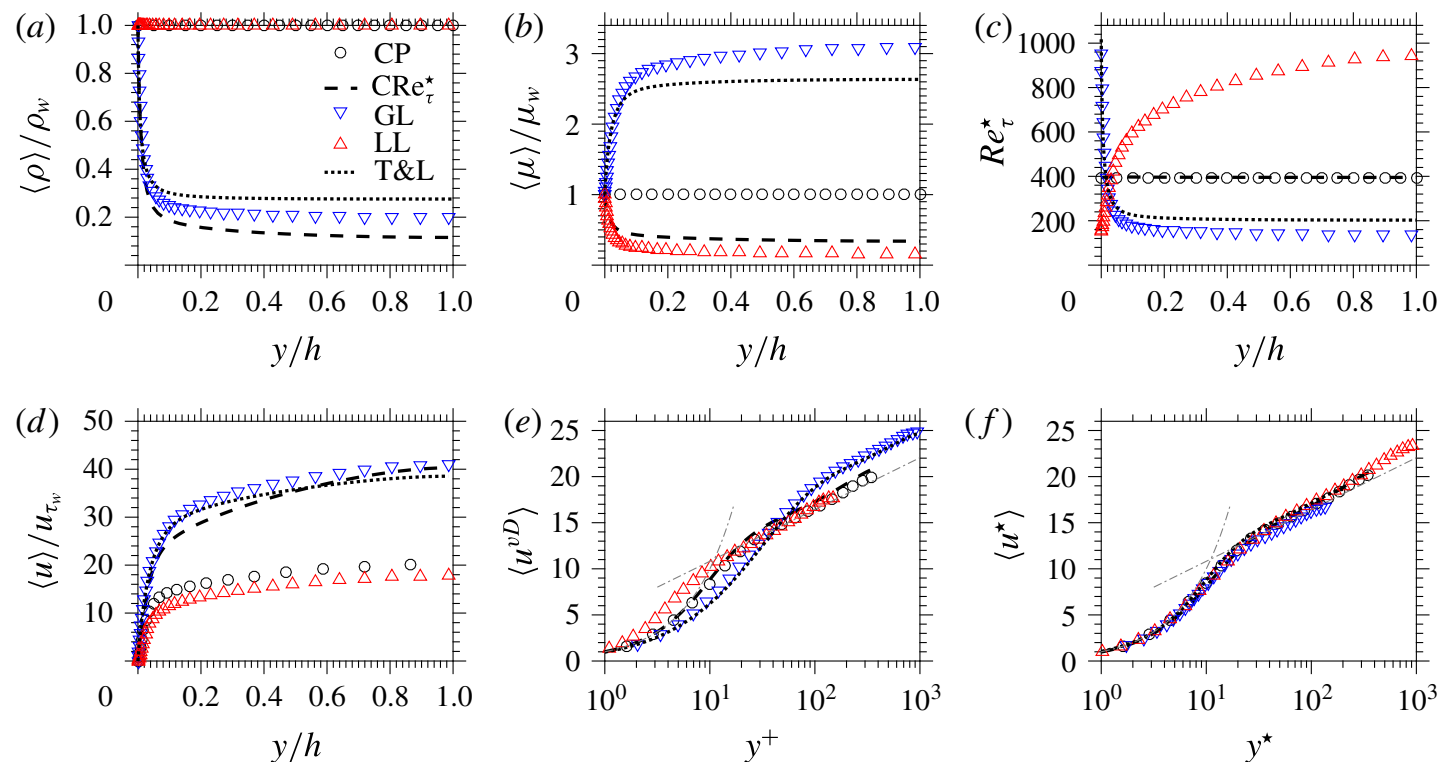


FIGURE 2. Averaged profiles for density (a), viscosity (b), semi-local Reynolds number (c), velocity (d), van Driest transformed velocity (e) and universal velocity scaling (f) for the DNS cases presented in table 1.

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# Summary and Open Issues

**Physical understanding of compressibility effects has improved**

**Significant progress in some areas with DNS, LES**

(limitations in  $Re$ , Many non-dimensional parameters)

**Theoretical framework still insufficient**

How does acoustic communication impact vortex dynamics?

