Quantized vortices following reconnections

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Superfluid helium

Two fluid model:

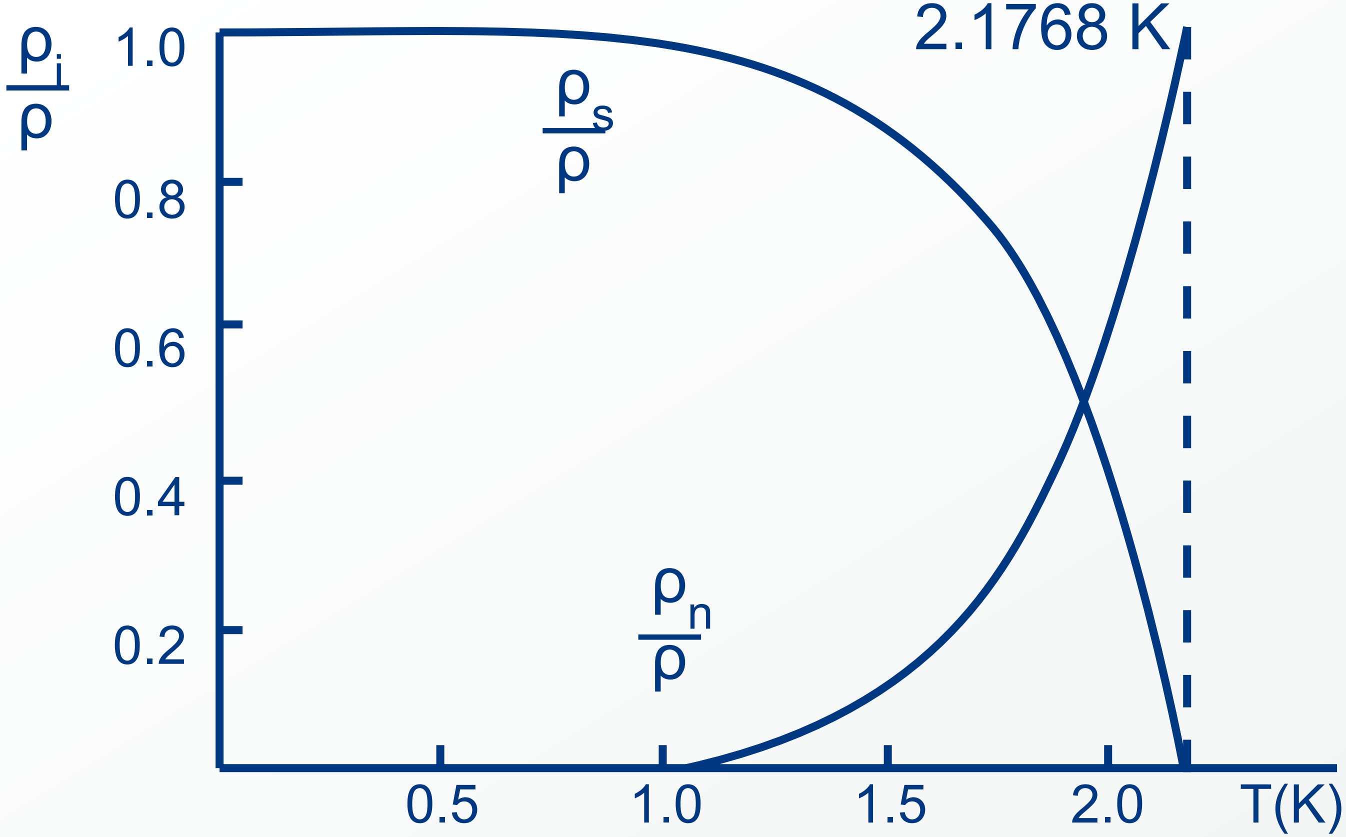
superfluid / *normal* component <u>similar</u> to classical inviscid / viscous *fluid*

$\rho = \rho_s + \rho_n$

$\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n = 0$

$\mathbf{v}_s$: Superfluid velocity

$\mathbf{v}_n$: Normal velocity



Quantized vortex - interactions

Superfluid helium flow is irrotational,

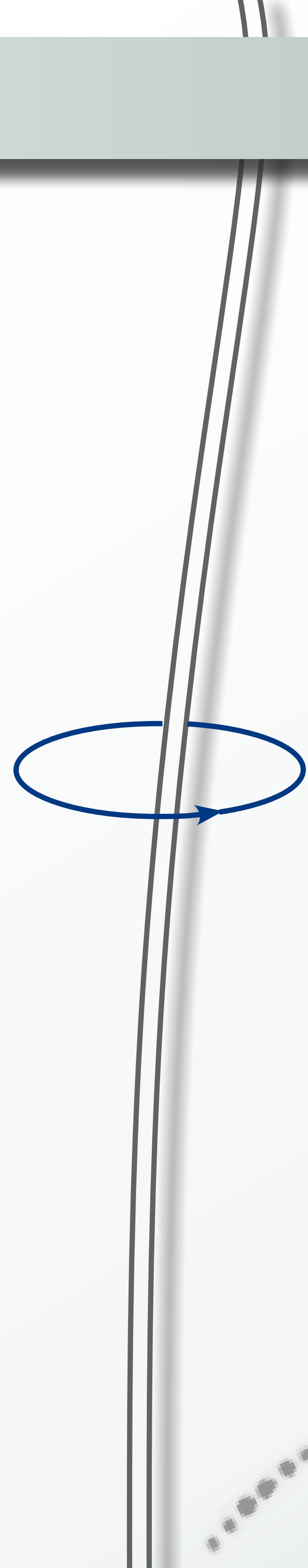
$$v_s = \kappa \nabla \varphi \qquad \nabla \times \mathbf{v}_s = 0$$

Quantized vortex - topological defect

$$v_s = \frac{\Gamma}{2\pi r} \qquad \Gamma = \kappa n \qquad n \in \mathbb{N} \qquad \kappa = \frac{h}{m_4}$$

m_4: mass ^{4}He atom   h: Planck constant

ends at walls, ring, knot



Quantized vortex - interactions

Evolving set of quantized vortices: **quantum turbulence**

code by Angheluta, Jeraldo, Goldenfeld, PRE 2012

Similarities with classical turbulence

pressure drops along pipes

Walstrom, Maddocks, Van Sciver, Cryogenics 1988

drag on a sphere

Smith, Hilton, Van Sciver Phys. Fluids 1999

vorticity decay

Smith, Donnelly, Goldenfeld, Vinen, PRL 1993

Kolmogorov energy spectrum

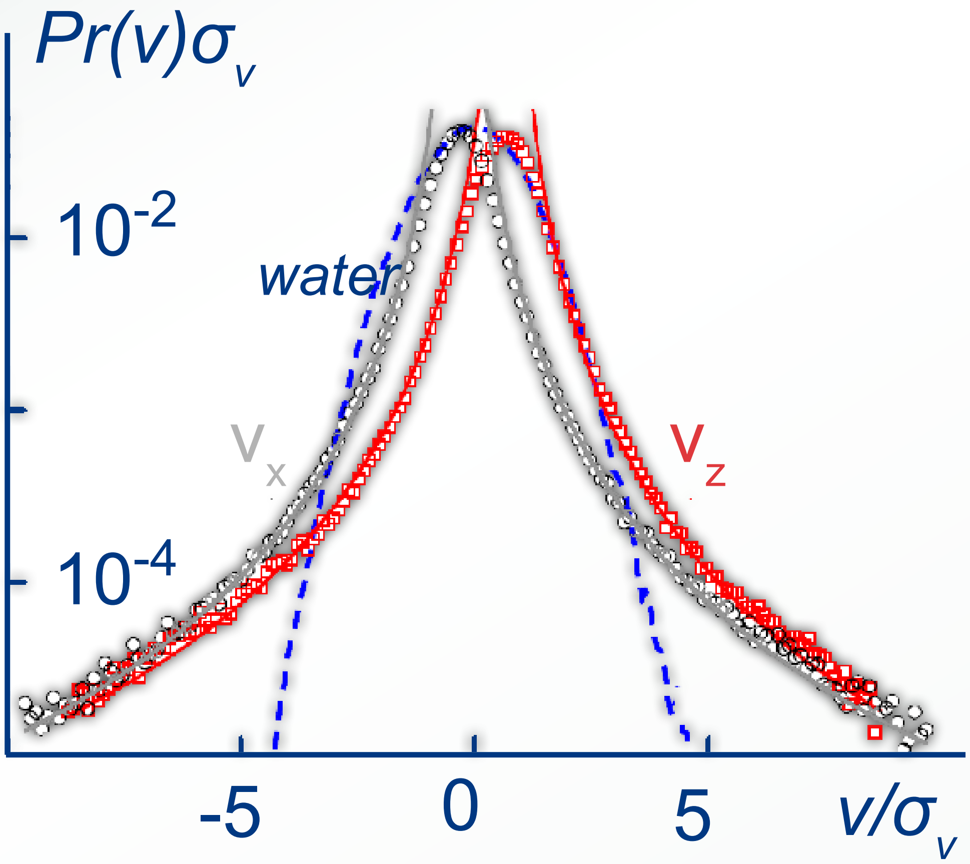
Nore, Abid, Brachet, PRL1997
Maurer, Tabeling EPL 1998
Salort et al. Phys. Fluids 2010

four-fifth law

Salort, Chabaud, Lévêque, Roche, EPL 2012

Classical vs Quantum Turbulence

Velocity distributions in quantum turbulence



Paoletti, Fisher, Sreenivasan, Lathrop, PRL 2008

strongly non-Gaussian
$1/v^3$ power-law tails

due to
reconnections
singular nature of the vortices

Gaussianity restored at scales bigger than the inter-vortex spacing

Baggaley, Barenghi PRE 2011
La Mantia, Skrbek EPL 2014

BEC simulations

White, Barenghi, Proukaki, Youd, Wacks PRL 2010

Quantum turbulence dissipation

At $0.6 < T < 2.17K$
Interaction vortices with normal component (finite Re)

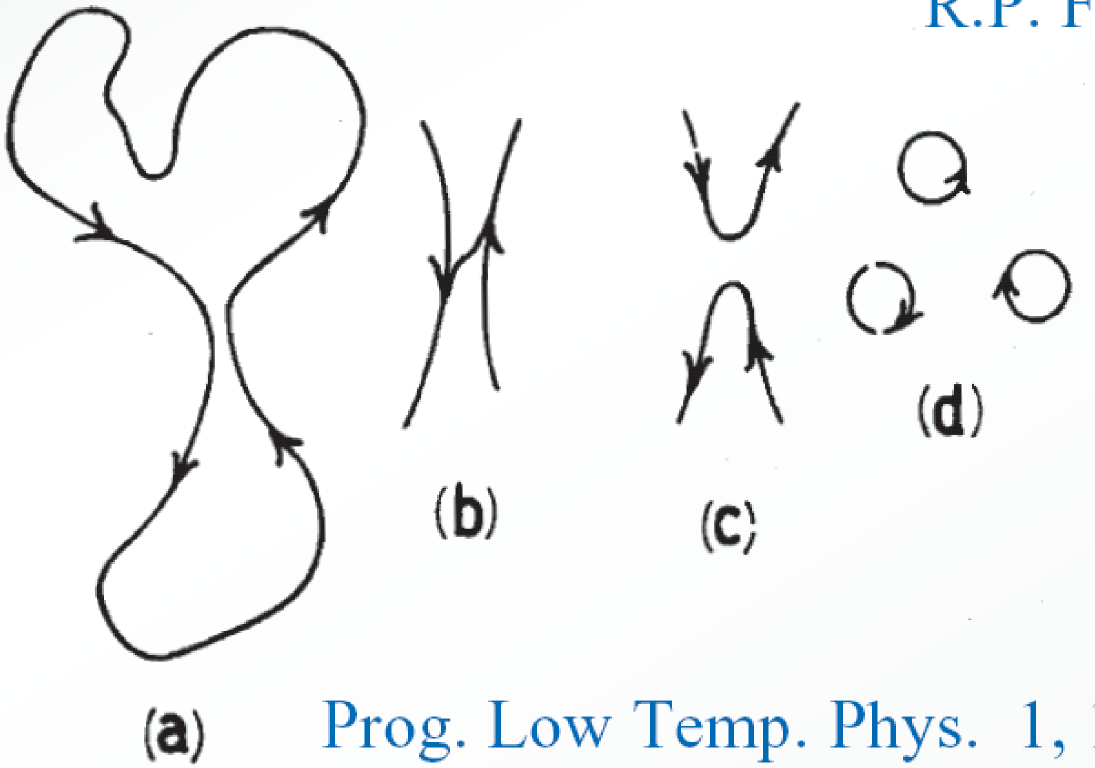
In the T->0 limit
dissipation still present

