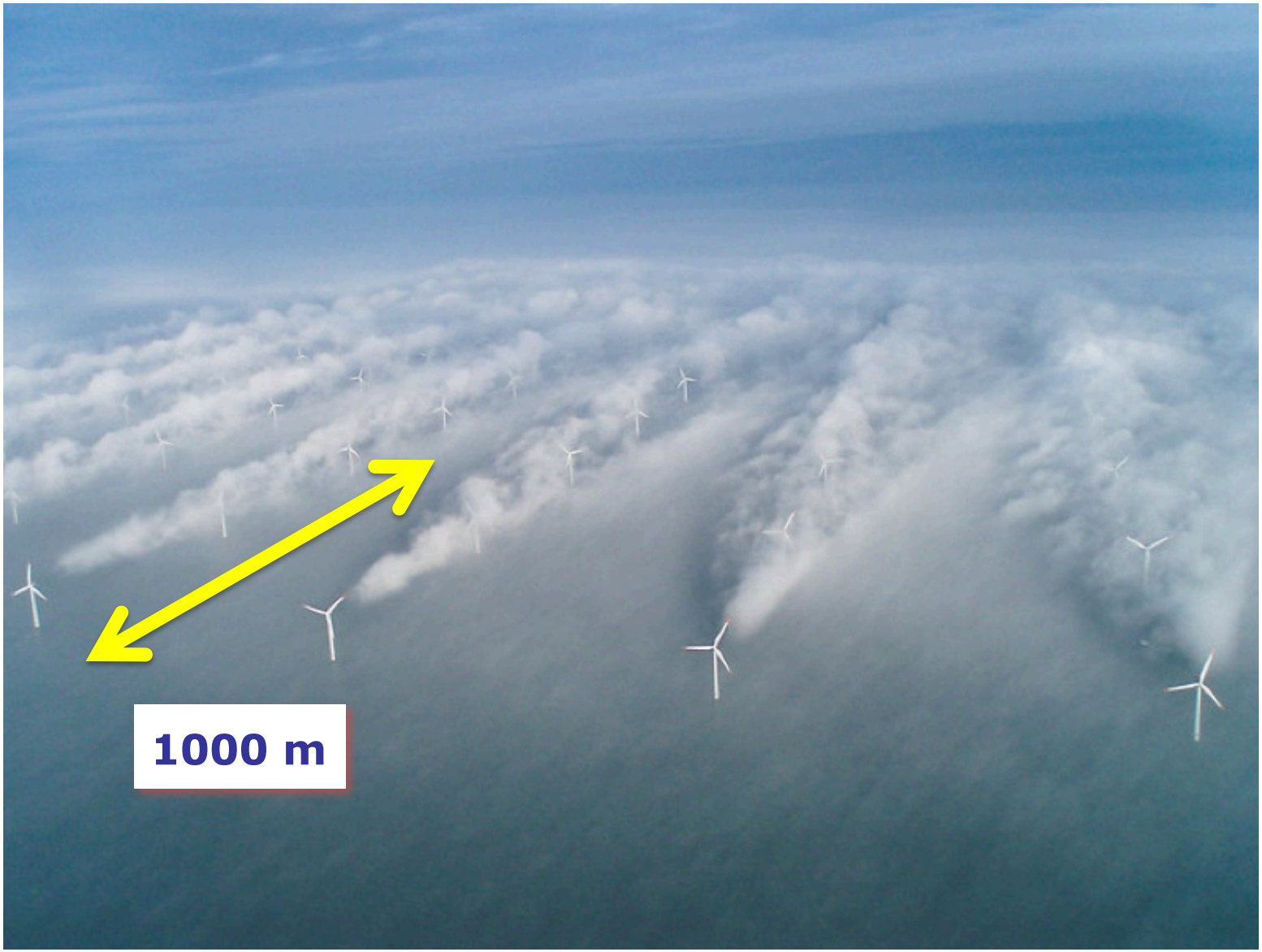
ICTS, Bangalore, January 2018

Windfarm turbulence at 10^{13} Angstroms or ~10^{-13} LightYears

Charles Meneveau

Mechanical Engineering & CEAFM, Johns Hopkins University

10^{13} Angstroms or ~10^{-13} LightYears = 1000 m



Horns Rev 1 owned by Vattenfall.
Photographer Christian Steiness

Close collaborators:

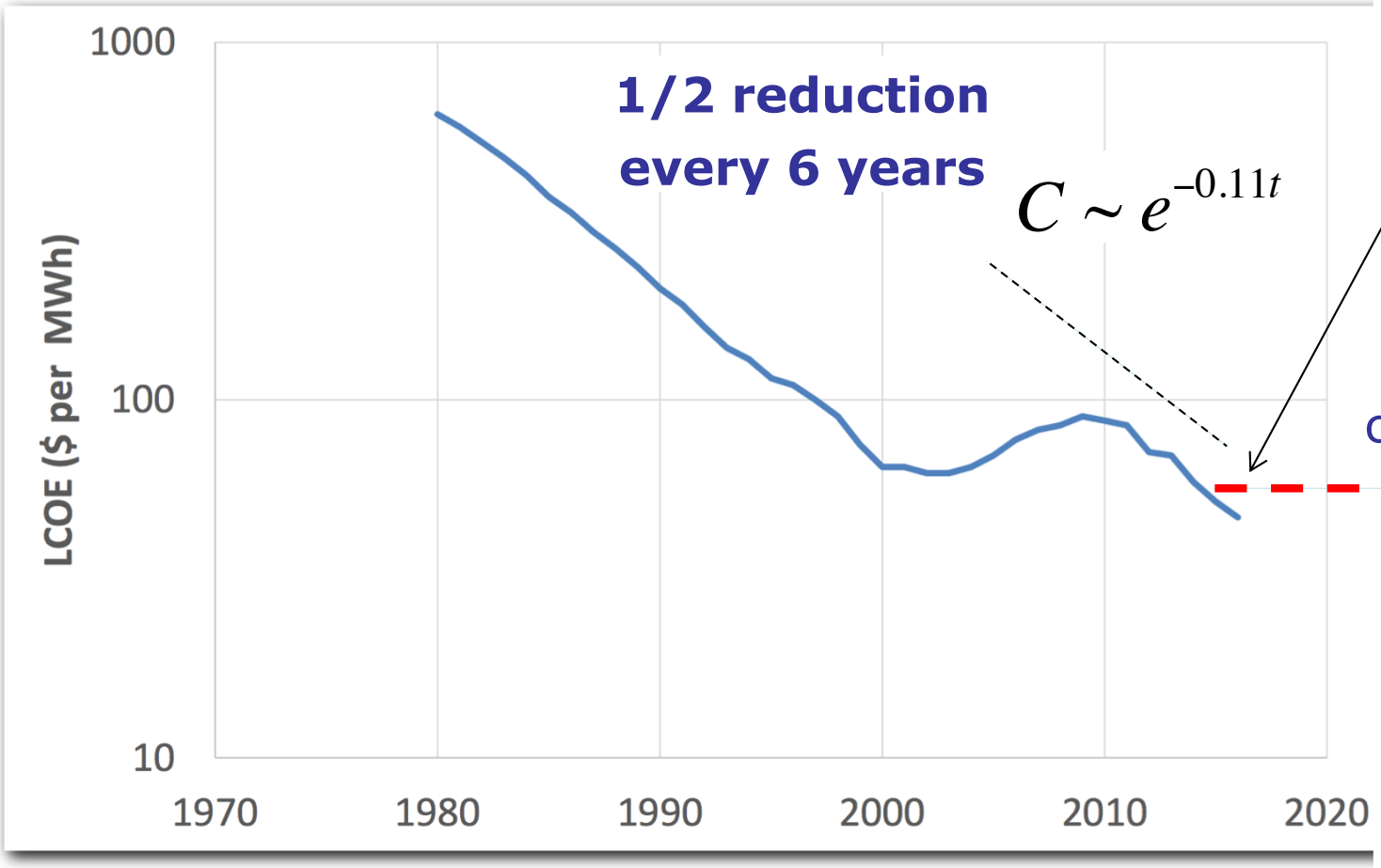
- Richard J.A.M. Stevens (now Twente U., NL) – LES + CWBL
- Juliaan Bossuyt (KU Leuven, JHU visiting student, B) – windtunnel
- Prof. Johan Meyers (KU Leuven, B) – LES
- Michael Wilczek (now MPI Göttingen, D) – spectral theory
- Dennice Gayme (JHU) – reduced models + control
- Marc Calaf (now U. Utah) – initial LES studies

Funding: NSF OISE-1243882 (WINDINSPIRE project)

Simulations: XSEDE, SARA (NL) & MARCC

Onshore wind (2017)
now the cheapest form of electricity (without subsidies)

The development in cost of wind energy, US market, average



natural gas
combined cycle:
$48-$78

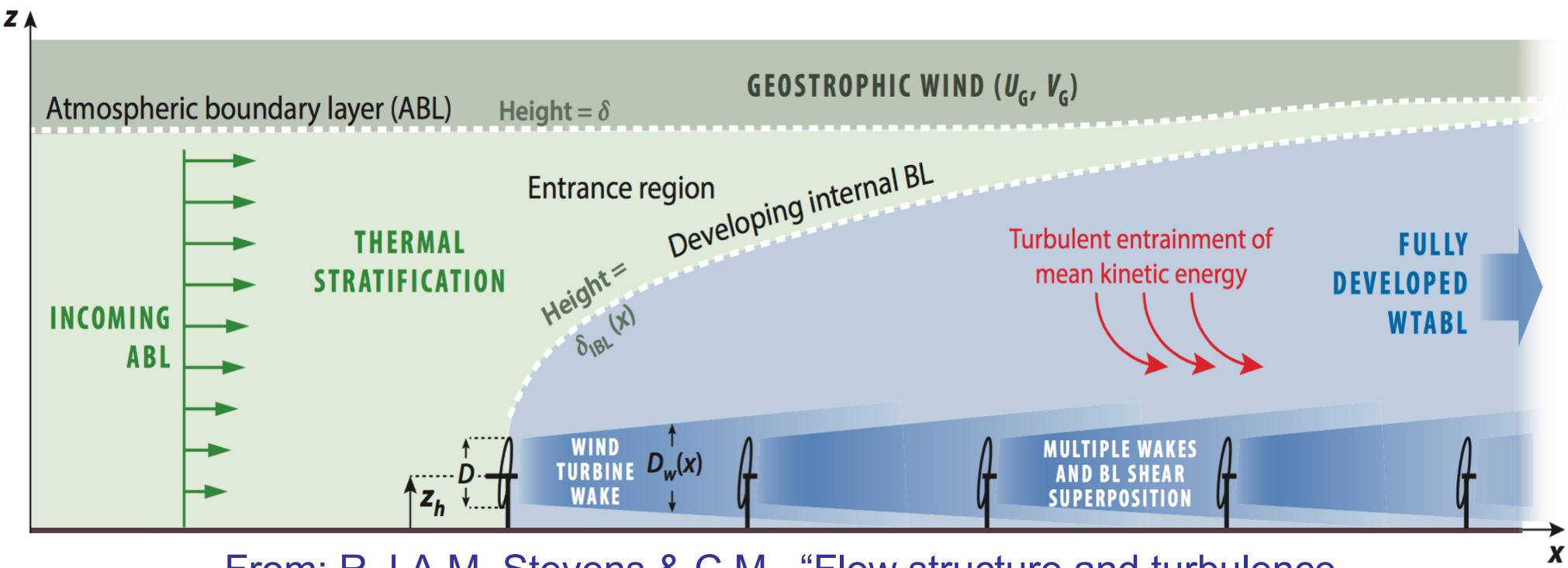
Source: 1980-2011: "Revolution Now", DoE, 2016; 2012-2016: Lazard

Source: Lazard Energy Analysis

Wind Farms: systems of gigantic turbomachines, interacting with geophysical phenomena

at "large scales": $\frac{1}{5}D < \ldots\ldots < H_{ABL}$

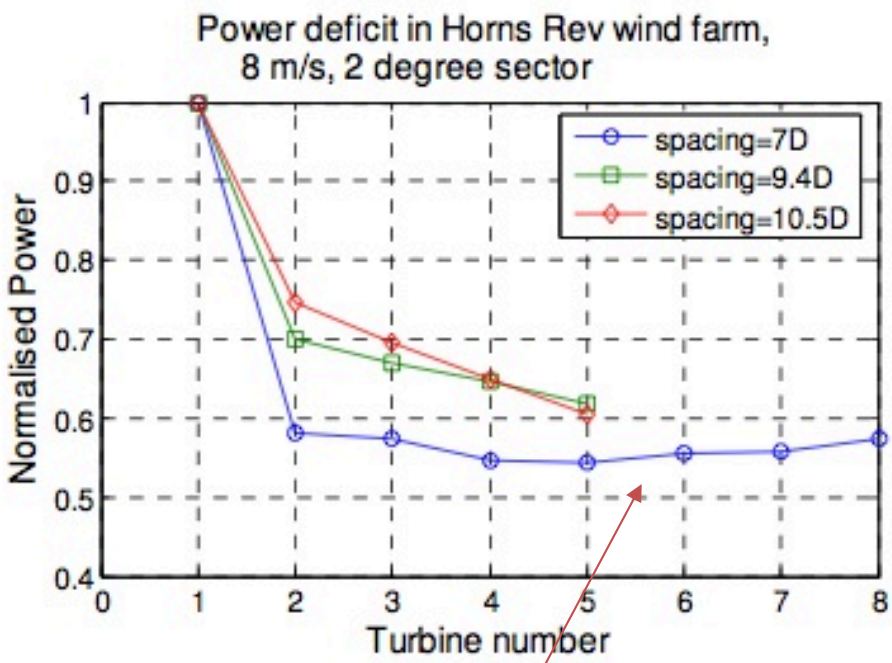
The windturbine-array boundary layer (WTABL)

