Universitas Carolina

Charles University in Prague

SINCE 1348

Introduction to quantum turbulence

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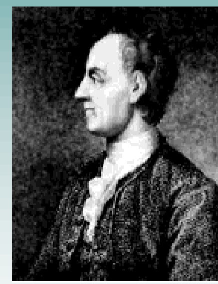
E. Varga, Š. Midlik, D. Duda

Turbulence - grand-challenge problem of our time, profound, difficult and important in a large variety of applications

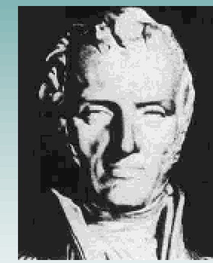
A long history…..

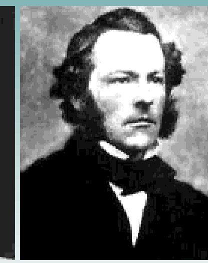
Leonardo da Vinci



Leonard Euler

$$\frac{\partial u}{\partial t} + (u\nabla)\,u + \nabla p = 0$$





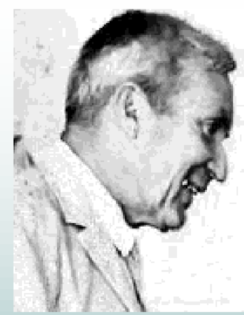
Claude Louis Marie Henri Navier

George Gabriel Stokes

$$\frac{\partial u}{\partial t} + (u\nabla)\,u + \nabla p = \nu \nabla^2 u$$

Osborne Reynolds

$$Re = \frac{\square}{\square}$$



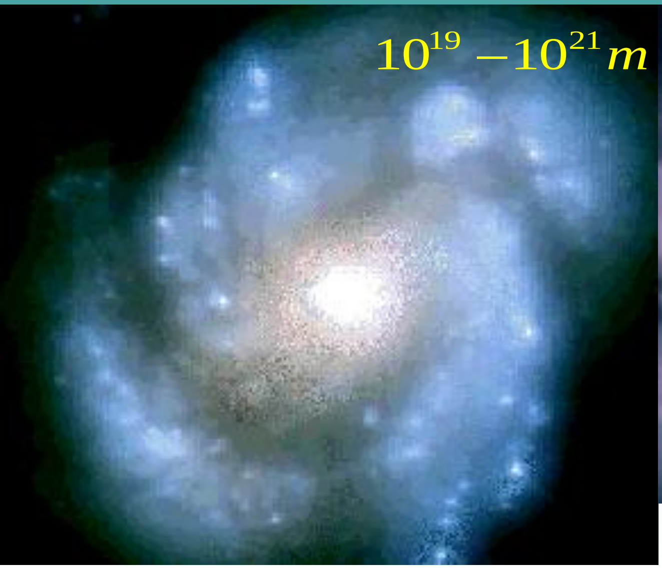
Andrey Nikolaevich Kolmogorov

K41

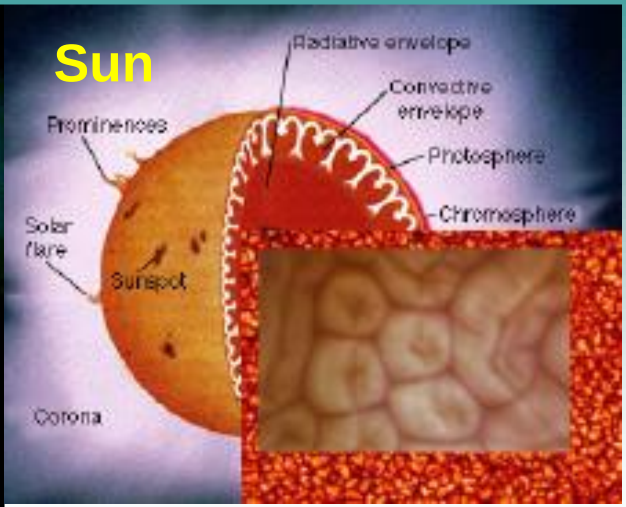
K62

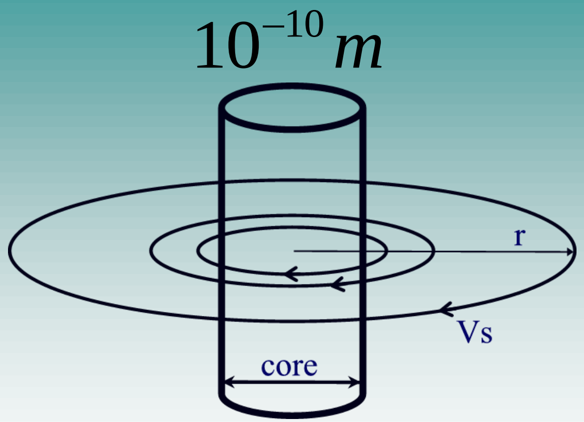
+ many more…

Characteristic length scales in turbulence



M100 Galaxy from the Hubble Space Telescope





Quantized vortex in He II

Quantum turbulence **occurs in** *quantum fluids*

- **Quantum fluids are so called because their physical properties cannot be explained by classical physics, they depend on quantum physics**

- **Quantum fluids (such as two stable isotopes of liquid helium at very low temperature) display superfluidity**

- **Quantum turbulence is concerned with turbulence in a superfluid: in a fluid in which flow is subject to severe quantum restrictions.**

- **Quantum turbulence can be defined loosely as the most general way of motion of a quantum fluid displaying superfluidity, that involves dynamical motion of tangles of thin quantized vortex lines**

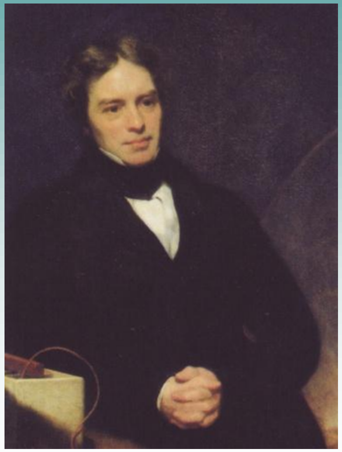
- **Low temperature physics, born on July 10, 1908 (the day of liquefaction of helium at 4.2 K) traditionally studies the properties of quantum fluids**

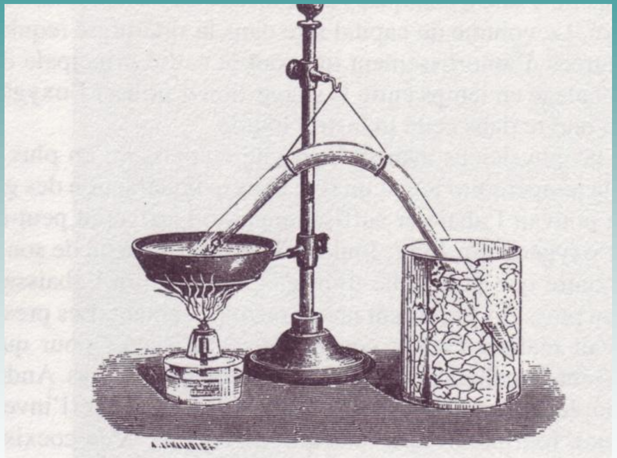
Low Temperature Physics, Cryogenics ----

Some basic facts....

Pre-history of helium liquefaction

Michael Faraday (1791 – 1867)

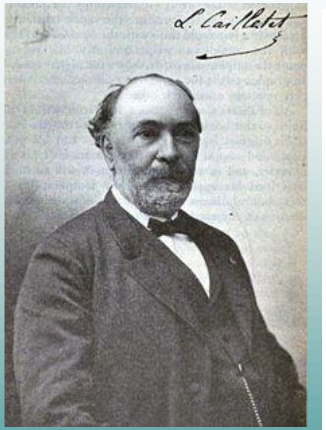


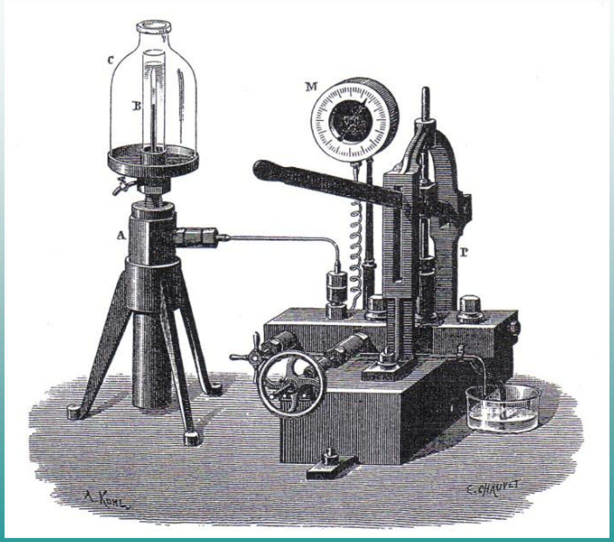


chlorine,
ammonia,
carbon
dioxide...

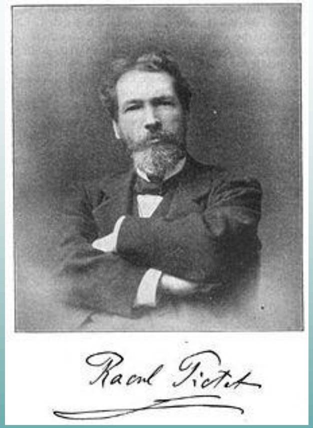
1877 – liquefaction of oxygen

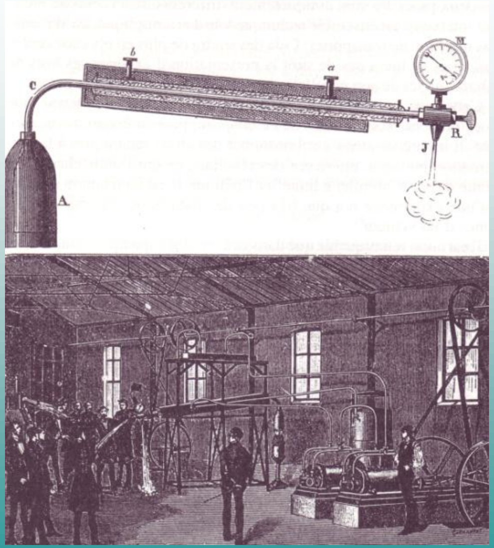
Louis – Paul Cailletet (1832 – 1913)





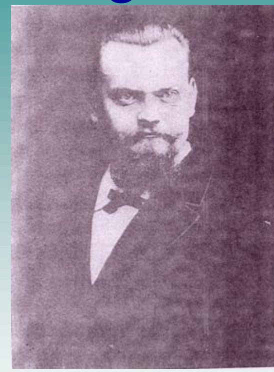
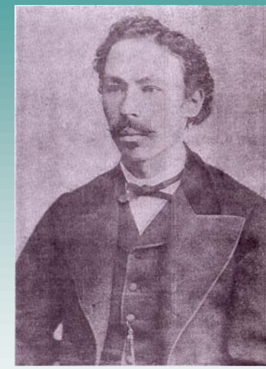
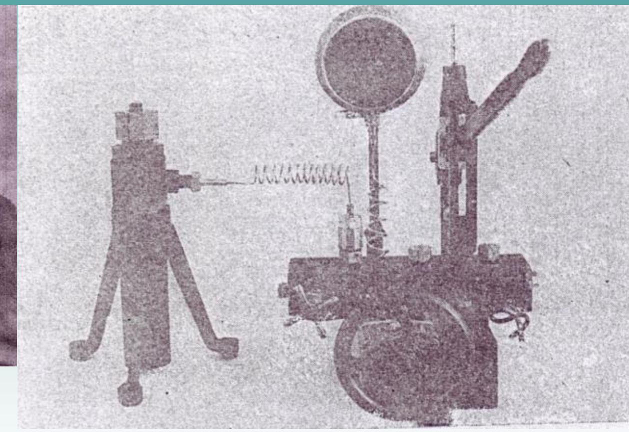
Raoul – Pierre Pictet (1846 – 1929)





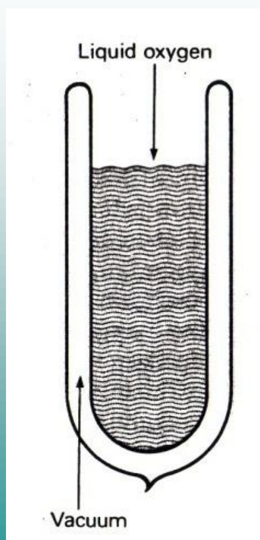
1883 liquefaction of nitrogen

University of Cracow
Zygmund Wróblewski (1845 – 1888)
Karol Olszewski (1846 – 1915)

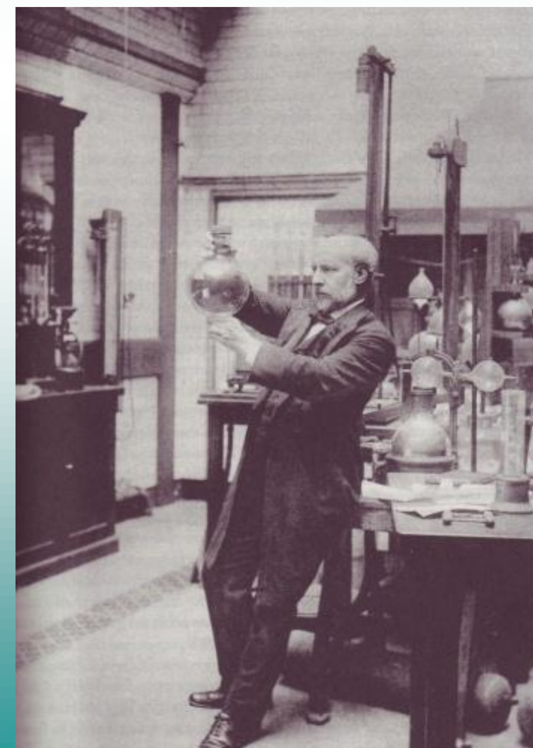




1898 liquefation of hydrogen

James Dewar (1842 – 1923)



1892 – invented vacuum – insulated vessels
- Dewar flask



July 10., 1908

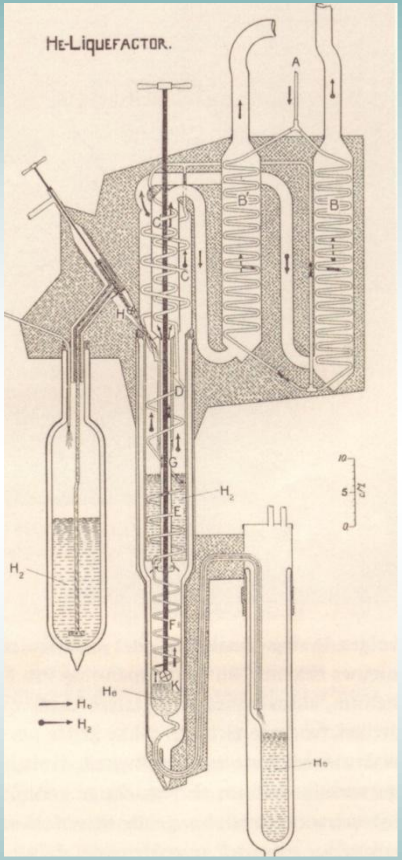
Helium liquified in Leiden - 4,2 K

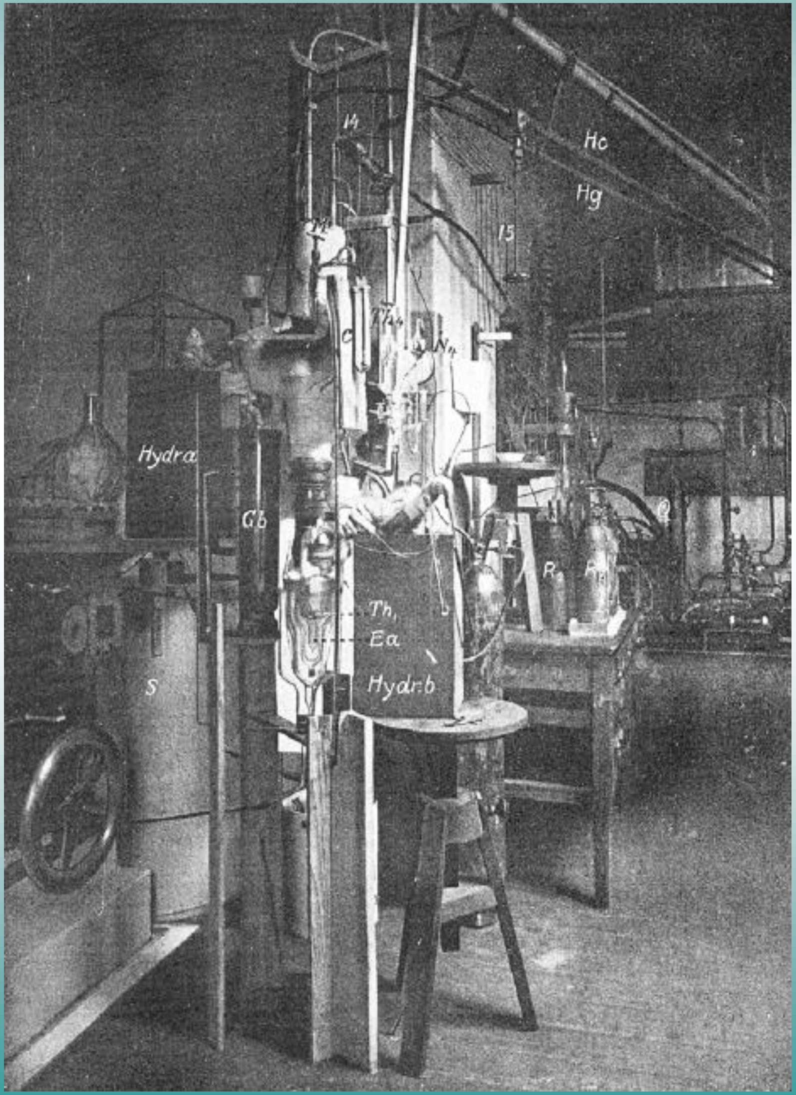
Beginning of low temperature physics

Heike Kamerlingh-Onnes
1853 – 1926 Leiden

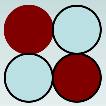
Nobel prize 1913

"for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium"