Theory: reconnections excites Kelvin waves
Kelvin waves cascade
energy emitted by phonons

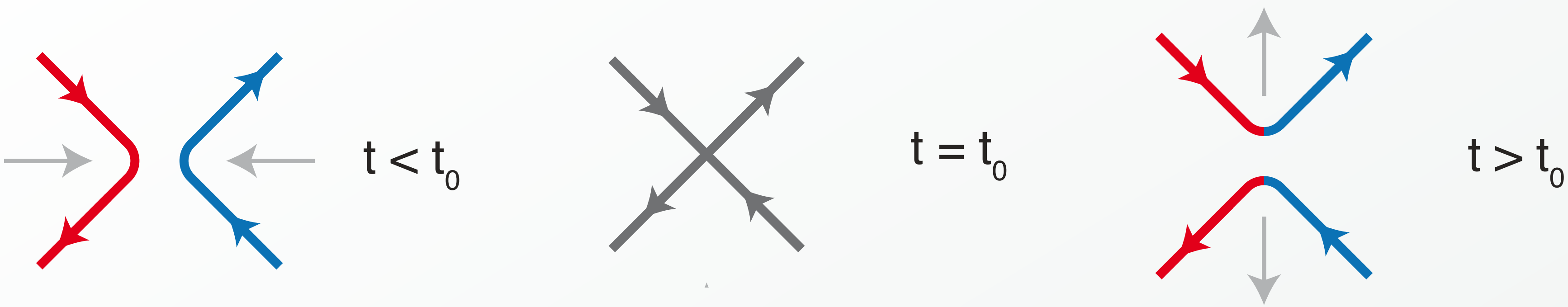
Svistunov, PRB 1995 - Barenghi, Physica D 2008

Vortex Reconnection

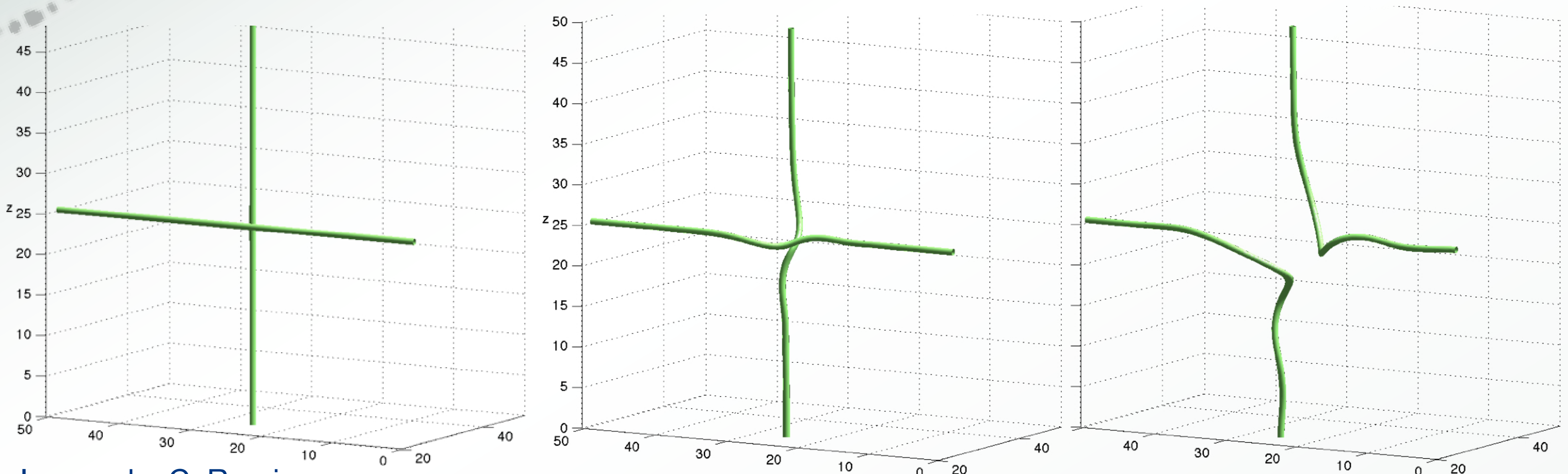
R.P. Feynman



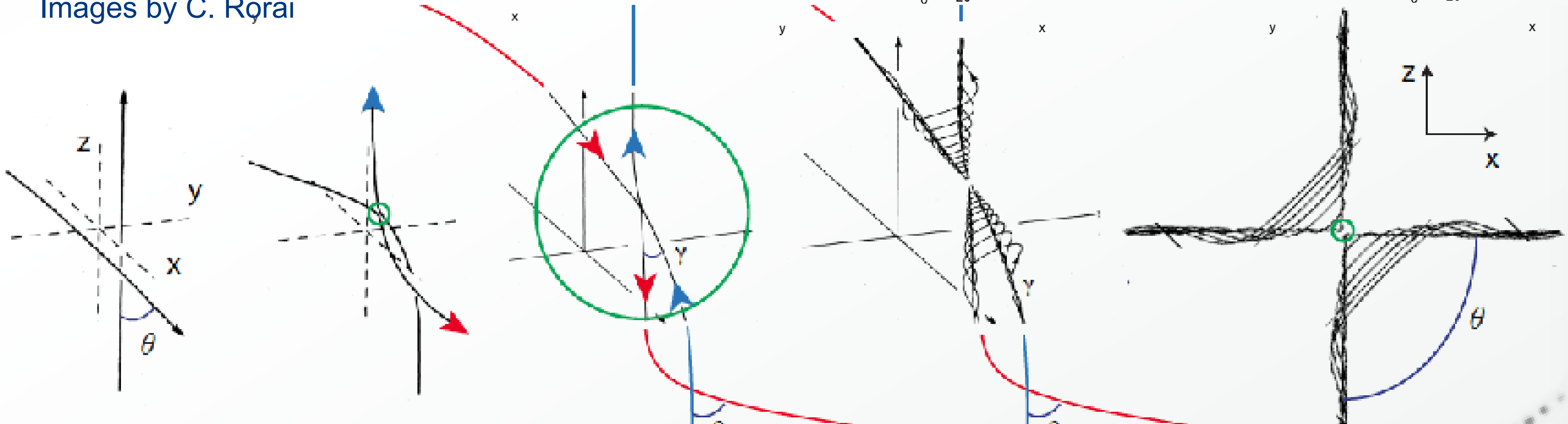
Prog. Low Temp. Phys. 1, 17 (1955)