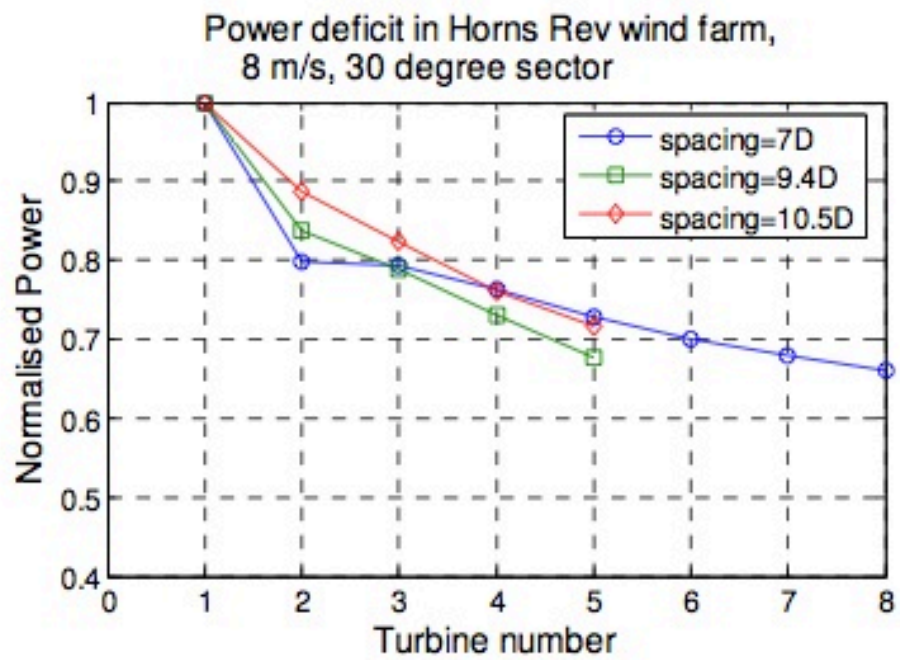


From: R.J.A.M. Stevens & C.M., "Flow structure and turbulence in wind farms", (2017), Annu. Rev. Fluid Mech. **49**, 311-339.

Related problem: Wind farm power degradation

(Wind farm operators: “10-15% wind farm underperformance problem”)





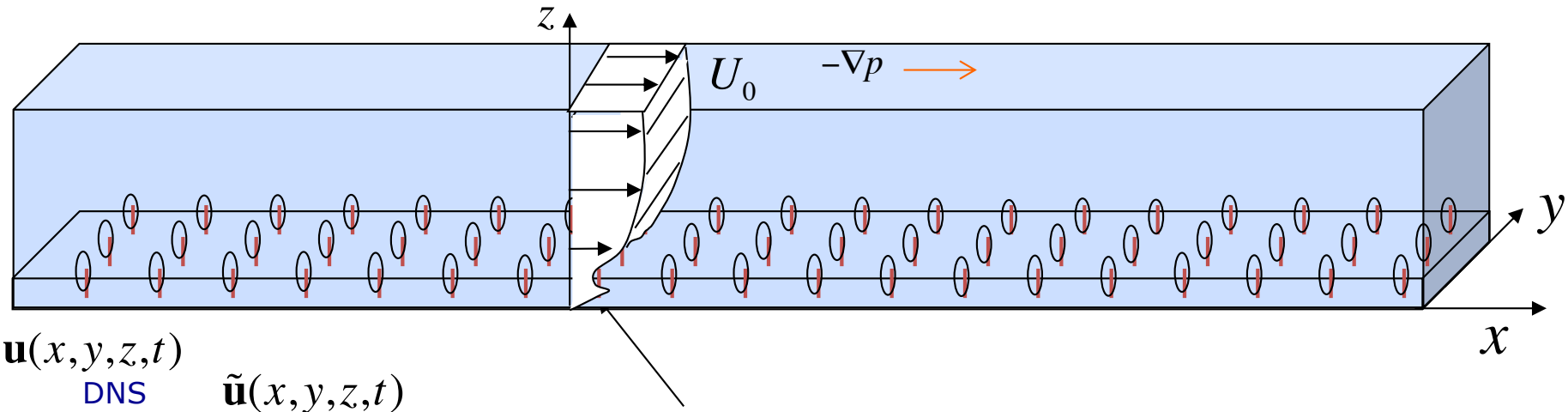
- asymptote ??
- how fast?
- is it really around 50%?
- mechanisms ?

“Modelling and measurements of wakes in large wind farms
Barthelemie, Rathmann, Frandsen, Hansen et al…
J. Physics Conf. Series 75 (2007), 012049

The "fully developed" pressure-grad-driven WTABL:

What is the generic structure of this specific type of boundary layer?

Pressure-gradient driven (half-channel flow)



$\mathbf{u}(x,y,z,t)$
DNS

$\tilde{\mathbf{u}}(x,y,z,t)$
LES

$\bar{\mathbf{u}}(x,y,z)$
RANS

$$U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$$

horizontal (canopy) average

What is the "averaged" velocity distribution? $U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$

Is there a "universal" WTABL profile?

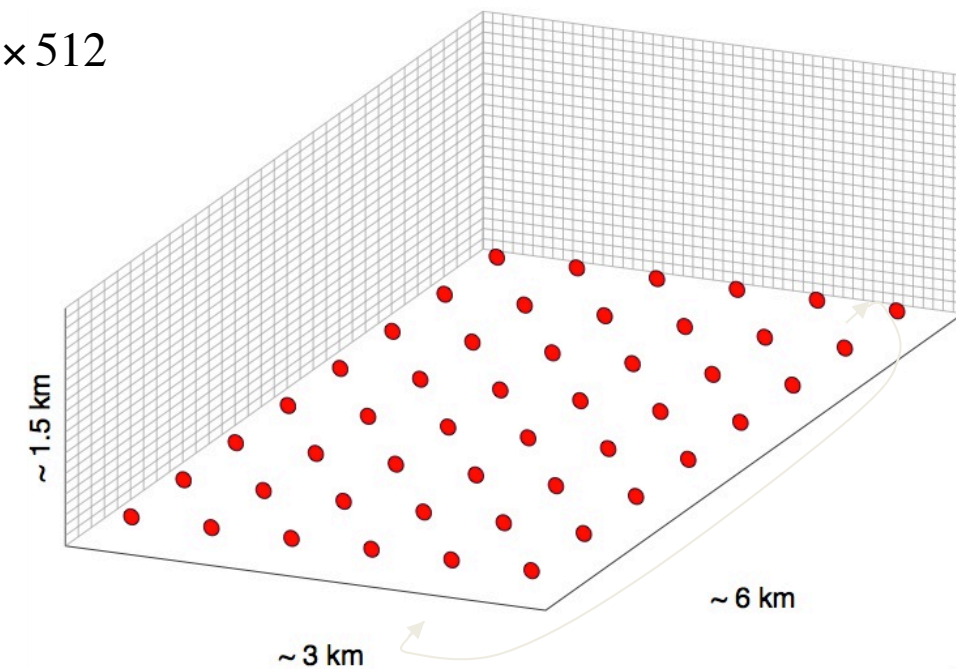
What are profiles of shear stresses? $\tau_{xz}(z) = -\langle \overline{u'w'} \rangle_{xy}$

LES of WTABL; typical simulation setup:

- LES code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128 \quad \rightarrow \quad 1024 \times 512 \times 512$$



- Horizontal periodic boundary conditions (for FD-WTABL, or precursor for developing)

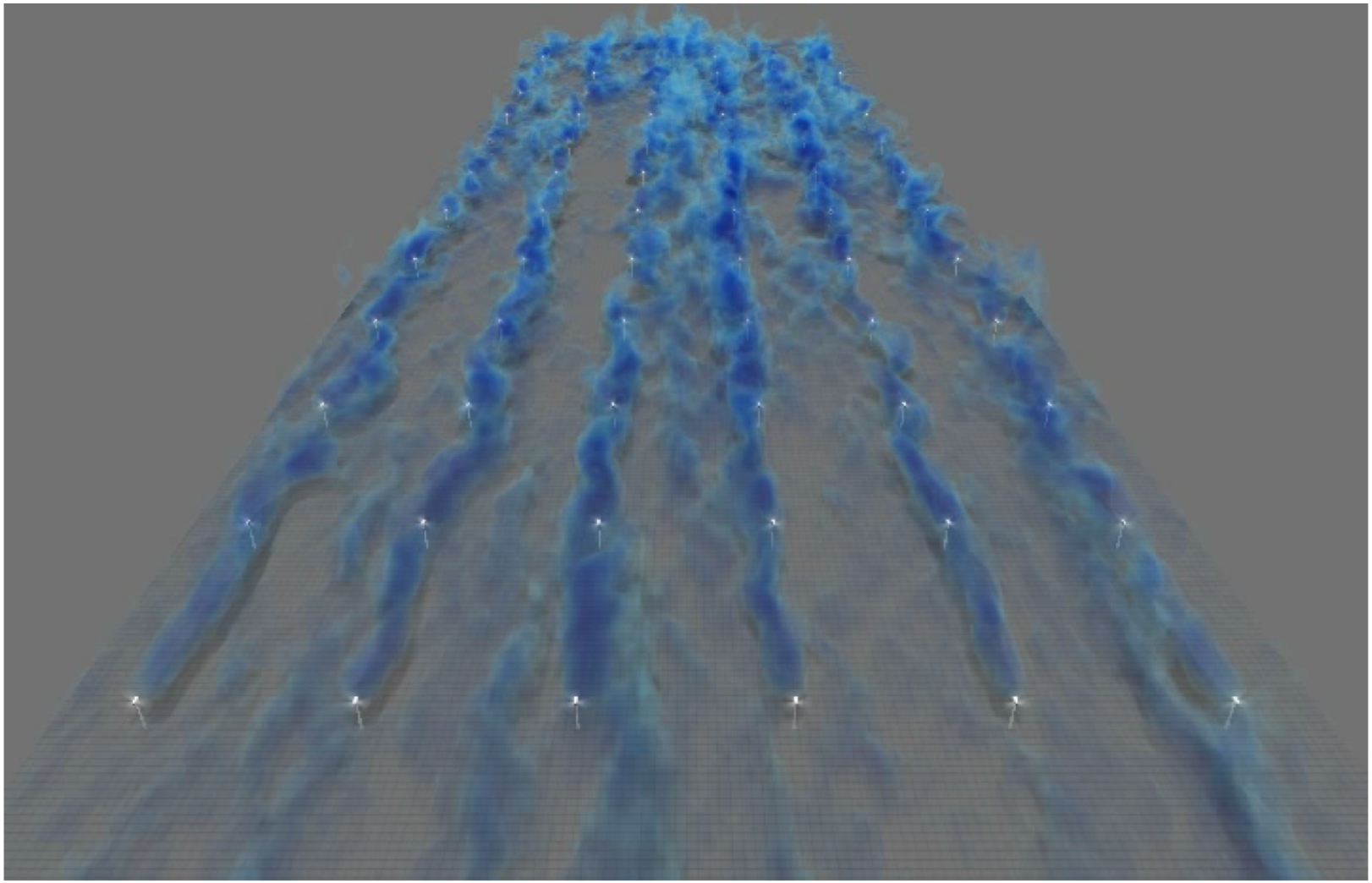
- Top surface: zero stress, zero w

- Bottom surface B.C.: Zero w + Wall stress: Standard wall function relating wall stress to first grid-point velocity

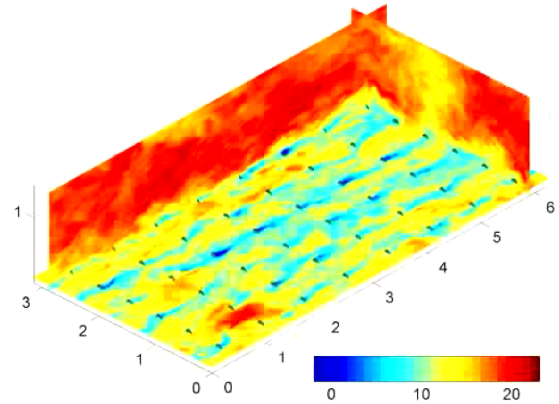
- Scale-dependent dynamic Lagrangian model eddy-viscosity closure (*no* adjustable parameters)

- Actuator disk model for rotor $$f_{Tx} = -\frac{1}{2} C_T \left(\frac{1}{1-a} \bar{U} \right)^2 \frac{\delta A_{yz}}{\delta V} = -\frac{1}{2} C_T' \bar{U}^2 \frac{\delta A_{yz}}{\delta V}$$

Developong case: ABL inflow (with R. Stevens)