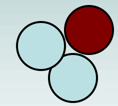
Helium



^{4}He

2 protons + 2 neutrons
2 electrons
Boson

Bose – Einstein quantum statistics



^{3}He

2 protons + 1 neutron
2 electrons
Fermion

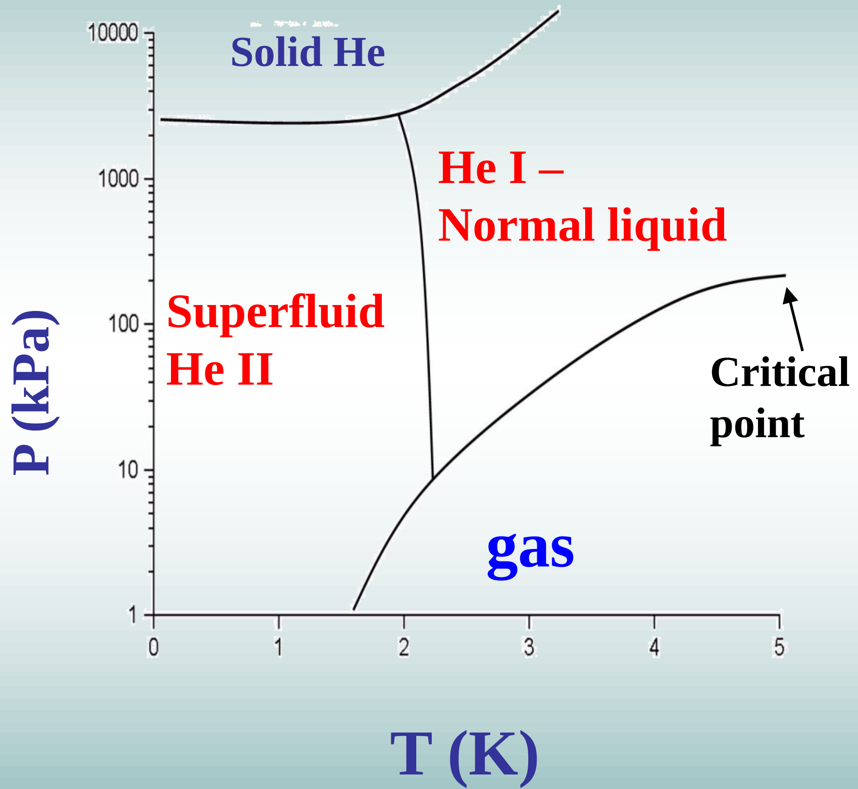
Pauli principle, Fermi- Dirac quantum statistics

Alcali atoms:
^{87}Rb, ^{85}Rb, ^{23}Na, ^{7}Li, + ^{1}H

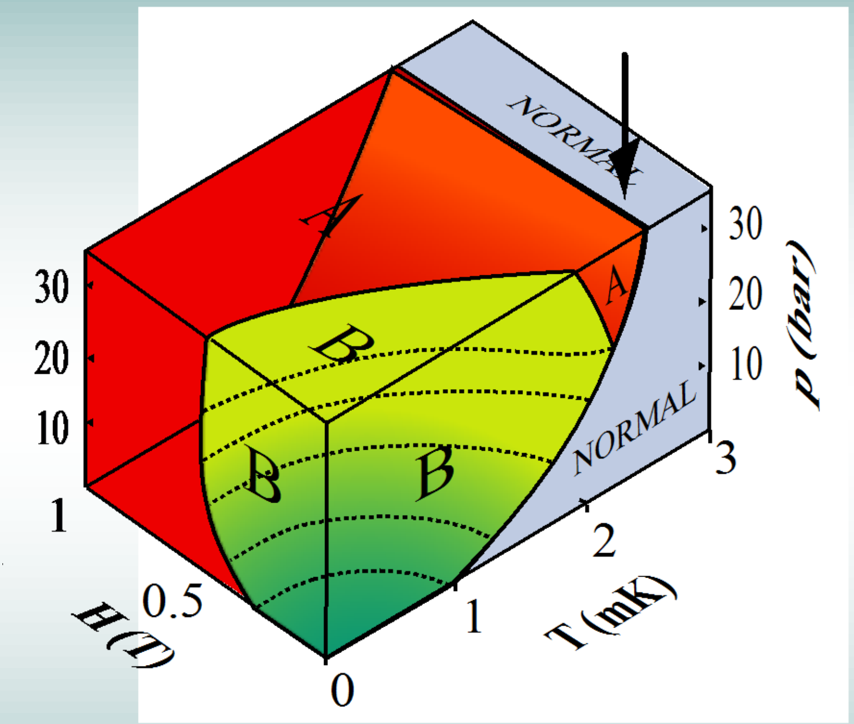
^{6}Li, ^{40}K, electrons in metals

At high temperature (300 K) both BE and FD → Boltzmann statistics
Both helium isotops behave almost as an ideal gas

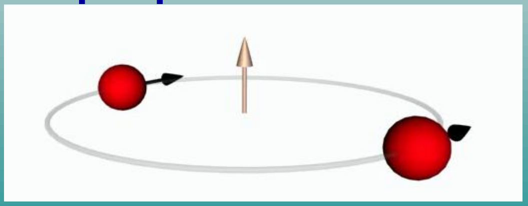
Phase diagram - ^{4}He



Phase diagram - ^{3}He



Cooper pair of 3He atoms



Measures of turbulence intensity

Reynolds number $Re = \frac{UL}{\nu}$

For isothermal flows

Rayleigh number $Ra = \frac{g\alpha\Delta T L^3}{\nu\kappa}$

for thermally driven flows in a gravitational field

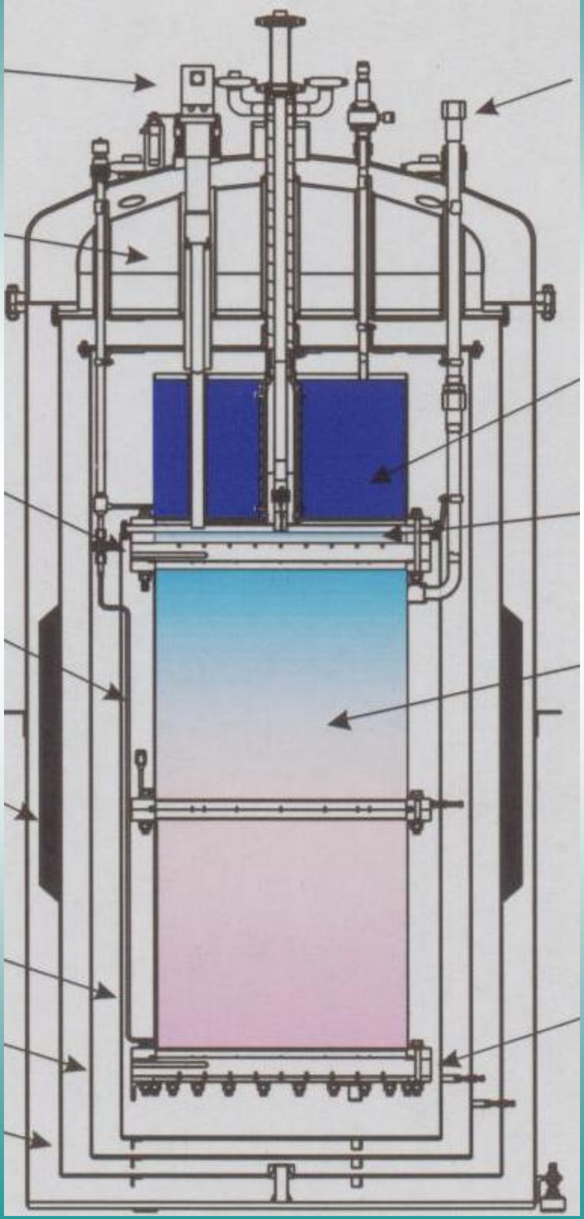
	Ra	Re
Sun	10^{21}	10^{13}
Ocean	10^{20}	10^{9}
Atmosphere	10^{17}	10^{9}
Navy (ship)		10^{9}
Aerospace (aircraft)		10^{8}-10^{9}

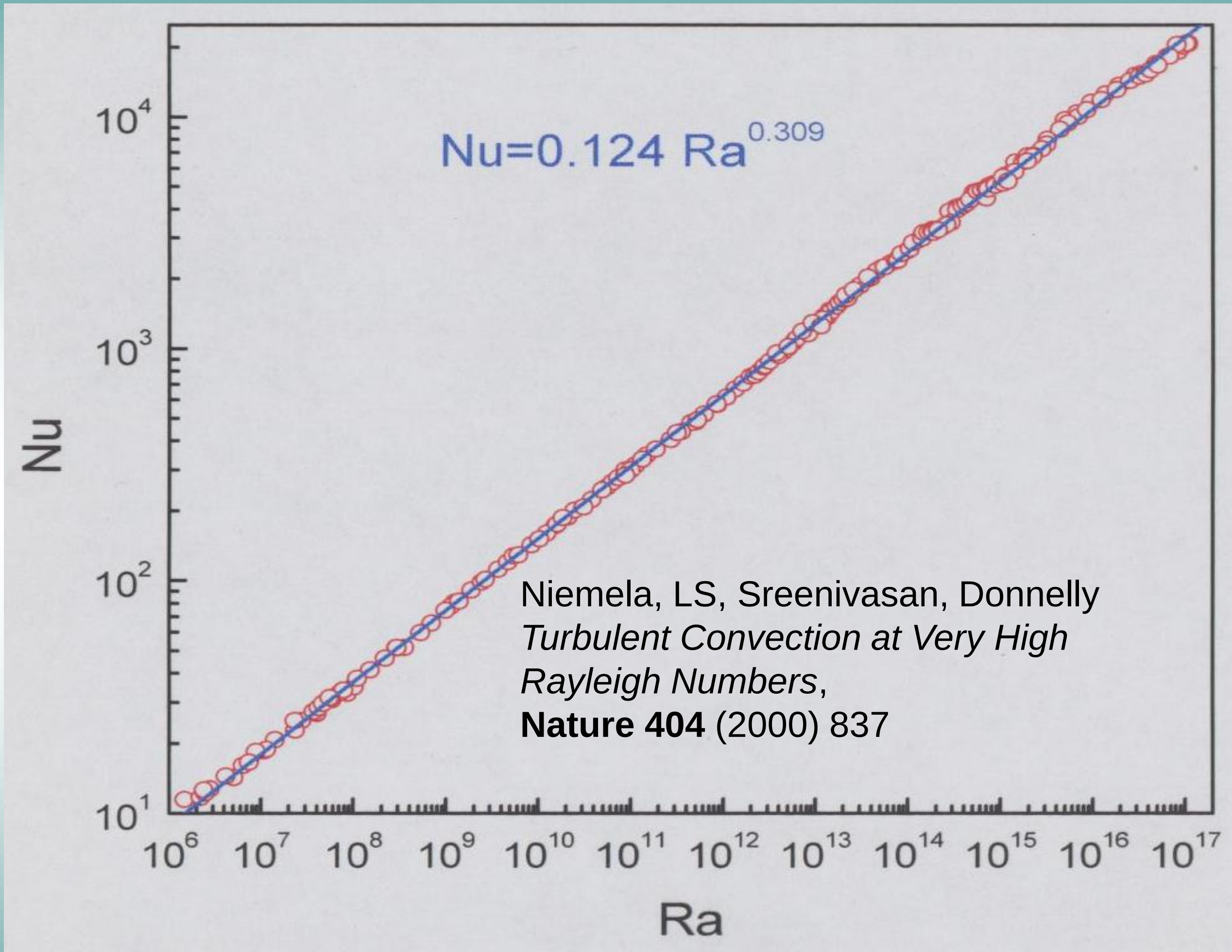
	T (p)	ν (cm²/s)	$\alpha/\nu\kappa$
air	20 C	0,15	0,122
water	20 C	$1{,}004\times10^{-2}$	14,4
Normal 3He	above Tc	~ 1, olive oil	
Normal fluid of 3He B	around 0.6 Tc	~ 0.2, air	
Helium I	2,25 K (SVP)	$1{,}96\times10^{-4}$	$3{,}25\times10^{5}$
Helium II	1,8 K (SVP)	$9{,}01\times10^{-5}$	X
He-gas	5,5 K (2,8 bar)	$3{,}21\times10^{-4}$	$1{,}41\times10^{8}$

- **Cryogenic He Gas, and normal liquid He I**

probably the best working fluids with tuneable properties (in situ) for the controlled, laboratory high Re and Ra turbulence experiments

Oregon/Trieste Cryogenic turbulent convection cryostat





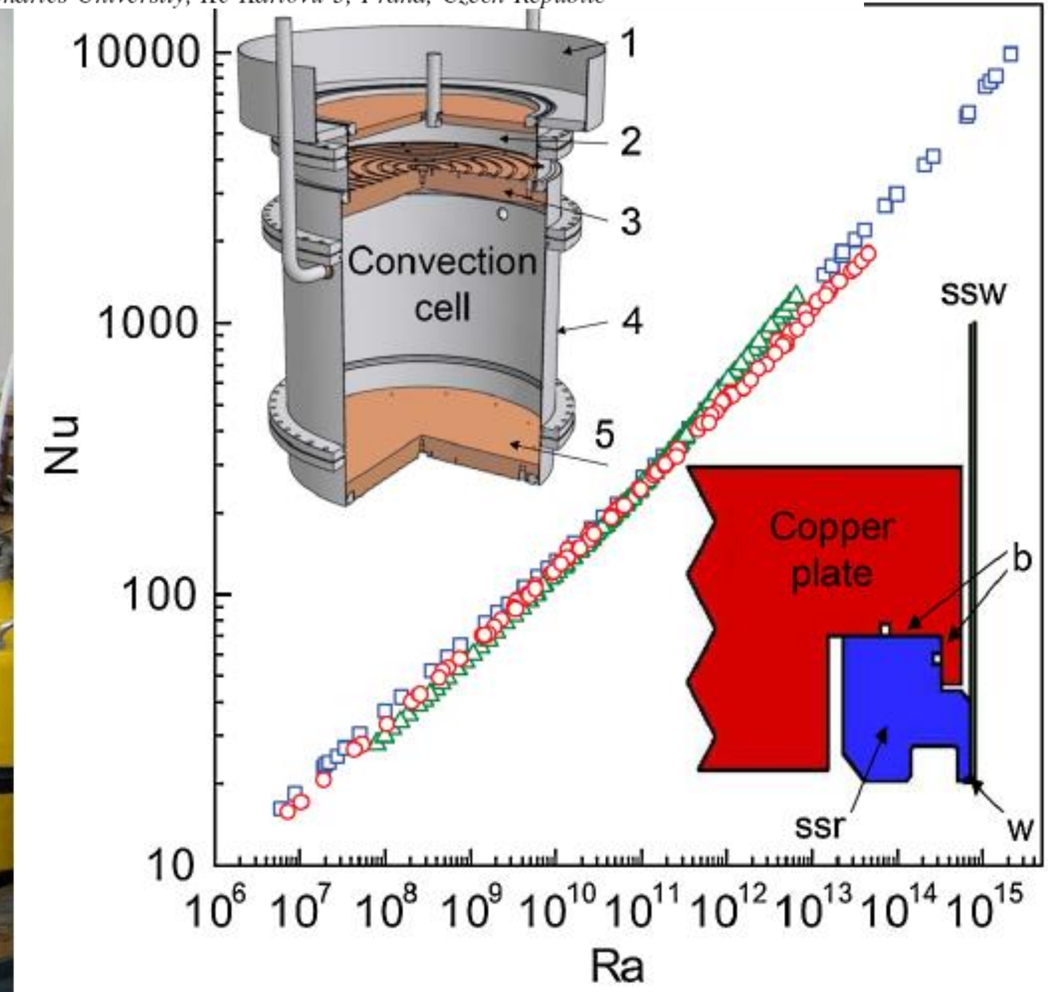
Niemela, LS, Sreenivasan, Donnelly
Turbulent Convection at Very High Rayleigh Numbers,
Nature 404 (2000) 837

Heat transfer efficiency in cryogenic turbulent convection

REVIEW OF SCIENTIFIC INSTRUMENTS **81**, 085103 (2010)

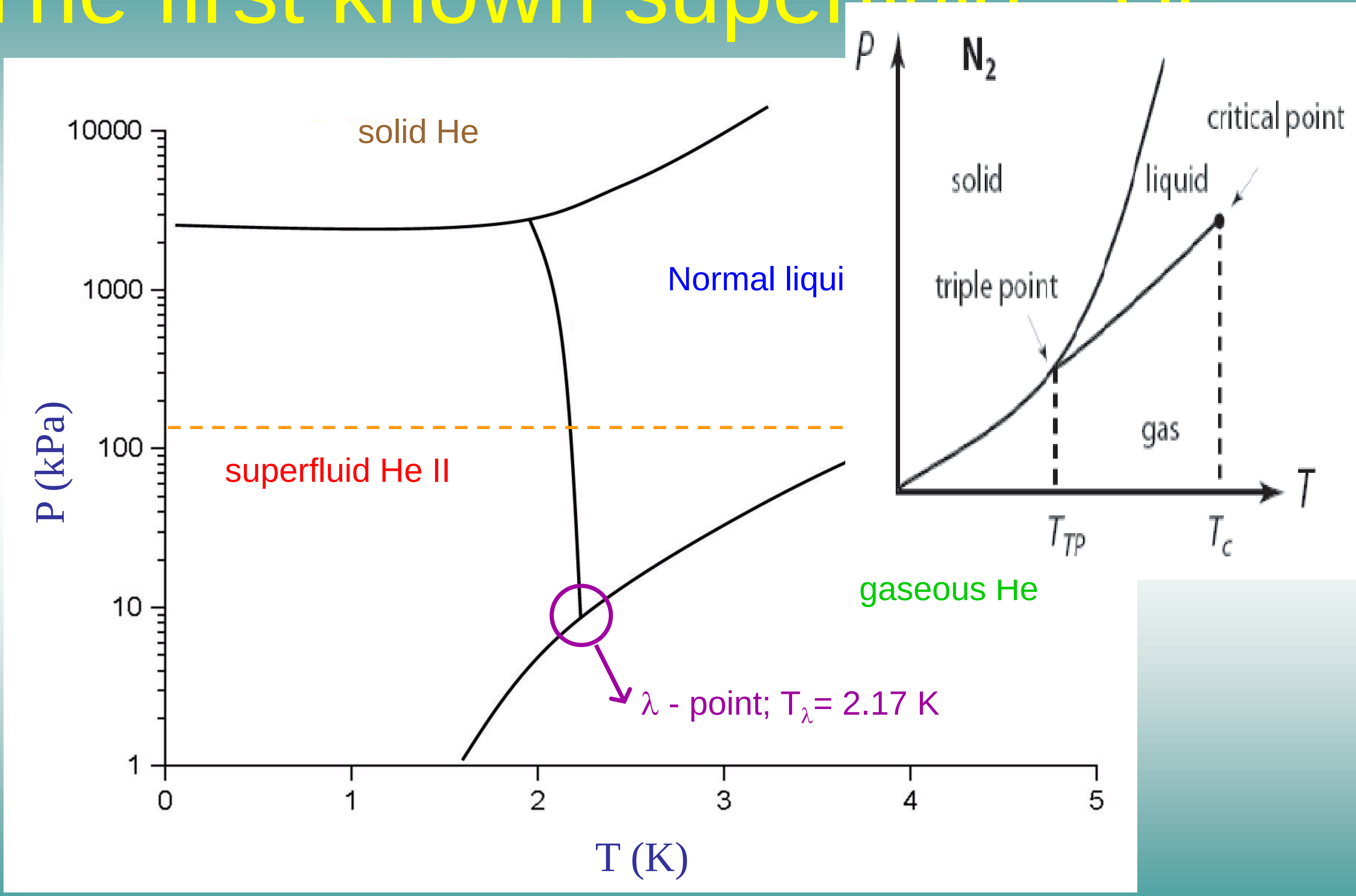
Helium cryostat for experimental study of natural turbulent convection