Vortex Reconnection



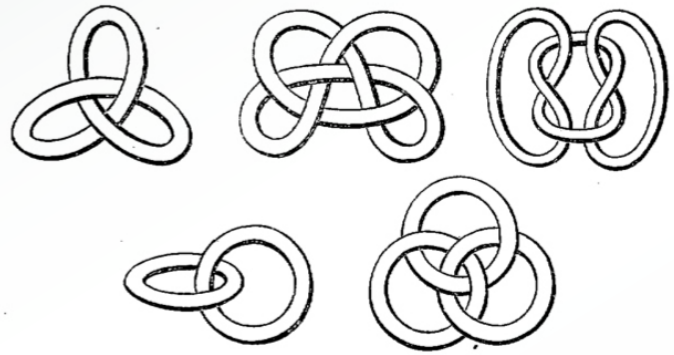
Images by C. Rorai



Images adapted from Schwarz, PRB 1985

Kelvin Wave

Goal: theory of vortex atoms



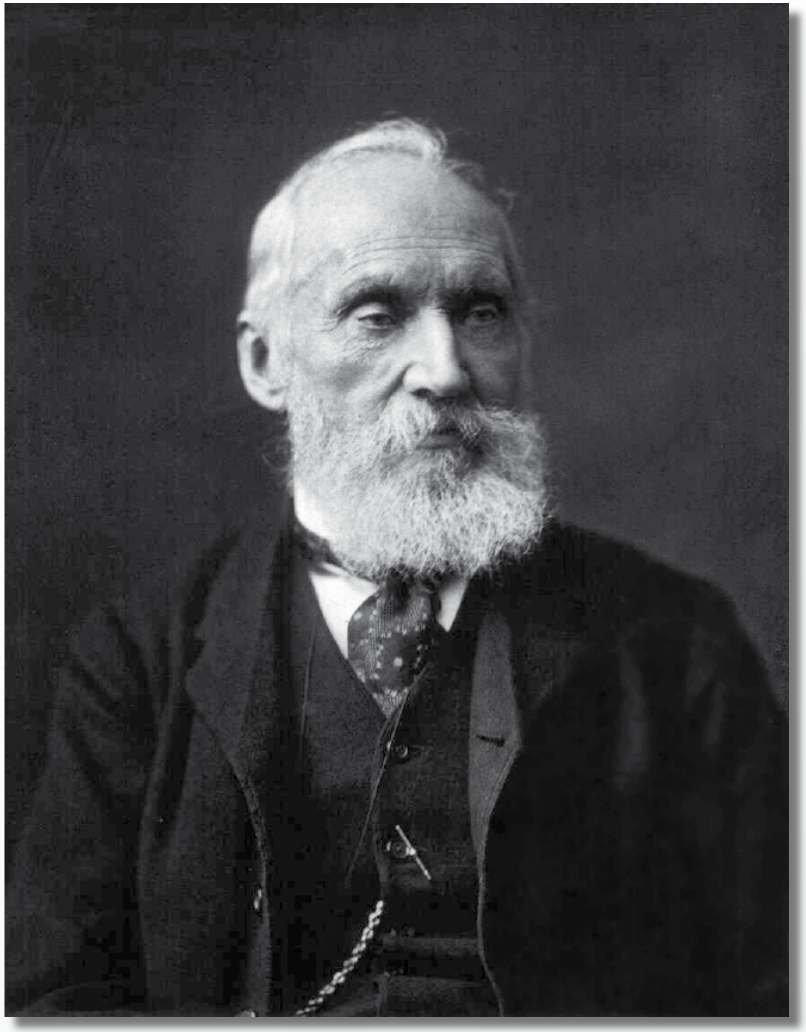
Propagation of a helical perturbation

on a

vortex with a **small core**

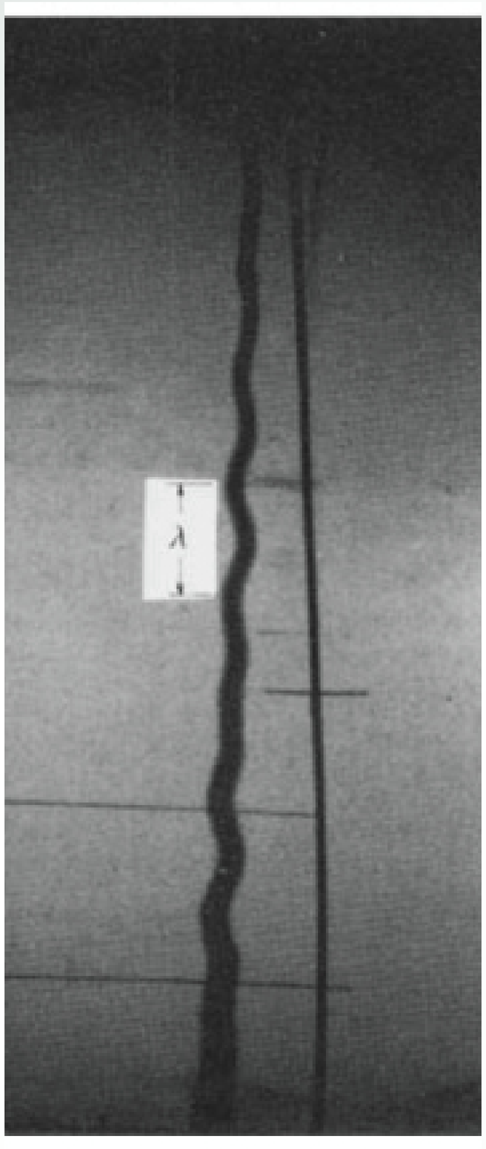
in an

incompressible **inviscid** fluid.



Vibrations of a columnar vortex, *Phil. Mag. 10, p. 155-168*, 1880, W. Thomson (Lord Kelvin)

Kelvin waves



Used to understand:

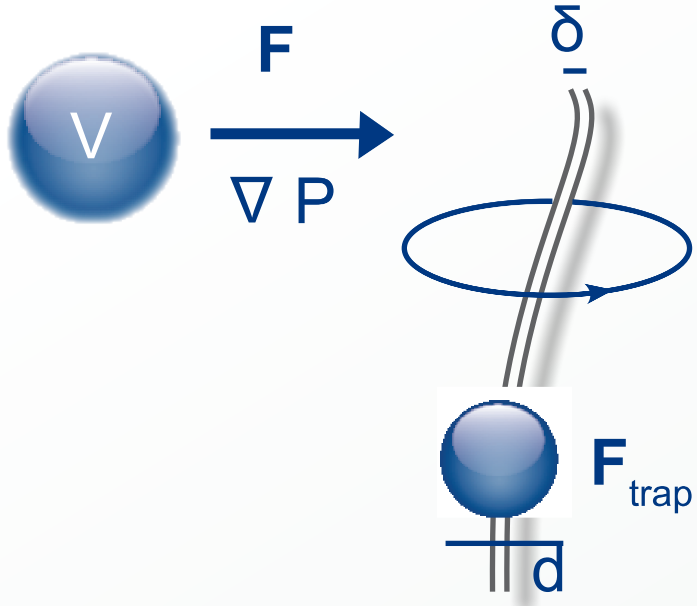
- airplane wakes
- tornadoes
- neutron stars dynamics
- related to whistler waves in plasmas

Detected only indirectly in quantum vortices:

- torsional oscillators — Hall, Proc. R. Soc. A 1958
- ions anomaly — Ashton, Glaberson, PRL 1979
- damping in BEC — Bretin et al. PRL 2003

Maxworthy, Hopfinger, Redekopp, JFM 1985

Visualization of quantized vortices in ^{4}He



$$P = -\frac{\rho_s \kappa^2}{8\pi^2 r^2} \qquad \mathbf{F} = \int \nabla P \, dV$$

Particles get trapped on vortex cores →

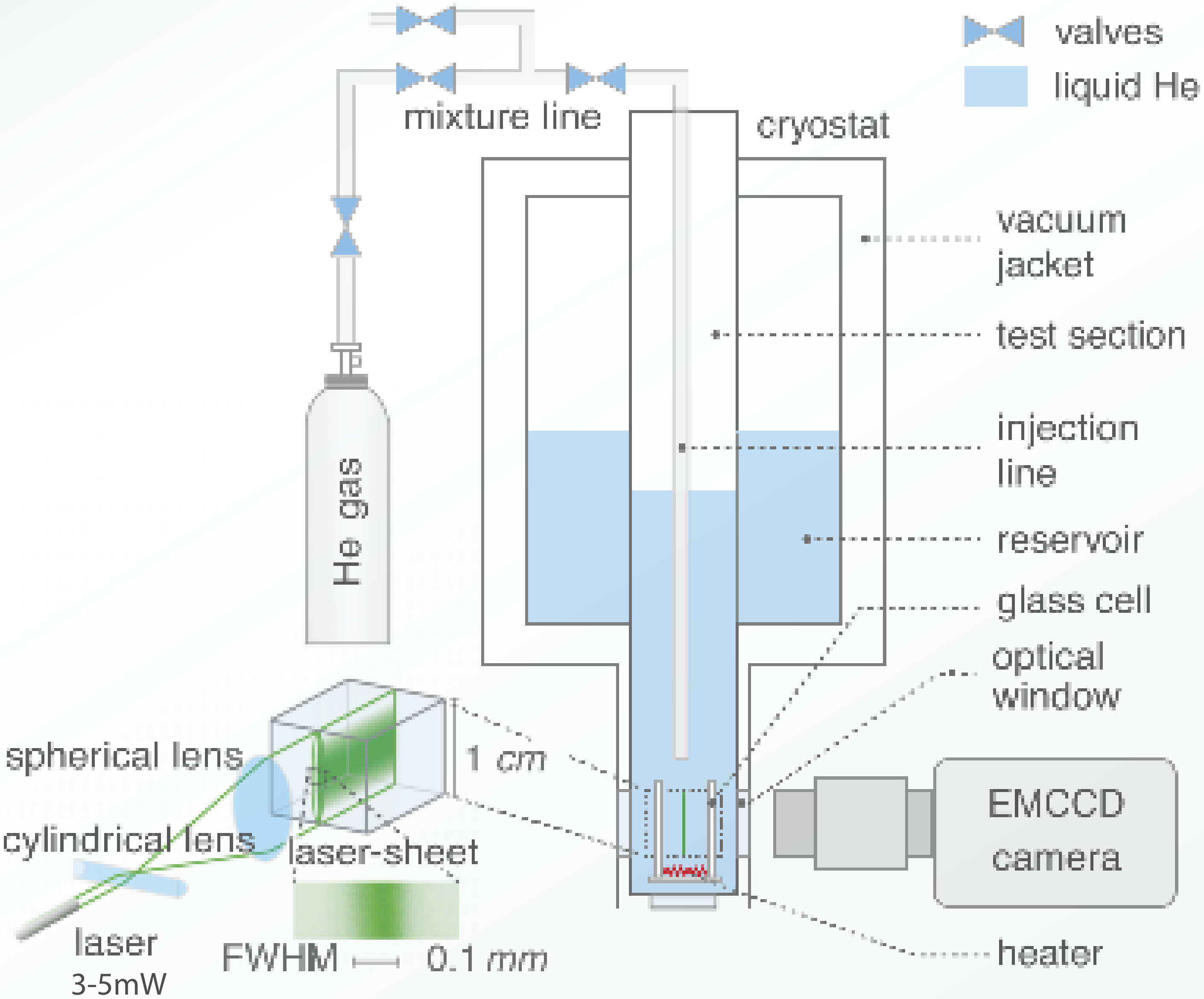
Ions in liquid He ⊖ 16 Å ⊕ 6 Å

Parks and Donnelly, PRL1966, Williams and Packard, PRL 1974

Solid hydrogen particles >1μm

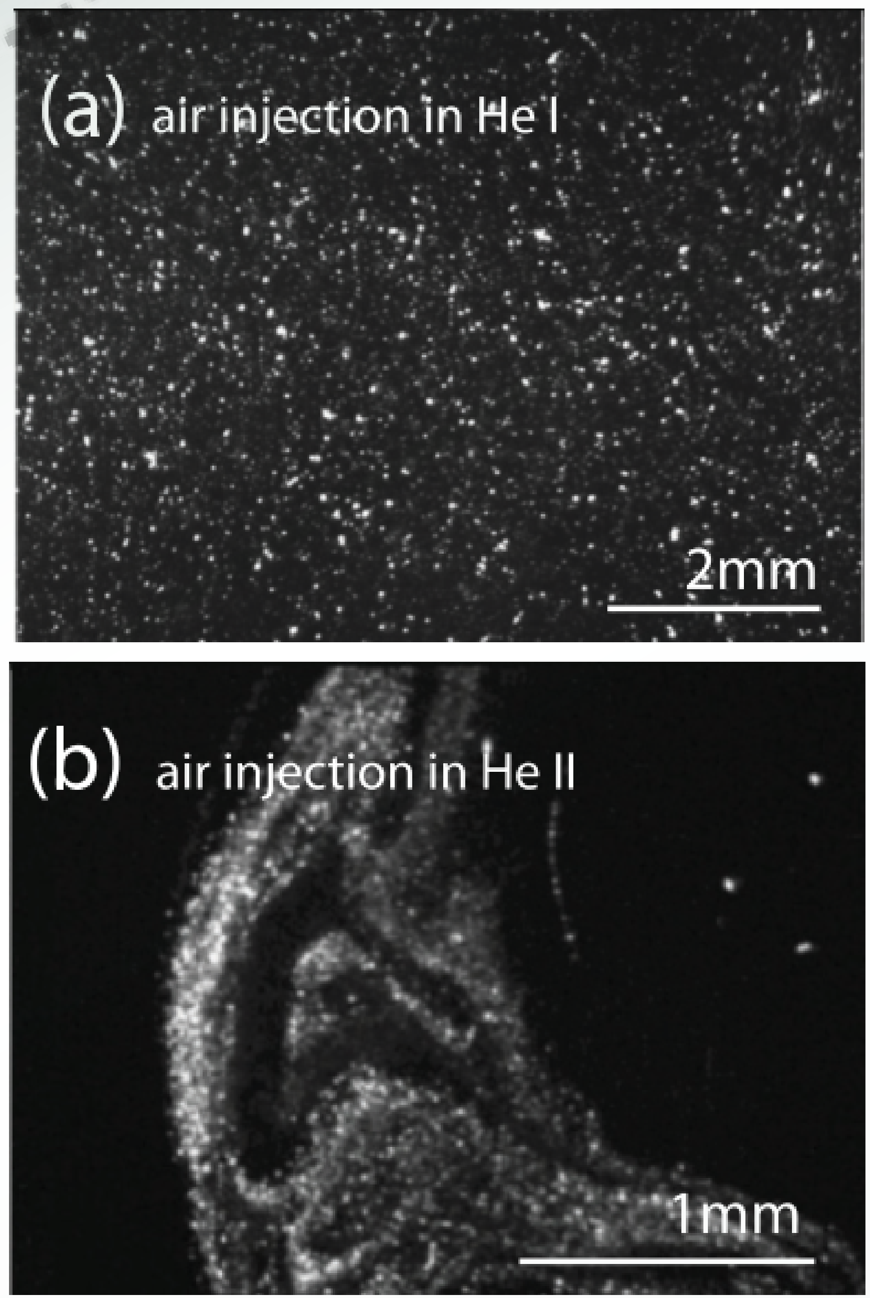
Bewley, Lathrop, Sreenivasan, Nature 2006