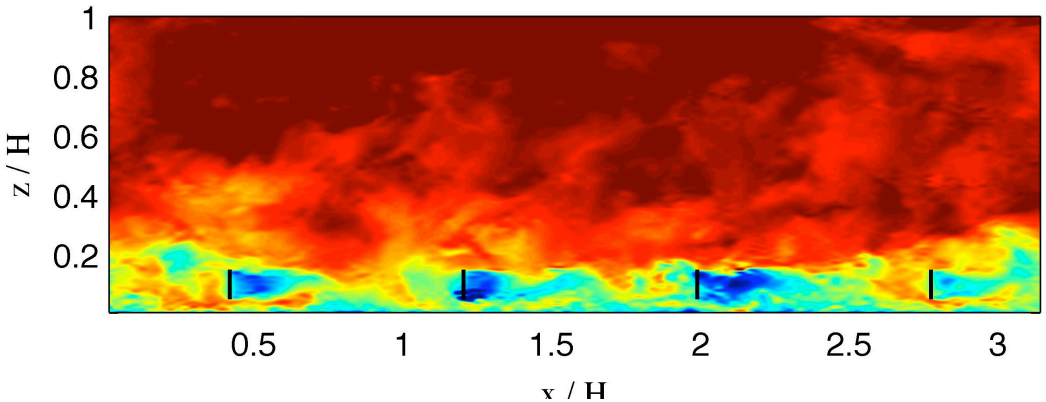


Suite of LES cases

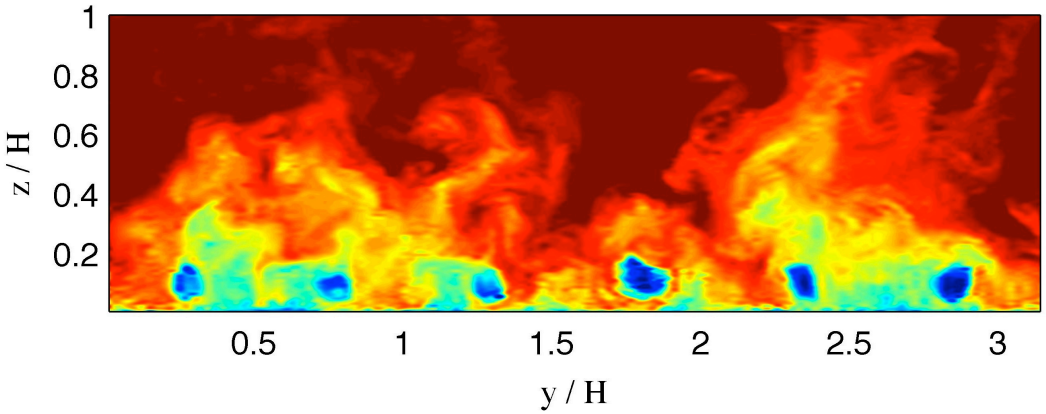


Instantaneous stream-wise velocity contours:

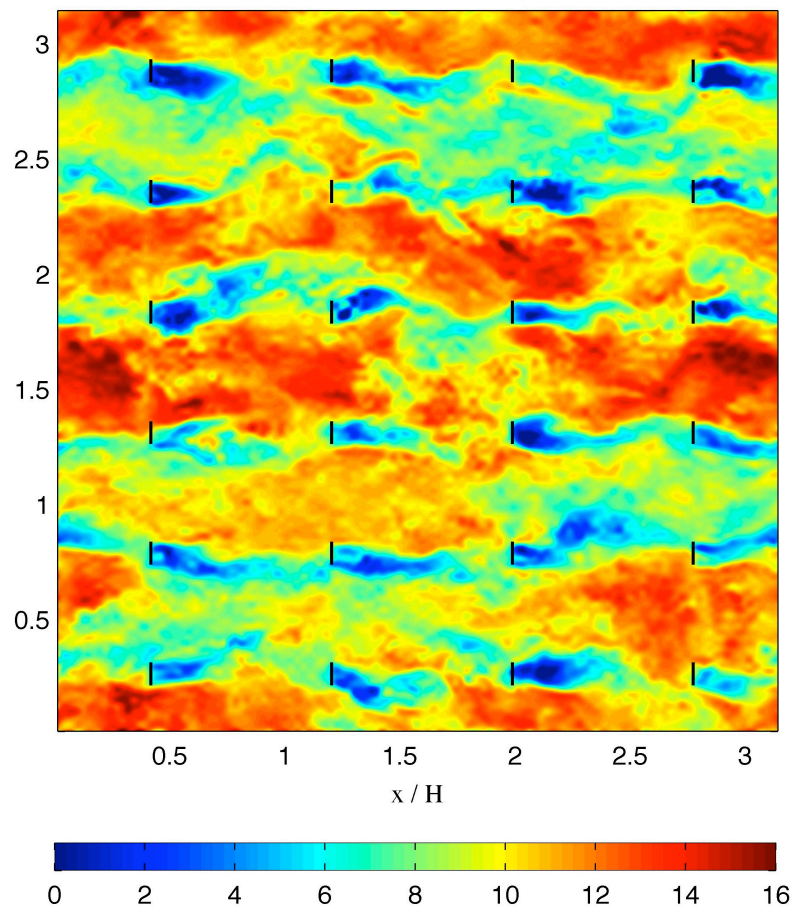
side-view



front-view



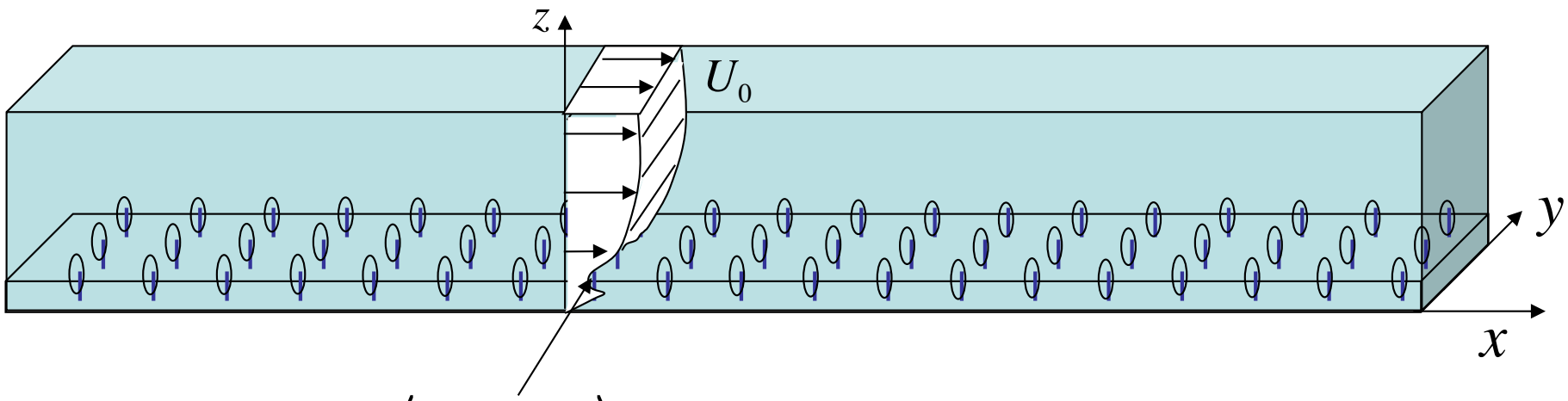
top-view



$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128$$

Mean velocity vertical profiles in the fully developed WTABL



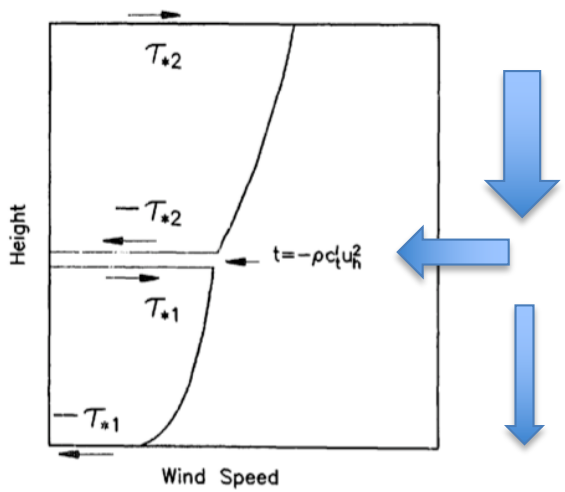
$$U(z) = \langle \overline{u}(x,y,z) \rangle_{xy}$$

horizontal (canopy) average

momentum theory

Horizontally averaged variables

Sten Frandsen,
(J. Wind Eng & Ind Appl 39, 1992)*:*



$$0 \approx -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dz}\left(-\left\langle \overline{u'w'} \right\rangle_{xy}\right) + \left\langle f_x \right\rangle_{xy}$$

Integrate in z-direction:

If top of WT canopy still

falls in the "surface layer", where $\frac{dp}{dx} z \approx 0$

$$-\left\langle \overline{u'w'} \right\rangle_{xy}(z_{top}) \approx -\left\langle \overline{u'w'} \right\rangle_{xy}(z_{bottom}) + \frac{1}{2} C_T \frac{A_{disk}}{A_{xy}} U_h^2$$

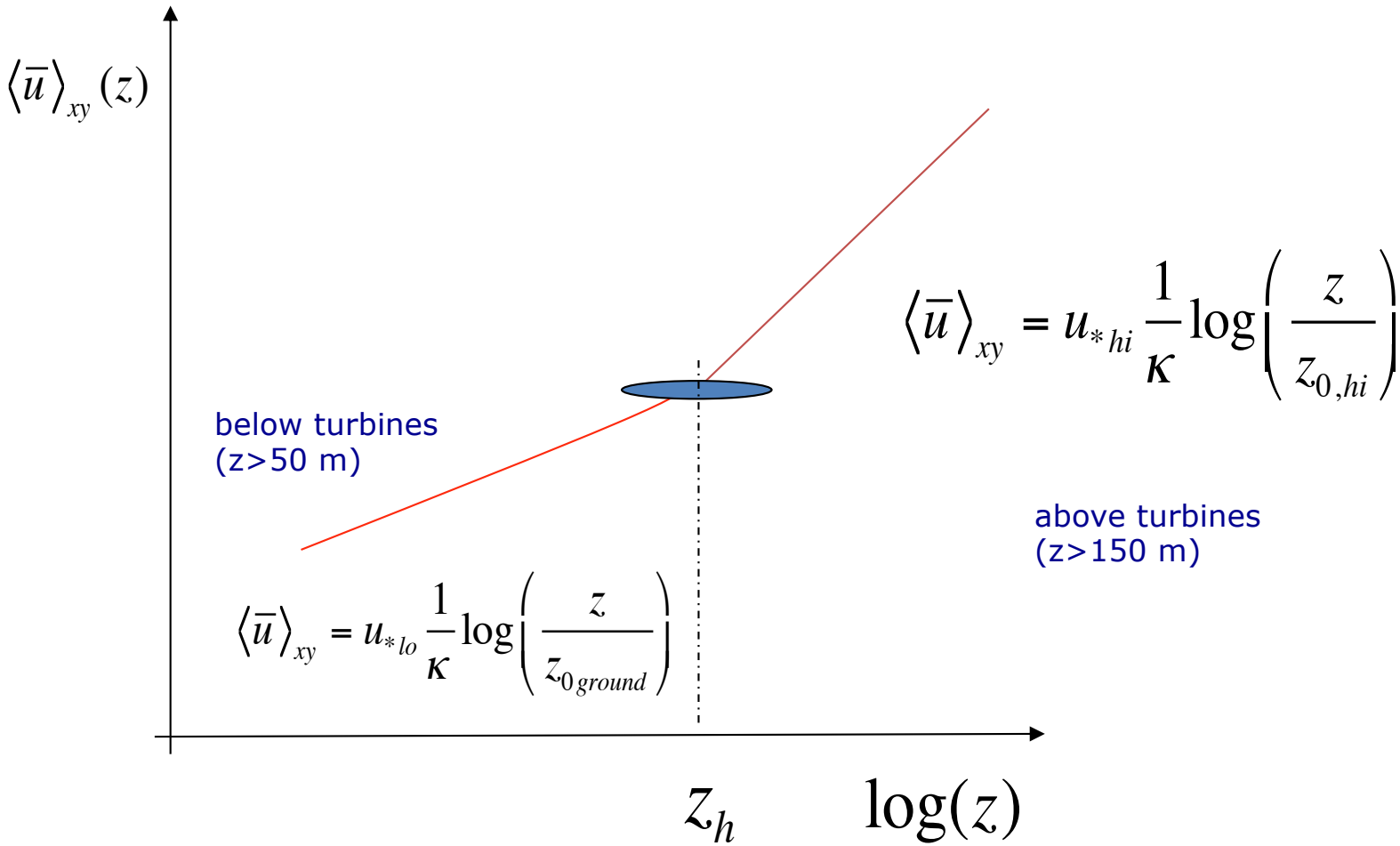
Analogy to energy cascade:
momentum flux toward the wall
(Sreenivasan , Jimenez AFM 2012)
Here: with extraction at hubheight

$$u_{*\,hi}^2 \approx u_{*\,lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2$$

