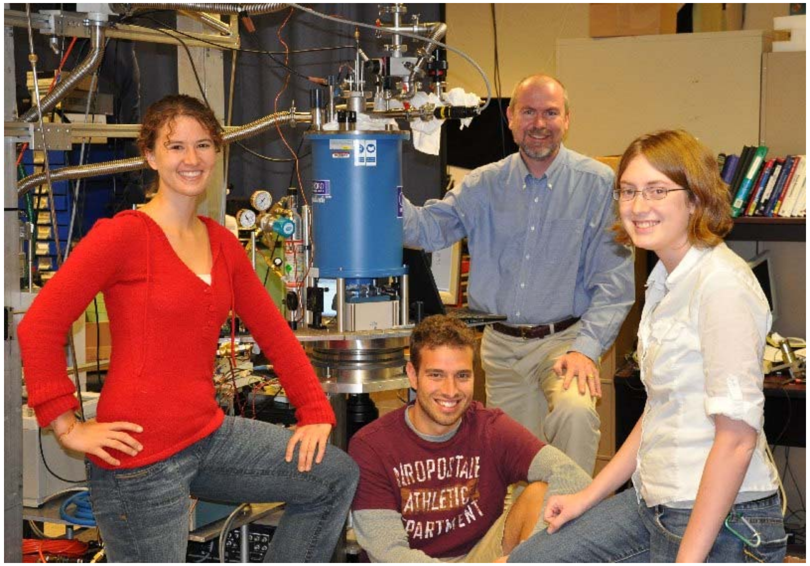
Helicity Dynamics

$$\int_v \vec{u} \cdot \vec{\omega} d^3 r$$



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K.R. Sreenivasan

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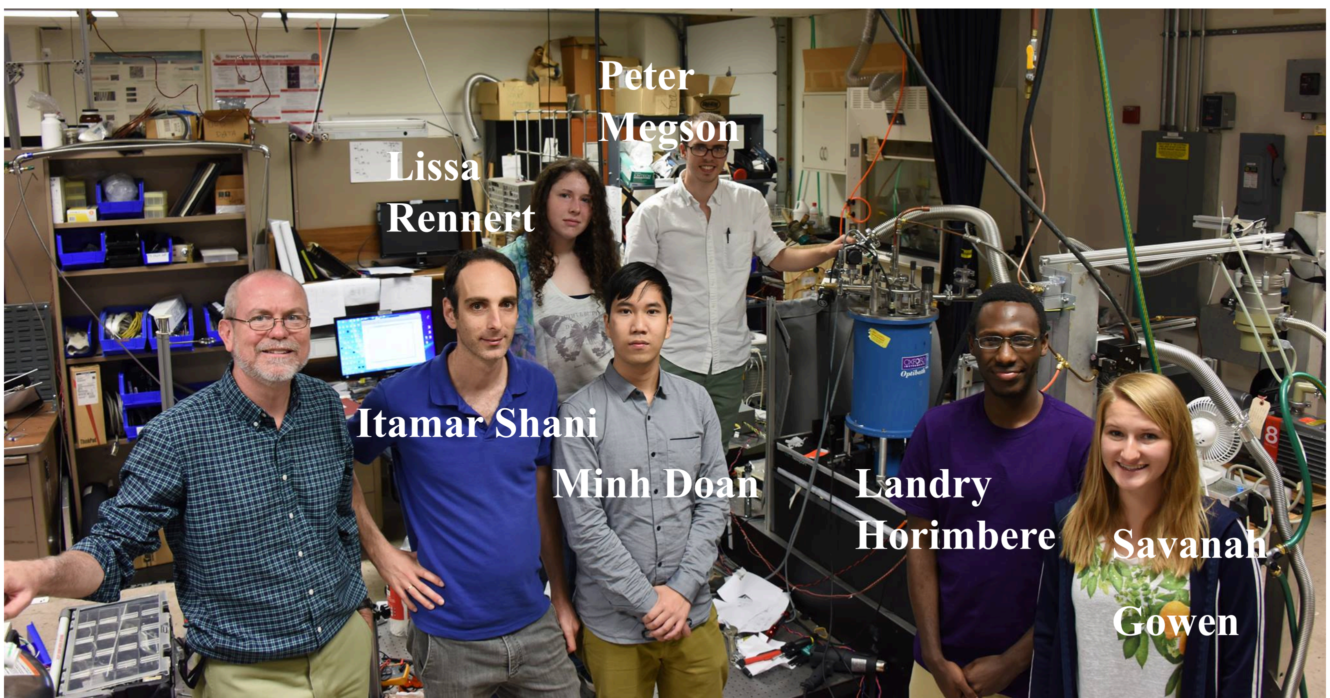

Michael Fisher

David Meichle

University of Maryland
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Helicity statics – background and definitions

- Ideal MHD
- Ideal fluids
- Quantum fluids

Helicity dynamics

- Classical fluids
- MHD reconnection

Helicity dynamics for quantum fluids

- Theoretical/computational approaches
- Novel concepts and constraints
- $Pr(h) \sim h^{-5/3}$ distribution

Helicity statics – background and definitions
Ideal MHD:
Woltjer PNAS 1958 "A THEOREM ON FORCE-FREE MAGNETIC FIELDS"

$$\int_v \vec{A}\cdot\vec{B}d^3r = \text{constant} \qquad (\vec{B}\cdot\hat{n})_{\partial v} = 0$$

Minimum energy solutions:

$$\nabla\times\vec{B} = \alpha\vec{B}$$

Berger and Field JFM 1984 "The topological properties of magnetic helicity"

$$\int_v \vec{A}\cdot\vec{B}d^3r = \text{constant} = \pm 2\Phi_1\Phi_2$$

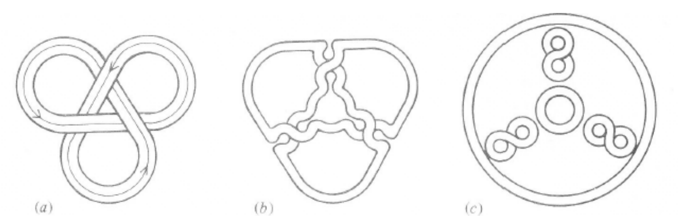
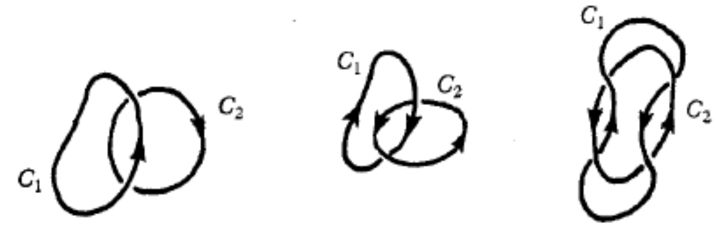


FIGURE 2. A trefoil knot with $H_K = -3\Phi^2$.