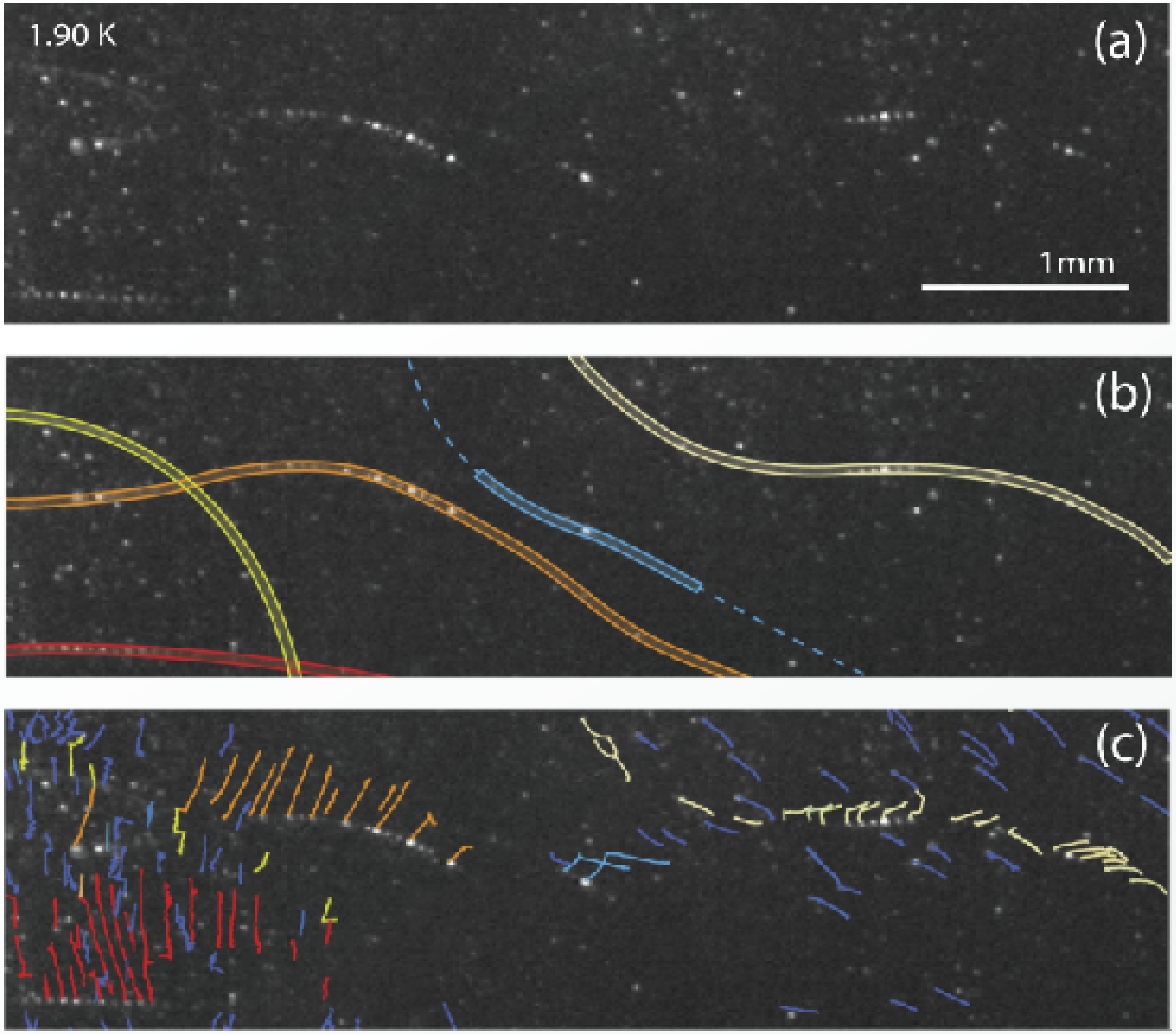
Experimental setup



Fonda, Sreenivasan, Lathrop, RSI 2016

Sub-micron air tracers injections in He II

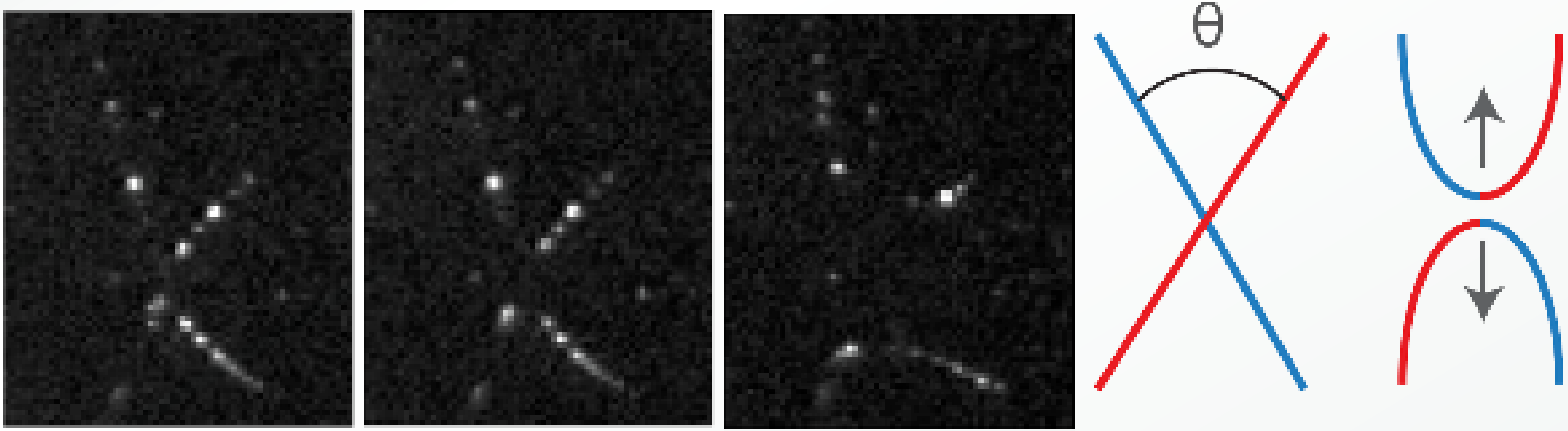




particles ~ 0.5 μm

Fonda, Sreenivasan, Lathrop, RSI 2016

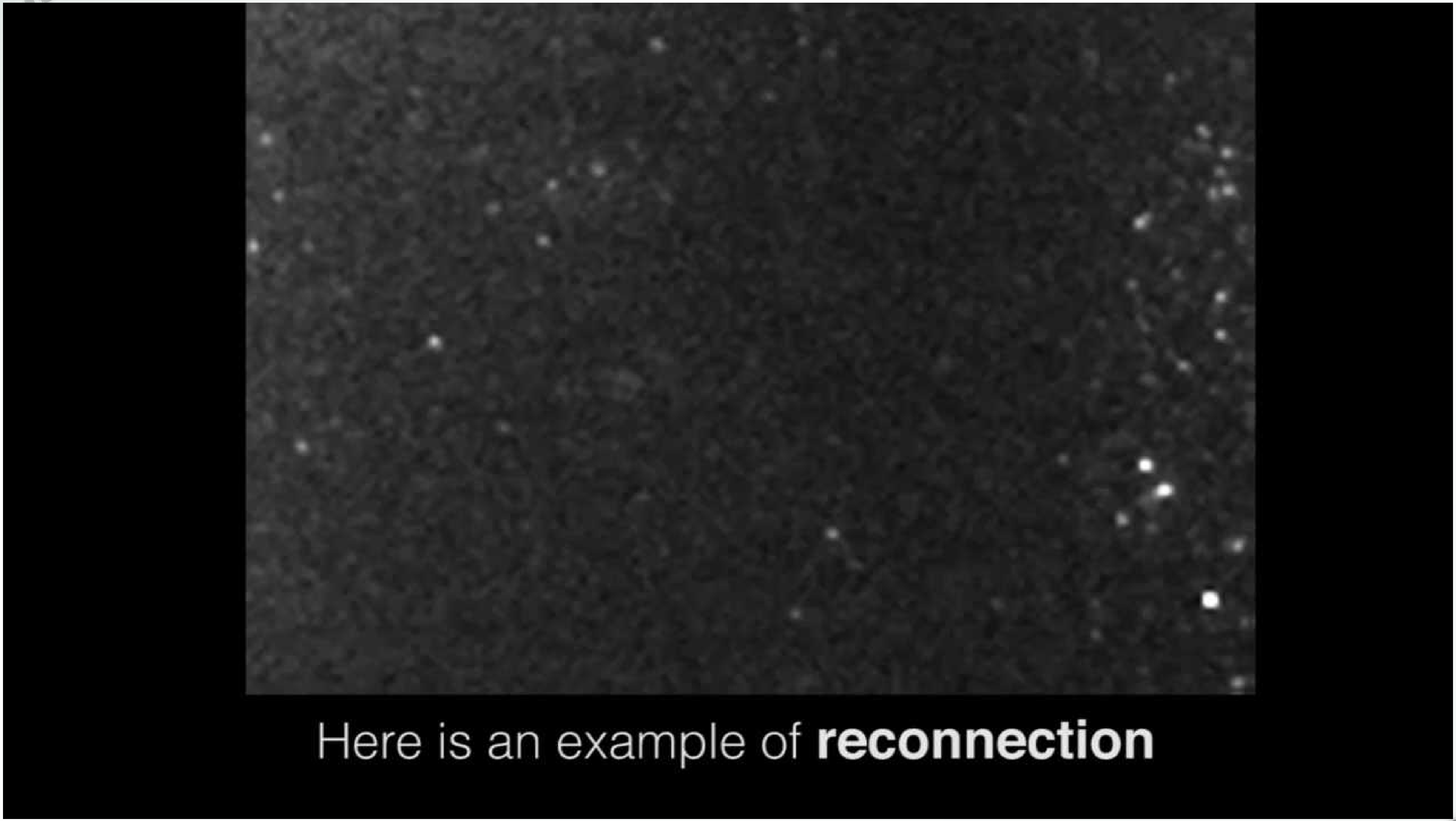
Vortex Reconnection