momentum theory

$$u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2$$

Frandsen 1992 (also Newman 1977): postulated the existence of 2 log laws



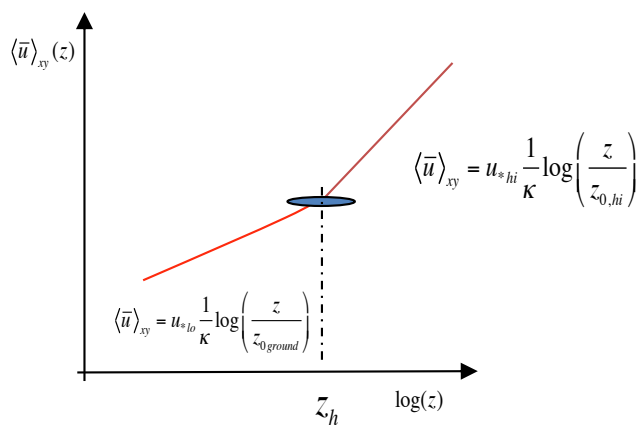
momentum theory

S. Frandsen 1992, Frandsen et al. 2006:

Knowns: u_{*hi}, $z_{0,ground}$, C_T, s_x, s_y

3 unknowns: $z_{0,hi}$, U_h, u_{*lo}

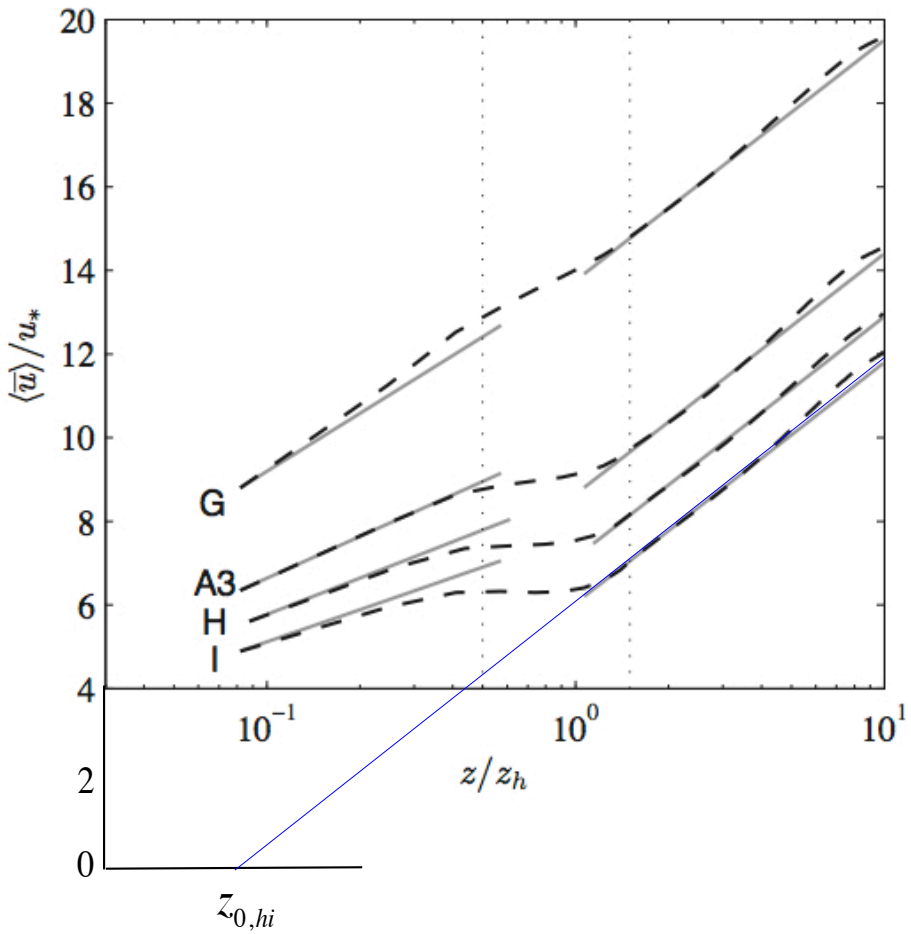
$$\begin{cases} u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2 \\ U_h = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) \\ u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) = u_{*lo} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,ground}}\right) \end{cases}$$



Solve for effective roughness:

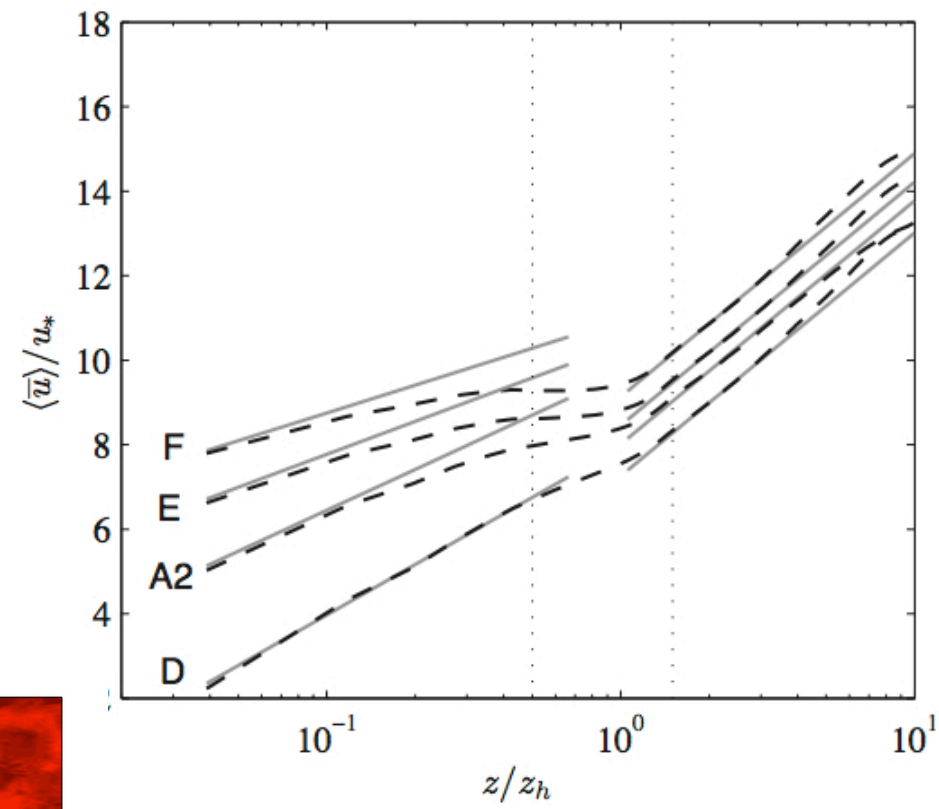
$$z_{0,hi} = z_h \exp\left(-\kappa \left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})} \right) \right]^{-1/2} \right)$$

From LES: Mean velocity vertical profiles in the FD-WTABL: confirmation of two log laws (Frandsen, 1992, 2006)

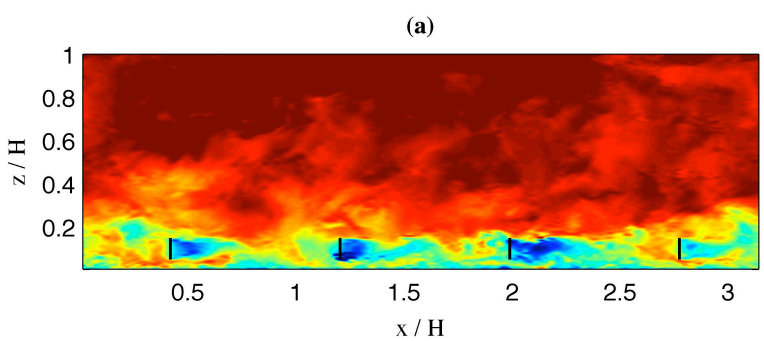


measure $z_{0,hi}$ from intercept

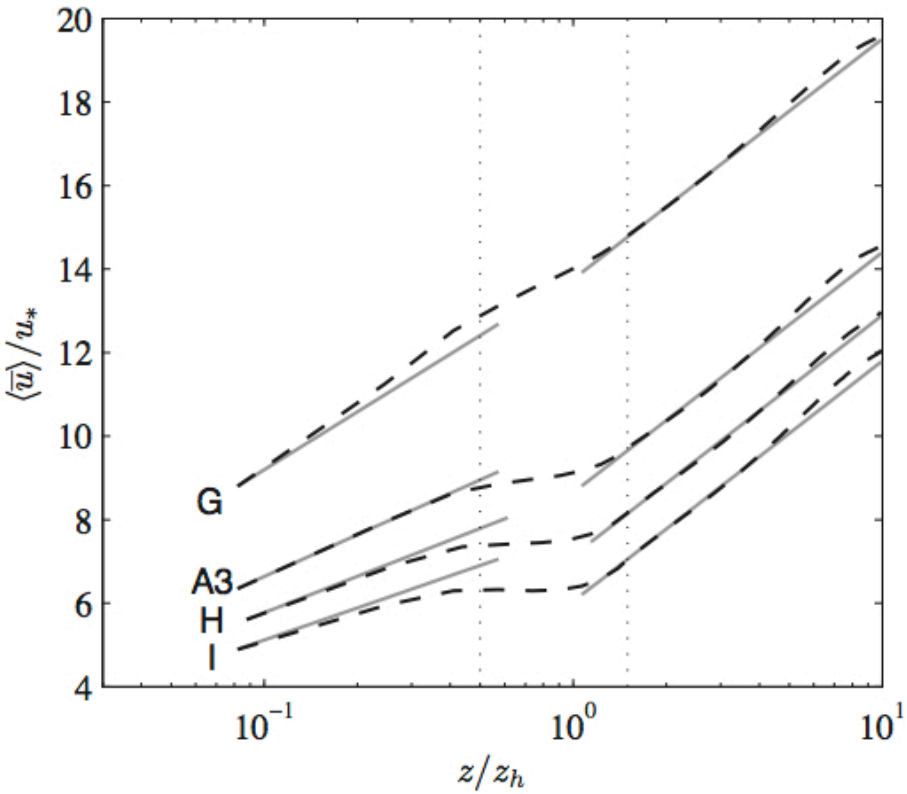
$$\langle \overline{u} \rangle_{xy} = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z}{z_{0,hi}} \right)$$



Calaf, Meyers & CM, PoF 2010



"Wake upgrade" to Frandsen's top-down model



$$\left(\kappa u_* z_h + \nu_w\right)\frac{\partial \langle \overline{u} \rangle}{\partial z} = u_*^2$$

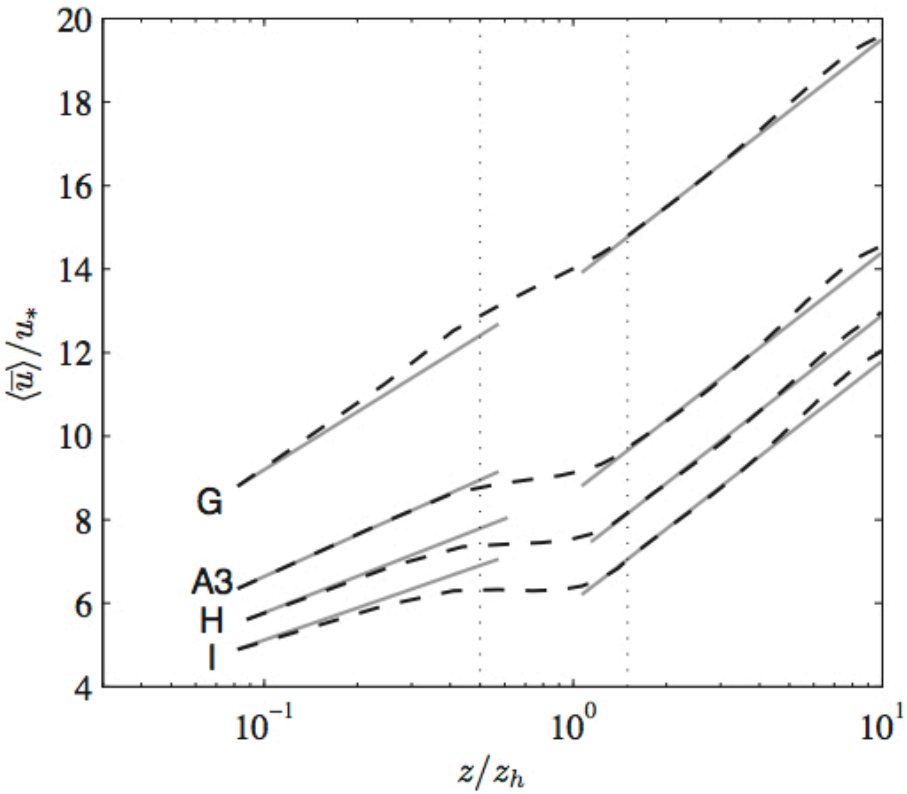
In wake, reduced slope:

$$\frac{\partial \langle \overline{u} \rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

$$\nu_w = \sqrt{\tfrac{1}{2} c_{ft}}\,\langle \overline{u} \rangle D$$

$$\nu_w^* = \frac{\sqrt{\tfrac{1}{2} c_{ft}}\,\langle \overline{u}(z_h) \rangle D}{\kappa u_* z_h} \approx 28\sqrt{\tfrac{1}{2} c_{ft}}$$

"Wake upgrade" to Frandsen's top-down model



$$\left(\kappa u_* z_h + \nu_w\right)\frac{\partial \langle \bar{u} \rangle}{\partial z} = u_*^2$$

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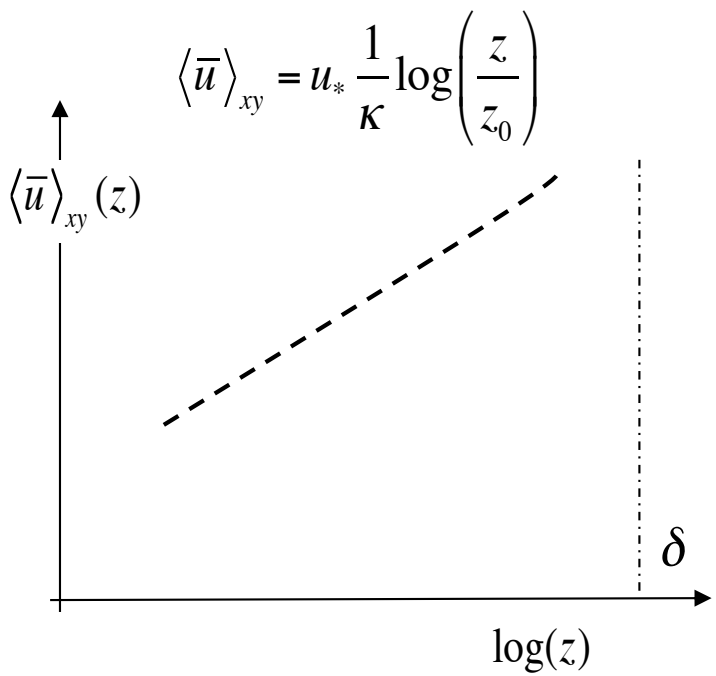
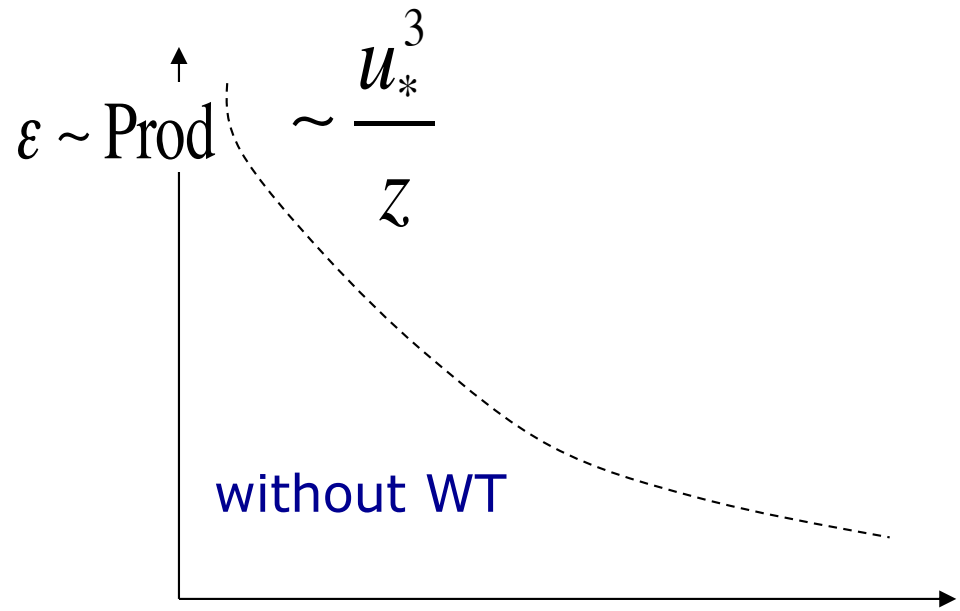
$$z_{0,hi} = z_h\left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

where $\beta = \dfrac{28\sqrt{\tfrac{1}{2} c_{ft}}}{1 + 28\sqrt{\tfrac{1}{2} c_{ft}}}$,