Vachaspati 2008
10.1098/rsta.2008.0074

Finn and Antonsen CPPCF 1985 "Magnetic Helicity: What Is It and What Is It Good For"
Helicity injection into various plasma configurations

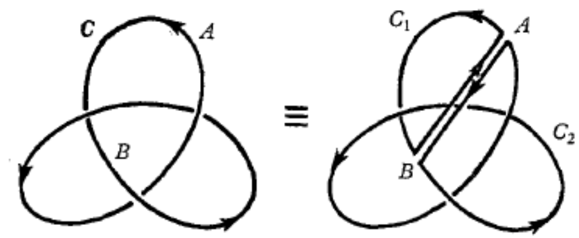
Helicity statics – background and definitions
Ideal fluid flow:



Moffatt JFM 1969 "The degree of knottedness of tangled vortex lines" (also Moreau 1961)

$$\int_v \vec{u}\cdot\vec{\omega}d^3r = \text{constant} \qquad (\vec{\omega}\cdot\hat{n})_{\partial v} = 0$$

$$= 2n\kappa_1\kappa_2 \quad n \text{ is the Gauss linking number}$$



$$\int_v \vec{A}\cdot\vec{B}d^3r = constant$$ ideal MHD

$$\int_v \vec{u}\cdot\vec{B}d^3r = constant$$ ideal MHD

Helicity statics – background and definitions
Quantum fluid flow:

$$\vec{j} = \frac{\hbar}{2mi}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*)$$

$$\vec{\Omega} = \nabla \times \vec{j}$$

$$h = \vec{j} \cdot \vec{\Omega}$$ Helicity density candidate

One long straight vortex along z-axis: $$\vec{\Omega}_1 = j_z \frac{\hbar}{m} \delta(x)\delta(y)\hat{z}$$

$$h = 0$$

$$\int_v \vec{j} \cdot \vec{\Omega} d^3 r = 0$$

Invariants of the nonlinear Schrodinger (GP) equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + g|\Psi|^2\Psi - \mu\Psi$$

1-D a countable infinite family of invariant densities from inverse scattering theory, mass, momentum, energy, …

2-D Invariant densities: mass, momentum, energy, and perhaps various integrals over the vorticity

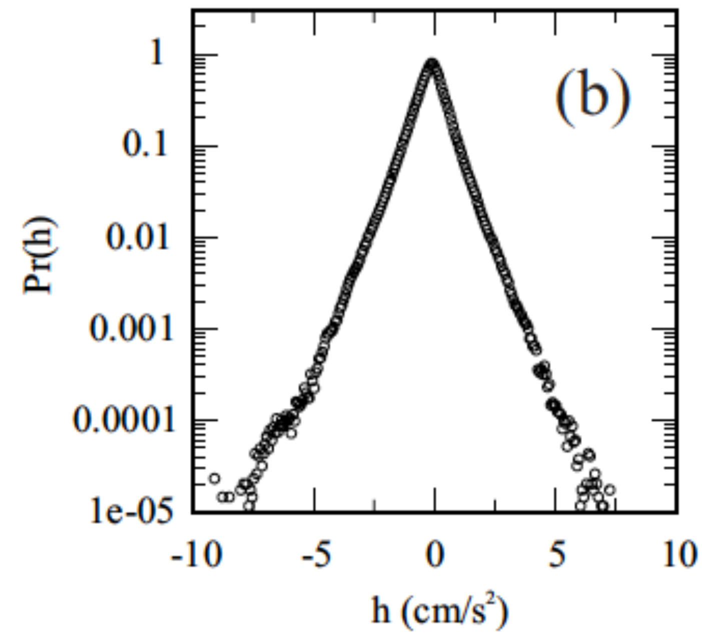
3-D Invariant densities: mass, momentum, energy, ?

Conjecture [dpl]: there is no *non-trivia*l conserved density corresponding to the Helicity

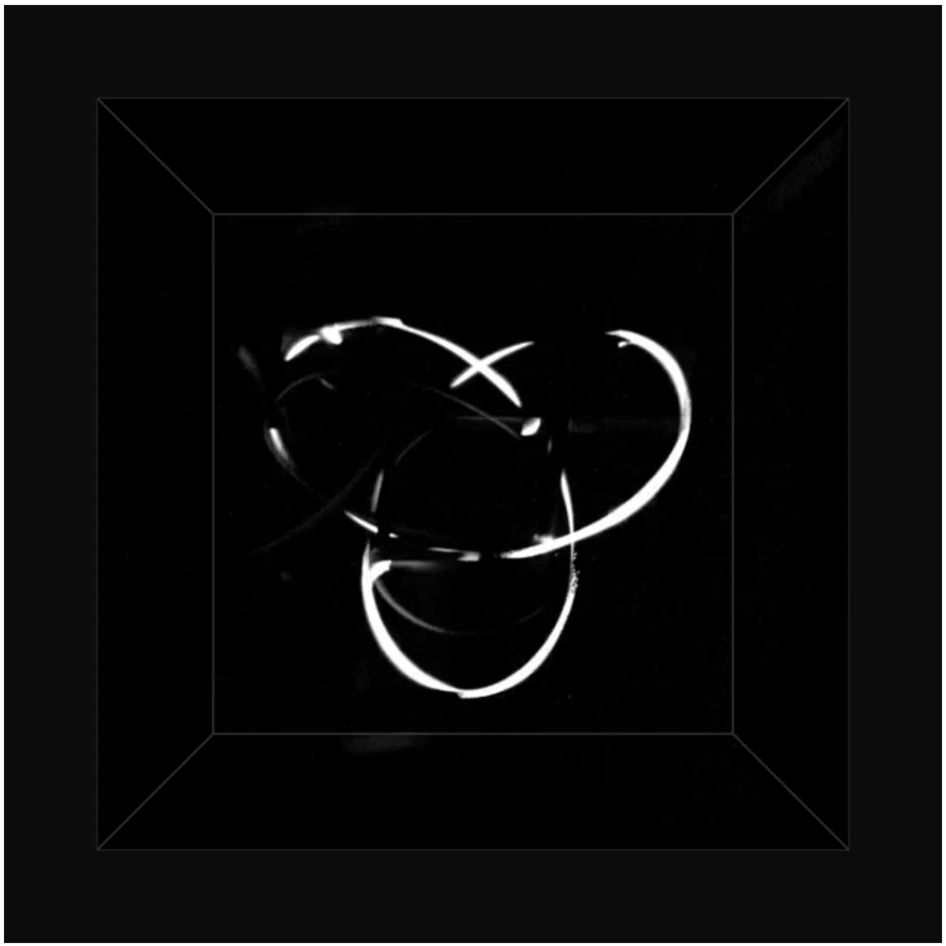
Helicity dynamics

Classical fluids

MHD reconnection



Benjamin Zeff Dissertation 2002



Creation and dynamics of knotted vortices
Dustin Kleckner and W.T.M. Irvine
Nature Physics 9, 253-258 (2013)