Engineering models incorporating physics of variable-density and compressibility

**Limited experiments with detailed turbulence measurements**

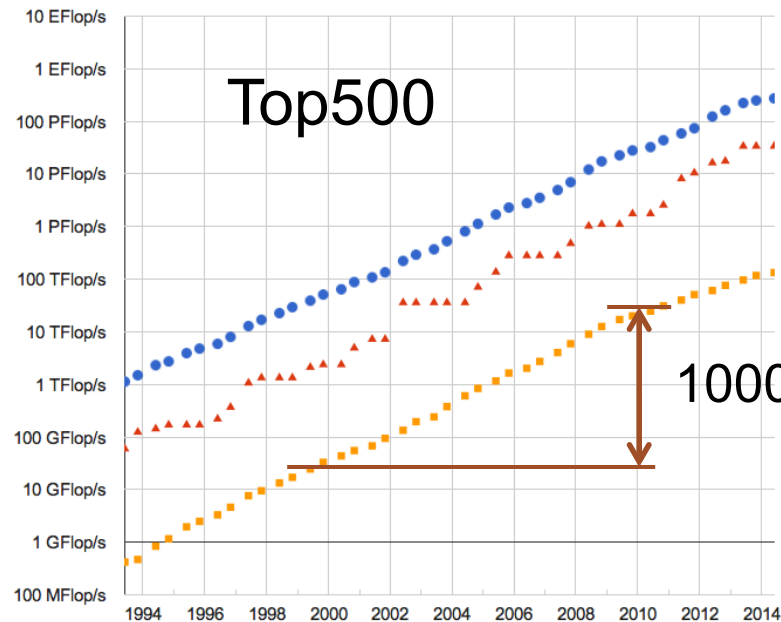
Inflow conditions, boundary conditions (confinement ...)

Confirmation of phenomena and scaling laws

observed in idealized simulations

# Open Issues: Shape of things to come ... ?

Performance Development



## HPC Hardware and Software

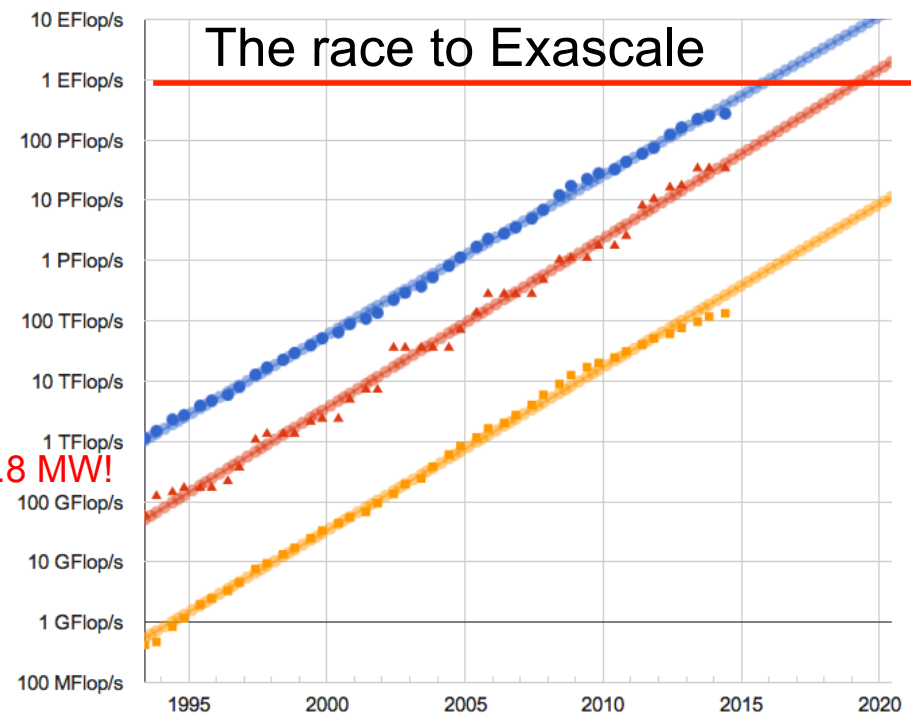
But this is not Moore's law at IC level  
Multicore chips, million cpu's,  
hyperthreading, memory hierarchies,...

No 1 system in 2014 with 33.8 petaflops with **17.8 MW!**  
**Need high Flops/watt**

Will today's petascale code (MPI+OpenMP) run  
on exascale system .... probably not!

Need to develop exascale ready computational frameworks

Projected Performance Development





Thank you!

Any questions ?

# The Challenge of High Reynolds Numbers

Spatial and temporal bandwidth increases with  $Re$

$\eta/L$  scales as  $Re^{-3/4}$

Computational cost (DNS) scales as  $Re^{9/4}$  times  $Re^{3/4}$  or  $Re^3$

Computational cost Wall resolved LES scales as  $Re^{13/7}$

Wall modeled LES reduces the cost to  $O(Re)$

Practically the computational cost restricts the resolution which can be afforded and thus the scale bandwidth.