P. Urban,[1,a)] P. Hanzelka,[1] T. Kralik,[1] V. Musilova,[1] L. Skrbek,[2] and A. Srnka[1]
[1]*Institute of Scientific Instruments, ASCR, v.v.i., Kralovopolska 147, Brno 612 64, Czech Republic*
[2]*Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, Praha, Czech Republic*



Phys. Rev. Lett. **107**, 014302 (2011); **PRL 109**, 154301 (2012); PRL **110**, 199402 (2013);
New J. Phys. **16**, 053042 (2014), J. Fluid Mech. **785**, 270282 (2015),

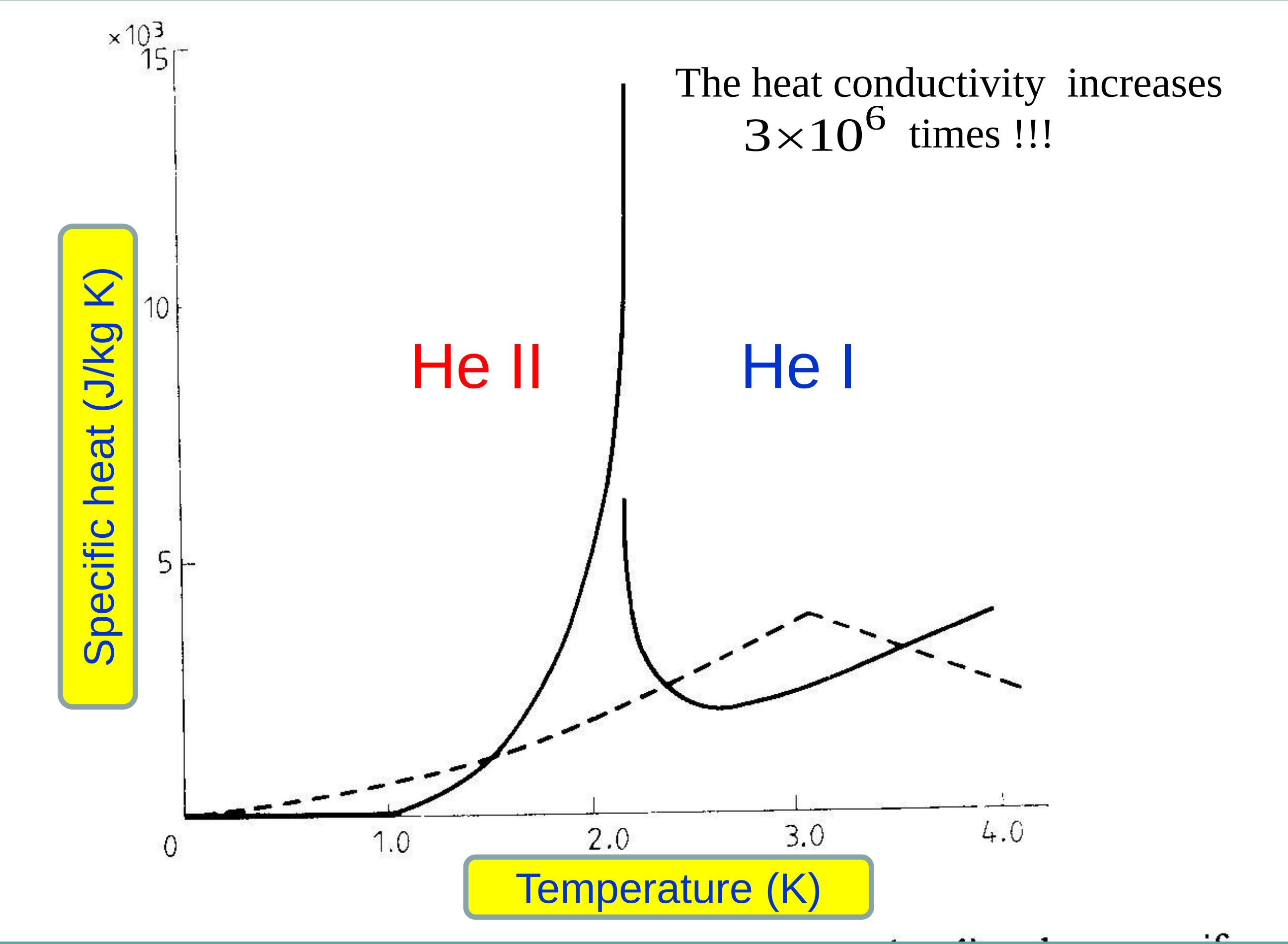
The first known superfluid: ^4He



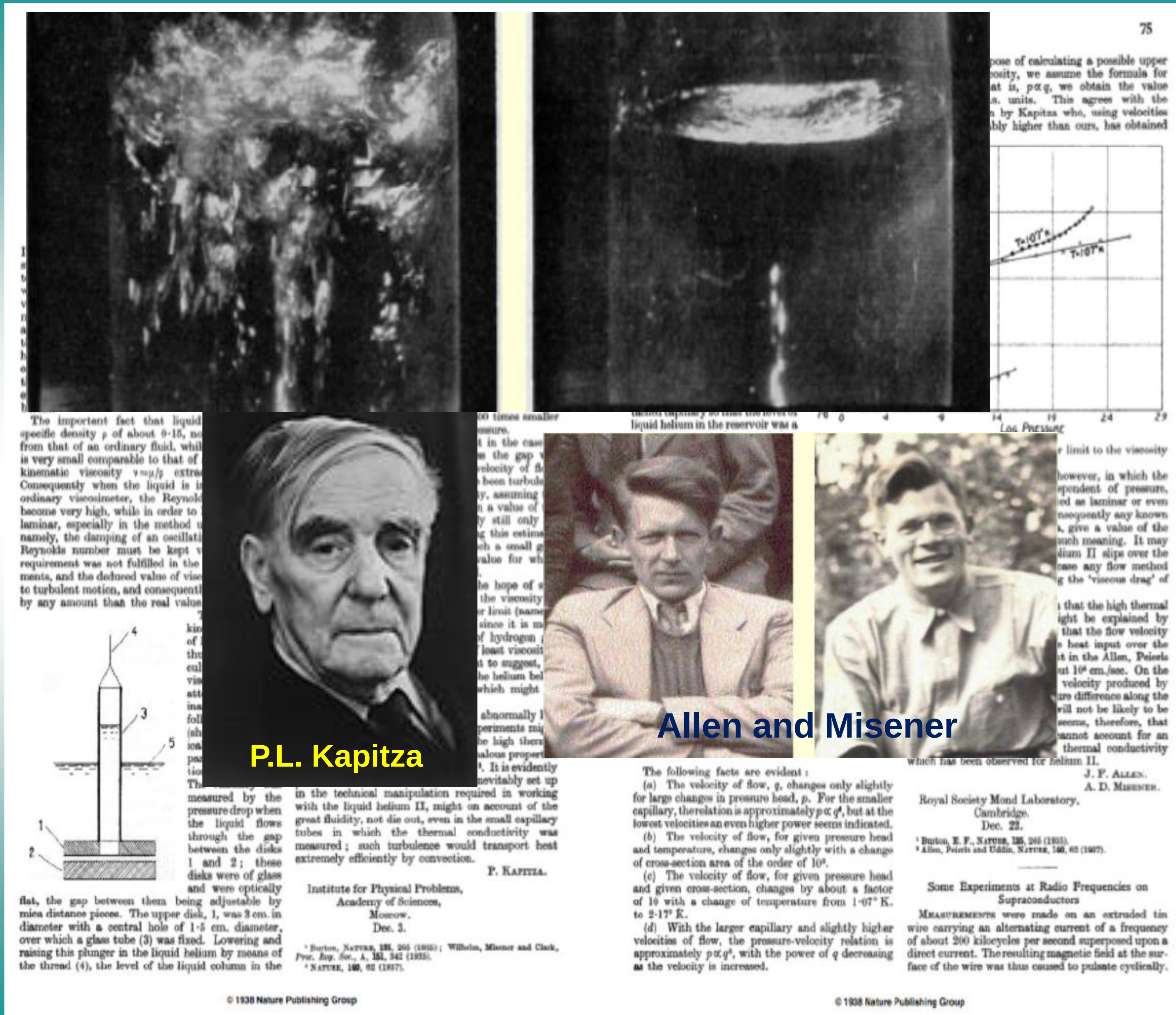
Various phases of liquid helium=quantum fluids

To explain its physical properties quantum mechanics is needed

Lambda transition-– thanks to the characteristic shape of the temperature dependence of the specific heat – liquid helium along the saturated vapour curve



Dashed line – theoretical dependence of the specific heat for the ideal Bose gas



P.L. Kapitza
Allen and Misener
P. Kapitza.
J. F. Allen.
A. D. Misener.
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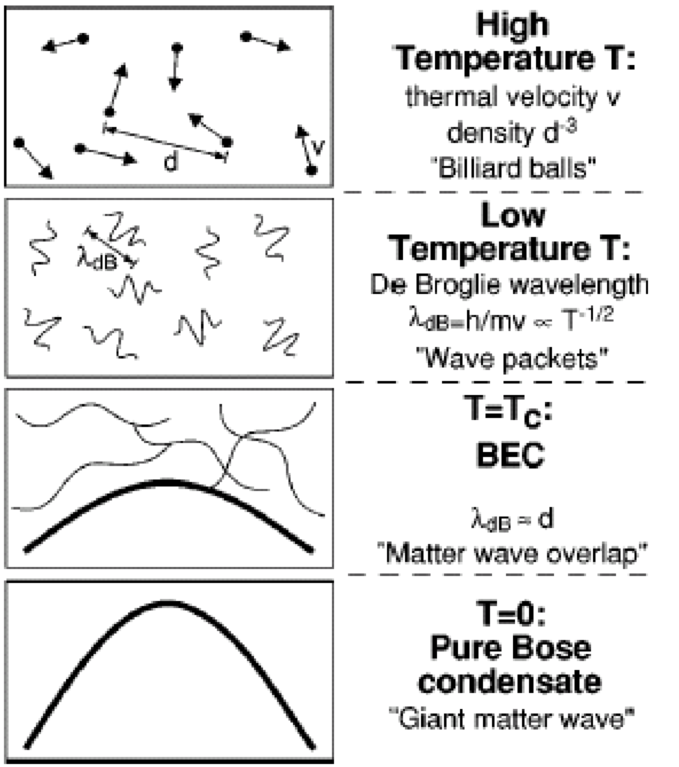
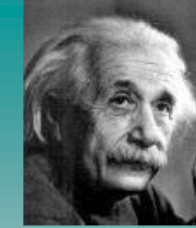
Ideal Bose gas



S. Bose



A. Einstein

Bose-Einstein quantum statistics

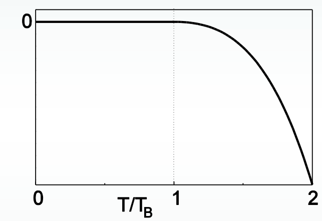
$$n_k = \frac{1}{\exp\left\{\dfrac{\varepsilon_k - \mu}{kT}\right\} - 1}$$

calibration

$$N = \frac{1}{V}\sum_{k=0}^{\infty} n_k$$

A. **Einstein 1924:** **(in 3D momentum space) below certain condensation temperature, macroscopically large number of particles will occupy the lowest energy state**

$$N = N_0 + \sum_{k=1}^{\infty} n_k = N_0 + \sum_{k=1}^{\infty} \frac{1}{\exp\left\{\dfrac{\varepsilon_k - \mu}{kT}\right\} - 1}$$



$$N = N_0 + \frac{4\pi}{(2\pi\hbar)^3}\int_0^{\infty} \frac{p^2}{\exp\left\{\dfrac{p^2}{2m_{He}} \Big/ kT_B\right\} - 1} dp = \frac{m_{He} kT_B}{2\pi^2\hbar^3}\sqrt{2m_{He}kT_B}\int \frac{\sqrt{z}\,dz}{e^z - 1}$$

N_0 → null

$$= \frac{\sqrt{\pi}}{2}\xi\left(3/2\right)$$

Rieman f-n

$$T_B = \frac{2\pi\hbar^2}{m_{He}k}\left(\frac{N}{\xi(3/2)}\right)^{2/3} \cong 3.15K$$

F. London

Experiment – He II

$$T_\lambda \cong 2.176\ K$$

Quantum mechanical description of He II

Macroscopic wave function $\Psi = \sqrt{\rho_s}\exp\{i\varphi(r,t)\}$ assuming incompressible flow

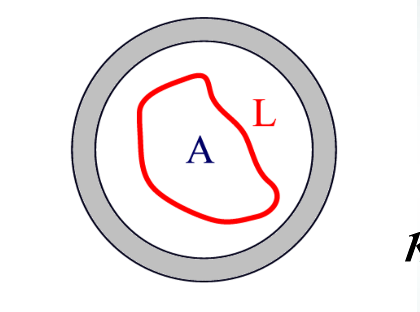
$$\hat{p} = i\hbar\nabla \longrightarrow \mathbf{v}_s = \frac{\hbar}{m_4}\nabla\varphi \longrightarrow curl\ \mathbf{v}_s = 0 \qquad div\ \mathbf{v}_s = 0$$

Maxwell's equations *in vacuo* for magnetic induction B are of the same form exactly: $div\, B = 0$; $curl\, B = 0$.

There is a striking and deep similarity between **superfluidity and electromagnetism !!!**

Circulation –singly connected region

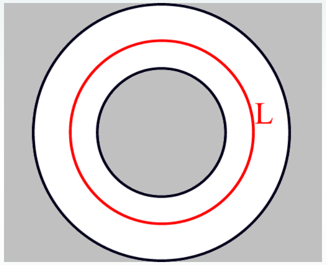
$$\Gamma = \oint_L \mathbf{v}_s d\ell = 0$$



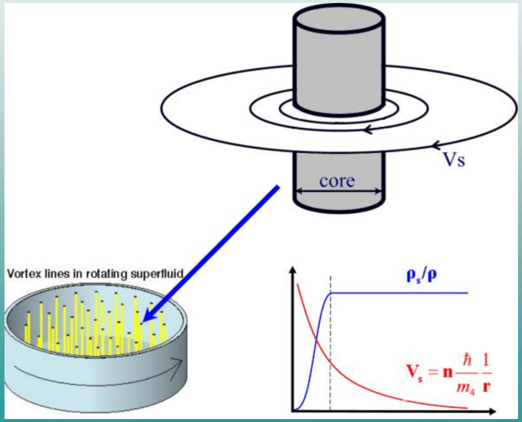
Circulation- multiply connected region

$$\Gamma = \oint_L \mathbf{v}_s d\ell = n\frac{h}{m_4} = n\kappa$$

$$\kappa \cong 10^{-7}\, m^2/s$$



Quantized vortices in He II



vorticity

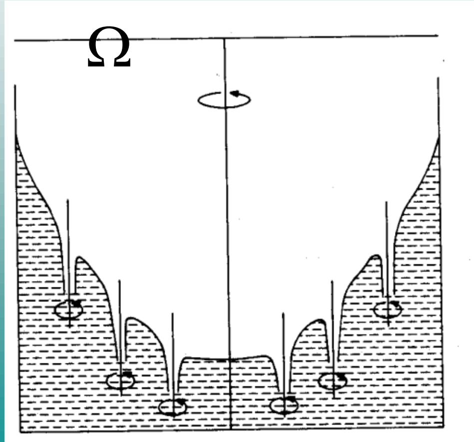
$$\omega_N = 2\Omega \cong \langle \omega_S \rangle \cong \kappa L$$

the superfluid velocity field is given by the Biot-Savart law where the integration takes place along all vortex lines.

$$v_s(r_0) = \frac{\kappa}{4\pi}\int \frac{(r - r_0)\times dr}{|\, r - r_0\,|^3},$$