Trefoil knotted vortex: William Irvine lab University of Chicago

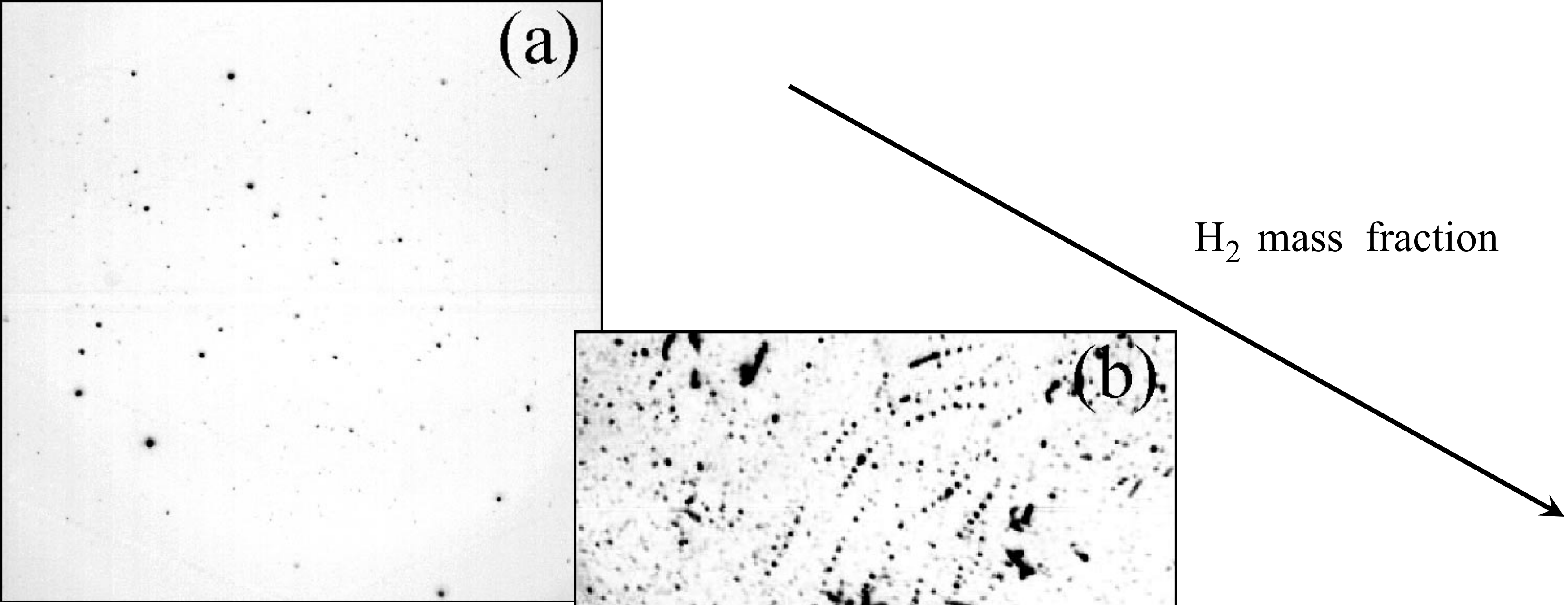
Helicity dynamics:

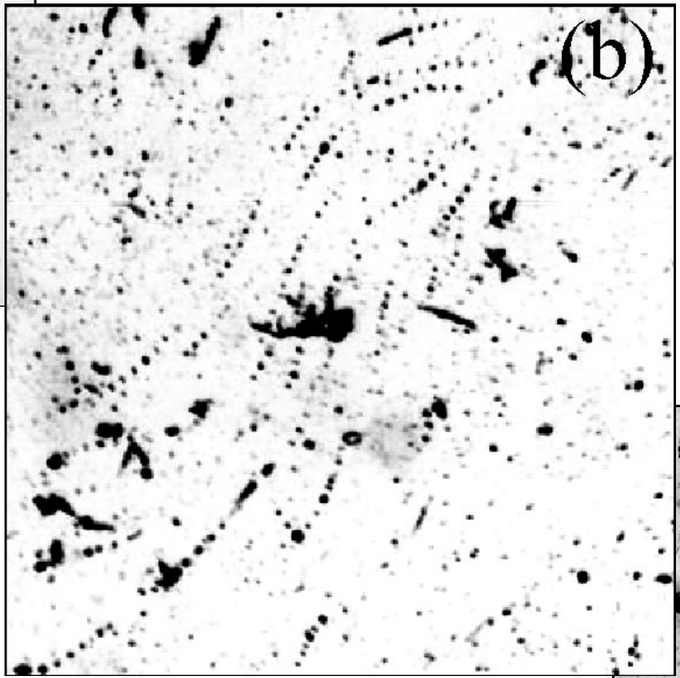
Classical fluids

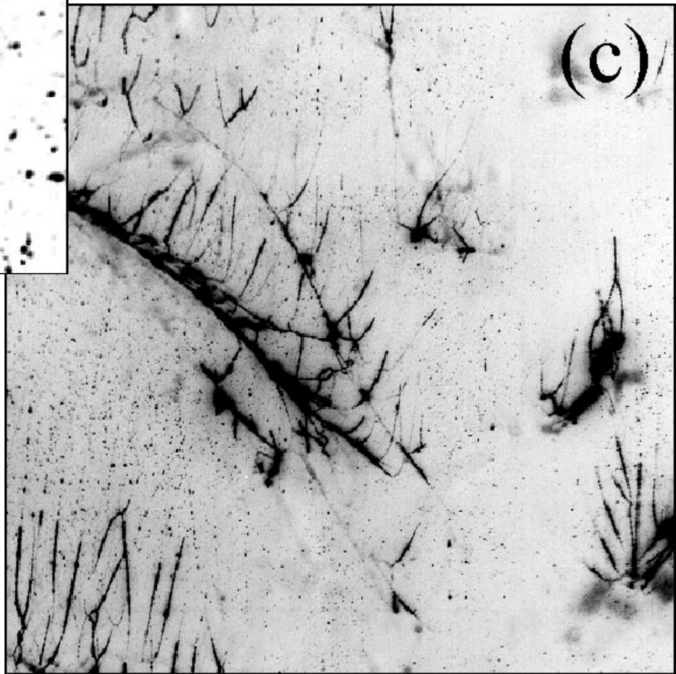
- Pipe flows
- Shear flows

Plasma dynamics

and Magnetohydrodynamics



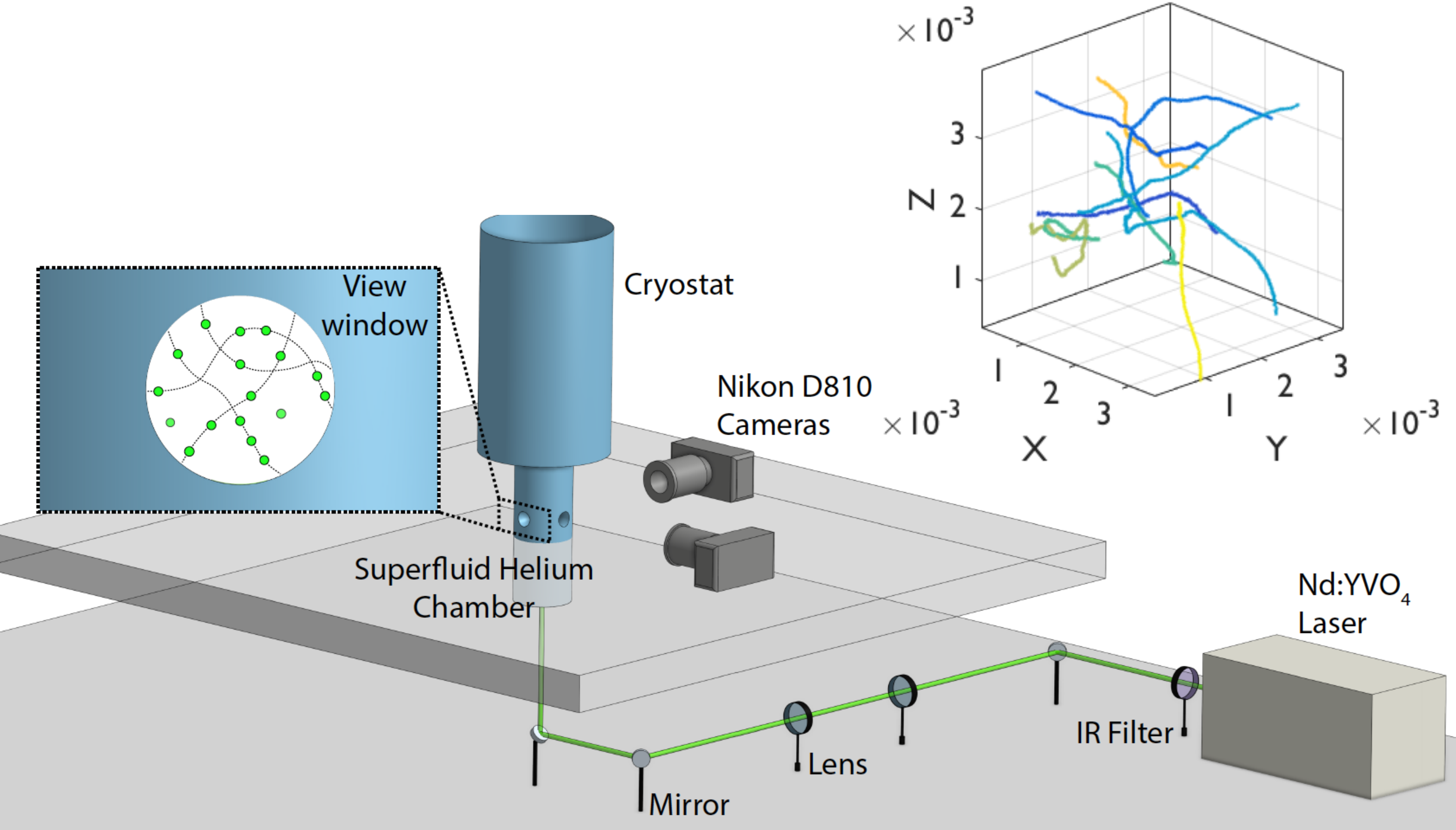




Smaller is better

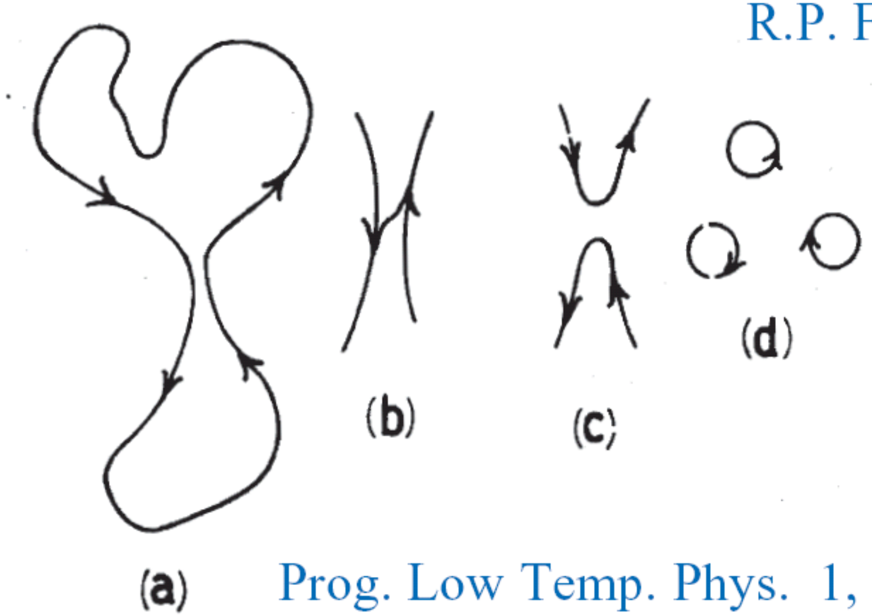
Fewer is better

Particles are not passive!



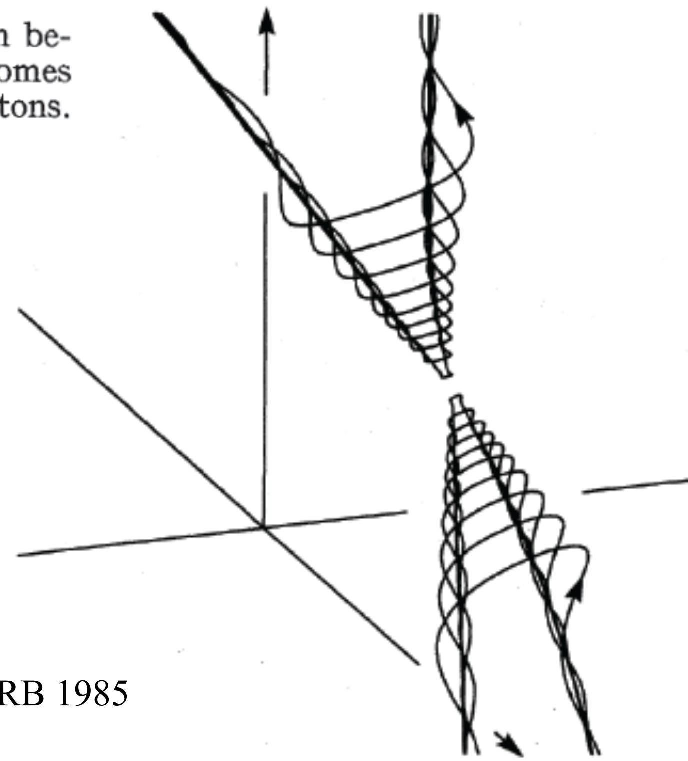
View
window
Cryostat
Nikon D810
Cameras
Superfluid Helium
Chamber
Nd:YVO4
Laser
IR Filter
Lens
Mirror
×10-3
Z
X
Y
1
2
3

R.P. Feynman



Prog. Low Temp. Phys. 1, 17 (1955)

Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.