Need for modeling in turbulence simulations relevant to high  $Re$  continues

# Compressibility Effects on Turbulence - Fundamentals

## How to characterize ?

- ❖ Effects on turbulence generation - flow instability and turbulence energetics
- ❖ Effects on turbulence cascade
- ❖ Effects on turbulence dissipation
  
- ❖ System scale effects (multiple-scale interactions)
  - Radiative damping
  - Flow-acoustic resonance

## Choice of variables ?

**Thermodynamic**  $\rho$ ;  $\log(\rho)$ ;  $v=1/\rho$ ;  $p$ ;  $\log(p)$ ;  $s$  (Entropy);  $B$  (Bernoulli enthalpy)

**Kinematic**  $U_i$ ;  $\rho U_i$ ;  $\sqrt{\rho} u_i$ ;  $\rho^{1/3} u_i$  (for spatial scale decomposition)



# Compressibility Effects on Turbulence - Fundamentals

## Scaling of spectra/structure functions ?

Vortical turbulence -- Kolmogorov scaling

Entropy fluctuations stirred by turbulence

Random nonlinear acoustic field

Random field of shocks

Pressure spectra -- hydrodynamic, pseudo-sound, sound

Density spectra – entropic, acoustic, pseudo-sound

# Aeroacoustics

Computational methods for predicting aerodynamic noise

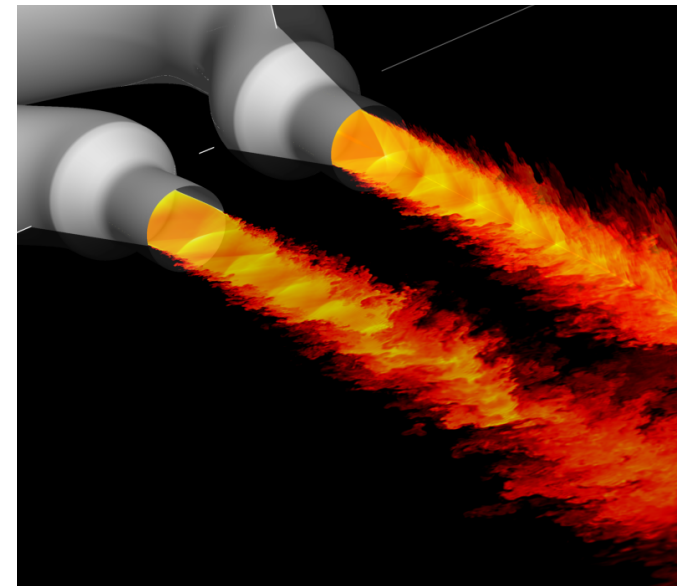
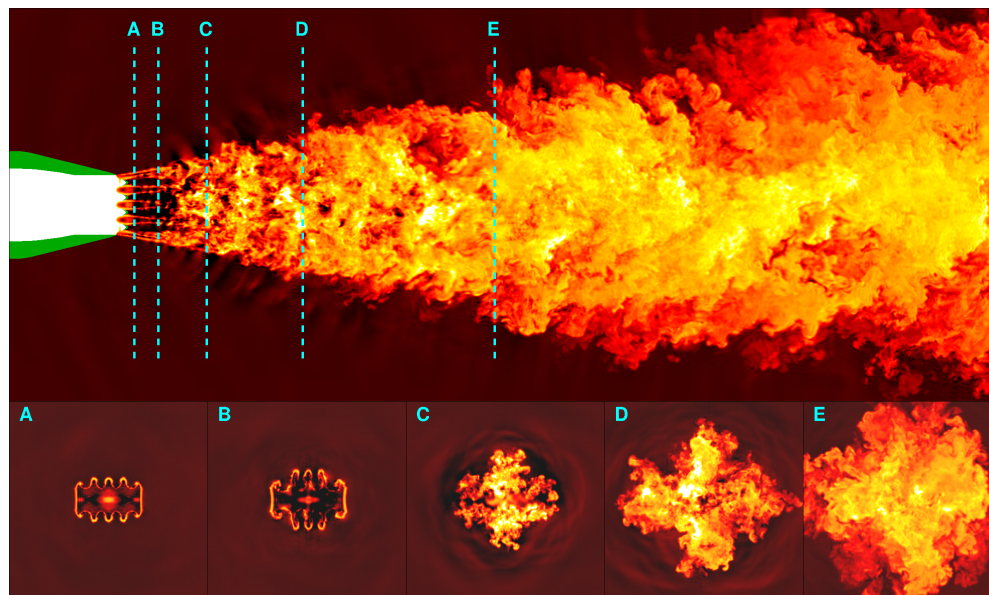
Physics of aerodynamic noise generation