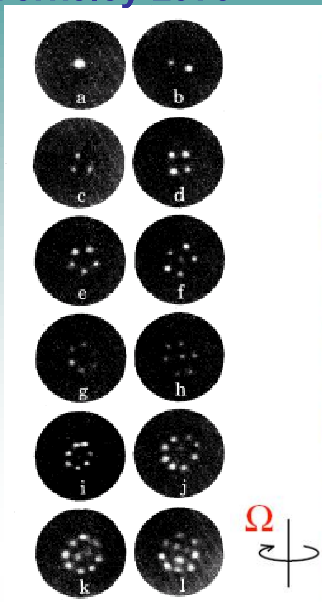
Rotating bucket of He II

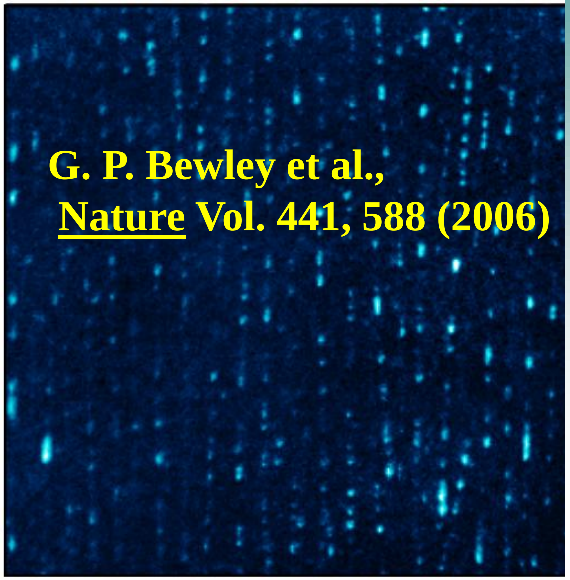
-thanks to the existence of rectilinear vortex lines He II mimics solid body rotation



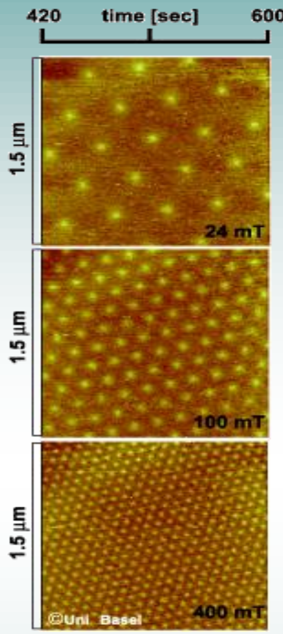
Images of quantized vortices in low temperature condensates

Rotating He-II
Berkeley 1979

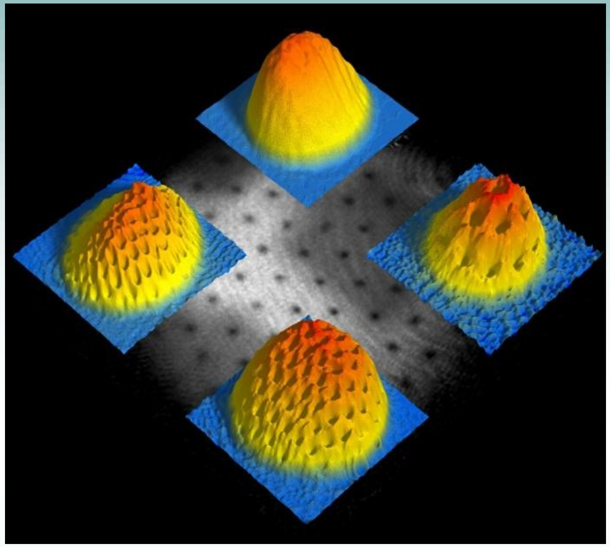




Superconductor
NbSe2
STM, Darmstadt



BEC
Vortex lattices
at MIT, 2001



Kelvin waves

Dispersion relation

$$\omega = \frac{\kappa k^2}{4\pi} \ln\left(\frac{1}{k\xi}\right)$$

Vortex nucleation

- **Intrinsic**

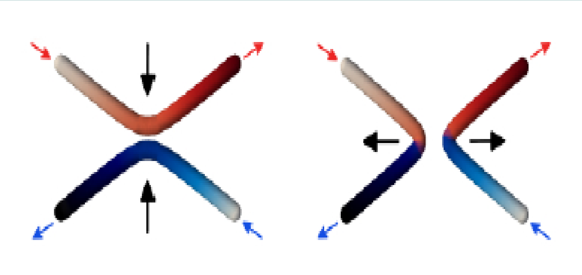
Moving ions
Kibble-Zurek scenario

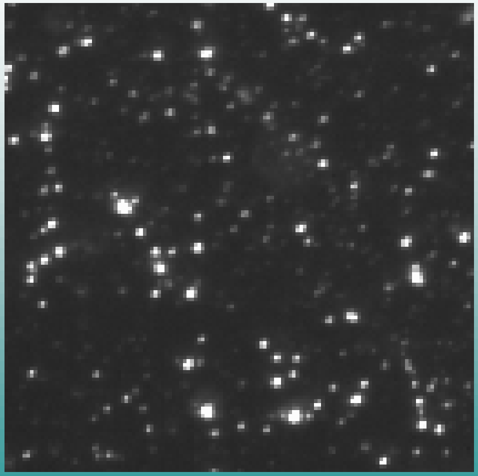
- **Extrinsic**

From existing seeds -
remnant vortices

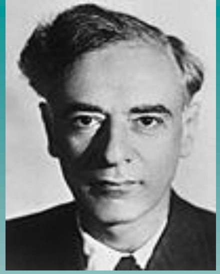
Reconnections

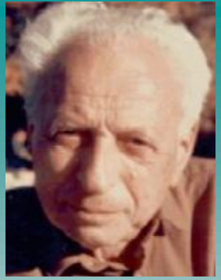
are allowed, important and frequent
First visualized by the Maryland group



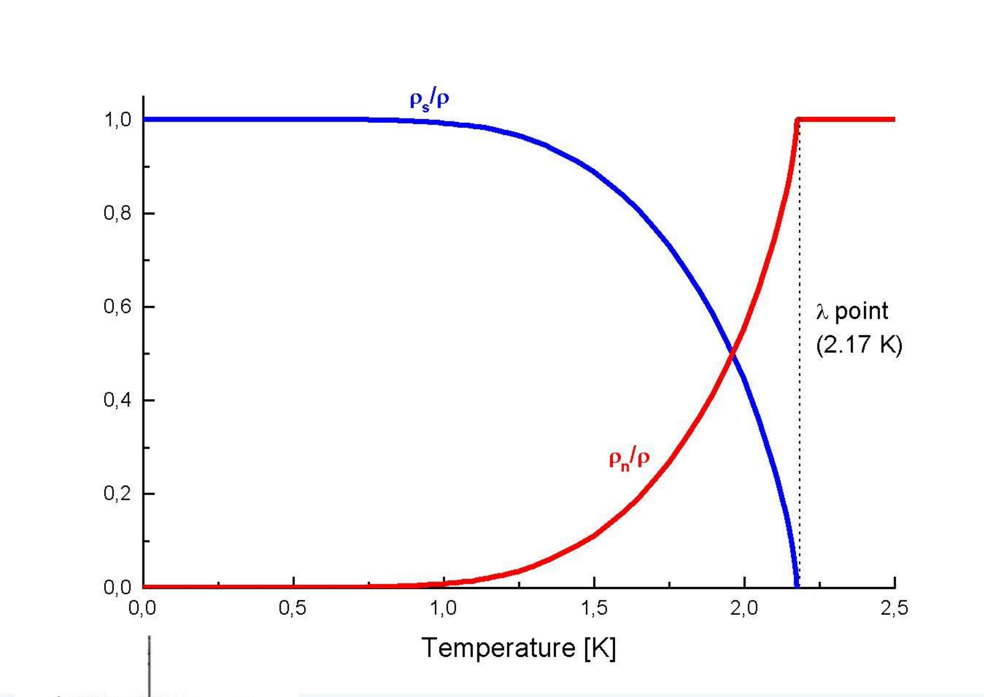


Two-fluid model (Landau)





L.D. Landau L. Tizsa



„First sound“

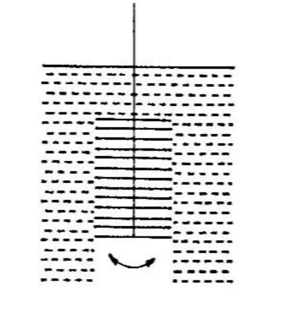
$$u = u_1; s' = 0; \rho' \neq 0; \nabla T = 0; \vec{v}_n = \vec{v}_s$$

Normal sound, i.e. density waves, propagates both in, He II and in He I

„Second sound“

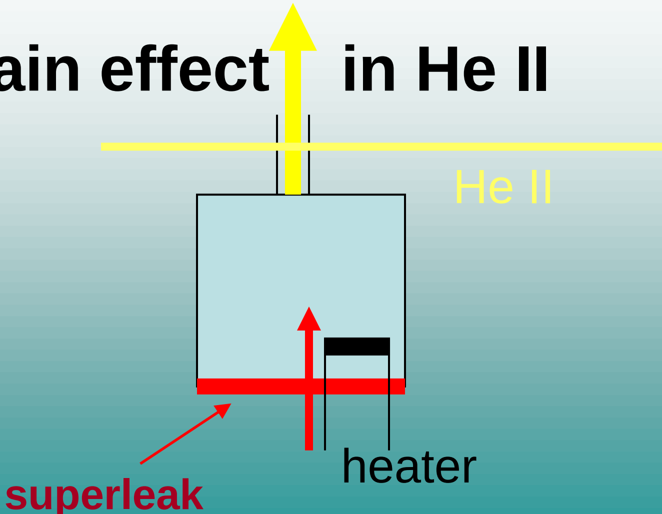
$$u = u_2; s' \neq 0; \rho' = 0; \nabla p = 0; \rho_n \vec{v}_n = \rho_s \vec{v}_s$$

Entropy (temperature) wave at constant density; normal fluid and superfluid oscillating in antiphase. No analogy in classical liquids. A powerful tool to detect quantized vortices.



Andronikashvili experiment

Fountain effect in He II



$$\rho_s \frac{D\mathbf{v}_s}{Dt} = -\frac{\rho_s}{\rho}\nabla p + \rho_s S \nabla T - \mathbf{F}_{ns}$$

$$\rho_n \frac{D\mathbf{v}_n}{Dt} = -\frac{\rho_n}{\rho}\nabla p - \rho_s S \nabla T + \eta \nabla^2 \mathbf{v}_n + \mathbf{F}_{ns}$$

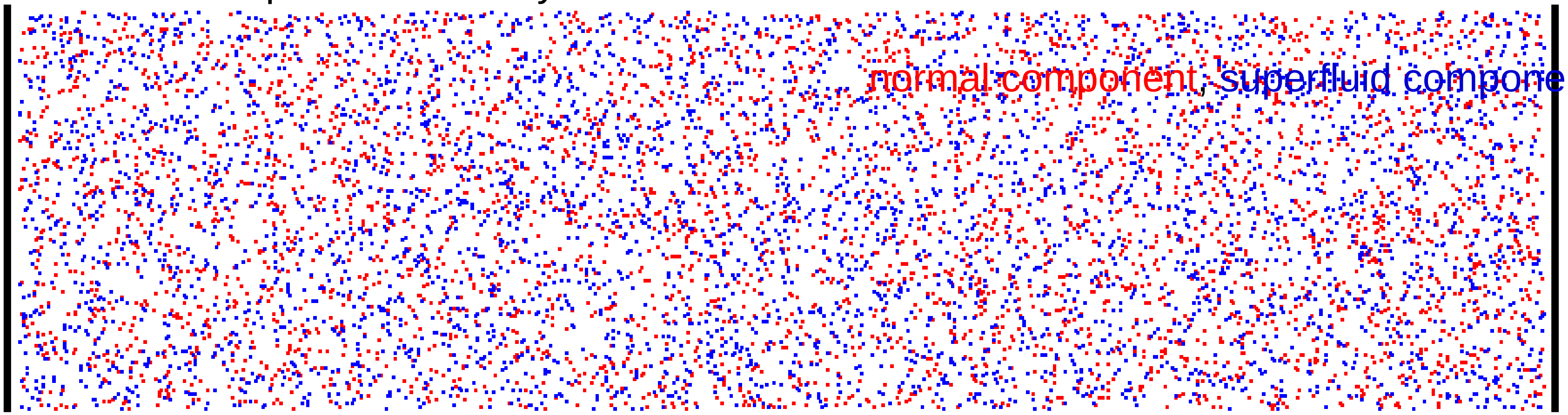
Mutual friction force

***L* voxtex line density**

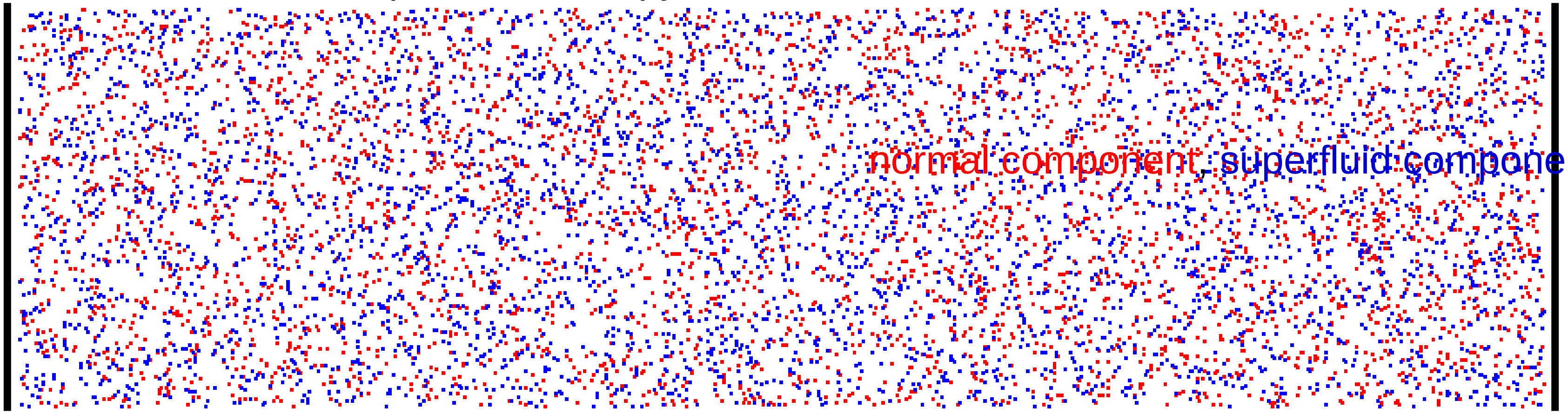
$$\mathbf{F}_{ns} = \alpha L \left| \mathbf{v}_s - \mathbf{v}_n \right|$$

He II: Sound modes

"First sound" = pressure/density wave



"Second sound" = temperature/entropy wave



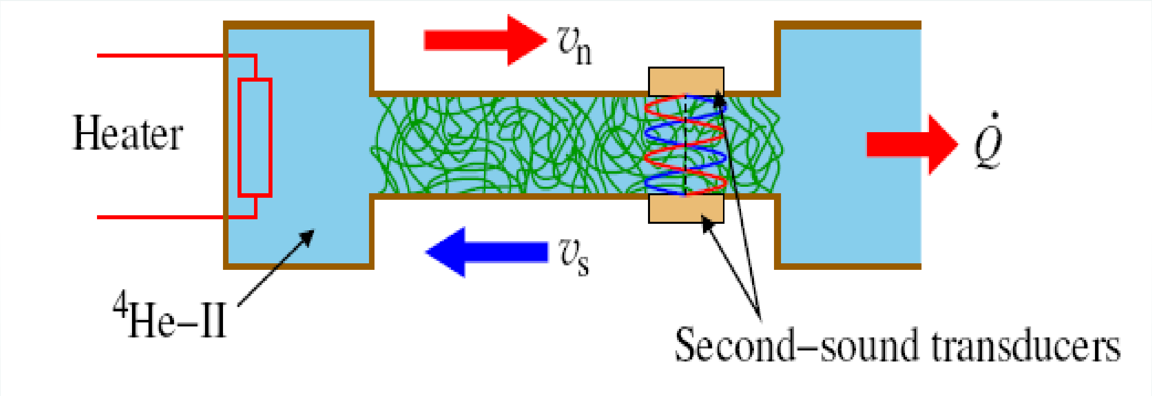
Second sound is attenuated at quantized vortices – can measure vortex line density.

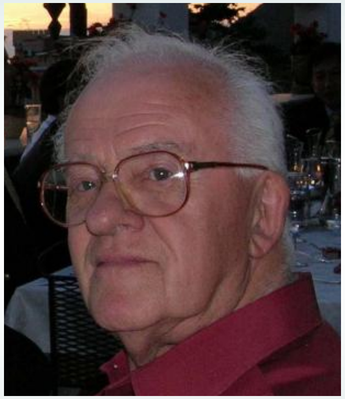
Second sound is overdamped in superfluid 3He phases

Historically, QT in He II was mentioned as a theoretical possibility by
R.P. Feynman, who recognized that QT ought to take the form of a random tangle of quantized vortices.