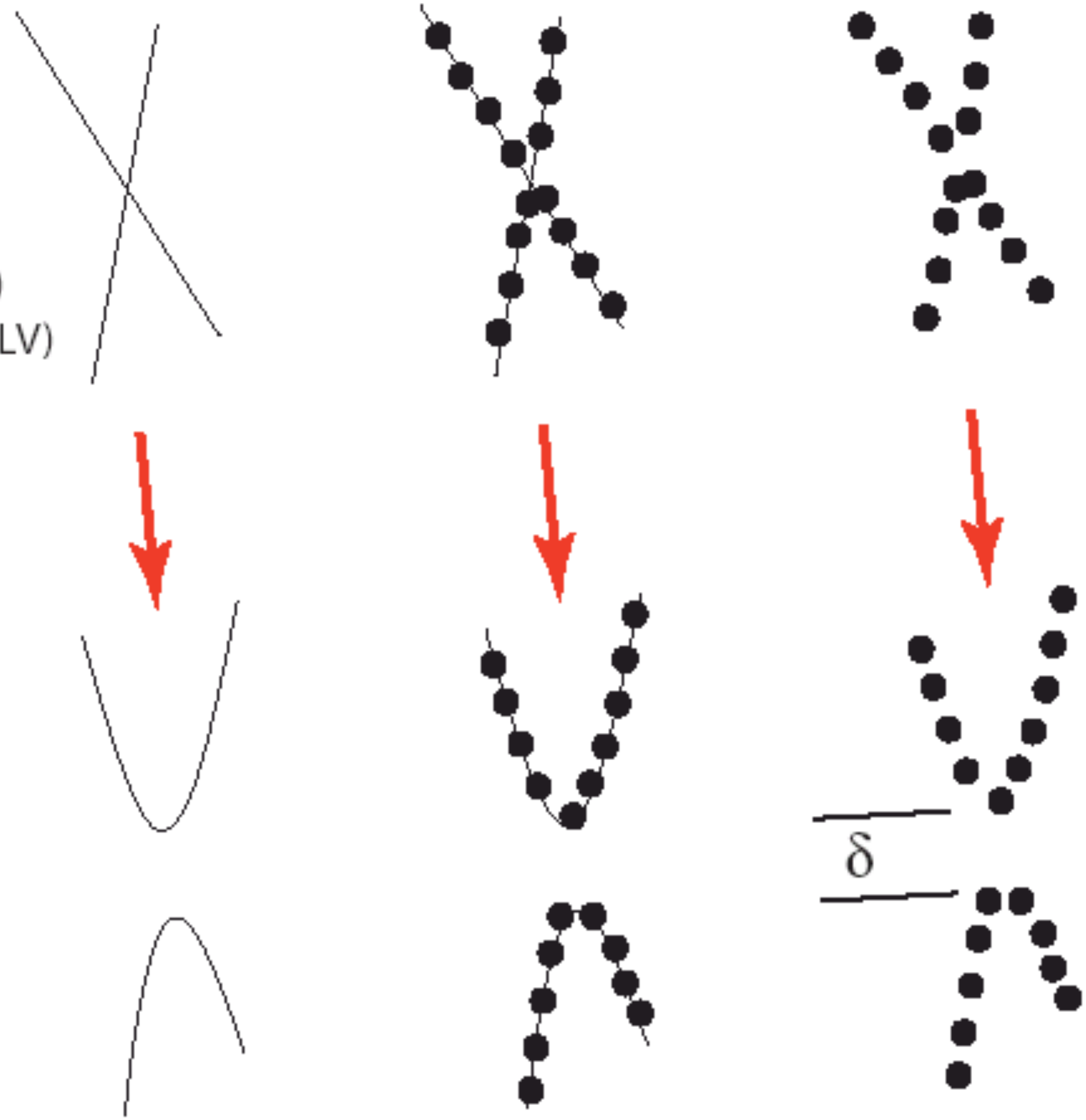
From Schwarz PRB 1985

Vortex reconnection

Theoretical work

Schwarz, PRB 1985 (LV)
de Waele and Aarts, PRL 1994 (LV)
Koplik and Levine, PRL 1993 (NLSE)
Tsubota and Maekawa, JPSJ 1992 (LV)
Nazarenko and West 2003 (NLSE)

Much more recent work!
Kerr PRL 2011 (NLSE)