Boundary layer concepts also enable us to understand fate of mean kinetic energy in the WTABL: $\frac{1}{2}\langle \bar{u} \rangle_{xy}^2$





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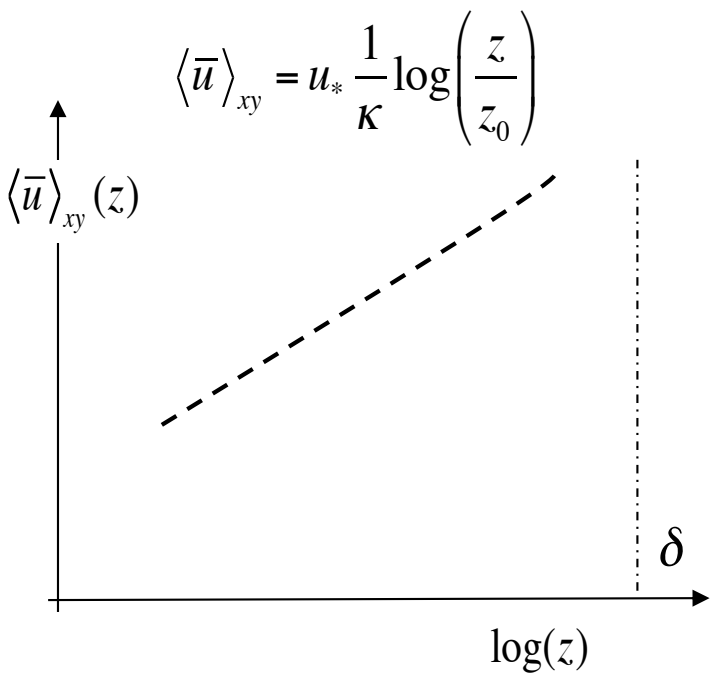
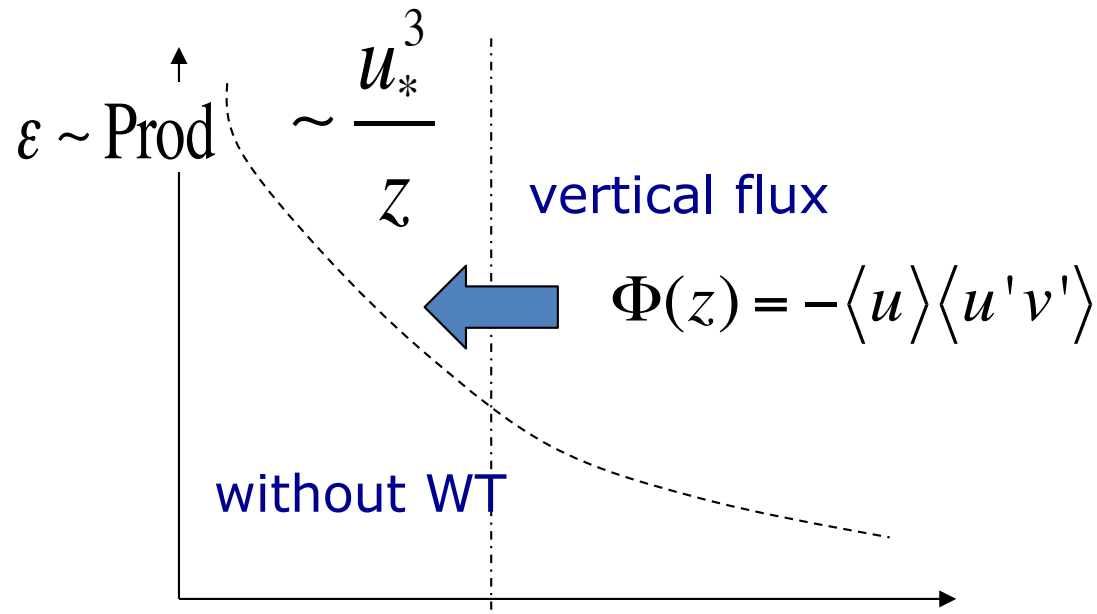
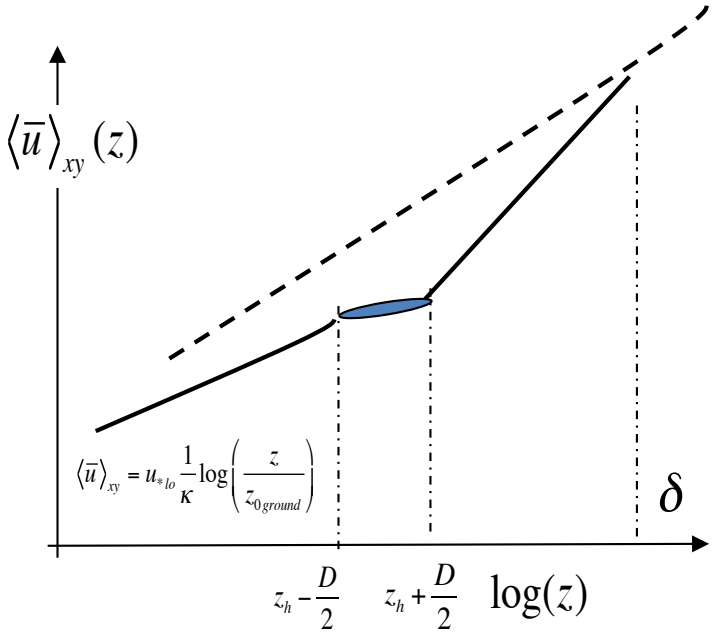
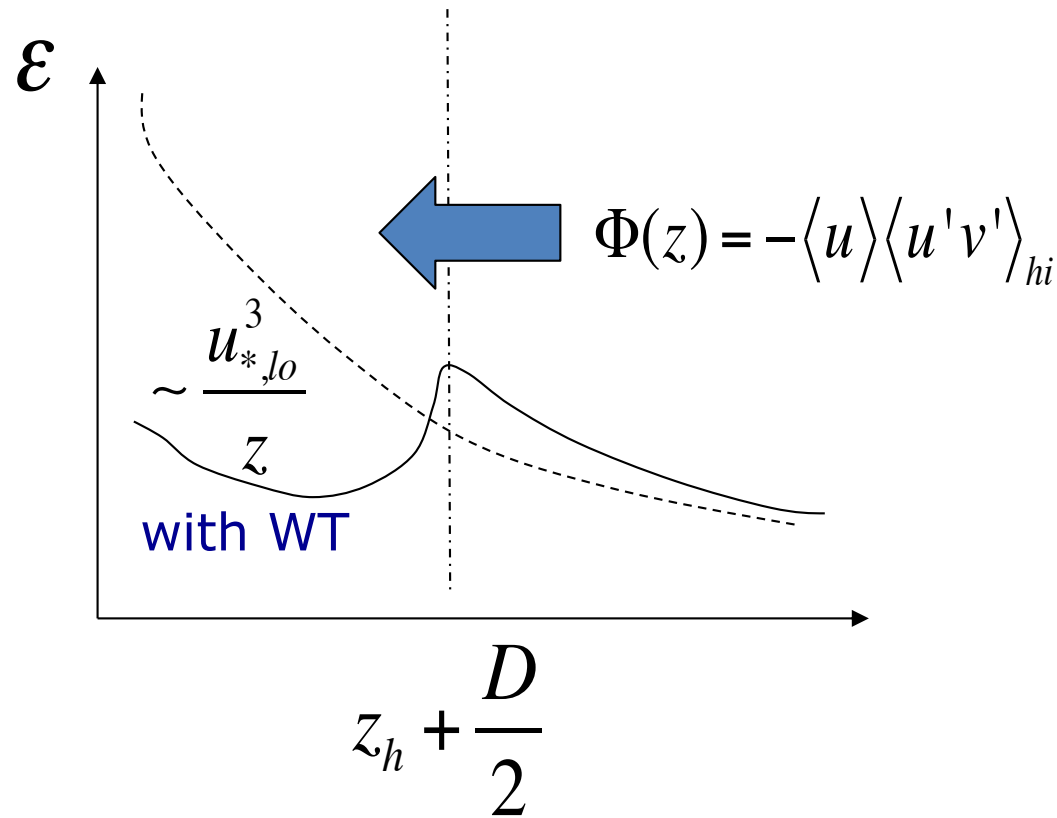




Mean flow KE dissipated in the boundary layer

Boundary layer concepts also enable us to understand fate of mean kinetic energy in the WTABL: $\frac{1}{2}\langle \bar{u} \rangle_{xy}^2$

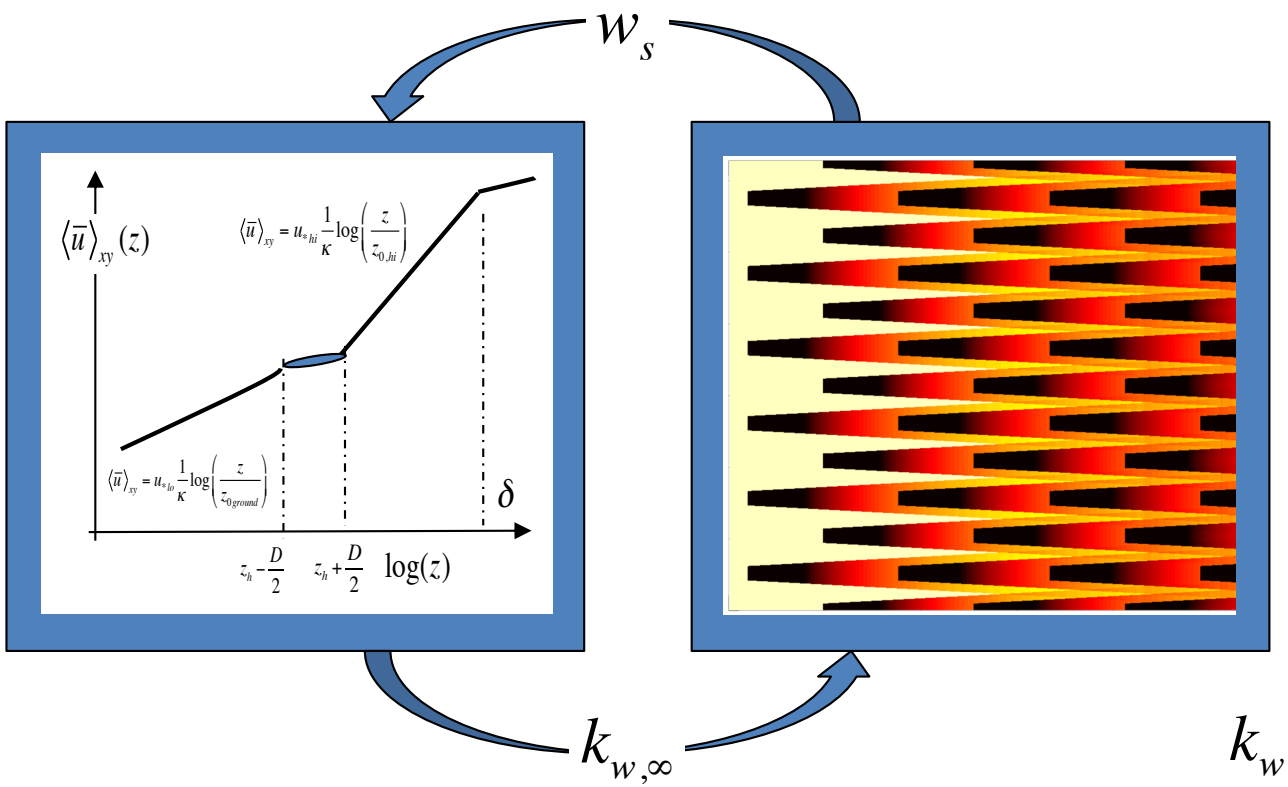




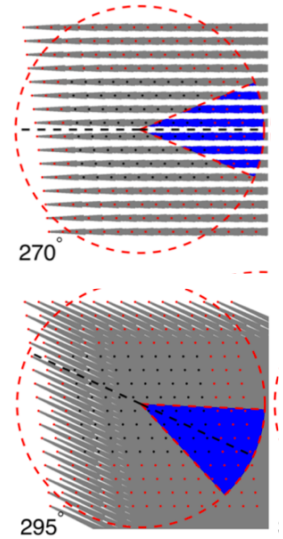
Instead of being dissipated entirely in BL,
mean KE extracted by turbines. Entrainment increased.