„ Application of quantum mechanics to liquid helium", *Prog. in Low Temp. Phys.,* vol. 1, (1955)

Experiment -thermal counterflow in He II- a form of motion peculiar to two-fluid superfluid hydrodynamics --no direct analogy in any ordinary viscous fluid





W.F. (Joe) Vinen, 60 years ago…

$$V_N \cong \frac{\dot{Q}}{AST\rho}$$

$$V_N \rho_N = V_S \rho_s$$

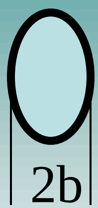
W.F. Vinen, Proc. Roy. Soc. A240 114, (1957)
W.F. Vinen, Proc. Roy. Soc. A240 128, (1957)
W.F. Vinen, Proc. Roy. Soc. A242 493, (1957)
W.F. Vinen, Proc. Roy. Soc. A243 400, (1958)

Experimental observation: quantized vortices attenuate second sound

$$L \propto \dot{Q}^2$$

Counterflow turbulence phenomenology (Vinen 1957)

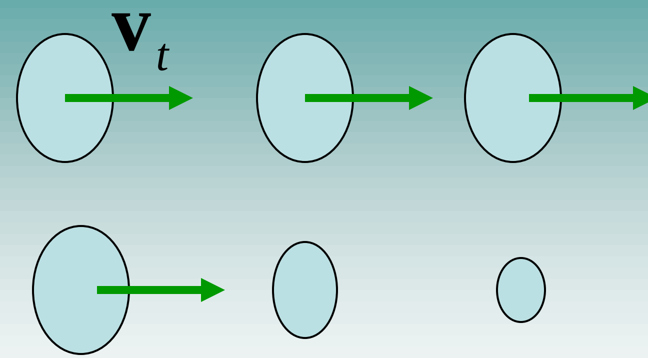
Vortex ring



$$\mathbf{v}_t = \frac{\kappa}{4\pi b}\left(\ln\frac{8b}{a} - \frac{1}{4}\right) \cong \frac{\kappa}{b}$$

$T \rightarrow 0$

Finite T



In counterflow, though, if $|\mathbf{v}_t| < |\mathbf{v}_n - \mathbf{v}_s| = V_{CF}$ rings with $b > b_c$ expand

Dimensional analysis and analogy with classical fluid dynamics leads to the Vinen equation:

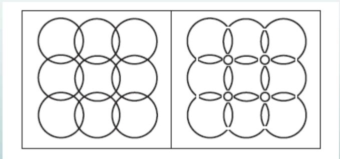
$$\frac{dL}{dt} = \chi_1 \frac{B}{2}\frac{\rho_n}{\rho} V_{CF} L^{3/2} - \chi_2 \frac{\hbar}{m_4} L^2$$

production **decay**

L – voxtex line density

Reproduced by Schwarz (1988) – computer simulations

reconnections



For steady V_{CF} there is a steady value of *L*

Early results reviewed by J.T. Tough **Turbulent states I, II, III**

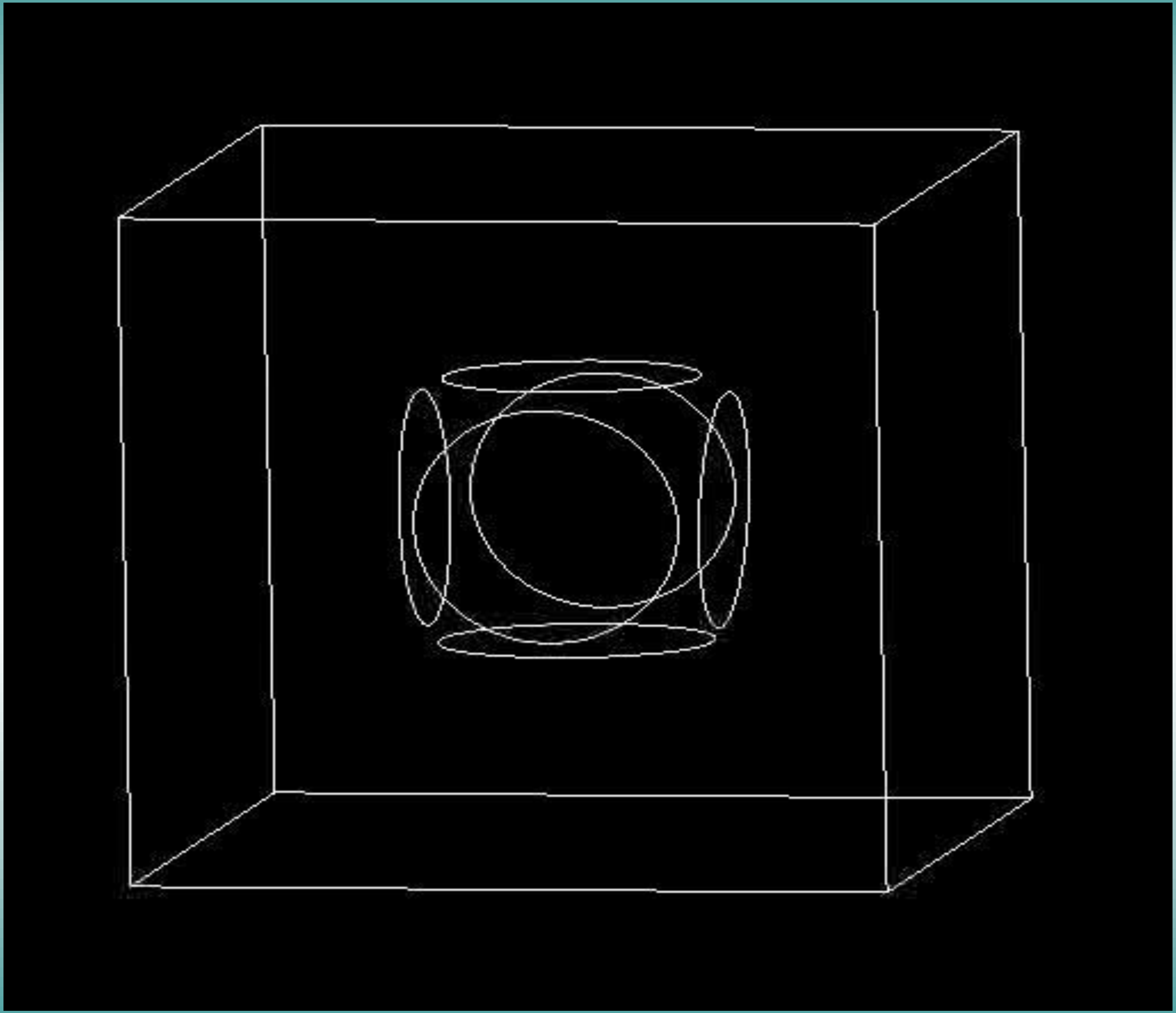
Decay of counterflow turbulence:

$$\frac{dL}{dt} = \chi_2 \frac{\hbar}{m_4} L^2 \longrightarrow L(t) \propto \frac{1}{t + t_{VO}}$$

Numerous experiments ???

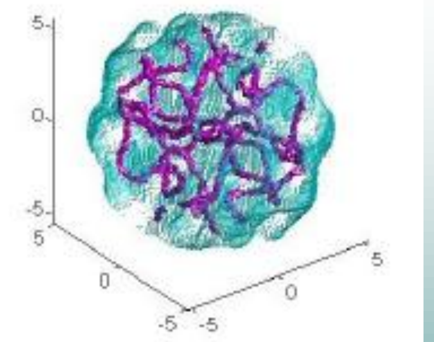
(Vinen, Schwarz, Milliken…)

Counterflow tangle (courtesy of M. Tsubota) He II



channel (Eltsov & al 2014)

3He-B



atomic BEC (White & al 2010)

Working fluids for quantum turbulence

T

^{4}He

^{3}He (B)

normal liquid He I

Classical Navier-Stokes fluid of extremely low kinematic viscosity

normal liquid ^{3}He

Classical Navier-Stokes fluid of kinematic viscosity comparable with that of air

Superfluid transition at Tc

a "mixture" of two fluids

He II –

normal fluid of extremely low kinematic viscosity

+

Inviscid superfluid

Circulation is quantized

$$\kappa = \frac{2\pi\hbar}{m_4} \approx 10^{-3}[cm^2/s]$$

superfluid ^{3}He B

normal fluid of of kinematic viscosity comparable with that of air

+

Inviscid superfluid

Circulation is quantized

$$\kappa = \frac{2\pi\hbar}{2m_3} \approx 0.66\times 10^{-3}[cm^2/s]$$

T → 0 limit

0

Pure superfluid

Pure superfluid

Classification scheme for QT

I. pure supefluid turbulence **^{4}He and ^{3}He B in the zero T limit**

no normal fluid

conseptually the simplest, experimentally the most difficult case

II. pure supefluid turbulence in a stationary normal fluid **^{3}He B at finite T**

thick normal fluid provides a unique frame of reference

mutual friction acts at all scales

III. QT in ^{4}He at finite T **experimentally the simplest, conseptually the most difficult case**

both NF and SF may or may not become turbulent

Particular cases of interest:

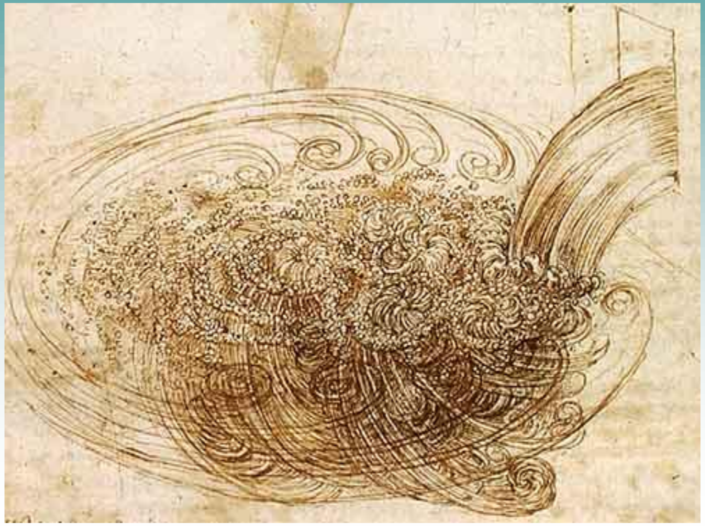
A counterflow QT

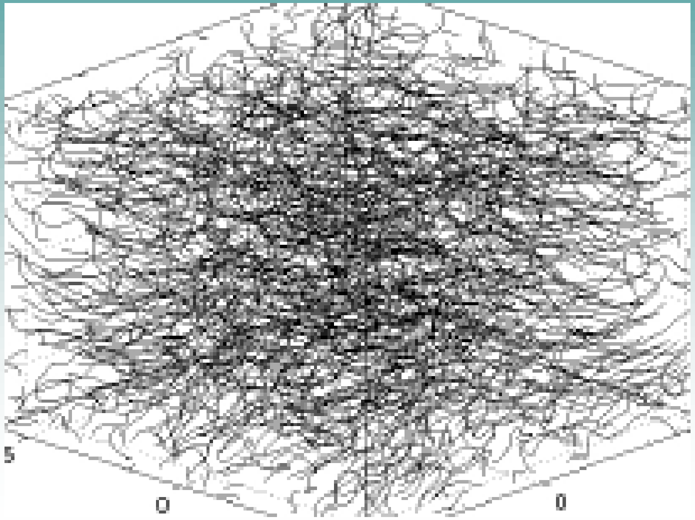
B co-flow QT

C QT in pure superflow

Classical versus quantum turbulence

in the zero temperature limit





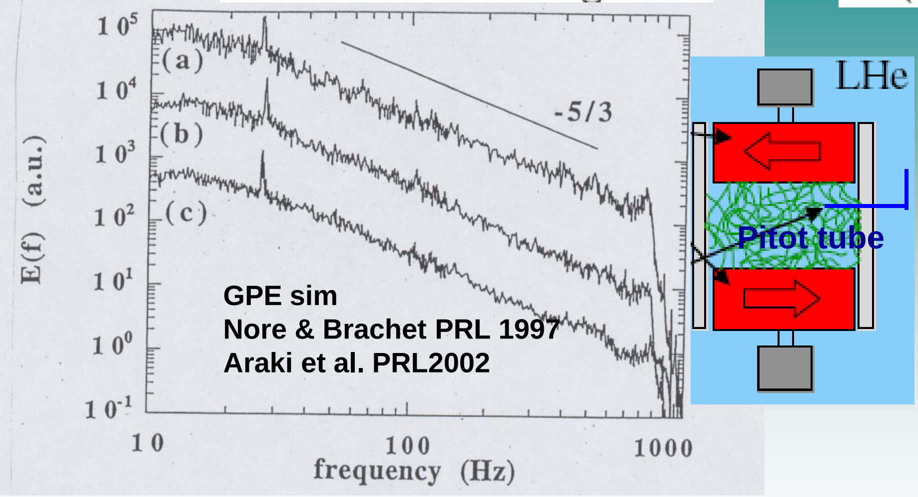
- Vortices are topologically unstable.
- It is difficult to identify them.
- Circulation differs from one vortex to another.

- Quantized vortices are topologically stable and all of them have the same circulation.
- Kelvin theorem is valid – circulation along a vortex tube is a conserved quantity.

Quantum turbulence (T→O) is simpler than classical turbulence

It makes a prototype of turbulence and as such ought to help to better understand the phenomenon of fluid turbulence in general.

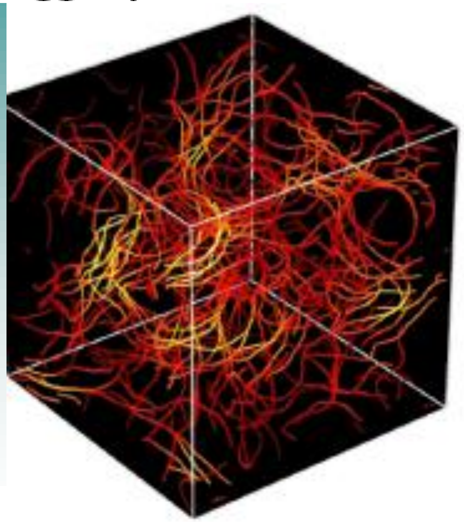
Mauerer & Tabeling 1998

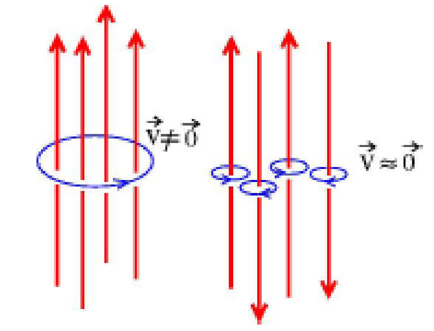


GPE sim
Nore & Brachet PRL 1997
Araki et al. PRL2002

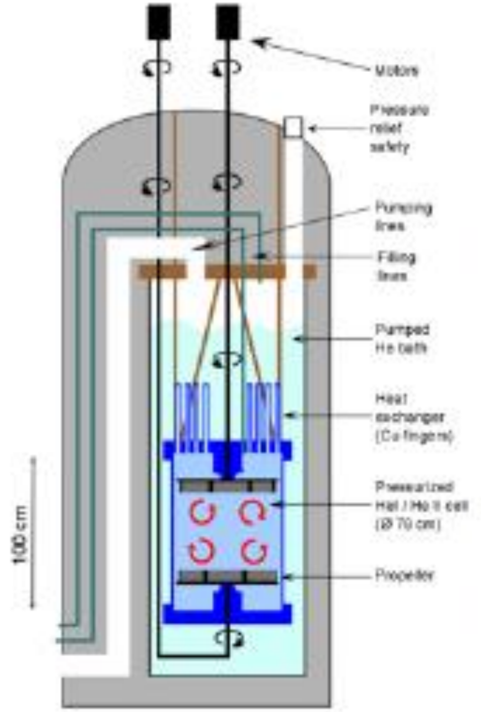
$E(k) \sim k^{-5/3}$ arises from polarisation of vortex lines