Modeling noise sources and the far-field radiated noise

## Supersonic Jet Noise

Noise predictions using accurate LES of noise producing flow

Approximate methods using source models



- NASA Nozzle -- NA2C3 geometry (2:1 aspect ratio)
- CharLES solver
- Up to 500M isotropic adaptively refined mesh
- BlueGene/P with up to 160K cores (ALCC program, ANL)

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**Crackle** : Annoying Component of Supersonic Jet Noise (Ffowcs Williams, 1975)

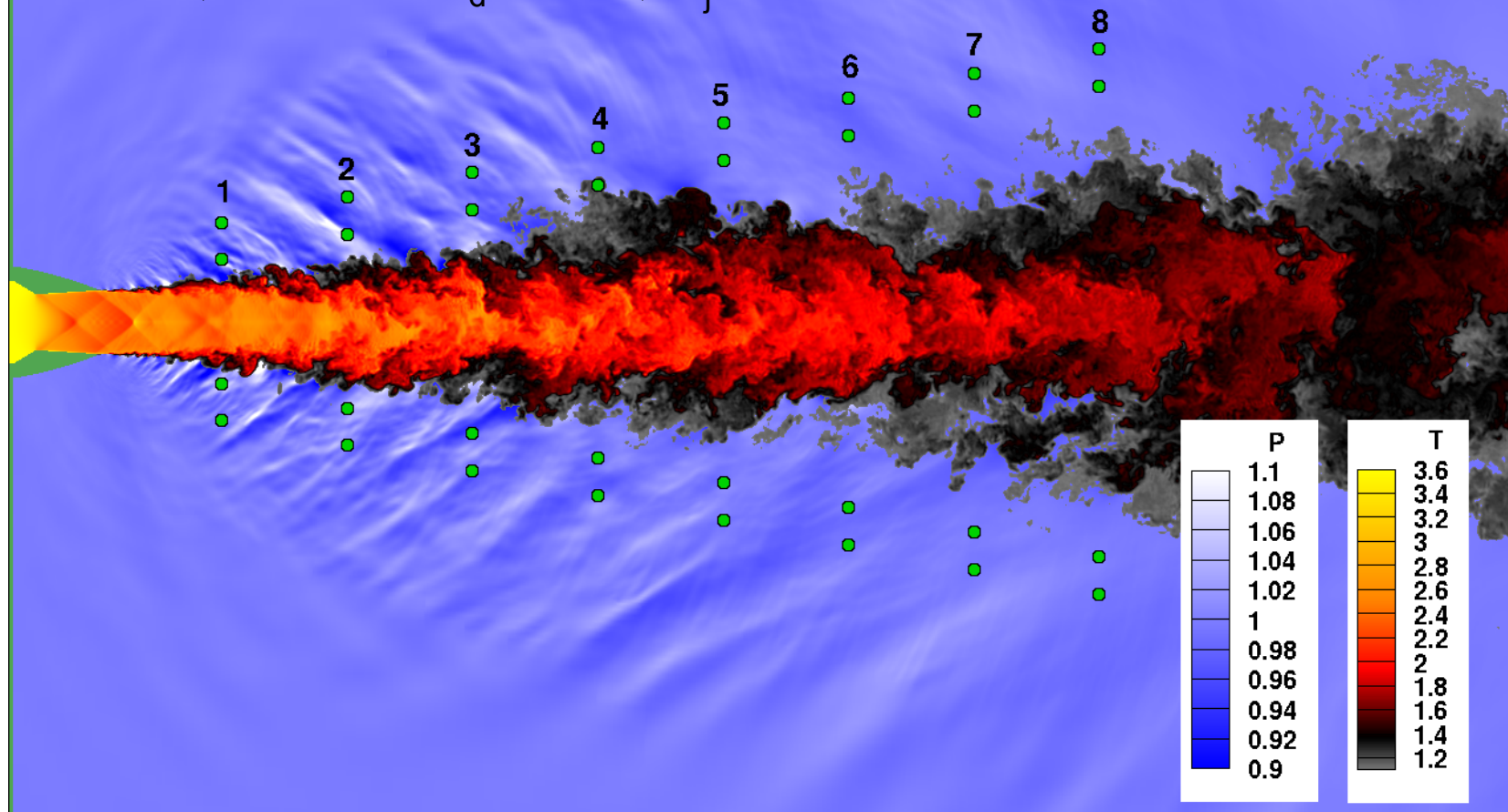
Intermittent, Steep N-wave signature, Skewness

What causes crackle ? Mechanism unknown –source nonlinearity vs non-lin. Propgn.