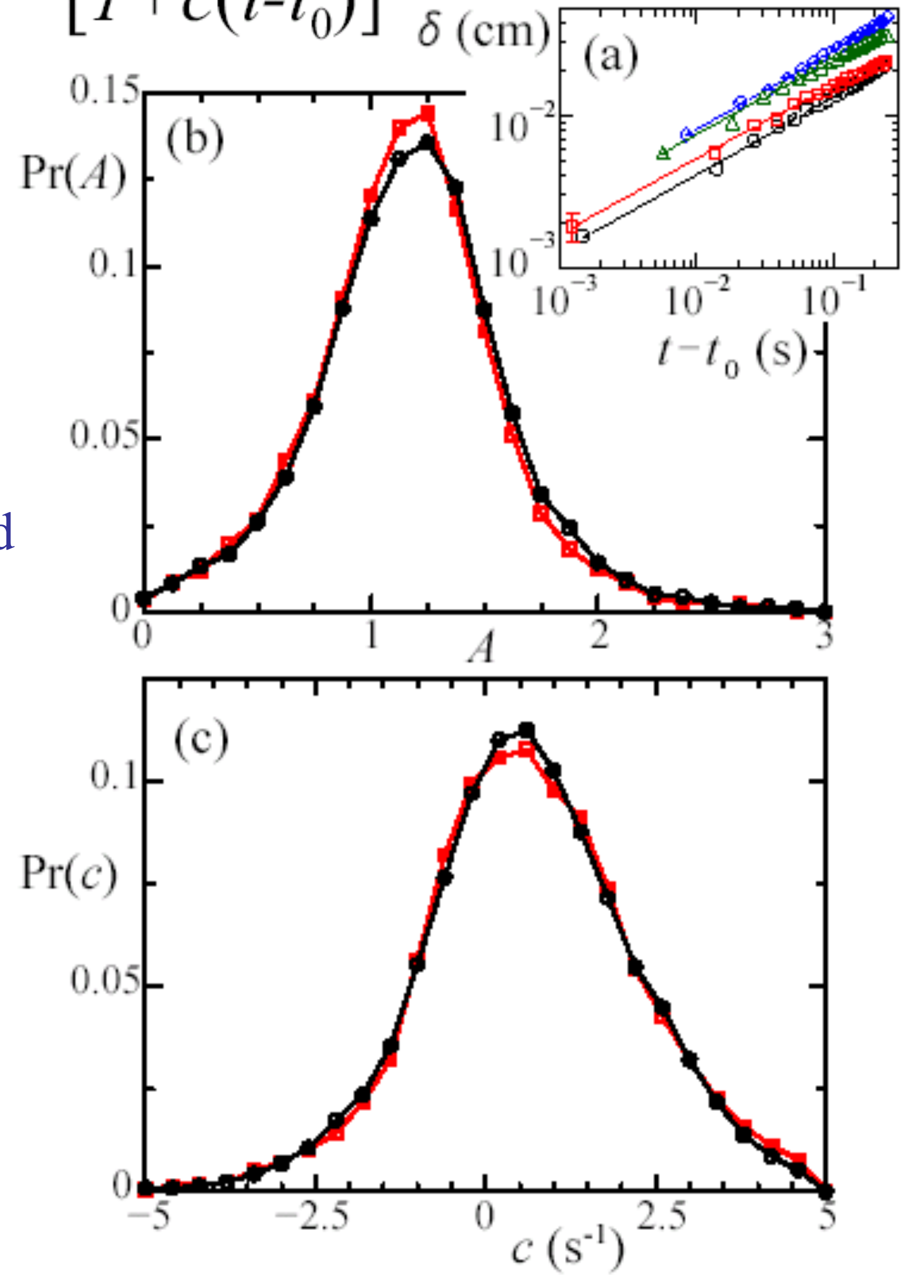


$$\delta \sim \kappa^{1/2}(t_0 - t)^{1/2}$$

$$\delta \sim \kappa^{1/2}(t - t_0)^{1/2}$$

Pre-reconnection: $\delta(t) = A[\kappa(t_0-t)]^{1/2}[1+c(t_0-t)]$

Post-reconnection: $\delta(t) = A[\kappa(t-t_0)]^{1/2}[1+c(t-t_0)]$



Only small pre- and post- differences

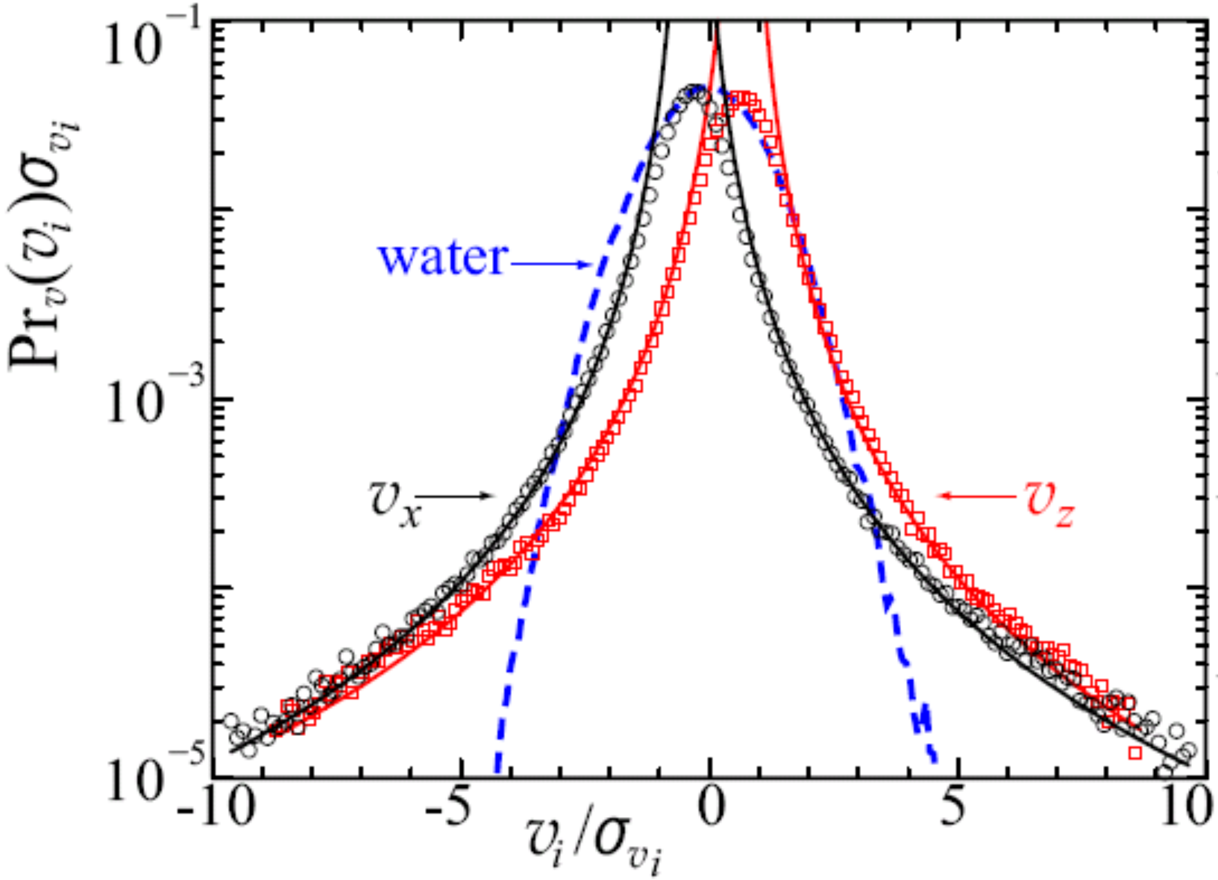
c may represent the affect of local strains

reconnection and ring collapse represented

NEARLY TIME REVERSIBLE!

M.S. Paoletti, M.E. Fisher, and D.P. Lathrop, "Reconnection dynamics for quantized vortices," Physica D (2010)

Velocity Statistics



$\delta = A\,\kappa^{1/2}\,(t-t_o)^{1/2}$

$\mathbf{v}(t) \sim \kappa^{1/2}\,|t-t_0|^{-1/2}$

$P(\mathbf{v})d\mathbf{v} = P(t)dt$

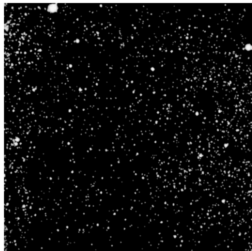
$P(\mathbf{v}) \sim \left|\frac{dt}{d\mathbf{v}}\right|$

$P(\mathbf{v}) \sim \left|\frac{1}{\mathbf{v}^3}\right|$

Reconnection produces predictable power-law velocity tails quite distinct from classical turbulence

M.S. Paoletti, M.E. Fisher, K.R. Sreenivasan, and D.P. Lathrop, "Velocity statistics distinguish quantum from classical turbulence," Phys. Rev. Lett. (2008)

Bagaley and Barenghi, PRE (2011).



Vortex Filament Models

- Local Induction Approximation (LIA)

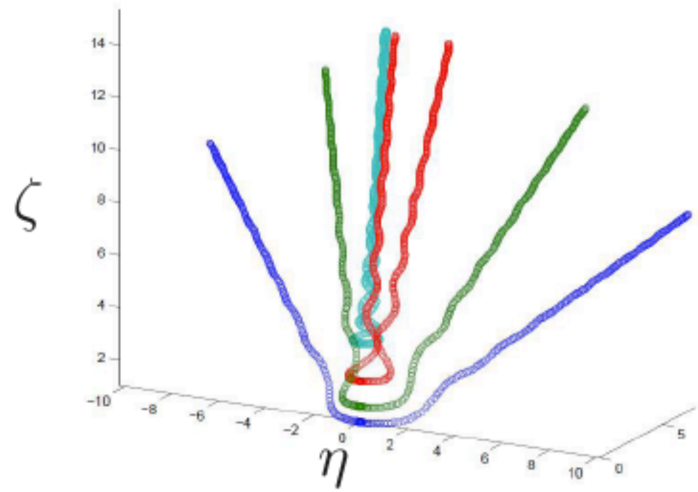
$$\frac{\partial \vec{s}(\sigma,t)}{\partial t} = \beta \frac{\partial \vec{s}(\sigma,t)}{\partial \sigma} \times \frac{\partial^2 \vec{s}(\sigma,t)}{\partial \sigma^2} + \alpha(T) \frac{\partial^2 \vec{s}(\sigma,t)}{\partial \sigma^2}$$

- LIA has one-parameter family of self-similar solutions in dimensionless similarity coordinates

- Adopt dimensionless similarity coordinates

$$\eta = (x - x_0)/\sqrt{\kappa(t-t_0)}$$
$$\zeta = (z - z_0)/\sqrt{\kappa(t-t_0)}$$

LIA Curves vs Vortex Angle



T. Lipniacki, "Evolution of quantum vortices following reconnection," European Journal of Mechanics - B/Fluids, vol. 19, pp. 361–378, May 2000.

S. Hormoz and M. P. Brenner, "Absence of singular stretching of interacting vortex filaments," Journal of Fluid Mechanics, vol. 707, pp. 191–204, Aug. 2012.