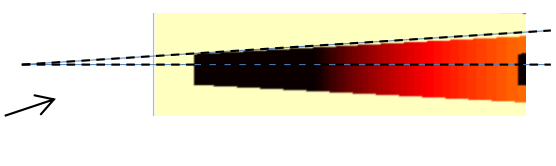
Coupled with "engineering wake" model (CWBL) for entrance effects and detailed placement

Stevens, Gayme & CM, Wind Energy 19, 2023-2040 (2016)



$$w_s = \frac{A_w}{A_T}$$





$$\frac{\ln(\delta / z_{0,\mathrm{lo}})}{\ln(\delta / z_{0,\mathrm{hi}})} \ln\left[\left(\frac{z_\mathrm{h}}{z_{0,\mathrm{hi}}}\right)\left(1+\frac{D}{2z_\mathrm{h}}\right)^{\beta}\right]\left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}}\right)\right]^{-1} = \frac{1}{N_d}\sum_{k=1}^{N_d}\left[1-2a\left(\sum_{j\in J_{A,k}}\left[1+k_{w,\infty}\frac{x_{T,k}-x_j}{R}\right]^{-4}\right)^{1/2}\right]$$

$$z_{0,hi} = z_h\left(1+\frac{D}{2z_h}\right)^{\beta}\exp\left(-\left[\frac{\pi C_T}{8\kappa^2 w_s s_x s_y}+\left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

Model comparisons with LES and field data (Horns Rev)

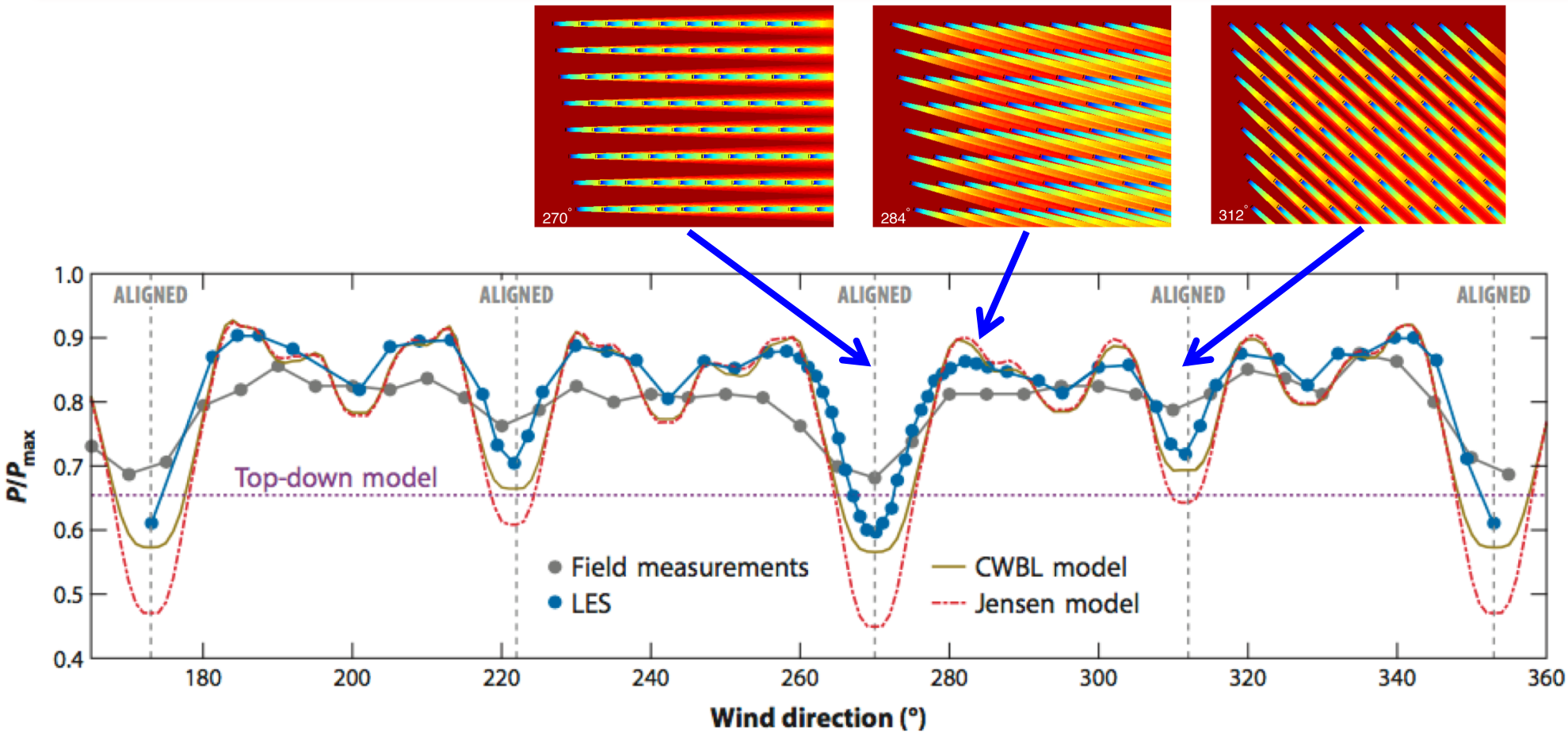


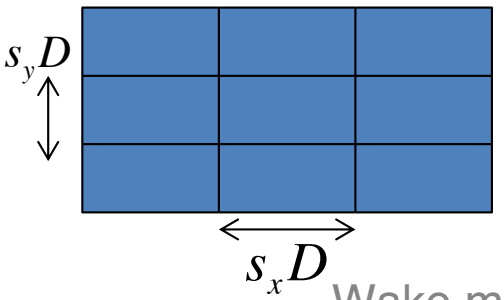
Figure 8

Normalized total power output P/P_{max} of Horns Rev wind farm as a function of the incoming wind direction, where P_{max} is the power of a non-wake-affected turbine times the number of turbines. The field measurement data are digitally extracted from the figure on page 25 of Peña et al. (2013). The large-eddy simulation (LES) results are from Porté-Agel et al. (2013); the coupled wake boundary layer (CWBL), Jensen, and top-down results are from Stevens et al. (2016b). Figure adapted with permission from Stevens et al. (2016b, figure 8).

Stevens & M, Annu Rev Fluid Mech (2017)

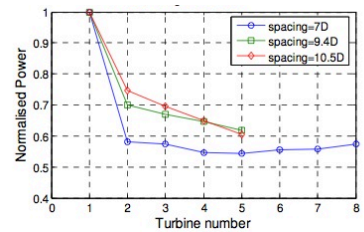
Model comparisons with LES

CWBL model: distinguishes well between aligned & staggered cases



$$\left(\frac{U_h(\infty)}{U_{h-0}}\right)^3$$

$$s = \sqrt{s_x s_y}$$

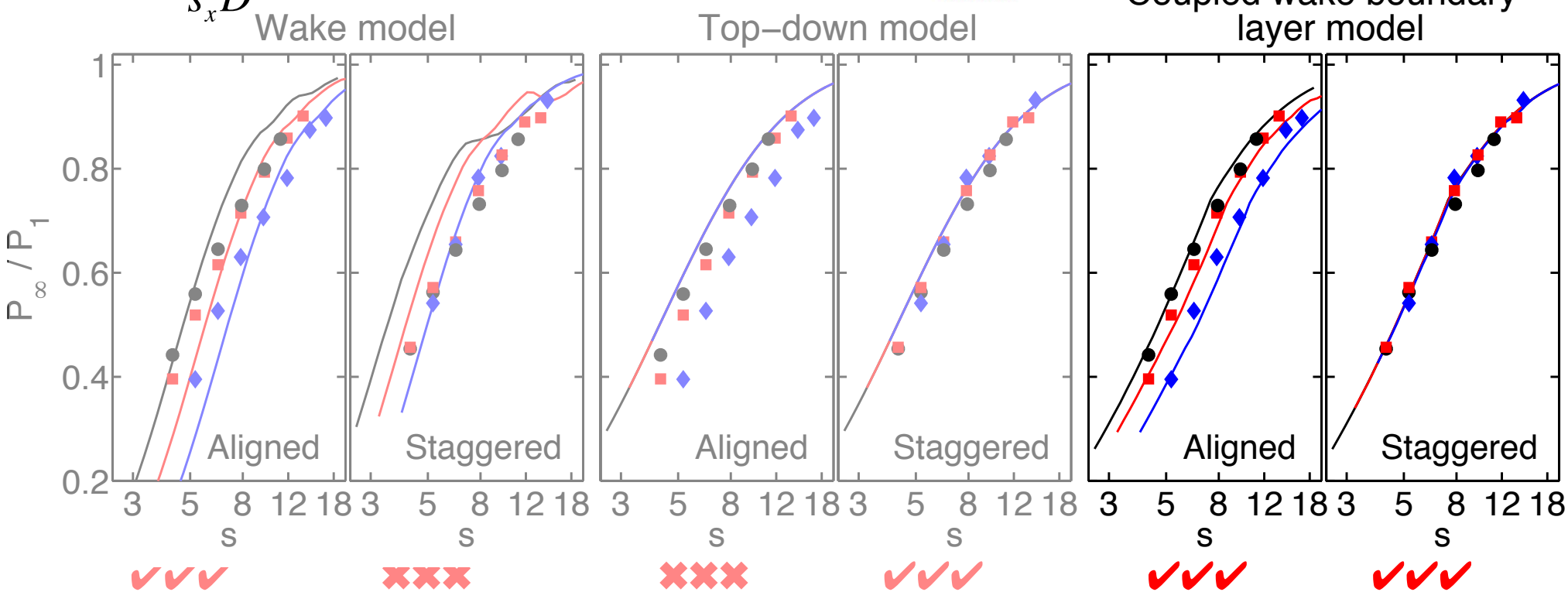


Color indicates spanwise spacing

s_y=3.49

s_y=5.23

s_y=7.85



Stevens, Gayme, Meneveau, WE (2016)

O : LES

------: CWBL

Fluctuations:

cost function:

- additional grid services, e.g. secondary frequency regulation and control ..
  (e.g. receding horizon control based on a dynamic wake model, C. Shapiro, CM & D. Gayme (Wind Energy 2017)

- increased O&M costs due to unsteady loading

We need analytical models of spectral properties as function of wind farm design parameters (spacing, etc..)

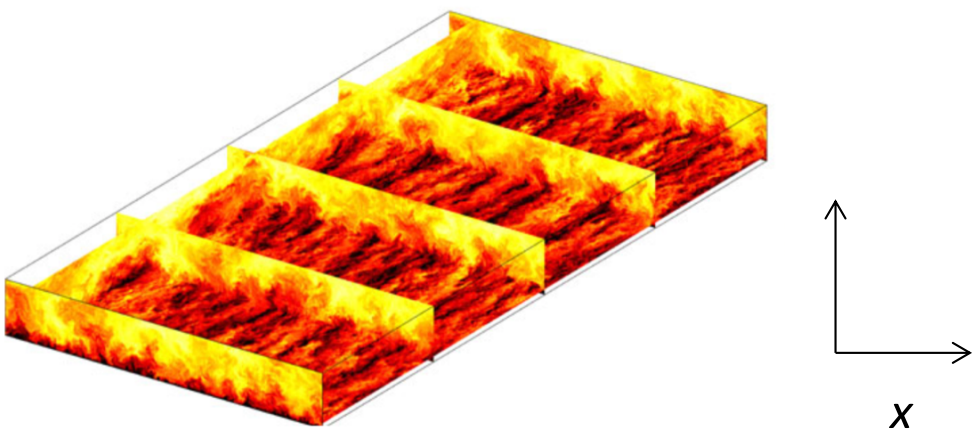
$$P_{tot}(t) = \sum P(t; s_x, s_y, z_h, layout, C_T ..)$$

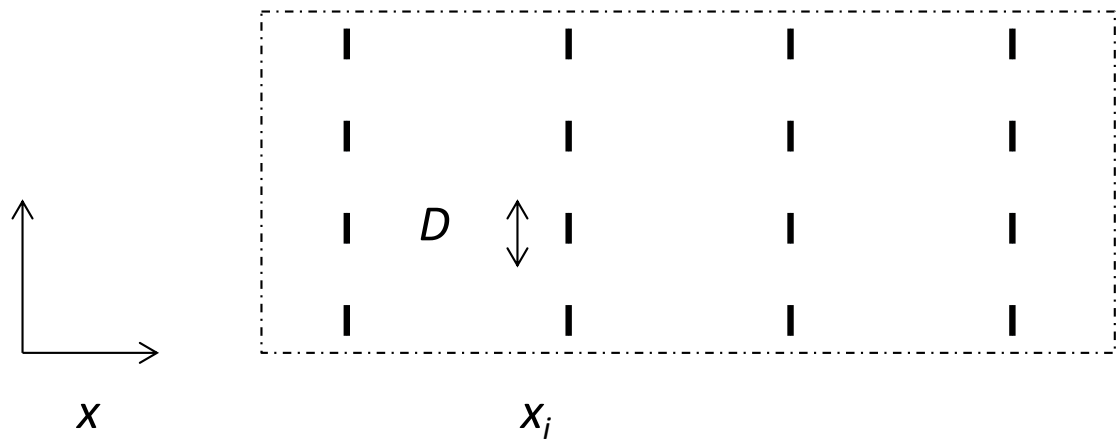
Interpret power as discrete sampling of TBL:

$$P_i(t) = \overline{P}_i + P_i'(t) \approx U^3 + 3U^2 u'(t) + h.o.t. \qquad P'_{\mathrm{WF}} = \bar{\sum} P'_i \approx C_2 \sum \langle u \rangle'_i$$