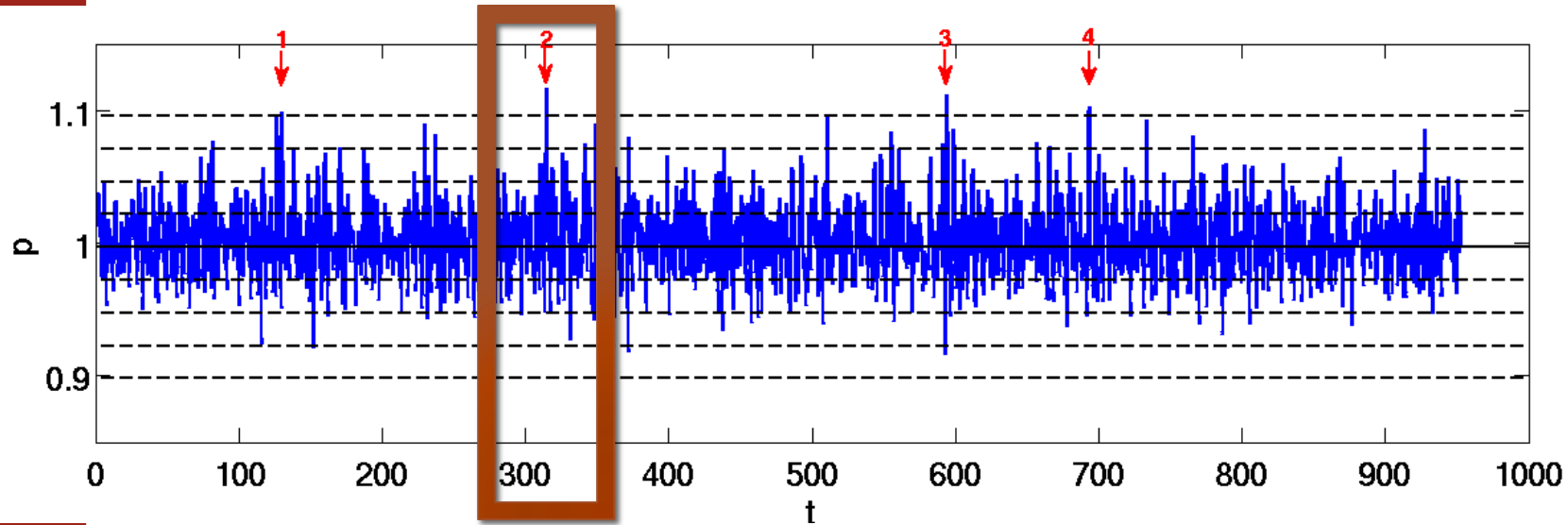
Nichols et. al. 2013, ASME J. Eng. Gas Turbines and Power 213, Vol. 135.

Flow visualization (207M mesh)

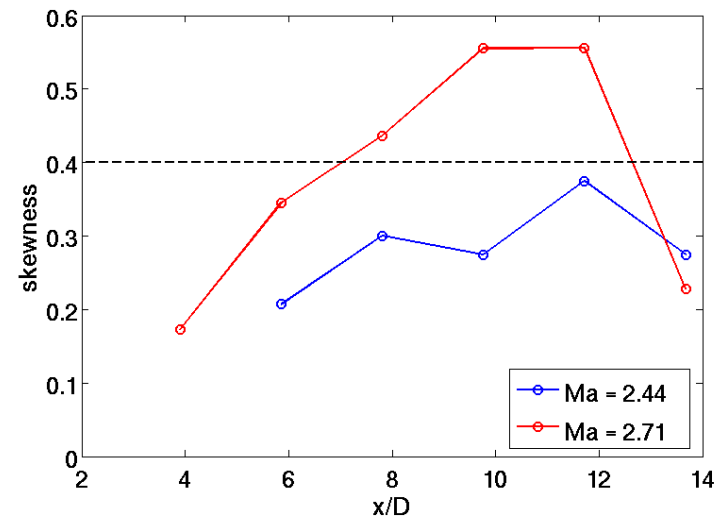
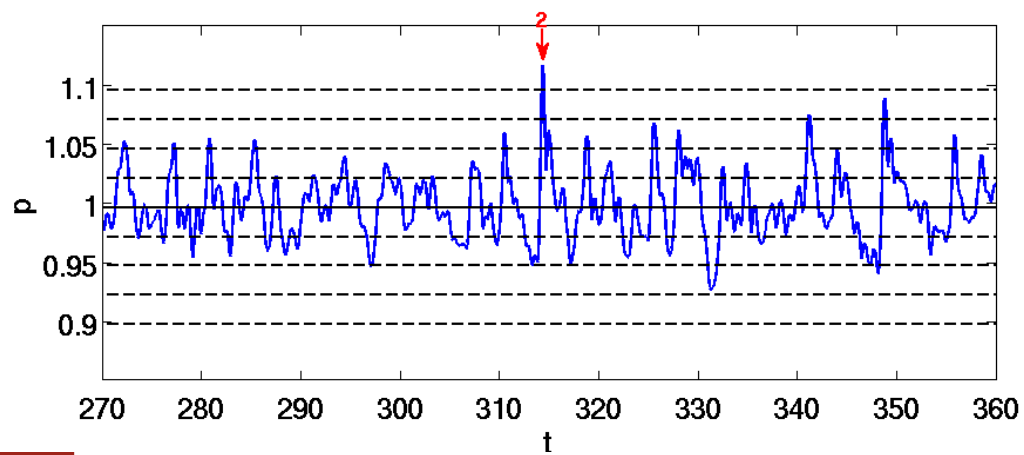
NPR= 4; TTR = 3.  $M_d = 1.65$ ,  $M_j = 1.56$