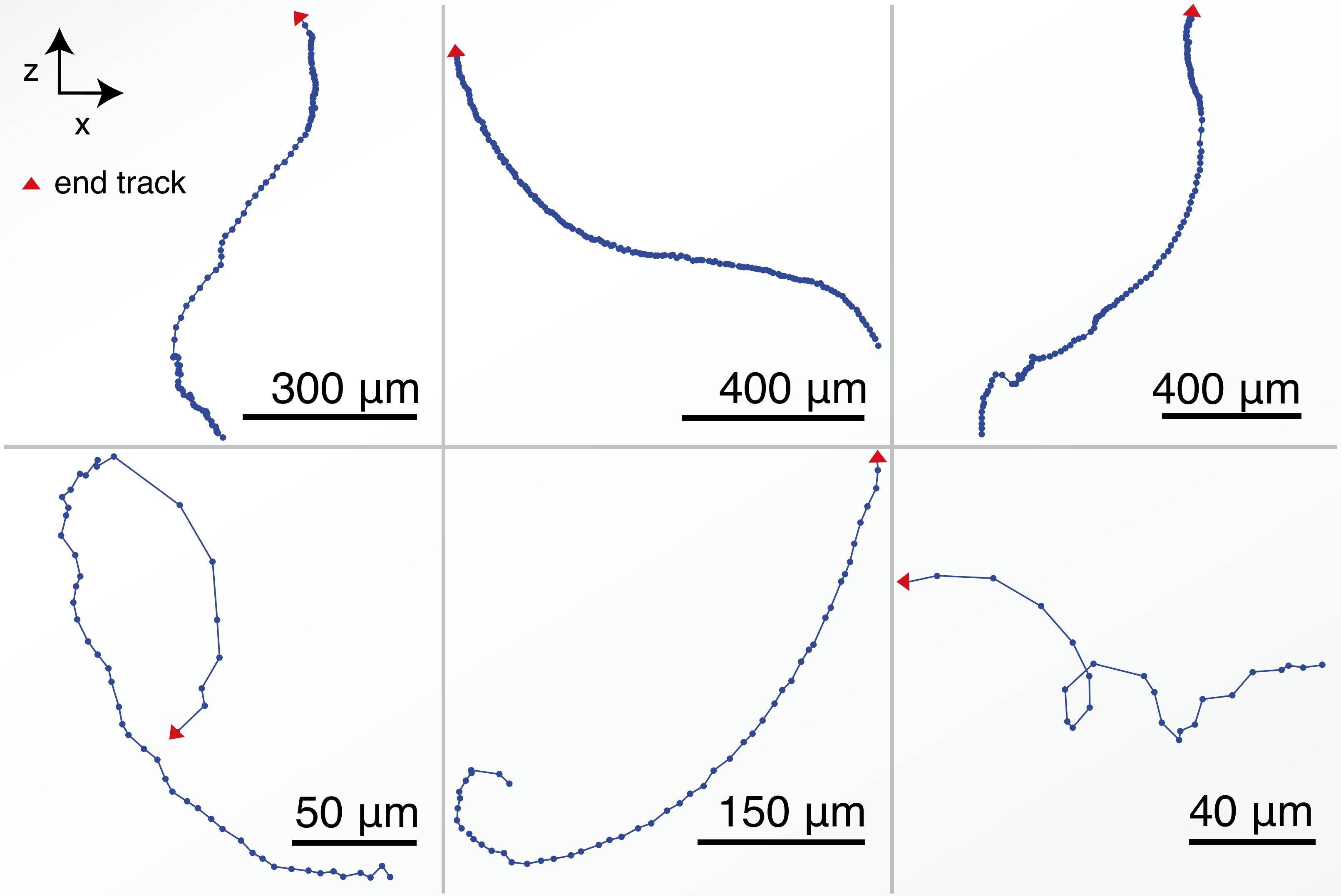
Bewley, Paoletti, Sreenivasan, Lathrop, PNAS 2008

Paoletti, Fisher, Lathrop, Physica D 2010

Reconnections and Kelvin waves

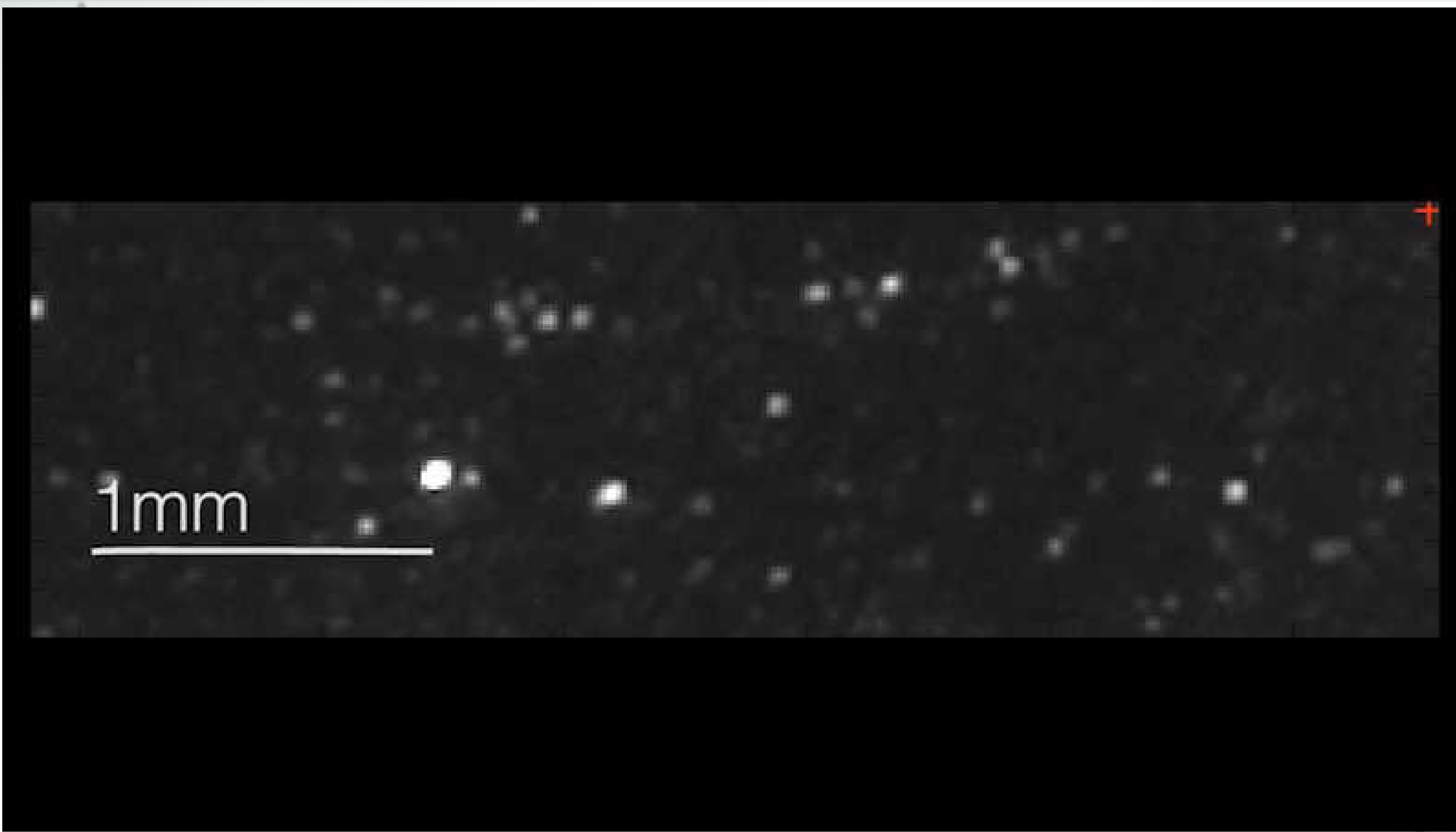


Wavy motion



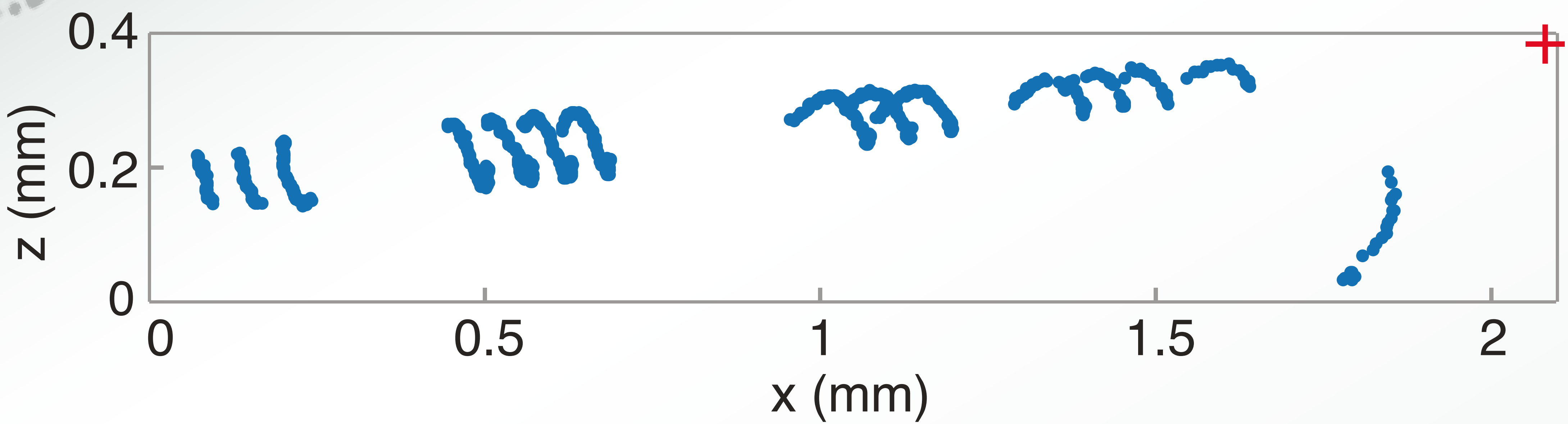
Fonda, Meichle, Ouellette, Hormoz, Lathrop, PNAS 2014