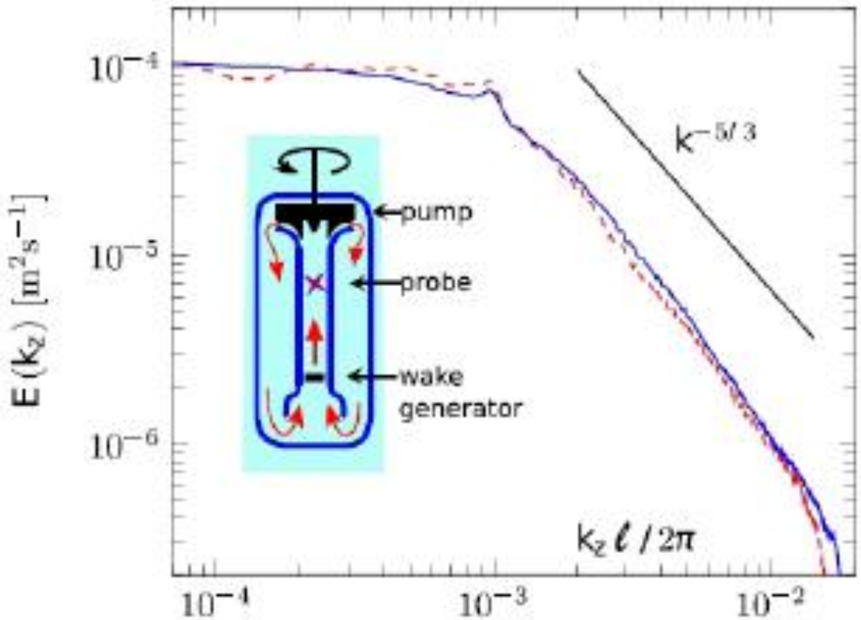
Baggaley, Laurie & CFB, PRL 2012



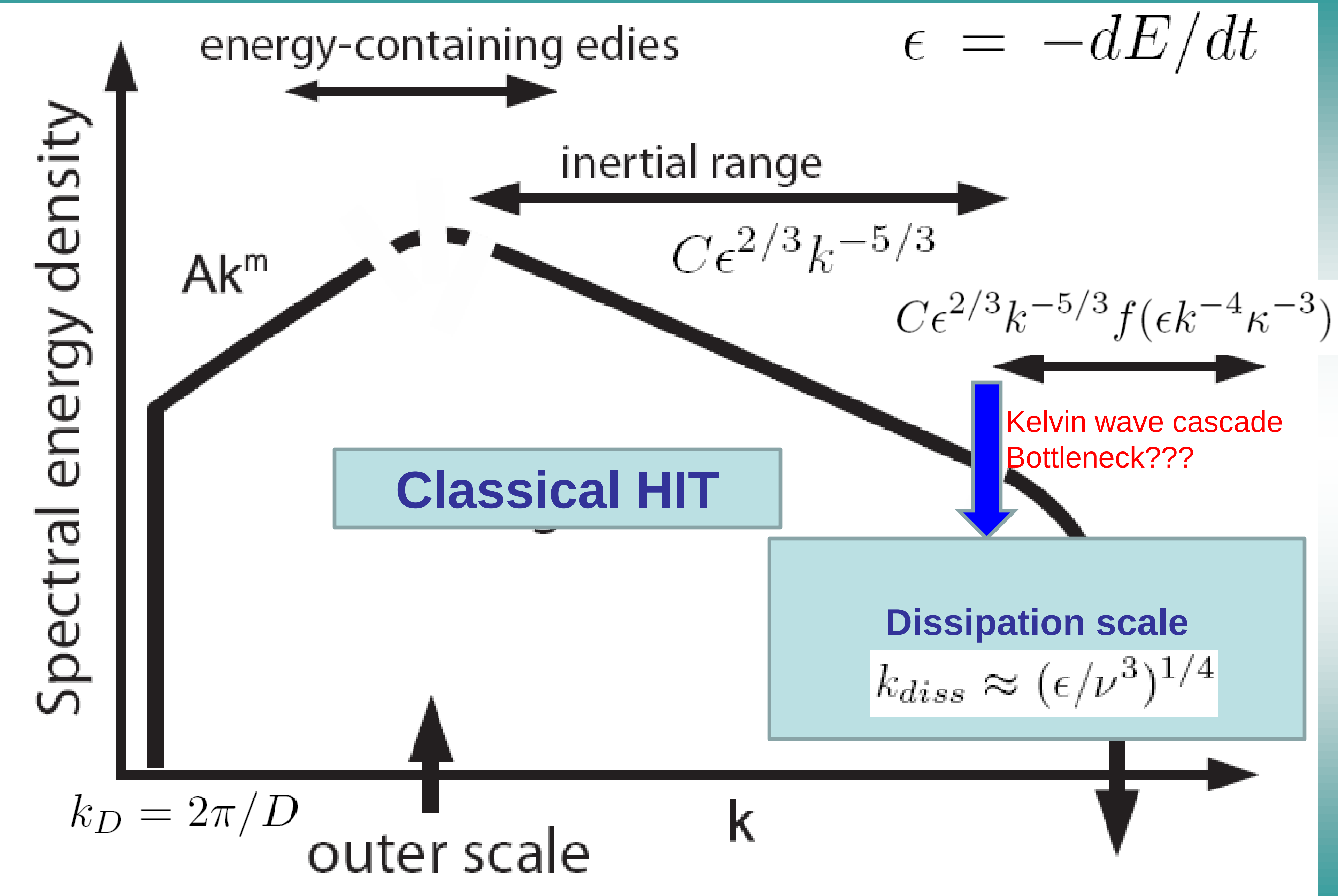


SHREK experiment, Grenoble





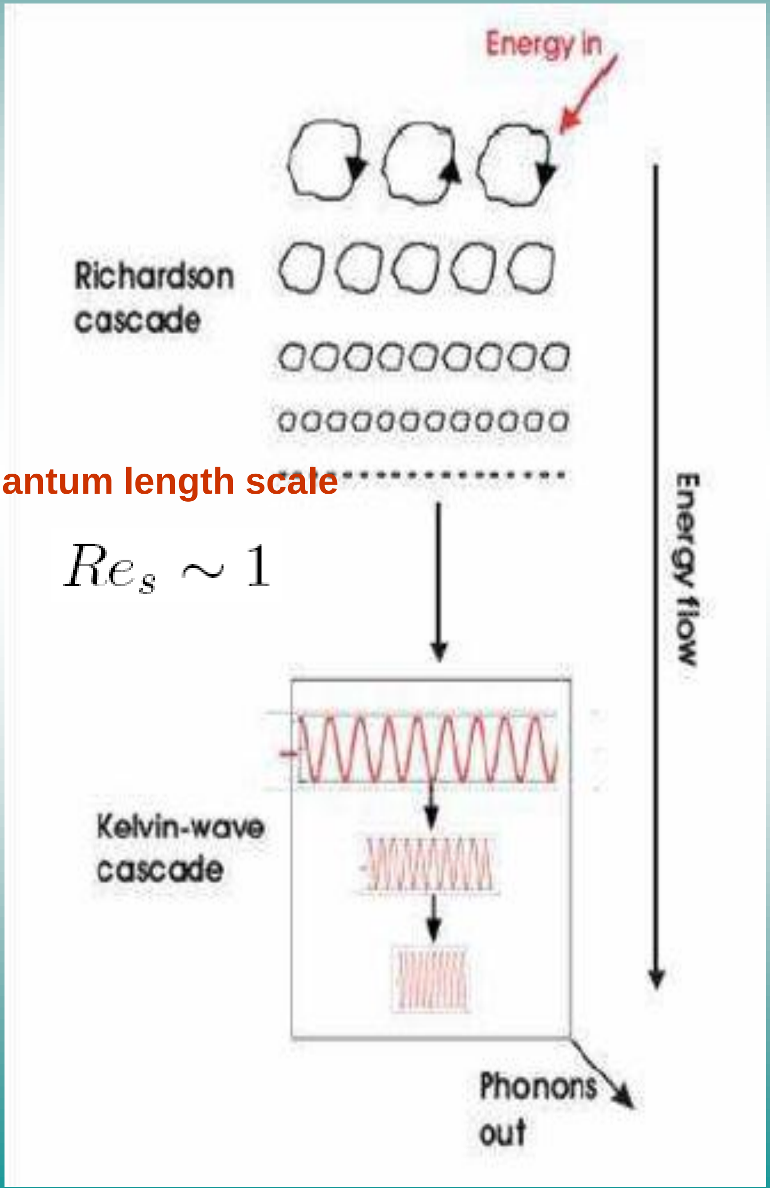
Salort & al 2012



e.g., mesh size of a grid

quantum turbulence in a pure superfluid (T ➡ 0)

„Vinen“ vs. „Kolmogorov“ QT



Quantum length scale

$Re_s \sim 1$

Superfluid Reynolds number

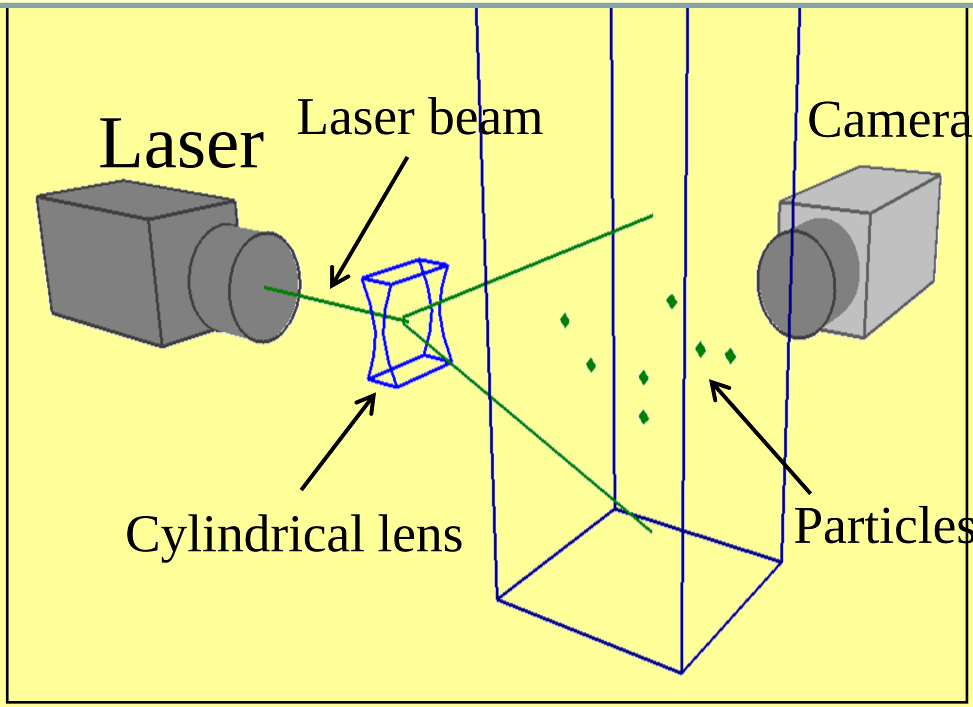
$$\mathrm{Re}_s = \frac{DV}{\kappa}$$

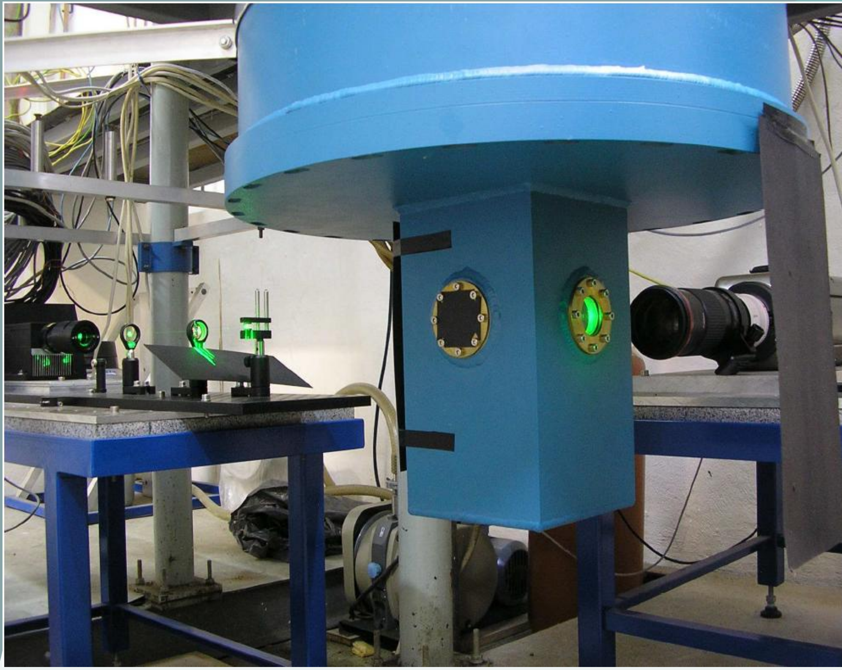
$Re_s \gg 1$

equivalent to $\kappa \propto \hbar \to 0.$

- **At finite temperature the situation becomes more complex, due to mutual friction coupling the normal and superfluid velocity fields.**
- **Both N and S components may serve as a source or sink for the motion in its counterpart**

Recent results on visualization of cryogenic helium flows
Prague Visualization Laboratory

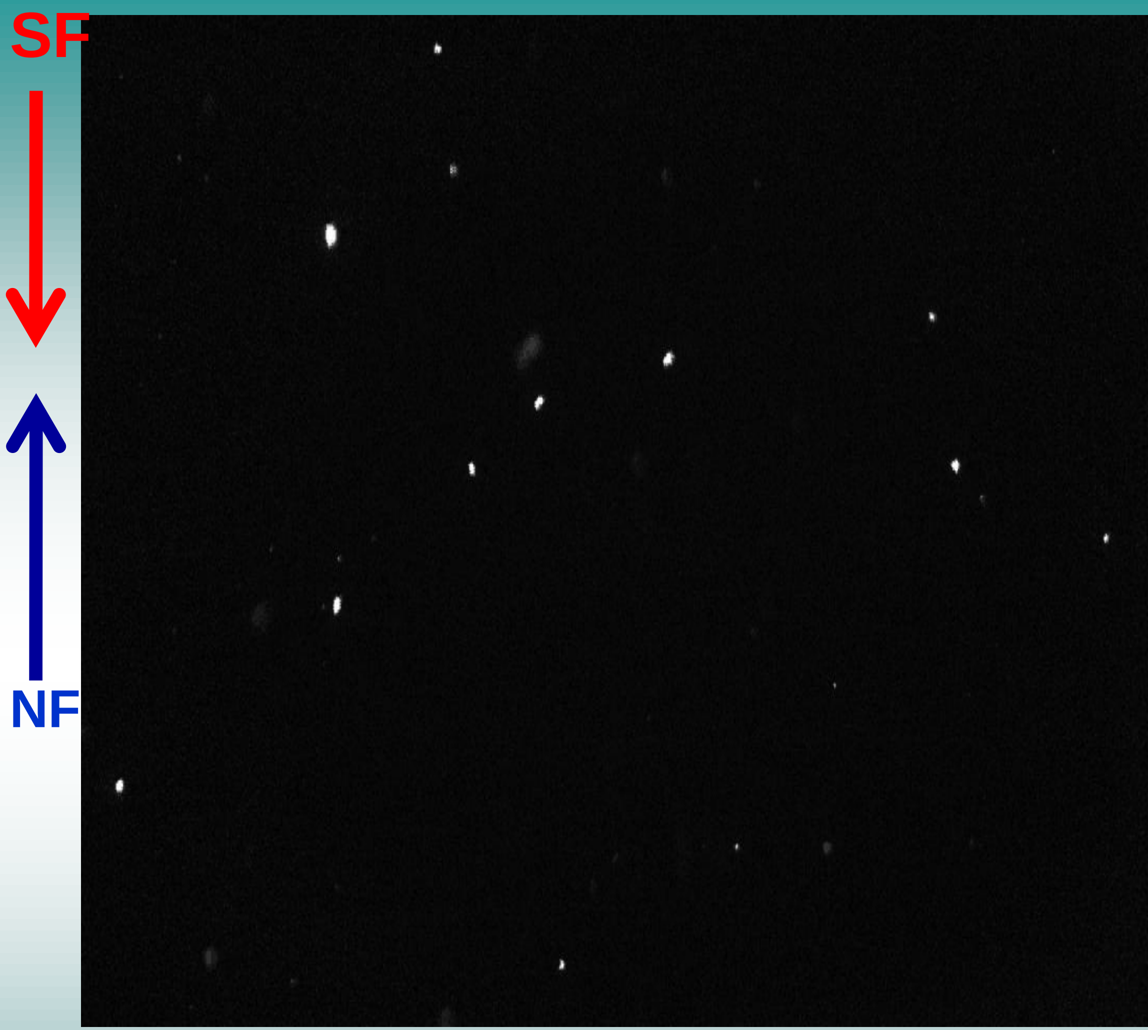


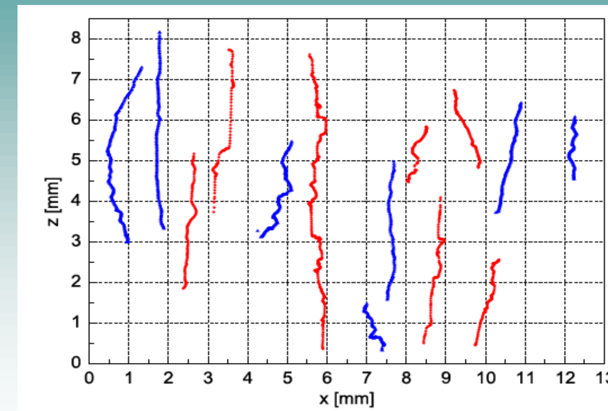


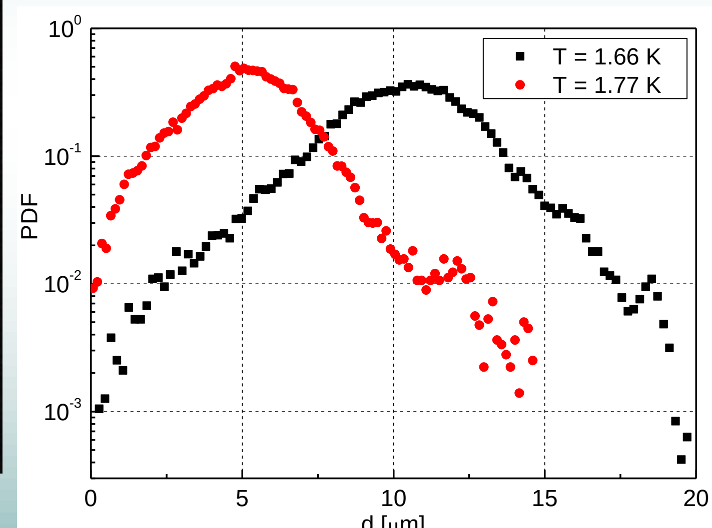
- **Custom-built low-loss cryostat with five sets of windows that minimise heat input into the helium bath, enabling horizontal as well as vertical optical access**
- **Continuous wave solid state laser, fast digital camera and relevant hardware and software to implement the PIV and PTV techniques for cryogenic flows analysis**

Micron-sized hydrogen/deuterium tracers

Example: thermal counterflow
Deuterium particles







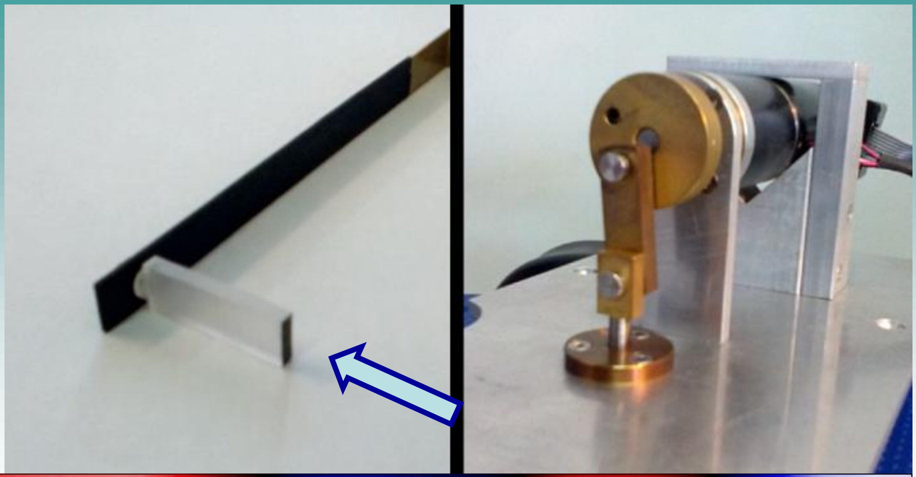
The particles' radii are calculated by assuming that the particles are spherical and that the buoyancy force is balanced by the Stokes drag

$$R_p = \sqrt{\frac{9\ \mu\ v_1}{2\ g\ (\rho - \rho_p)}}$$

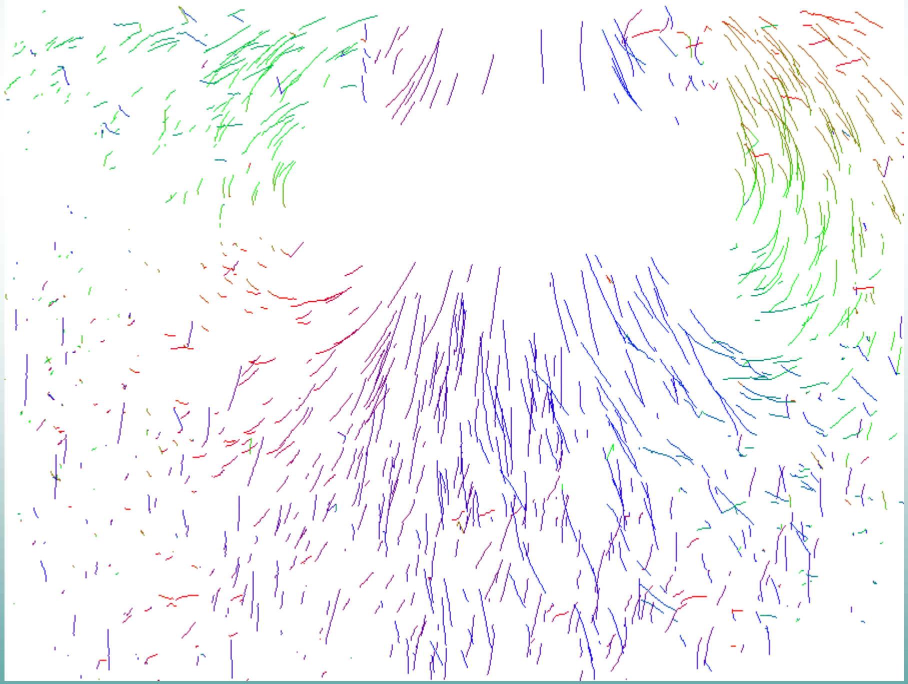
Oscillating cylinder of rectangular cross-section 3 x 10 mm