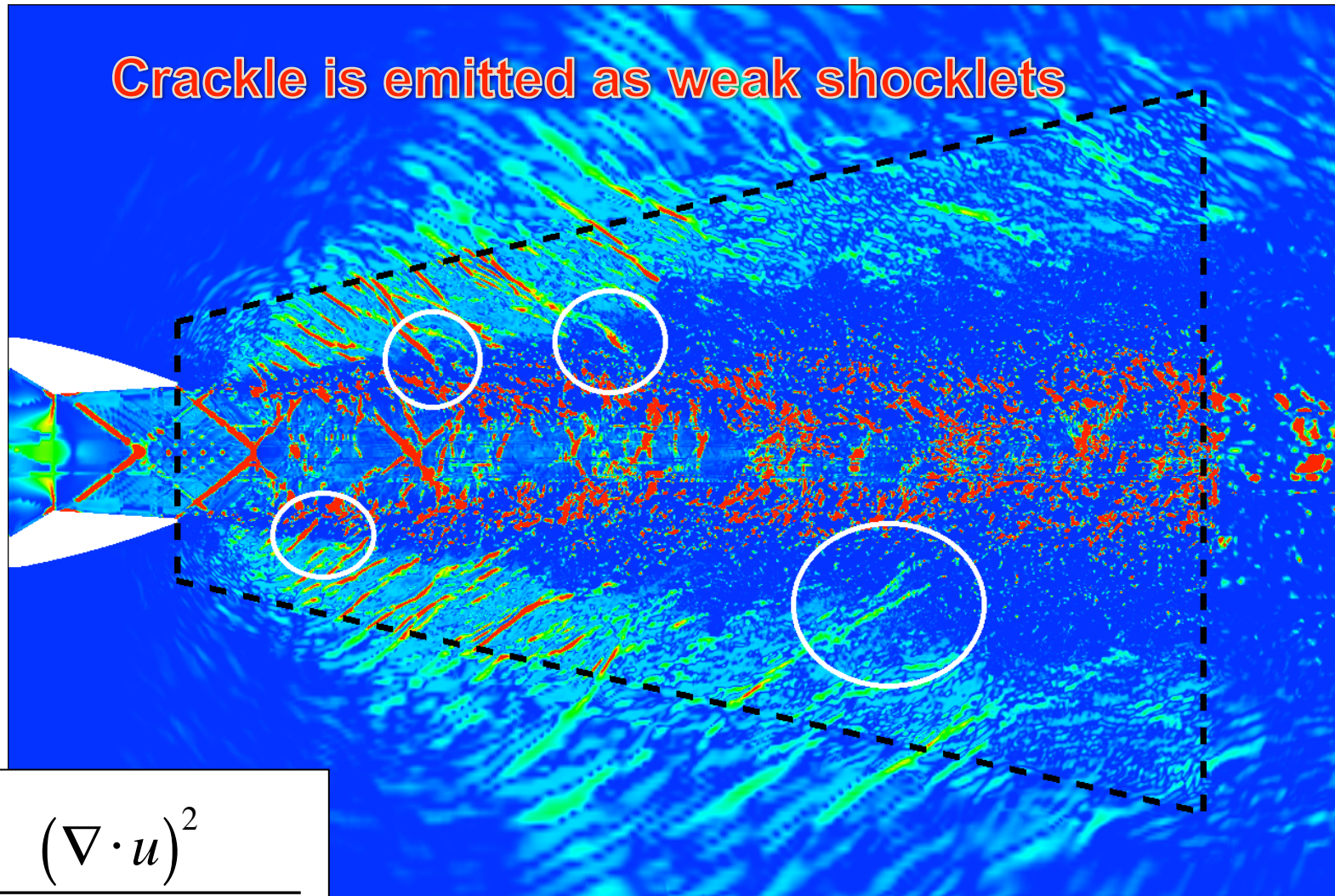
# Pressure signal (skewness 0.425)