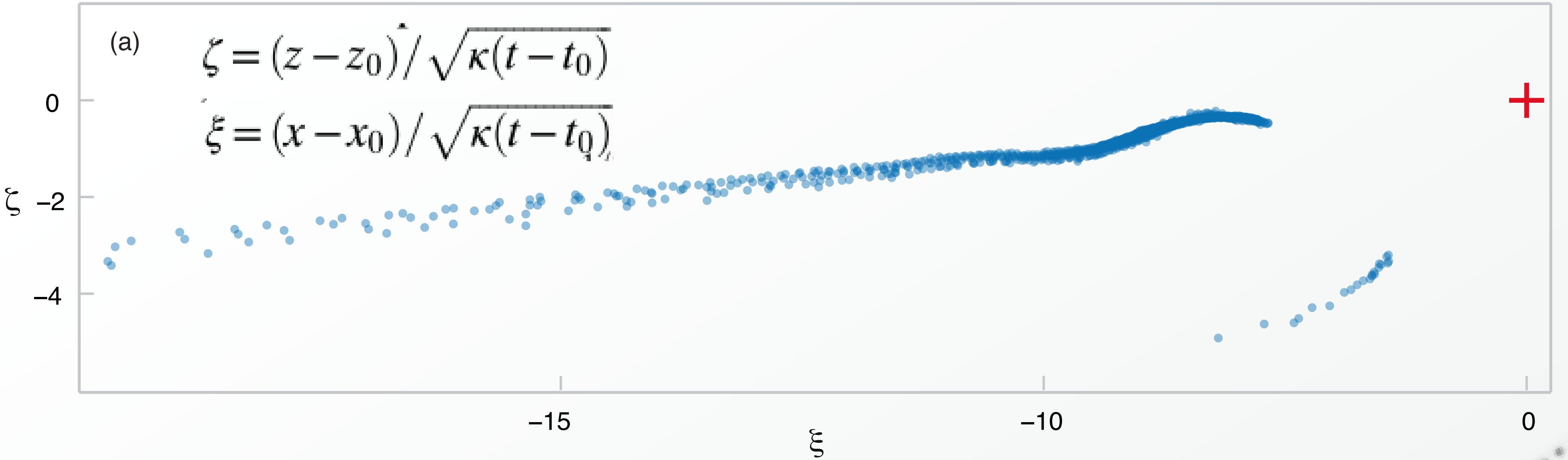
Direct visualization Kelvin Waves



Fonda, Meichle, Ouellette, Hormoz, Lathrop, PNAS 2014

Similarity Collapse



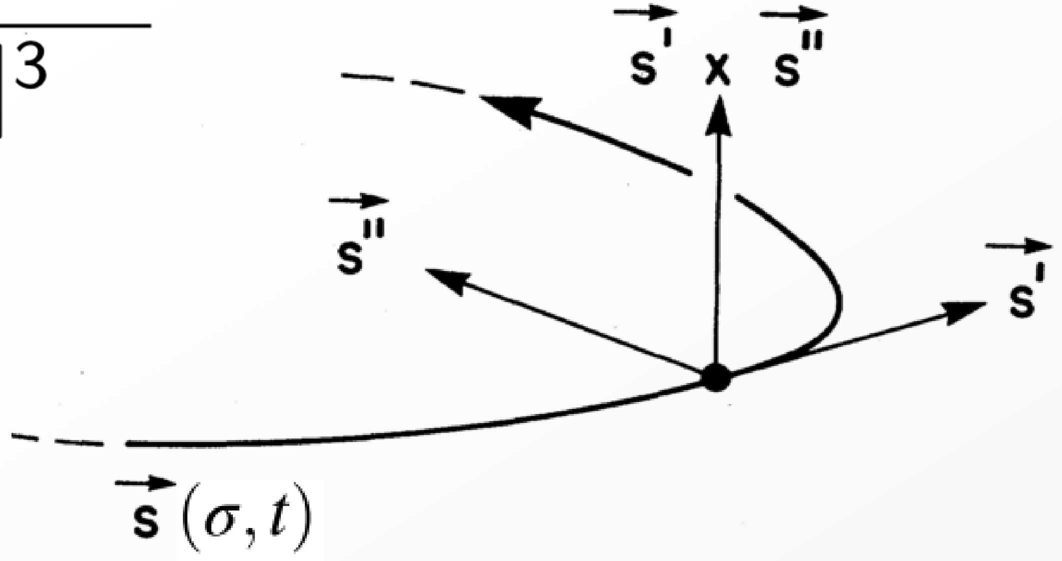


Fonda, Meichle, Ouellette, Hormoz, Lathrop, PNAS 2014

Local Induction Approximation (LIA)

Biot Savart neglecting non-local term = LIA

$$\frac{d\mathbf{s}}{dt} = \frac{\kappa}{4\pi}\oint \frac{(\mathbf{z}-\mathbf{s})\times d\mathbf{z}}{|\mathbf{z}-\mathbf{s}|^3}$$



Da Rios equations:

$$\frac{d\mathbf{s}}{dt} \approx \beta\mathbf{s}' \times \mathbf{s}''$$

Rediscovered multiple times in past century

Schwarz, K., PRB 31(9), (1985)

Rios, L.S., Rendiconti del Circolo Matematico di Palermo, 22(1), (1906)

LIA - BS similarity solutions

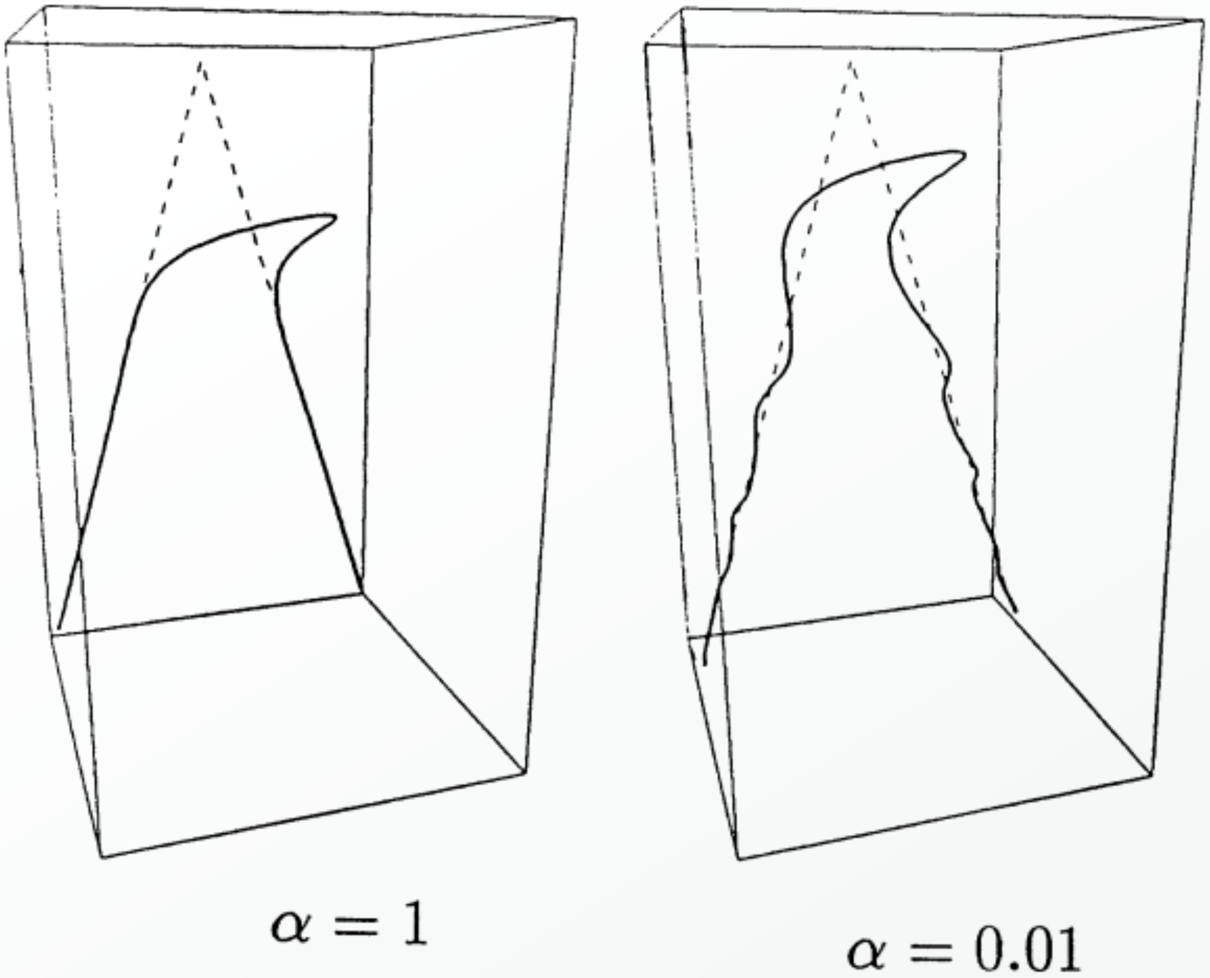
$$\frac{ds}{dt} \approx \beta s' \times s'' + \alpha s''$$

α = mutual friction

Similarity assumption

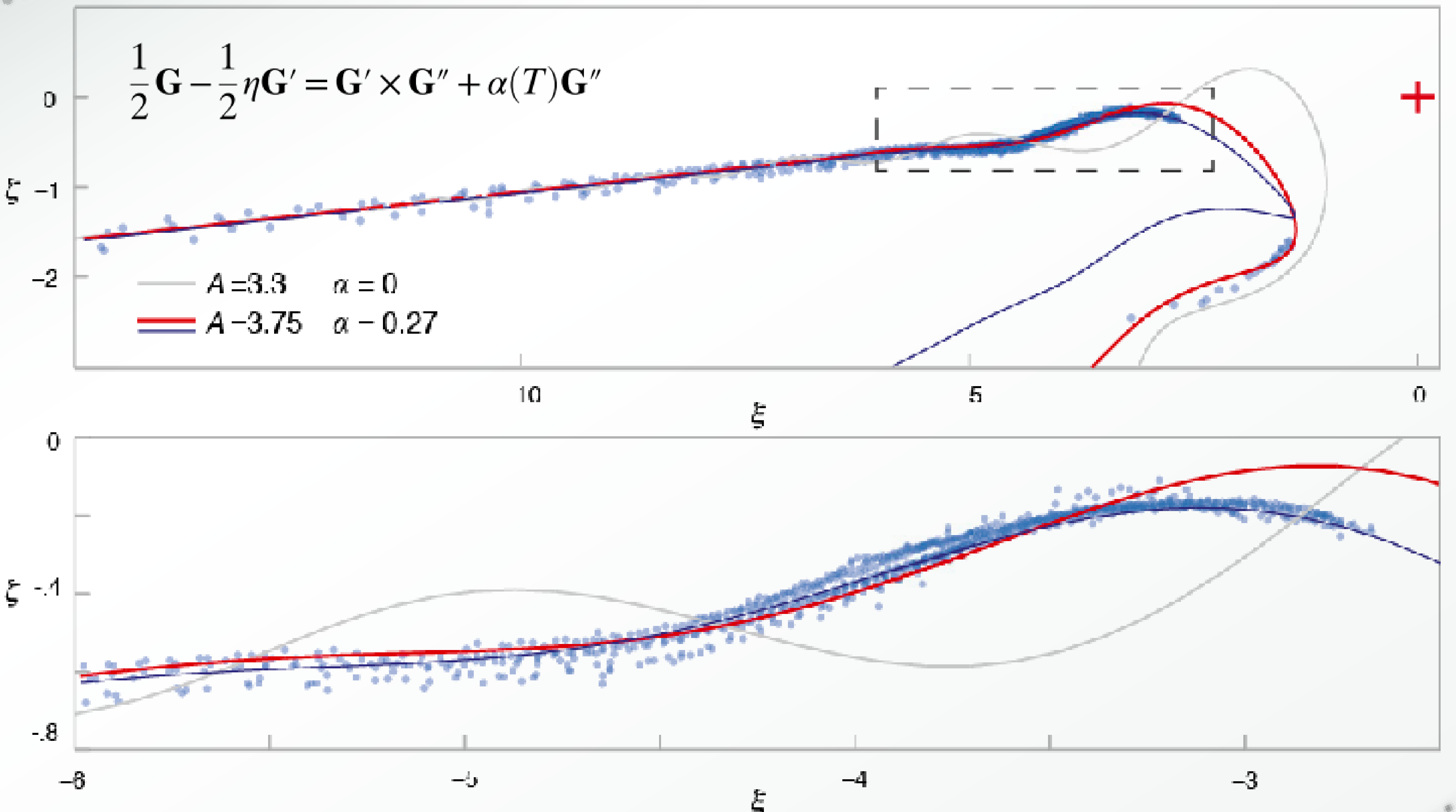
$$\eta = \sigma / \sqrt{\beta t}$$

$$s(\sigma, t) = \sqrt{\beta t}\, G(\eta)$$



Gutierrez, S., Rivas, J. & Vega, L., Comm. Part. Diff. Eq. , 2003
Lipniacki, T., Eur.J.Mech.B-Fluids, 2000