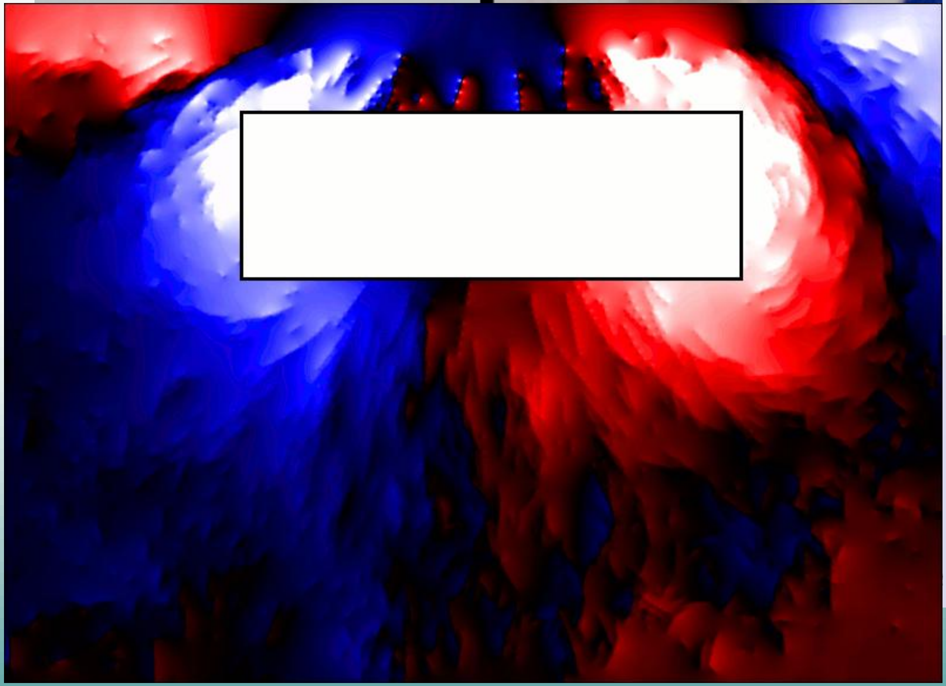
Oscillations:
frequency 0.5 Hz, amplitude 5 mm

20 s entire video,
camera frequency 100 Hz (exposition time 5 ms),
phase averaged,
trajectories of min 5 points shown ,
laser power 1.05 W



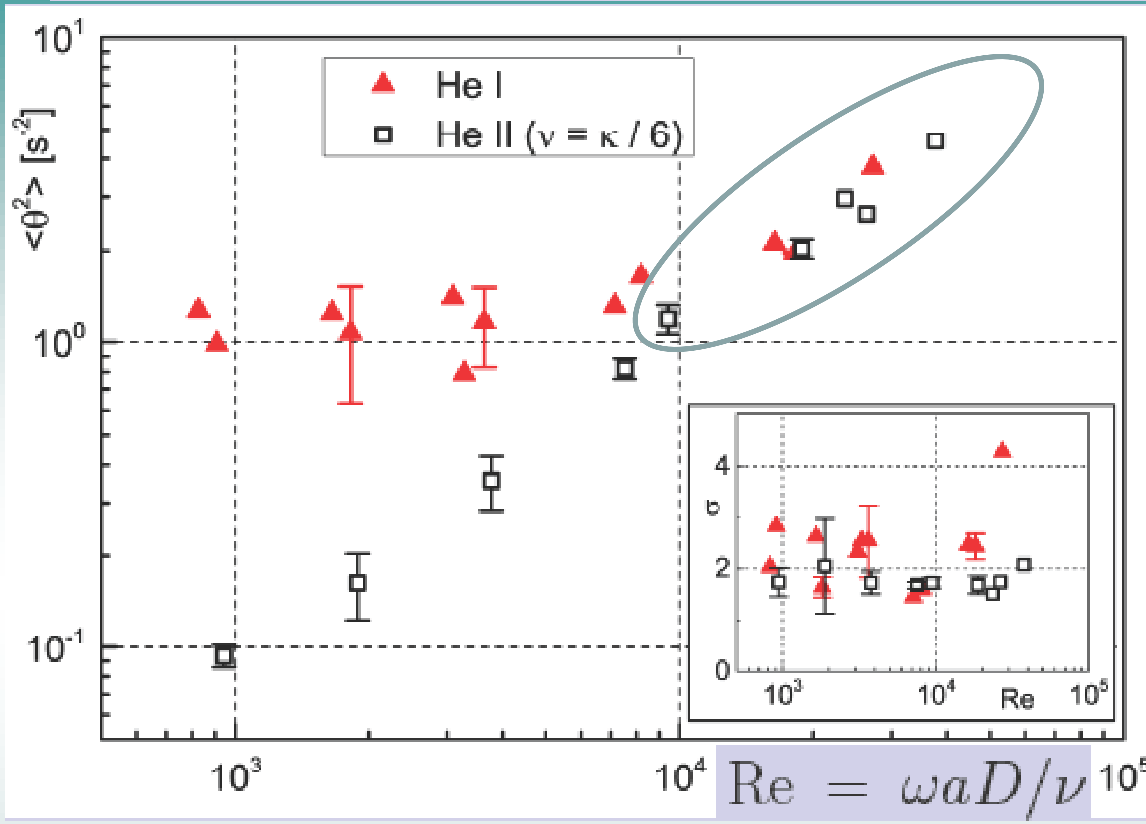
T=1.24 K





For details, see: D. Duda, P. Svancara, M. La Mantia, M. Rotter, and LS: Visualization of viscous and quantum flows of liquid 4He due to an oscillating cylinder of rectangular cross section, PRB 92, 064519 (2015)

$\langle\theta^2\rangle$: ensemble average of the θ^2 parameter



Kinematic viscosity of He II taken as:

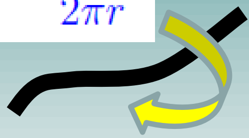
$$\nu \approx \kappa / 6$$

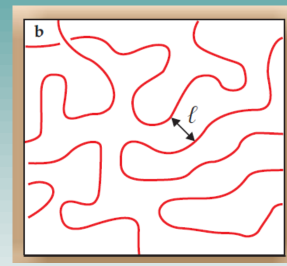
At large enough length scales (larger than Kolmogorov dissipation length and quantum length scale - average distance between quantized vortices) He I and II behave similarly.

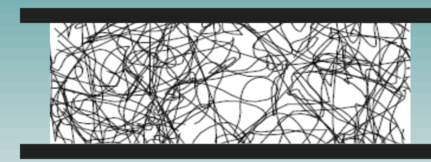
At smaller length scale, there is a clear difference . Why???

Characteristic length scales in 4He turbulence

$$v_s = \frac{\kappa}{2\pi r}$$







$\xi \approx 10^{-8}\ cm$

Vortex core size

$\ell = 1/\sqrt{L} \approx 100\ \mu m = 10^{-2}\ cm$

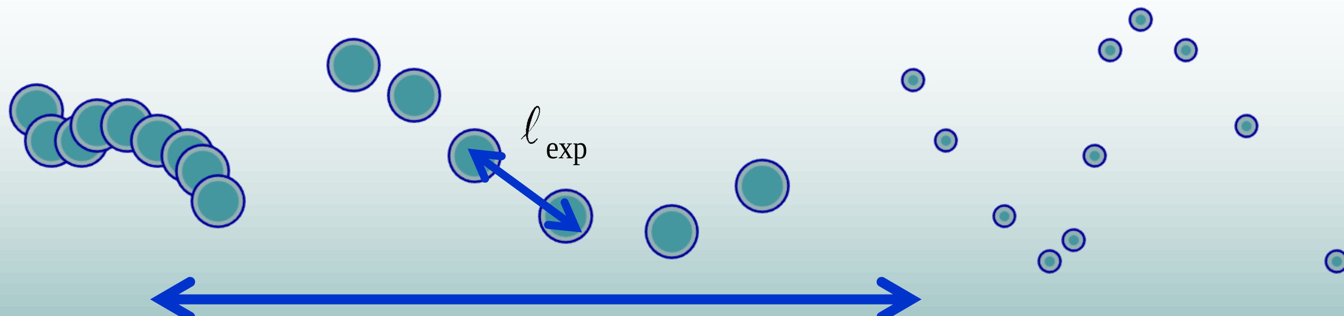
Mean intervortex distance

quantum length scale

$D \approx 1\ cm$

Outer scale

Scales experimentally accessible by particle tracking



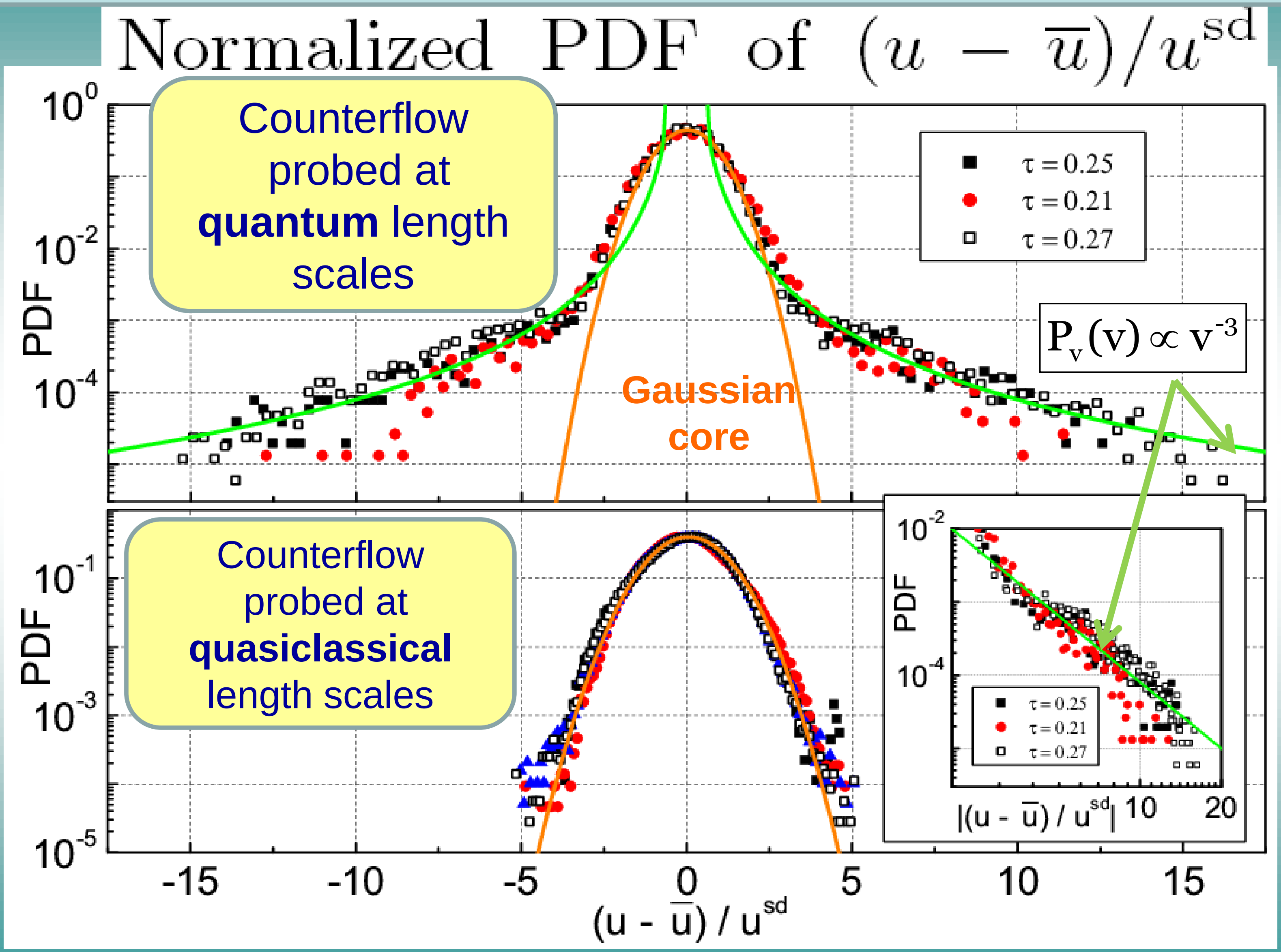
$\ell_{exp} \approx \ell/10$

$\ell_{exp} \approx 10\ \ell$

Vinen (ultraquantum) QT → crossover → ???

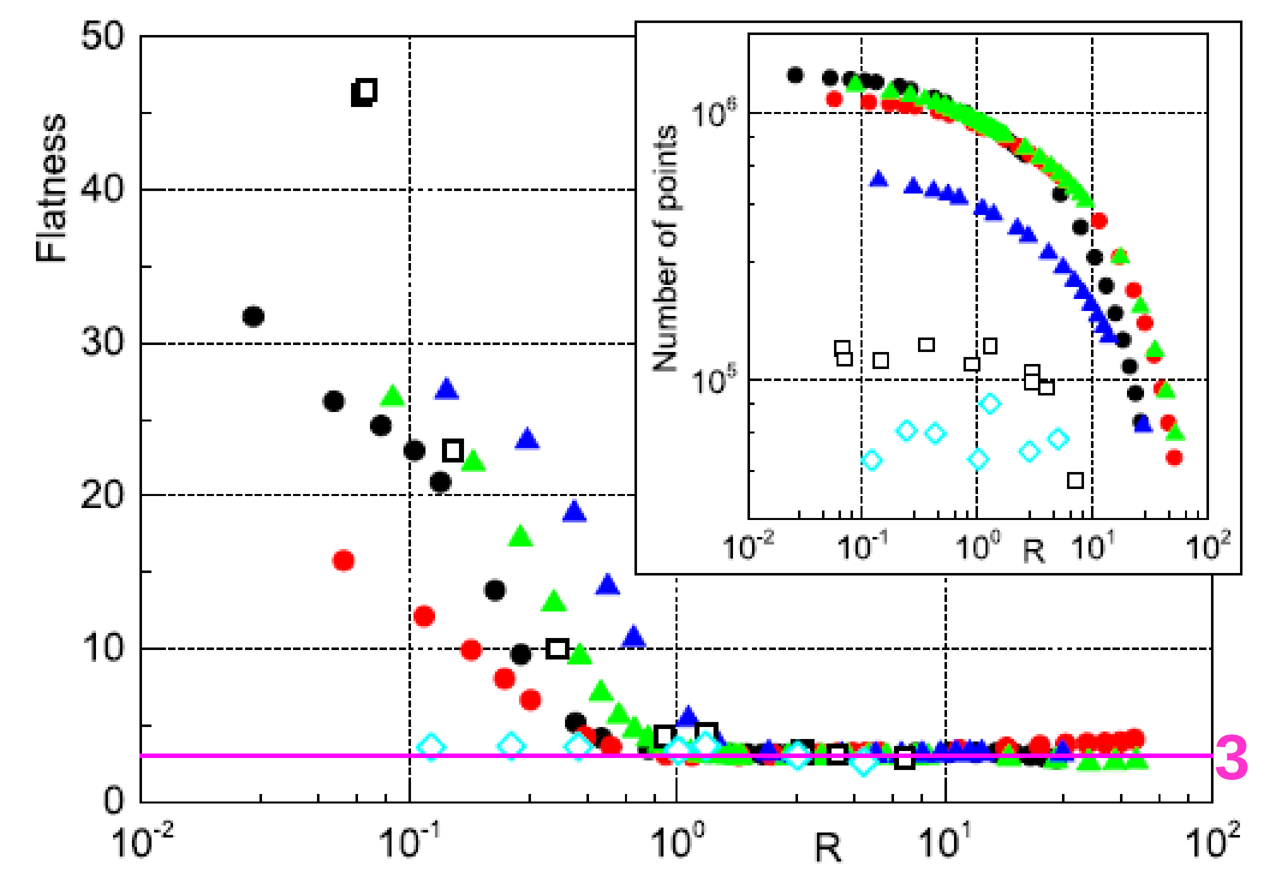
Kolmogorov (quasiclassical) QT

Quantum, or classical turbulence ?



Quantum, or classical turbulence ?

The answer depends on the scale at which the quantum flow is probed !!!