(Zoom)



## Shock sensor (207M)



$$\frac{(\nabla \cdot u)^2}{(\nabla \cdot u)^2 + \Omega^2 + \varepsilon}$$

(Ducros et al., 1999; Bhagatwala & Lele, 2009)

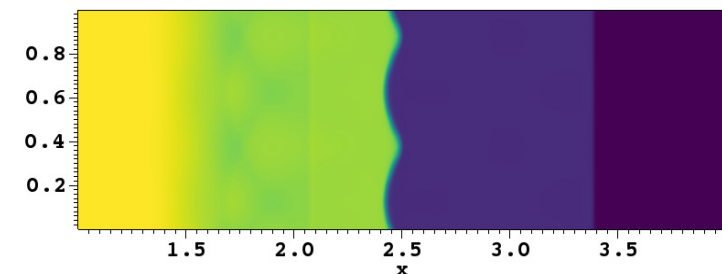
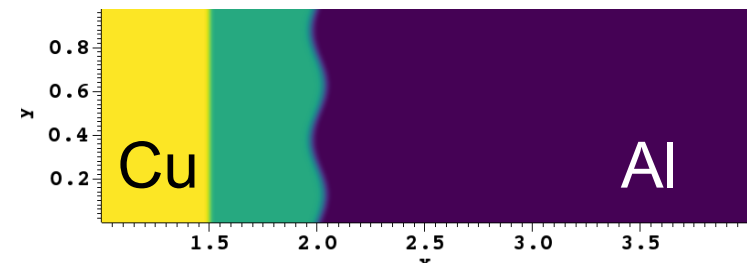
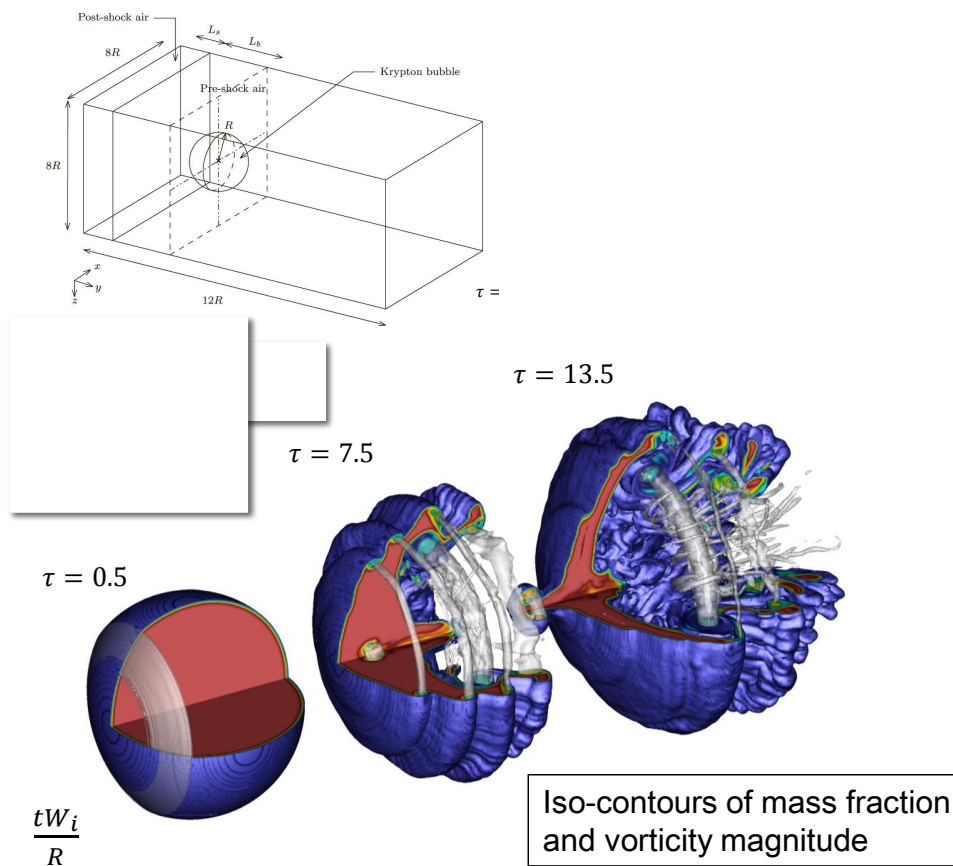