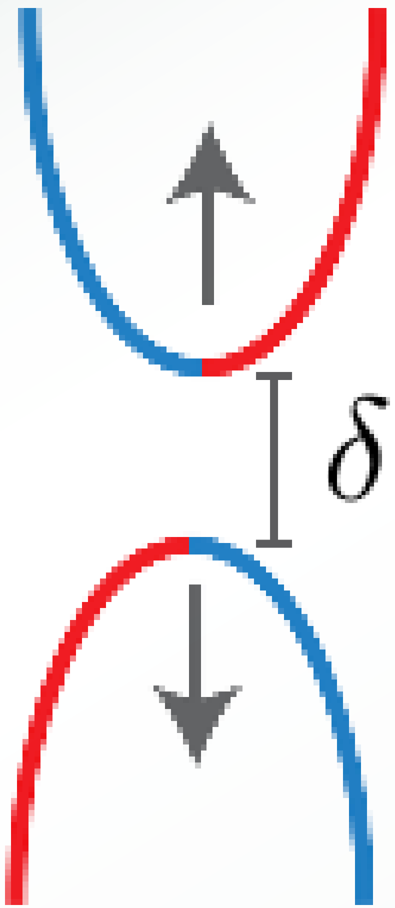
Curve fits: LIA and Biot-Savart

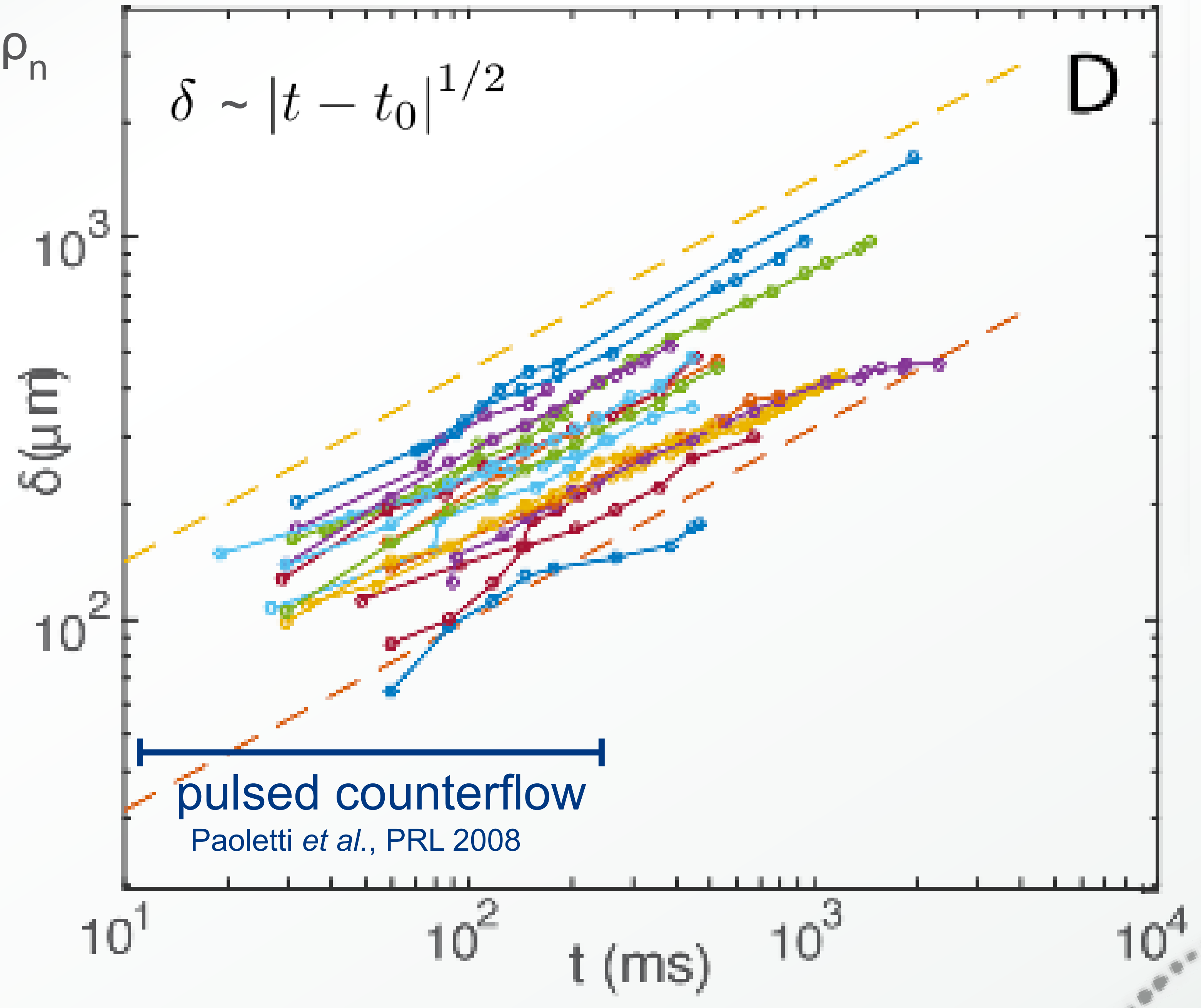


Fonda, Meichle, Ouellette, Hormoz, Lathrop, PNAS 2014

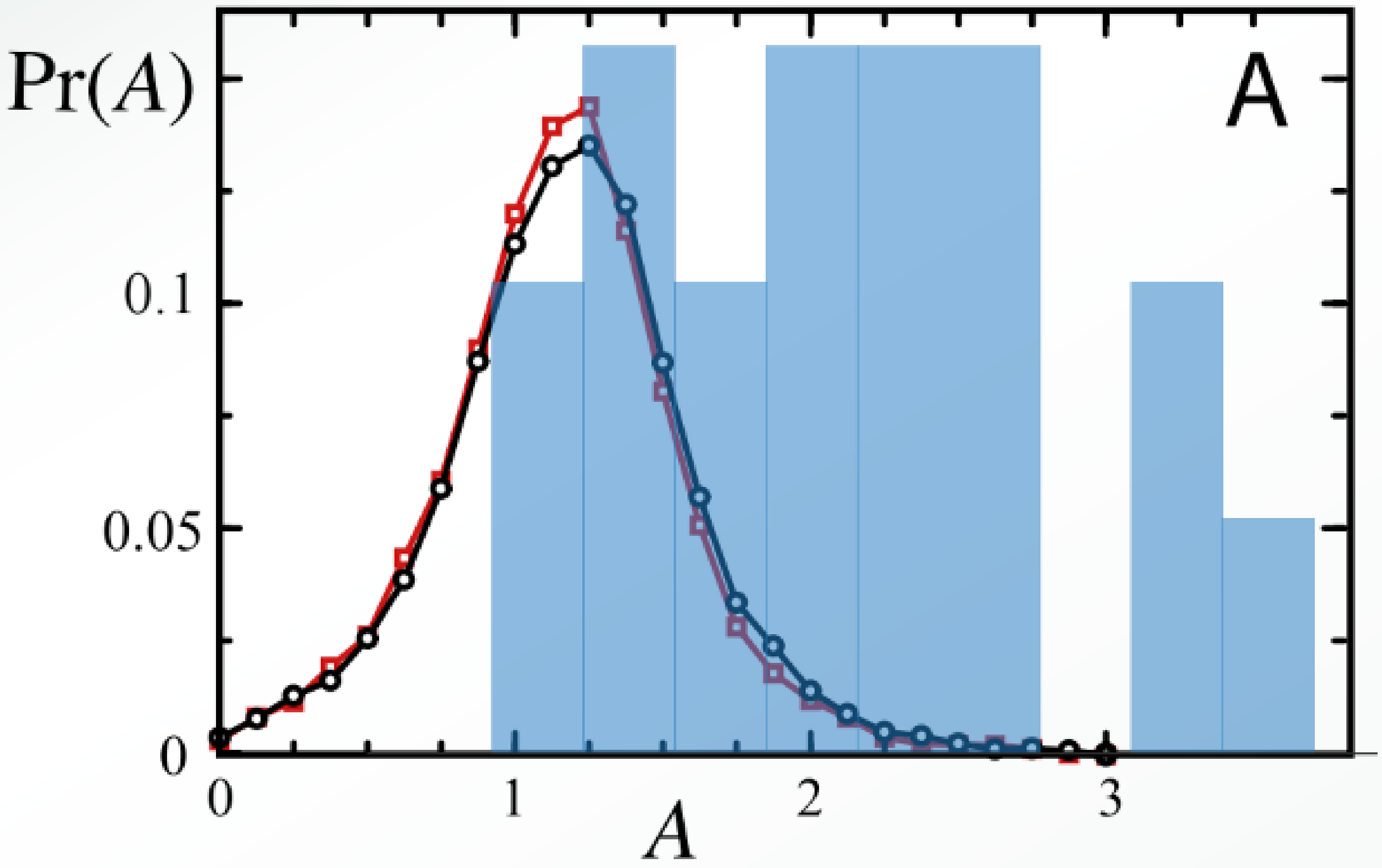
Inter-vortex distance

T = 1.9 - 2.0 K, $\rho_s \sim \rho_n$





Pre-factor A



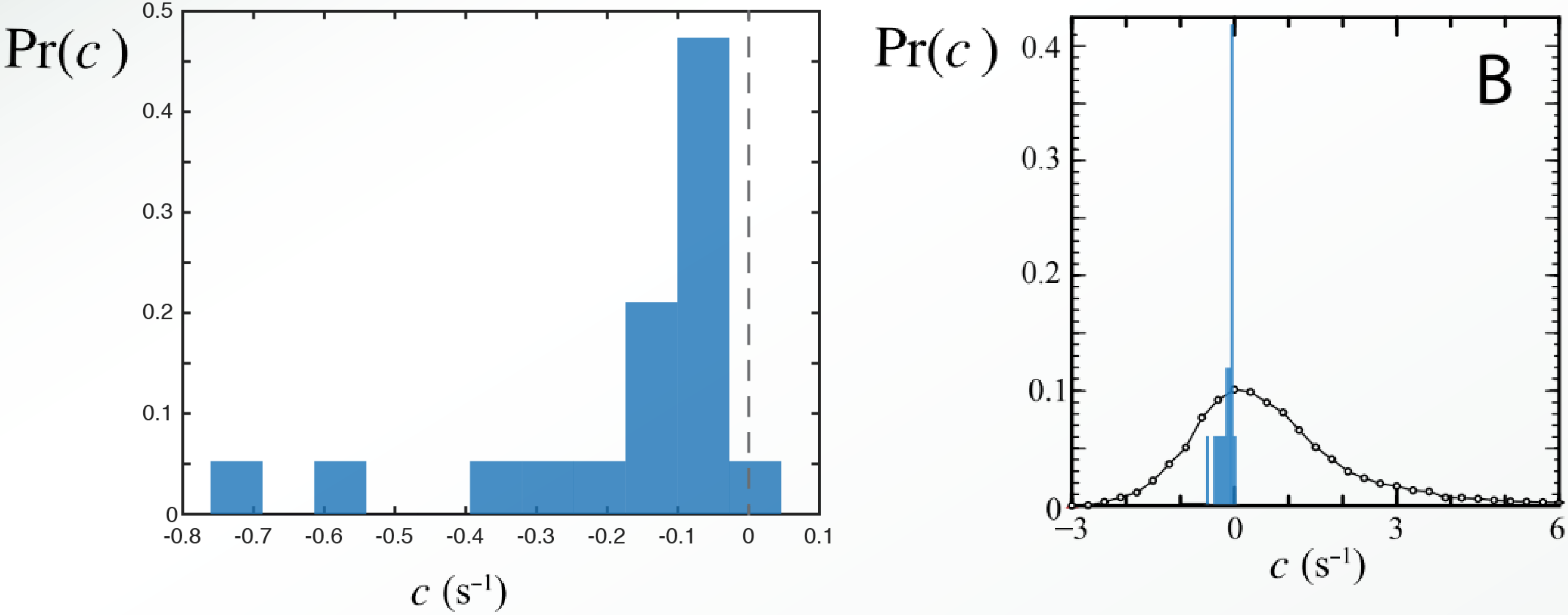
pulsed counterflow

Paoletti *et al.*, Physica D 2010

quiet system

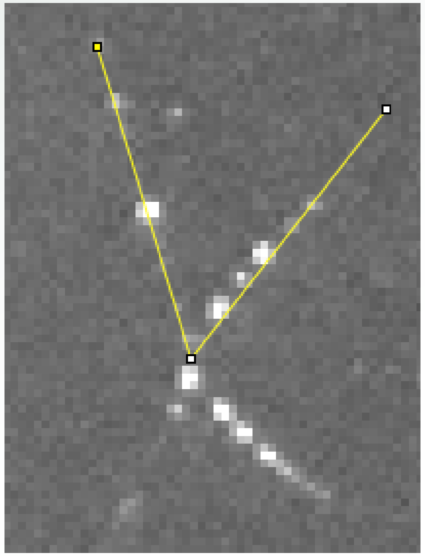
$$\delta(t) = A\left(\kappa|t - t_0|\right)^{1/2}\left(1 + c|t - t_0|\right)$$