(linearlization, but see Bandi (PRL 2017)

"Transfer function": $g(x,y) = \sum_{i=1}^{N} \delta(x - x_i) \frac{1}{D} H\left(\frac{D}{2} - |y - y_i|\right).$





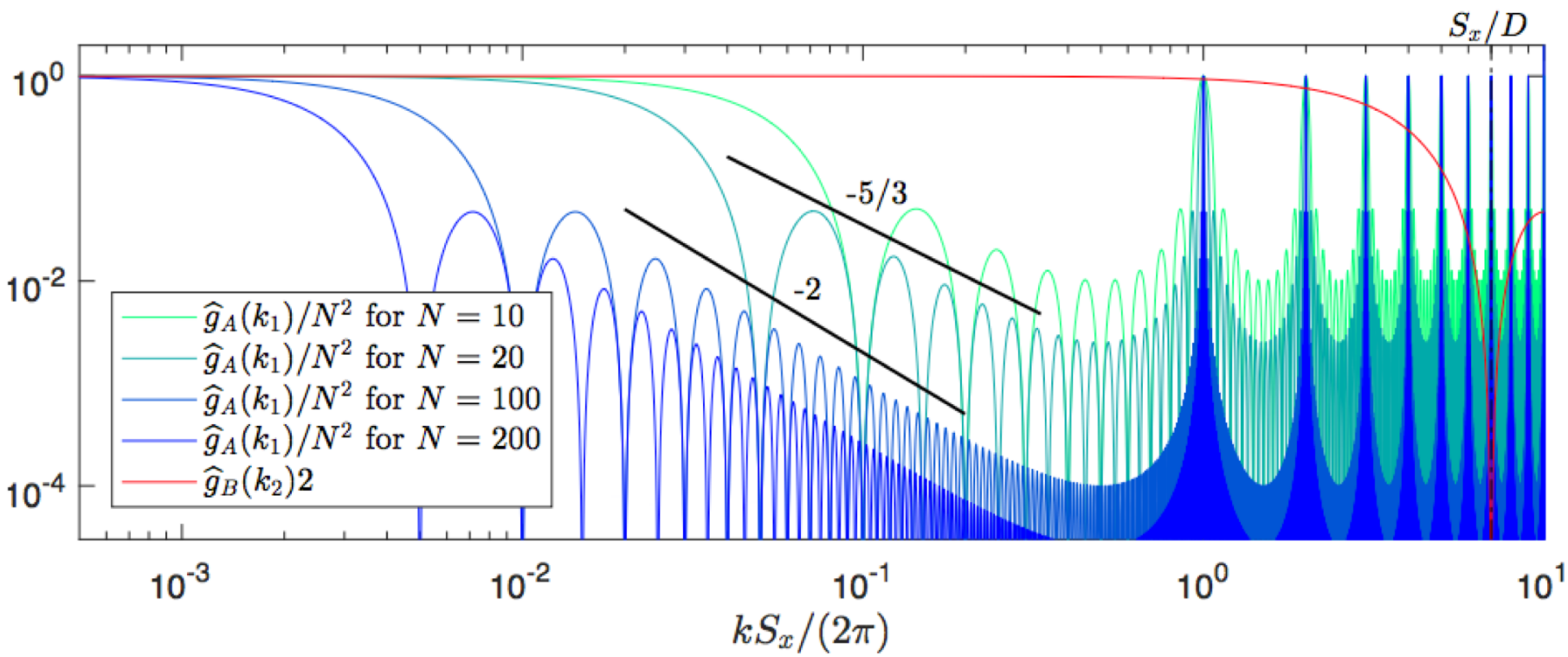
$$E_{P'}(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{|\hat{g}(k_1,k_2)|^2}_{} \underbrace{E_{11}(k_1,k_2,\omega,z_h)}_{} \, dk_1 dk_2$$

Needed:

Transfer function of turbine array (spacing, layout)

$$g(x,y) = \sum_{i=1}^{N} \delta(x - x_i) \frac{1}{D} H\left(\frac{D}{2} - |y - y_i|\right).$$

$$|\widehat{g}(k_1,k_2)|^2 = \left(\frac{\sin\left(k_2 \frac{D}{2}\right)}{k_2 \frac{D}{2}}\right)^2 \left(\sum_{i=1}^{N}\sum_{j=1}^{N} \cos\left(k_1\left(x_i - x_j\right) + k_2\left(y_i - y_j\right)\right)\right).$$



Analytical model for wave#-freq spectrum of BL turbulence $E_{11}(k_x,k_y,w;z_h)$:

$$E_{11}(k_x,k_y,\omega;z)=\left\{\left[1-\theta_\alpha\right]A\left[\left(\frac{1}{H}\right)^4+k_x^4\right]^{-1/4}+\theta_\alpha\frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\Gamma\left(\frac{5}{6}\right)}C_K\varepsilon^{2/3}\left[1-\frac{8}{11}\frac{k_x^2}{k^2}\right]k^{-8/3}\right\}\left[2\pi\sigma^2(z)\right]^{-1/2}\exp\left[-\frac{(\omega-\mathbf{k}\cdot\mathbf{U})^2}{2\sigma^2(z)}\right]$$

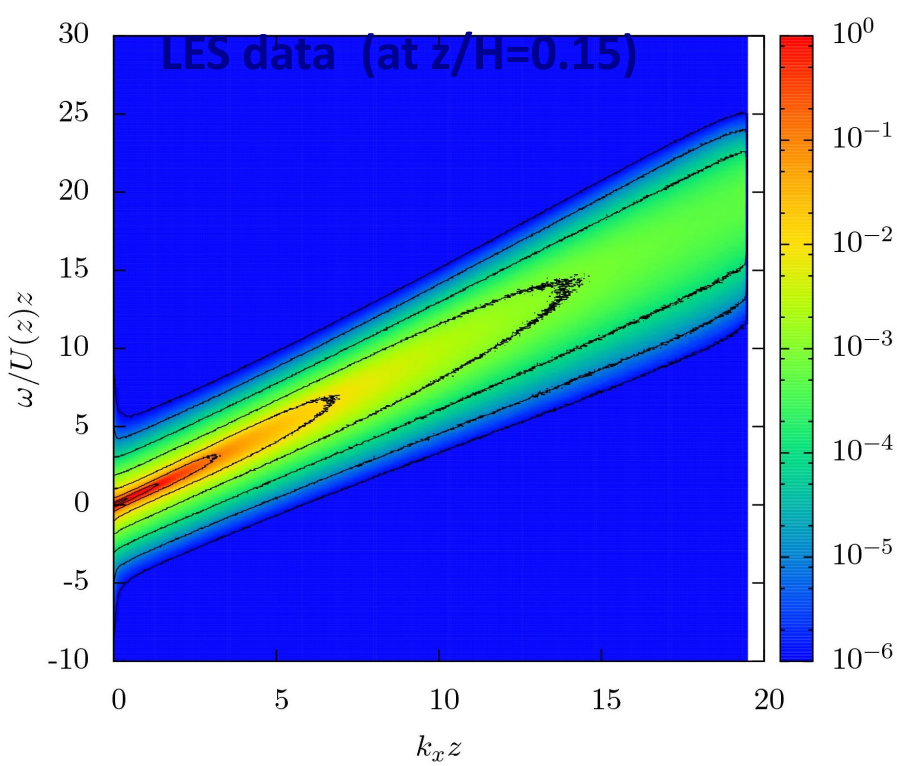
$$U(z)=\frac{u_*}{\kappa}\log\left(\frac{z}{z_0}\right)$$

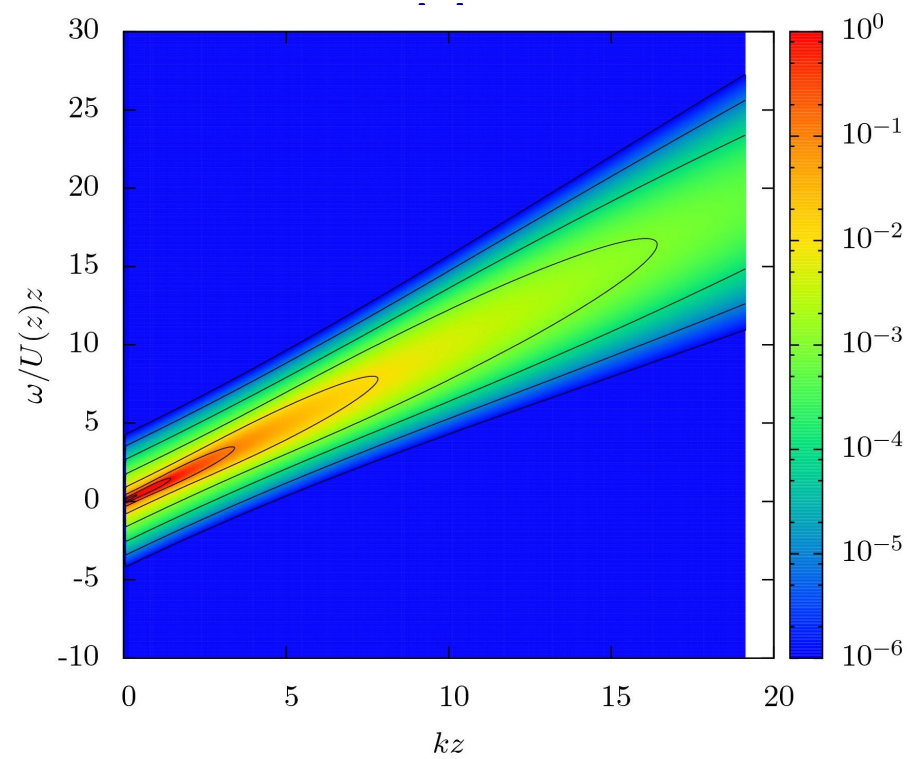
$$\left\langle v_x^2\right\rangle=u_*^2\left[B-A\log\left(\frac{z}{H}\right)\right]$$

Wilczek et al. JFM 2015

$$\sigma^2(z)=\left\langle(\mathbf{v}\cdot\mathbf{k})^2\right\rangle=\left\langle v_x^2\right\rangle k_x^2+\left\langle v_y^2\right\rangle k_y^2$$

$$\sigma^2(z)=\left\langle v_x^2\right\rangle\left[k_x^2+Ck_y^2\right],\quad A=0.96,\quad B=2.41,\quad C=0.33,\quad \kappa=0.4$$





Wind tunnel tests in a "micro-windfarm" in the Corrsin wind tunnel: (Juliaan Bossuyt's thesis, KU Leuven)

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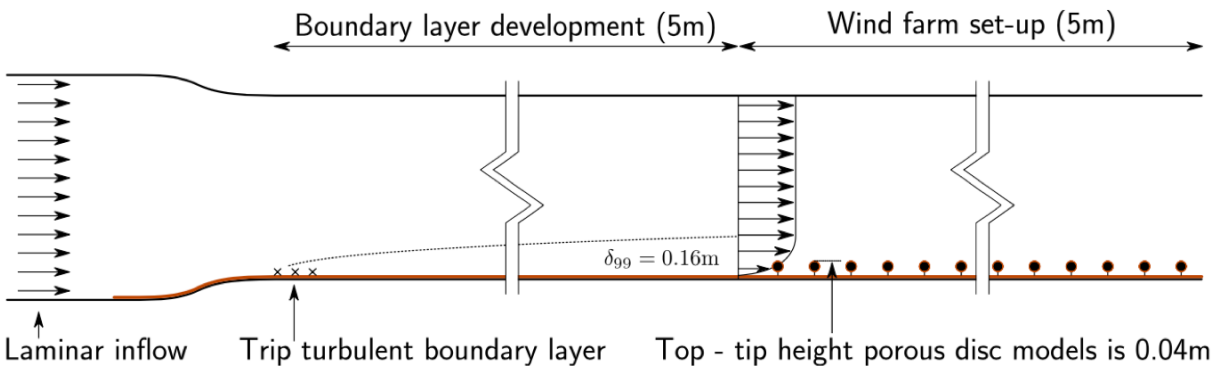
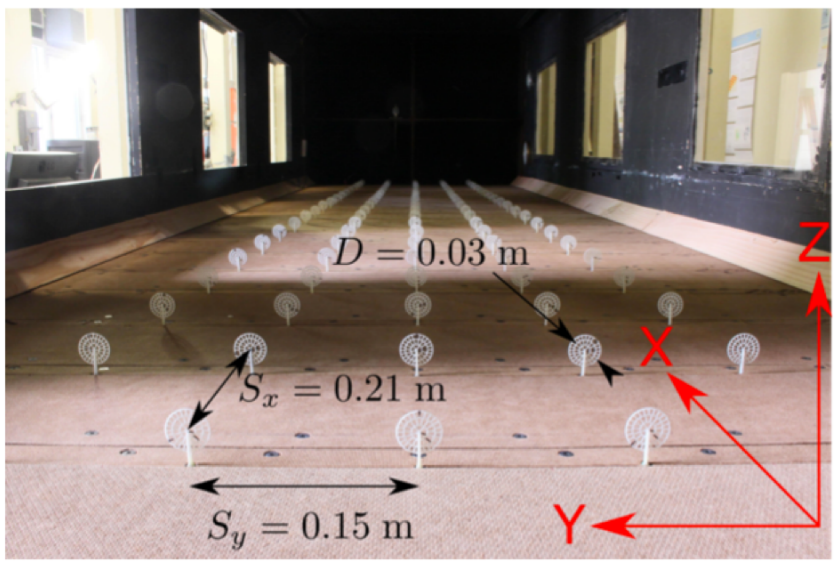
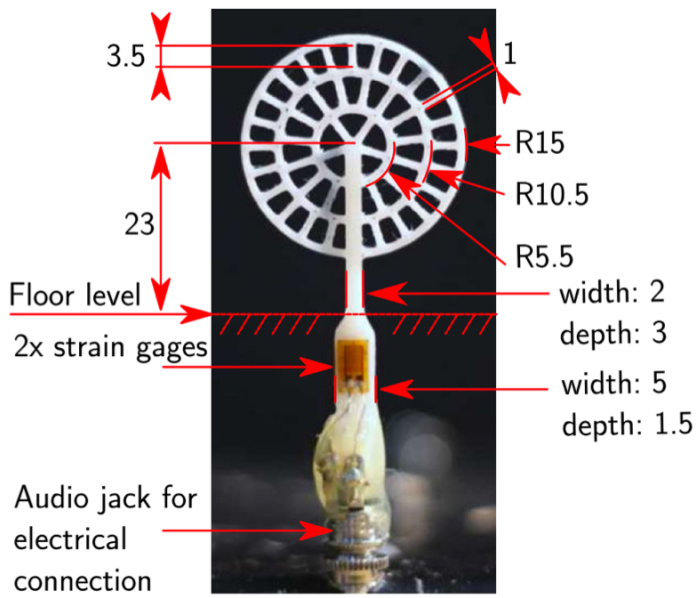