# Main Research Areas: Fluid Dynamics - Turbulence

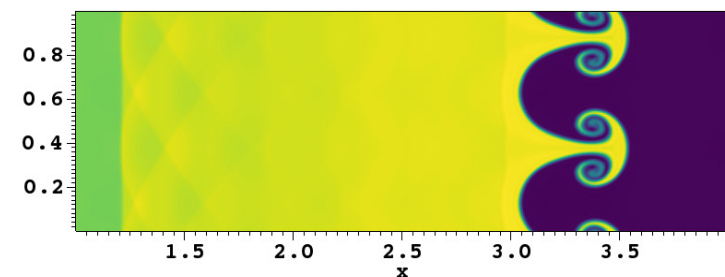
- › Shock-induced instability and multi-material mixing

Numerical methods for shocks/interfaces  
Non-linear elastic/plastic deformations

Shock driven instability  
at solid-solid interface



(b)

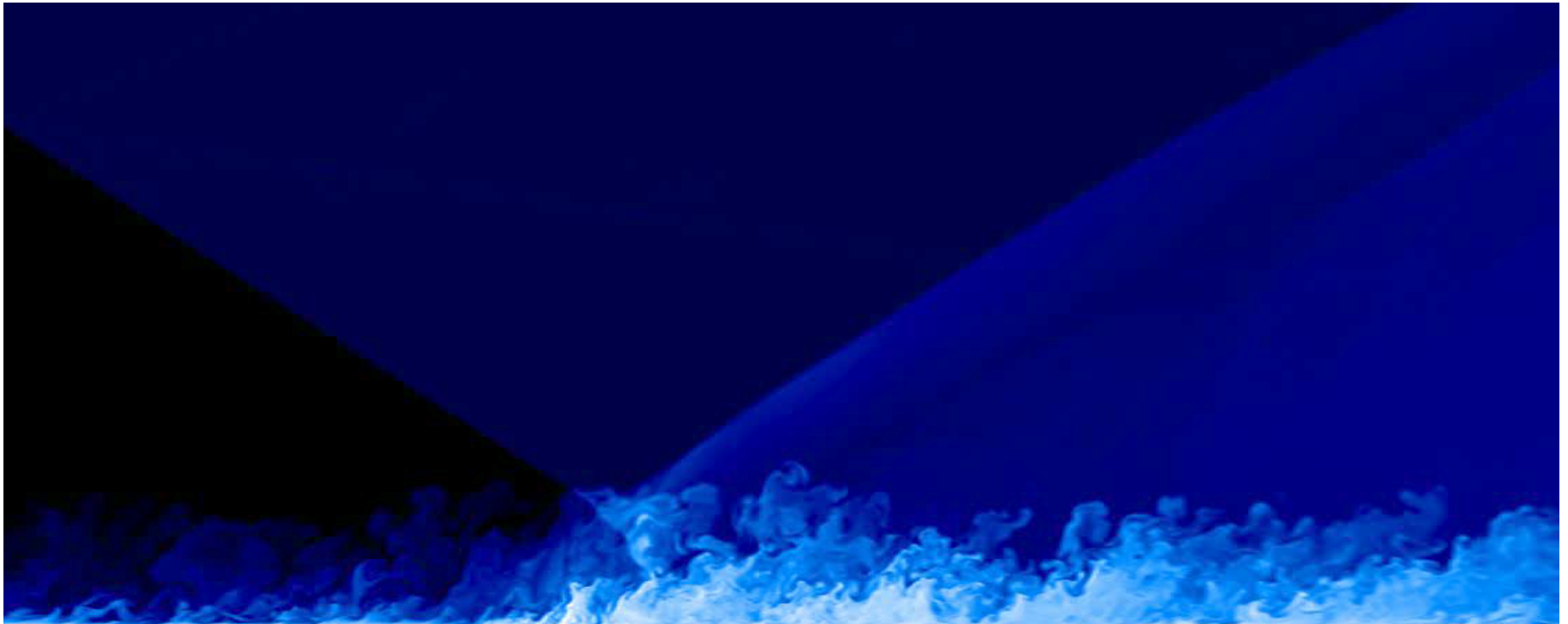


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# Compressible turbulence, Shock-Boundary layer Interactions

## Capturing shock waves and turbulence

**LES** results Morgan, Kawai & Lele AIAA 2011-3431, *J. Fluid Mech.* 2013, Vol 729



$M = 2.28$ ;  $Re_\theta = 2300$ ; wedge angle = 8 deg Expt: IUSTI, Marseille

Temperature contours

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