Single monochromatic Kelvin wave:

- Transverse
- Helical
- Propagating
- Dispersion relation known
- Helicity density

$$\omega = \frac{\kappa k^2}{4\pi}\left[\ln\left(\frac{1}{ka}\right)+c\right]$$

$$h = \vec{u} \cdot \vec{\omega}$$

$$h = u_z \kappa \delta^2(xy)$$

$$h = u_z \kappa / \pi a^2$$

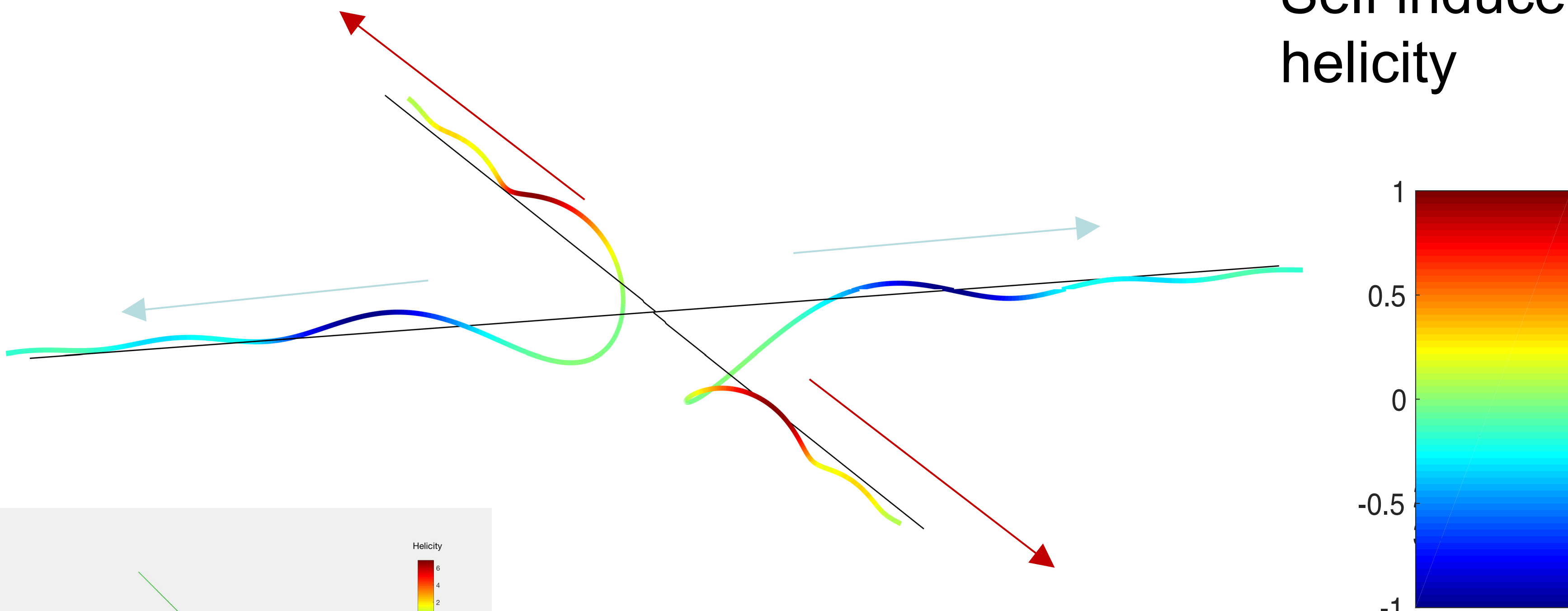
} Long straight vortex on z-axis

$$h = \frac{2\kappa^2 b^2 k^3}{\pi a^2} ln(ka)$$

Kelvin wave amplitude b and wavenumber *k*

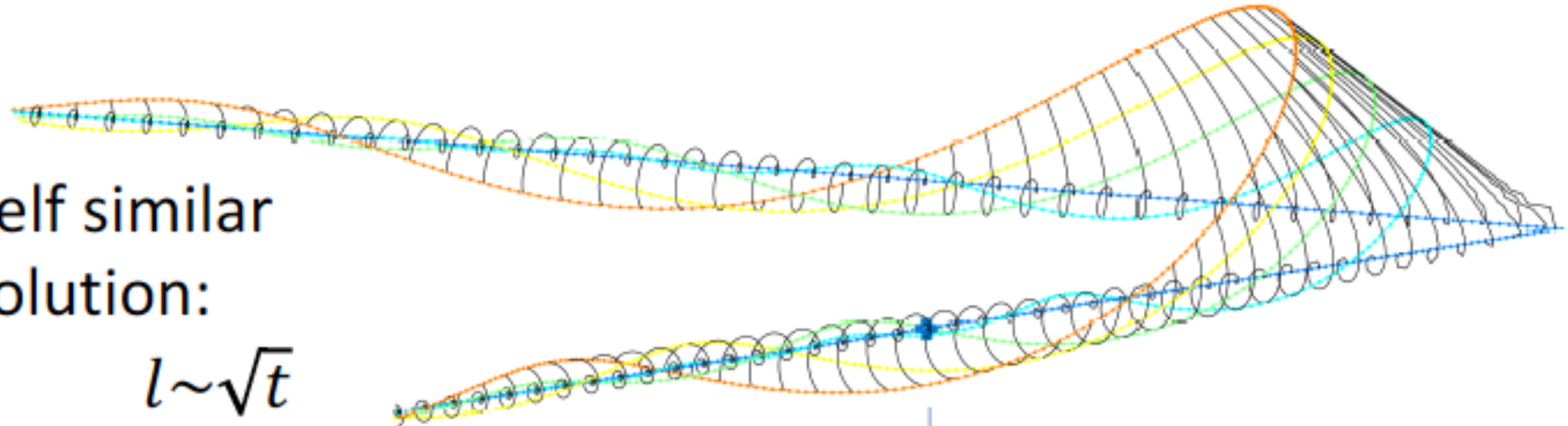
Reconnection generates helicity quadrupole

Self induced helicity



Self similar solution:

$$l \sim \sqrt{t}$$



Helical motion of a tracer particle on a reconnecting vortex

Local Helicity measure

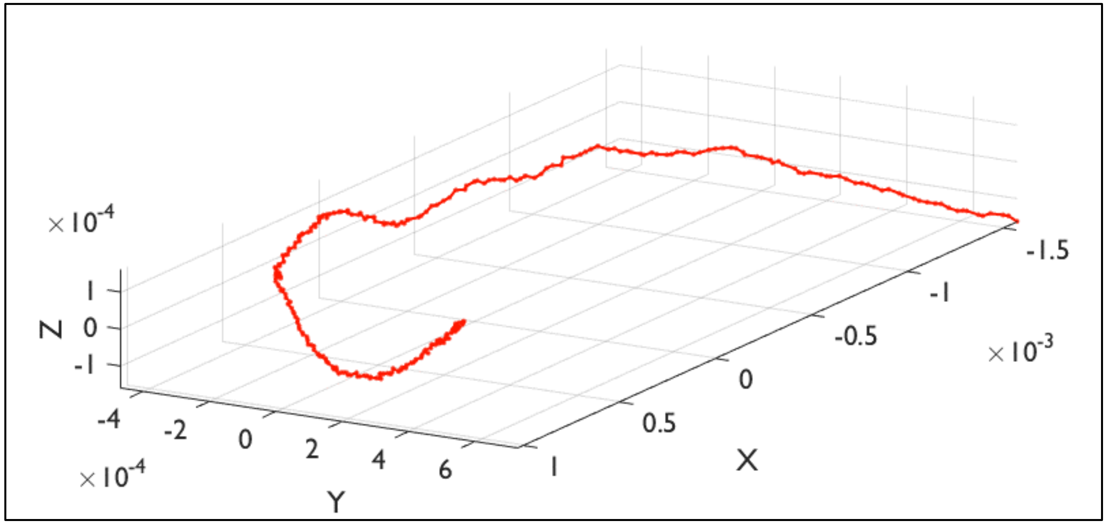
$$h = \vec{u} \cdot \vec{\omega}$$

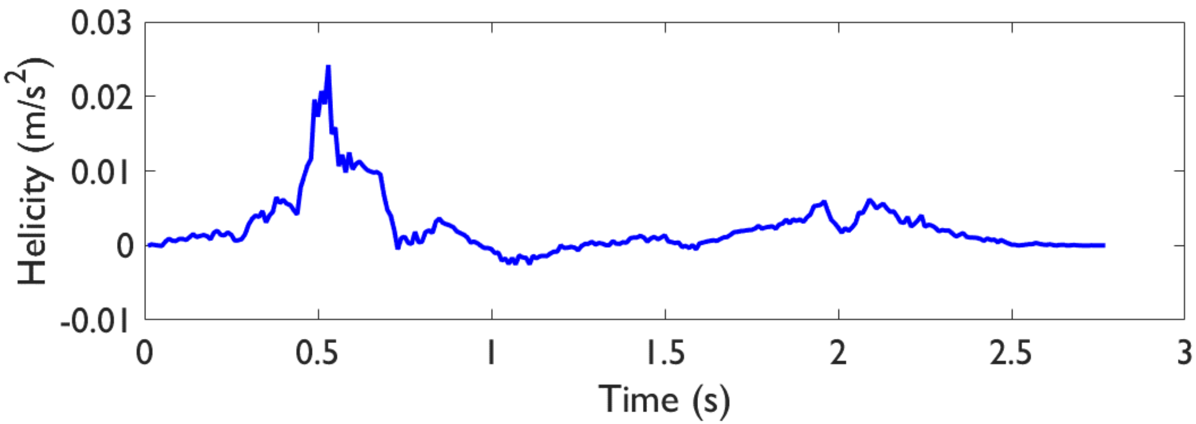
Principle component Analysis – long axis

$$h_{pca} = u_z \dot{\Theta}$$

Very noisy

$$h_{rp} = r u_z \dot{\Theta} / < r >$$





Local Helicity Statistics

$$h = \vec{v} \cdot (\nabla \times \vec{v})$$

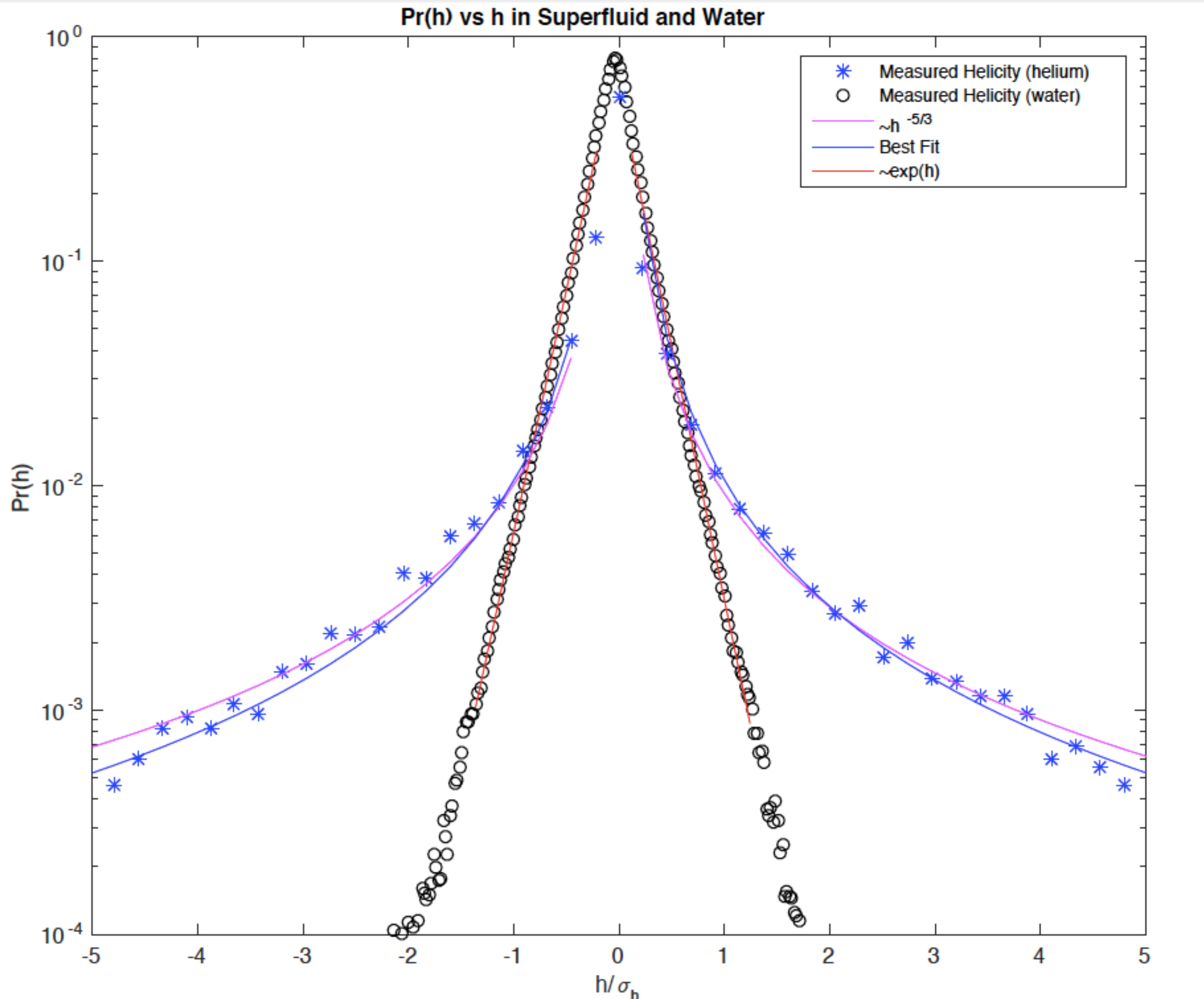
$$h \sim \kappa^{1/2}(t - t_o)^{-3/2}$$

$$P(h)dh = P(t)dt$$

$$P(h) = \left|\frac{dt}{dh}\right|$$

$$P(h) \sim \kappa^{1/3} h^{-5/3}$$

Reconnection produces predictable
Power-law helicity tails
Heavy tailed – ultraviolet divergence of the variance



Pr(h) vs h in Superfluid and Water
Measured Helicity (helium)
Measured Helicity (water)
~h -5/3
Best Fit
~exp(h)
Pr(h)
h/ σ_h
10^0
10^-1
10^-2
10^-3
10^-4
-5
-4
-3
-2
-1
0
1
2
3
4
5

Helicity statics – background and definitions

Ideal MHD

Ideal fluids

Quantum fluids

Helicity dynamics in classical fluids and plasmas

Helicity dynamics for quantum fluids

Role in relaxation of quantum turbulence

Dissertations: complex.umd.edu

Youtube channel: Lathrop Lab

Bewley, Lathrop, and Sreenivasan Nature 2006

Paoletti, Fisher, Sreenivasan, and Lathrop, PRL 2008

Paoletti, Fisher, and Lathrop, Physica D 2008

Paoletti and Lathrop, Ann. Rev. of Cond. Matter Phys. 2011

Meichle, Rorai, Fisher, and Lathrop, PRB 2012