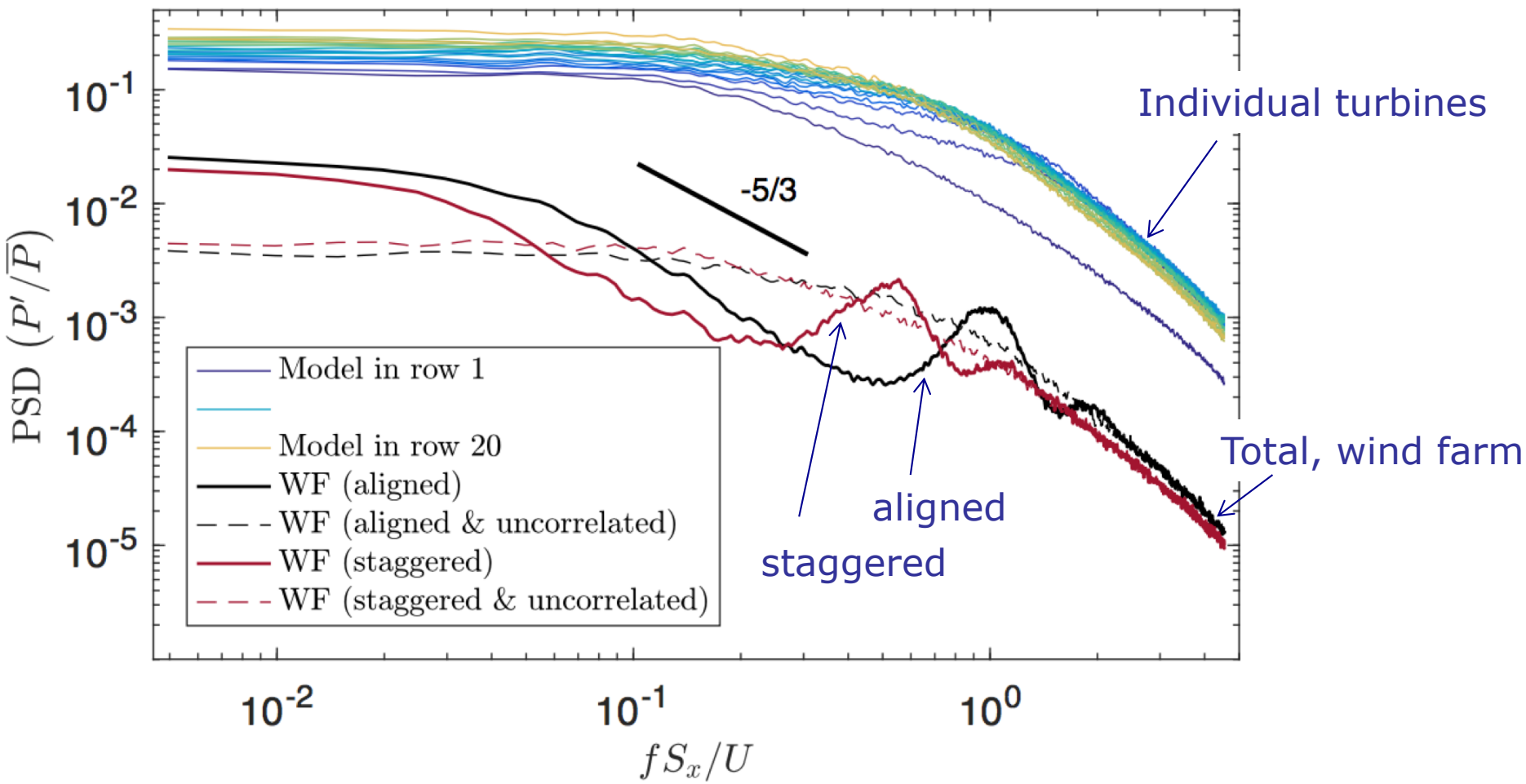


Flyby over micro-windfarm in Corrsin wind tunnel at JHU (staggered)

Power-spectral density of "power" fluctuations:

$$F_i(t) = \frac{1}{2}\rho C_T A \langle u(t) \rangle_i^2 \qquad P_i(t) = \frac{1}{2}\rho C_P A \langle u(t) \rangle_i^3 \approx \frac{C_P}{C_T}(\tfrac{1}{2}\rho A)^{-1/2} F(t)_i^{3/2}$$

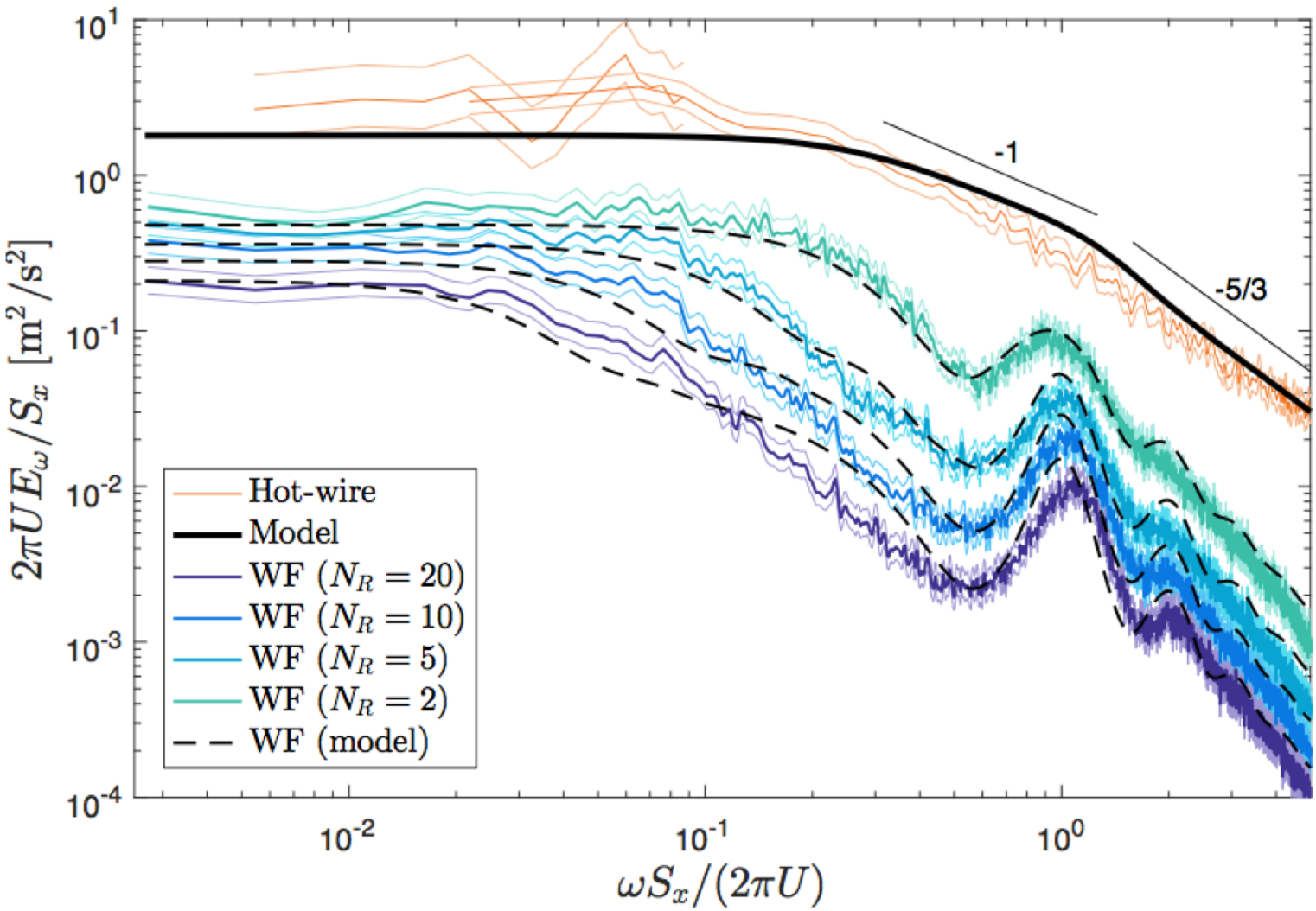
(~ valid in region II)



Bossuyt et al. JFM (2017)

Comparisons of measured and model spectra

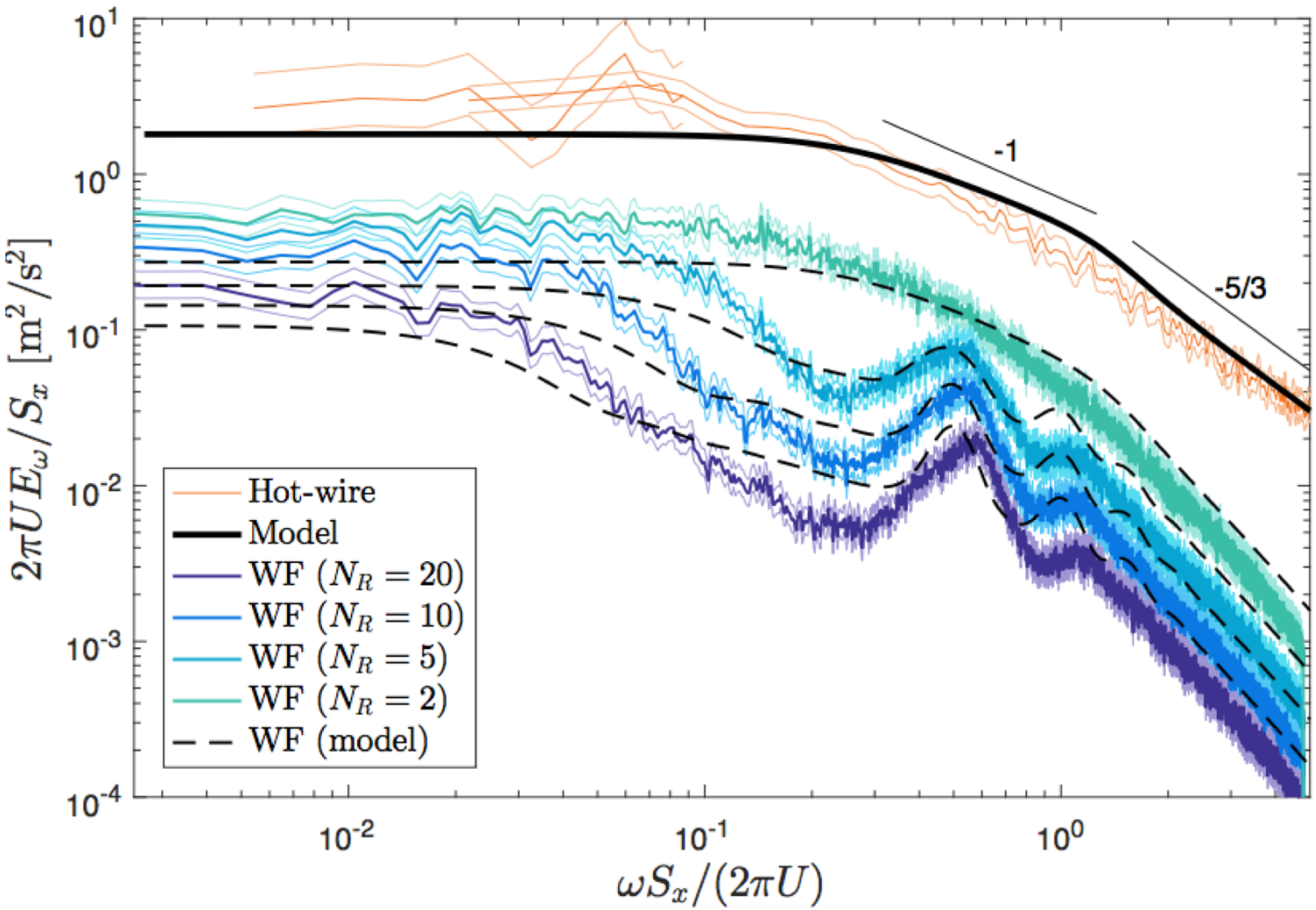
Aligned array (lines: data, dashed line: model)



Bossuyt et al. JFM (2017)

Comparisons of measured and model spectra

Staggered array (lines: data, dashed line: model)



Bossuyt et al. JFM (2017)

Closing thoughts:

- As we modify our energy infrastructure, wind energy will continue to increase & turbulence research plays a critical role for wind energy.
- Important for both for maximizing the resource utilization as well as minimizing impacts
- In these applications we found “very classical” approaches (momentum theory, eddy viscosity, log law, wake growth) useful if properly adapted to the new conditions.
- We must continue to teach and actively absorb these fundamentals (even if they are not able to predict details and decimal places)