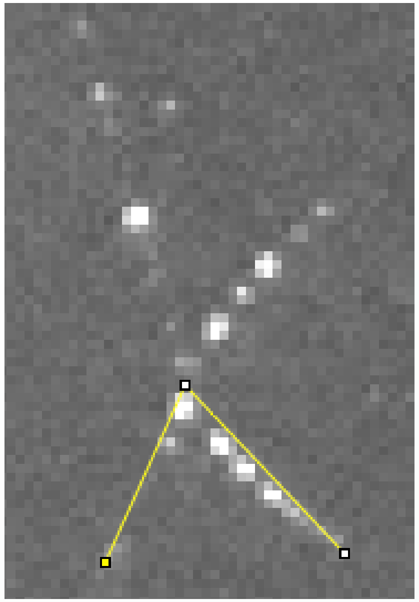
Correction factor c

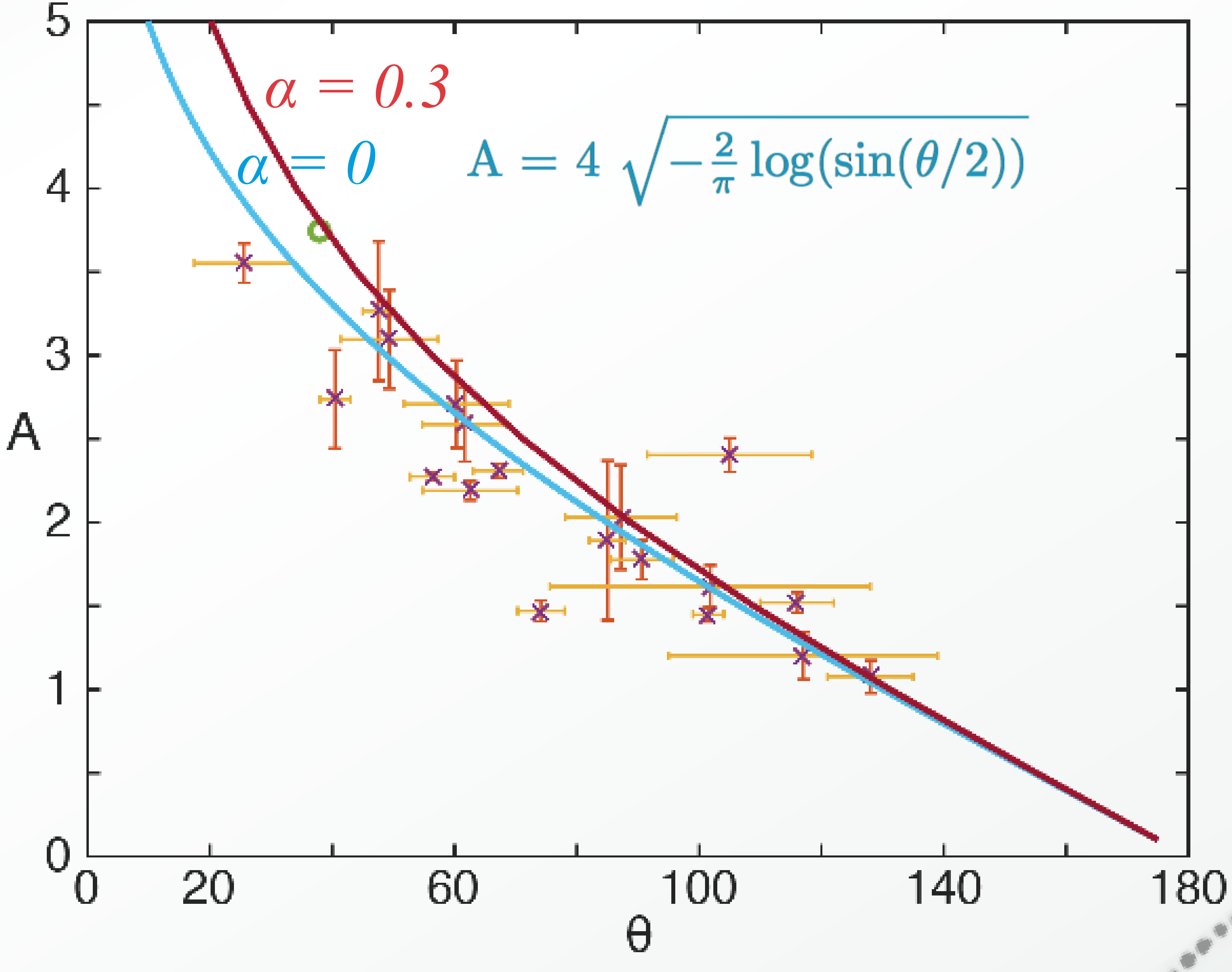


$$\delta(t) = A\left(\kappa|t-t_0|\right)^{1/2}\left(1+c|t-t_0|\right)$$

Angle dependance





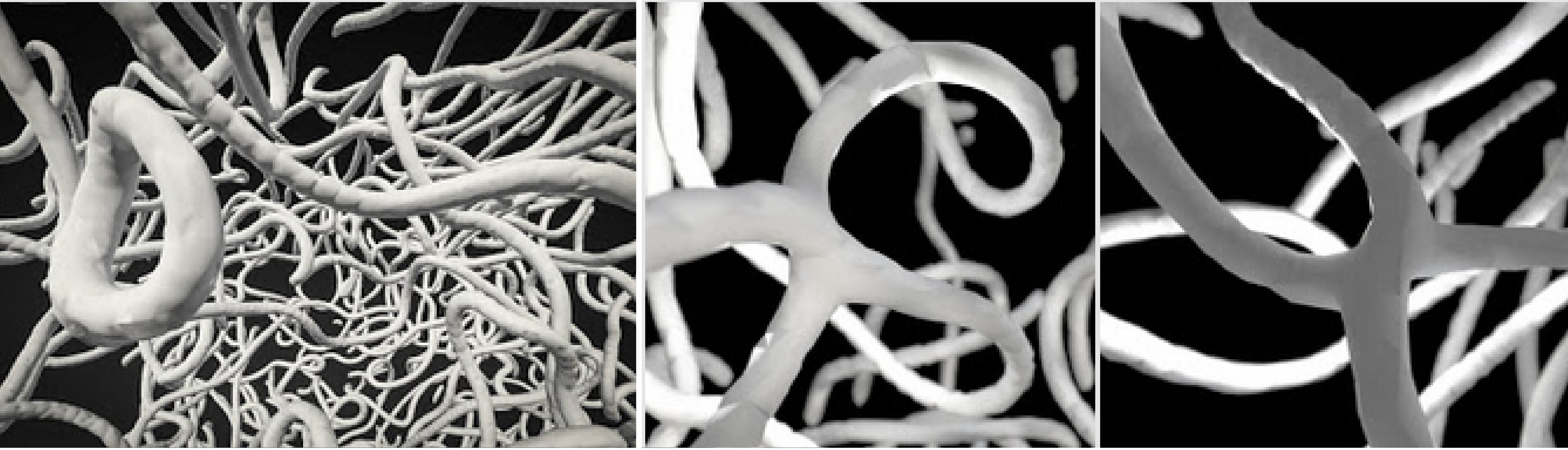


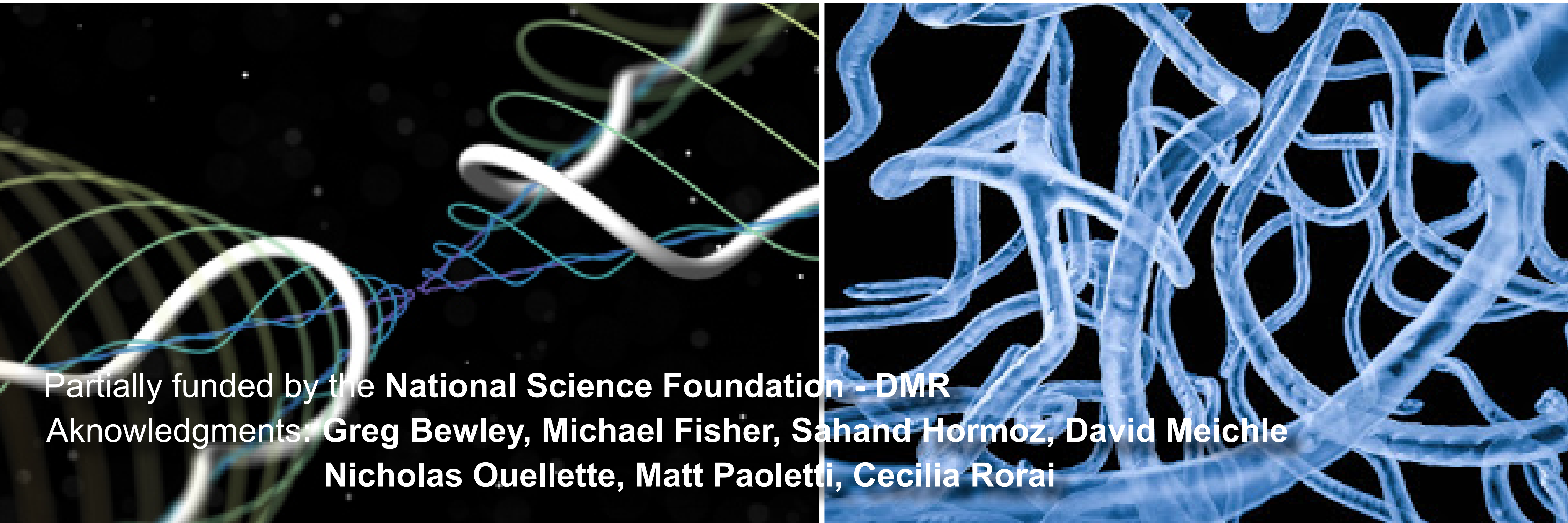
Conclusions

For long straight vortices on the scale of these experiments:

- the dynamics of the reconnection can be described by the self-similar solutions of the LIA or Biot-Savart equations.
- two approaching vortices locally arrange themselves in an antiparallel configuration. this distortion in our case can be neglected.
- the distribution of the correction factor *c* compared to that for the pulsed counterflow experiments suggests that *c* depends on the influence of neighboring vortices.
- reconnection behavior for long straight vortices is very different than that of the reconnections resulting from the decay of a vortex tangle.

Thank you!





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