Vinen (ultraquantum) QT → crossover → Kolmogorov (quasiclassical) QT

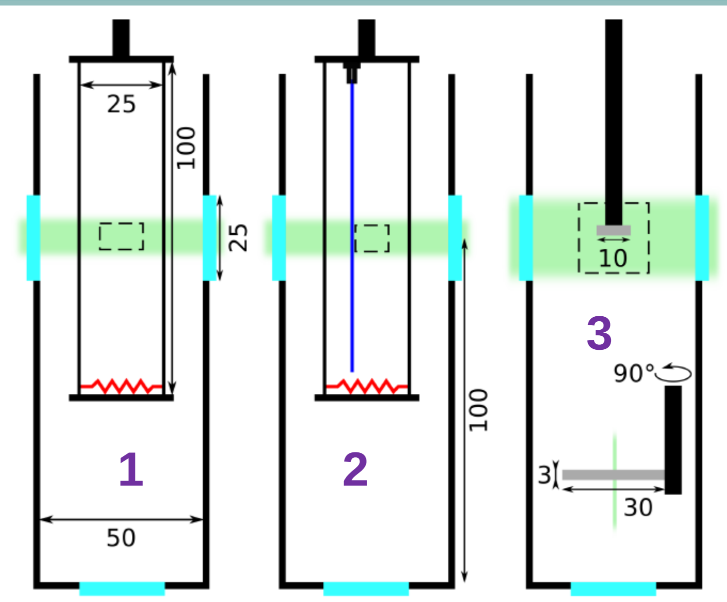
For details, see M. La Mantia, LS: Quantum or classical turbulence? EPL **105,** 46002 (2014)

M. La Mantia, LS: Quantum turbulence visualized by particle dynamics PRB **90,** 014519 (2014)

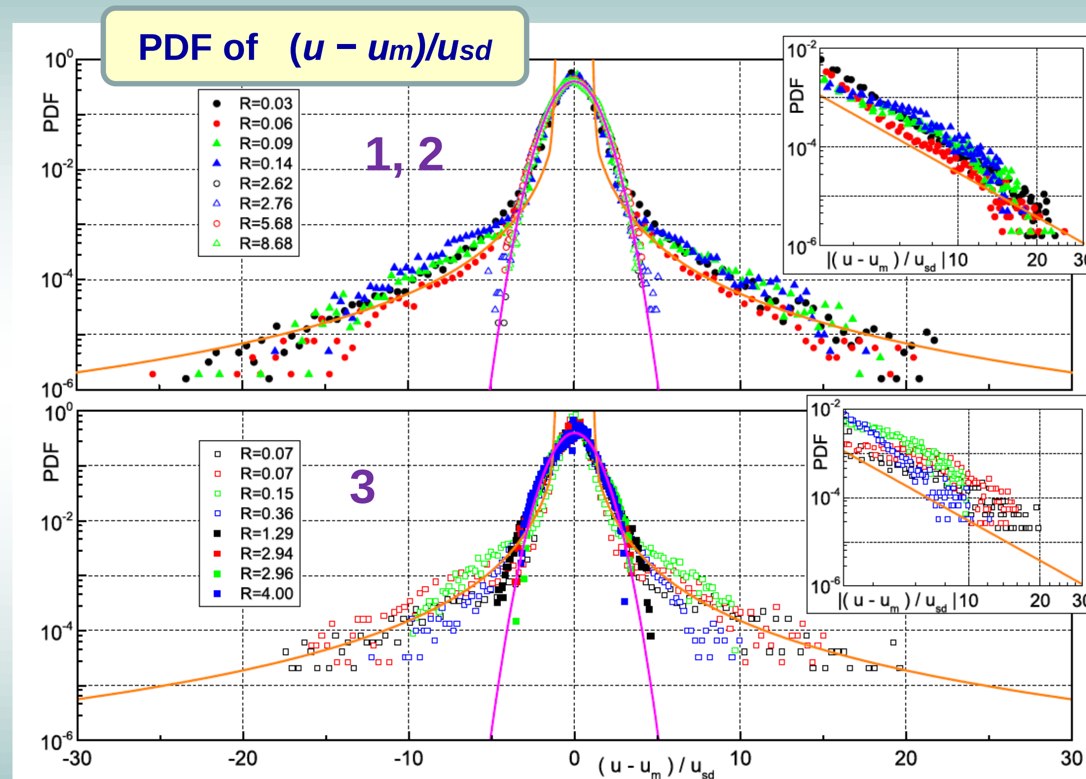
Small-scale universality in quantum turbulence

thermal counterflow **oscillating cylider**

bulk close to the wall (rectangular cross-section)



- **Small-scale universality is observed in classical turbulent flows of viscous fluids, as it emerges from the pioneering work of Kolmogorov**



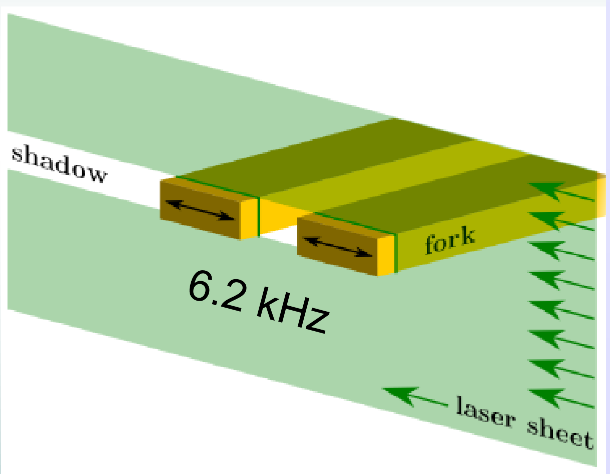
For details, see
M. La Mantia, P. Svancara, D. Duda, and LS:
Small-scale universality of particle dynamics in quantum turbulence PRB **94,** 184512 (2016)

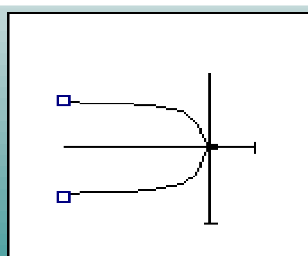
- **Small-scale universality is observed in quantum turbulence, similarly as in classical (viscous) fluid turbulence**
- in viscous flows, these small scales are still larger than the Kolmogorov length scale
- in quantum flows these small scales are smaller than the quantum scale, the average distance between quantized vortices (fluid motion may exist all the way down to the size of the quantized vortex core).

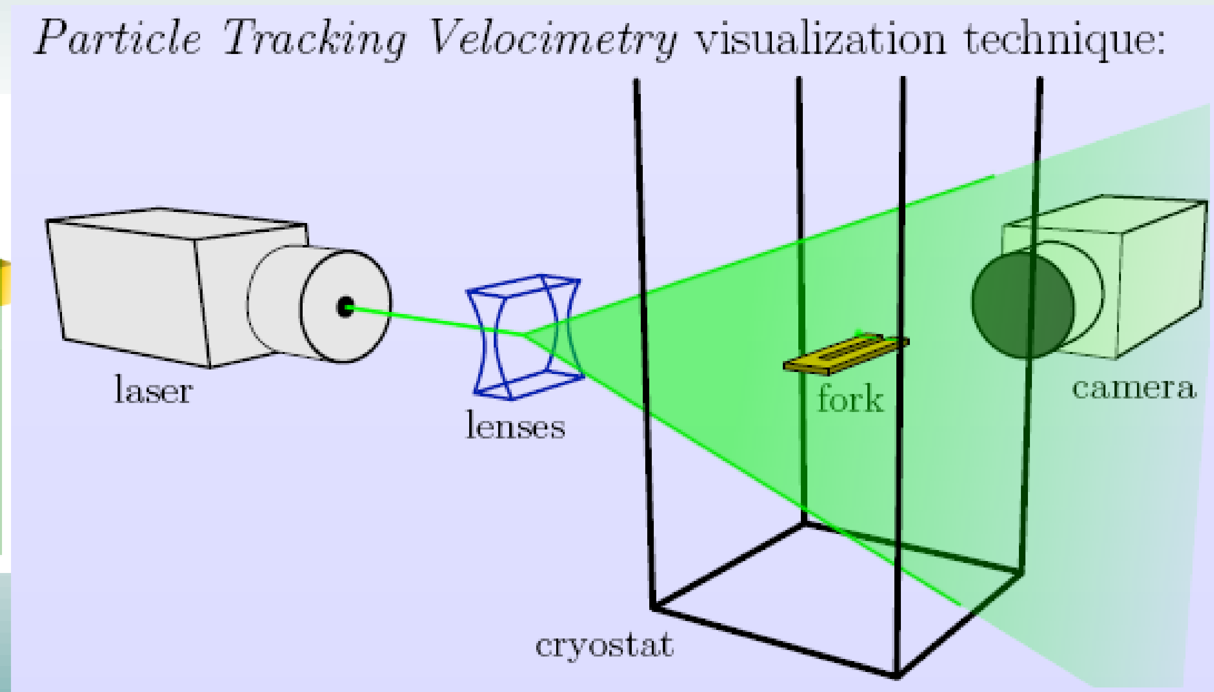
Visualization of Streaming Flow due to Quartz Tuning Fork Oscillating in Normal and Superfluid 4He

Streaming flow - steady part of oscillatory flows of classical viscous fluids due to vibrating obstacles

- **Can we observe it in He I, classical viscous liquid, in flow due to vibrating quartz fork?**
- **Does streaming occur in He II, which is a quantum liquid displaying two-fluid phenomena, superfluidity and macroscopic quantum effects such as quantization of circulation (quantized vortices)?**



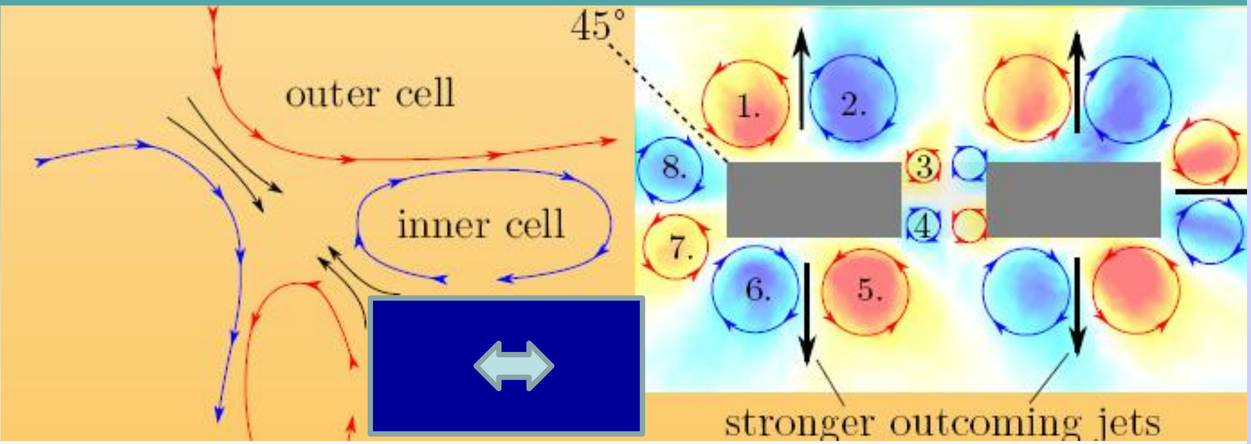


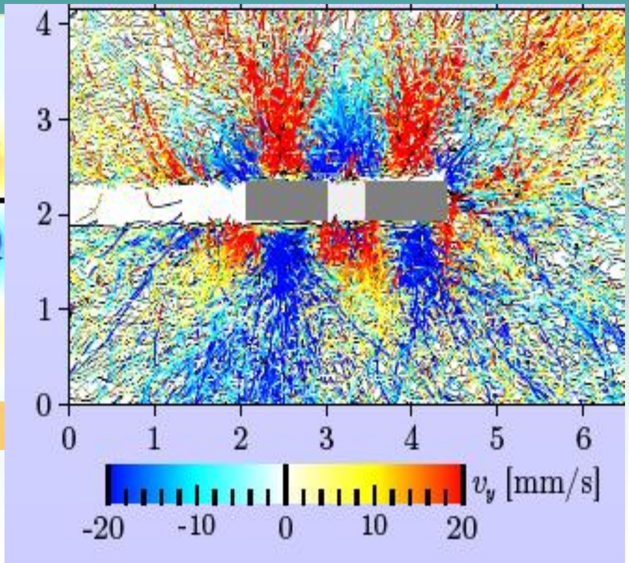


Solid deuterium particles 4-8 microns in size

1 Mpx 800 Hz CMOC camera, resolution 10 microns

Pseudovorticity **Particle trajectories**



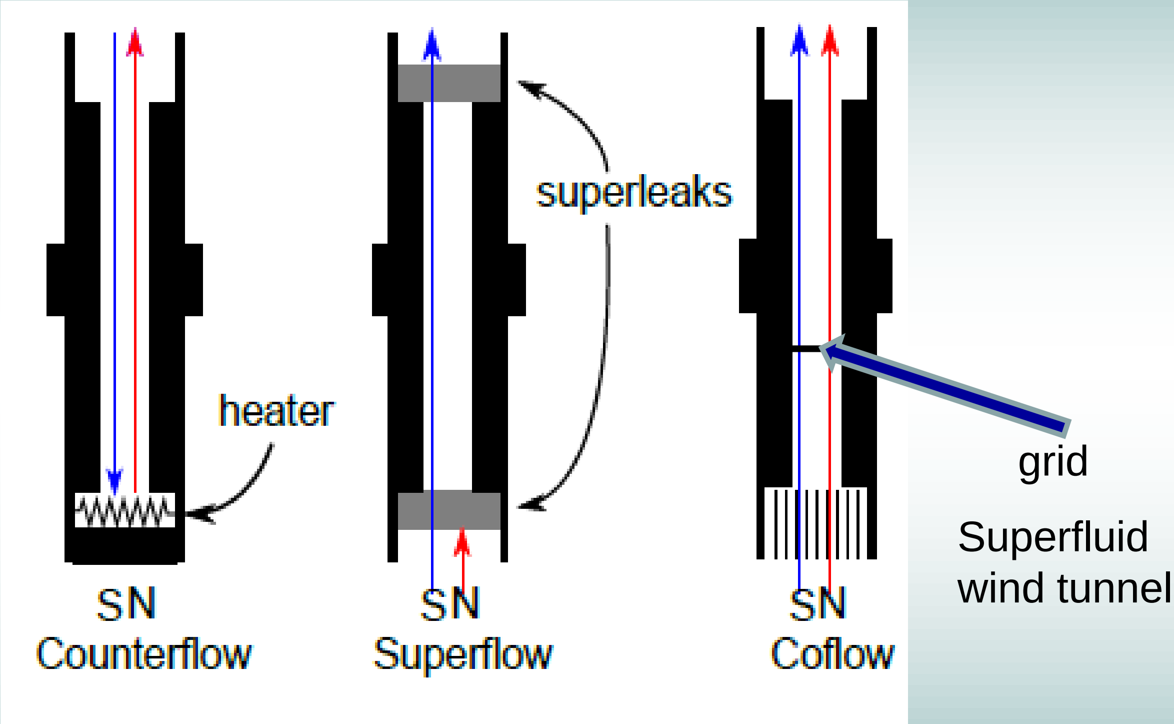


For details, see D. Duda, M. La Mantia, and LS:
Streaming flow due to a quartz tuning fork oscillating in normal and superfluid 4He
Phys. Rev. B **96**, 024519 (2017)

- **Steady nonlinear streaming flow due to vibrating quartz fork is clearly observed in He I**
- There are 8 streaming cells, identified as outer cells due to their orientation, around each prong produced by its corners, inner cells of thickness of order of our resolution, are invisible
- **For the first time, nearly identical streaming patterns are found in superfluid He II, probed at length scales exceeding the quantum length scale, where He II behaves as a single component quasiclassical fluid**
- **Streaming flow in the neighbouring bulk might affect the performance of forks as sensors in practical applications**

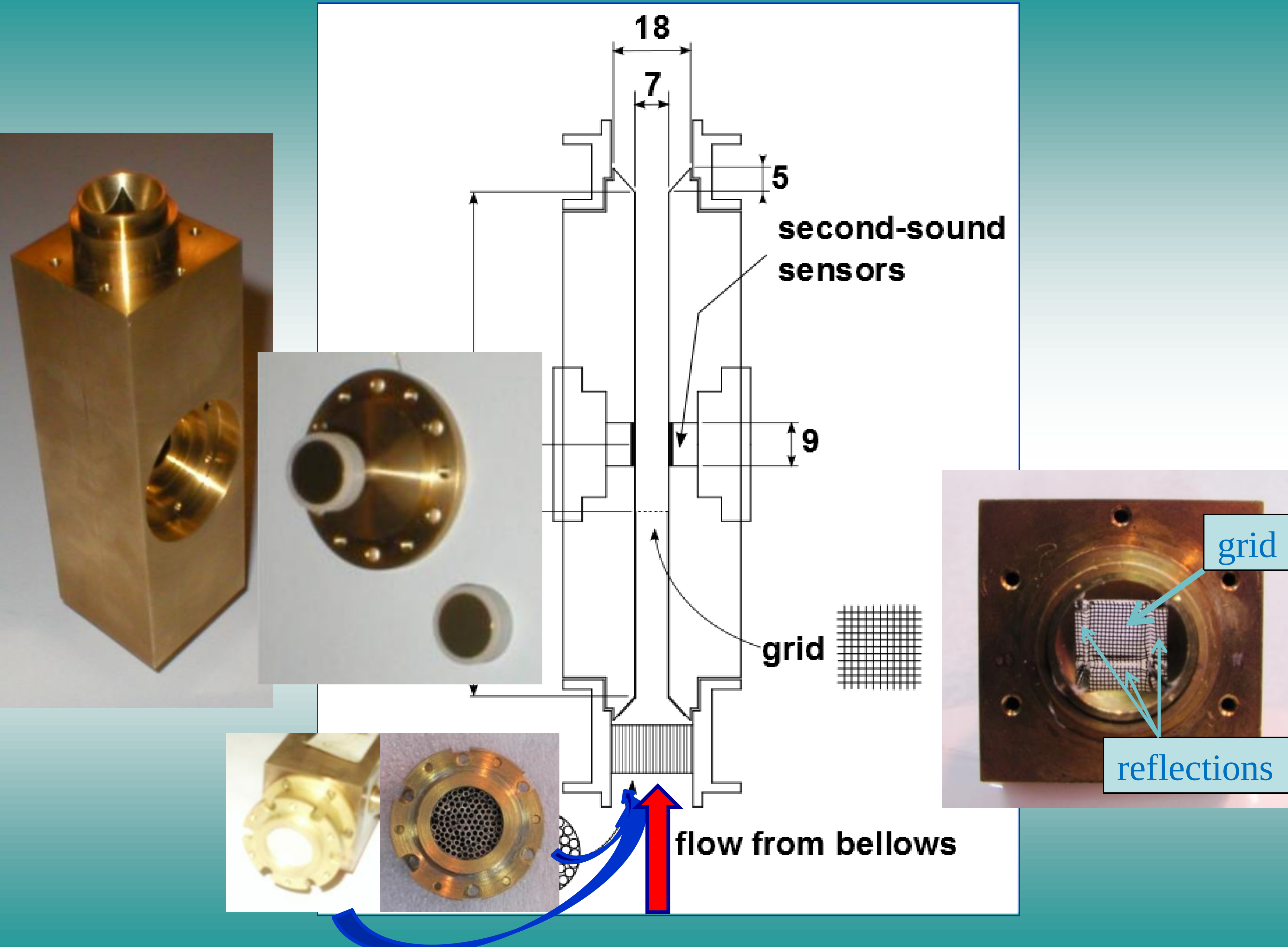
Quantum turbulence in He II
-various types of two-fluid steady-state and decaying channel flows investigated by Second Sound

1.35 K< T < 2.15 K   two channels: 7x7 and 10x10 mm

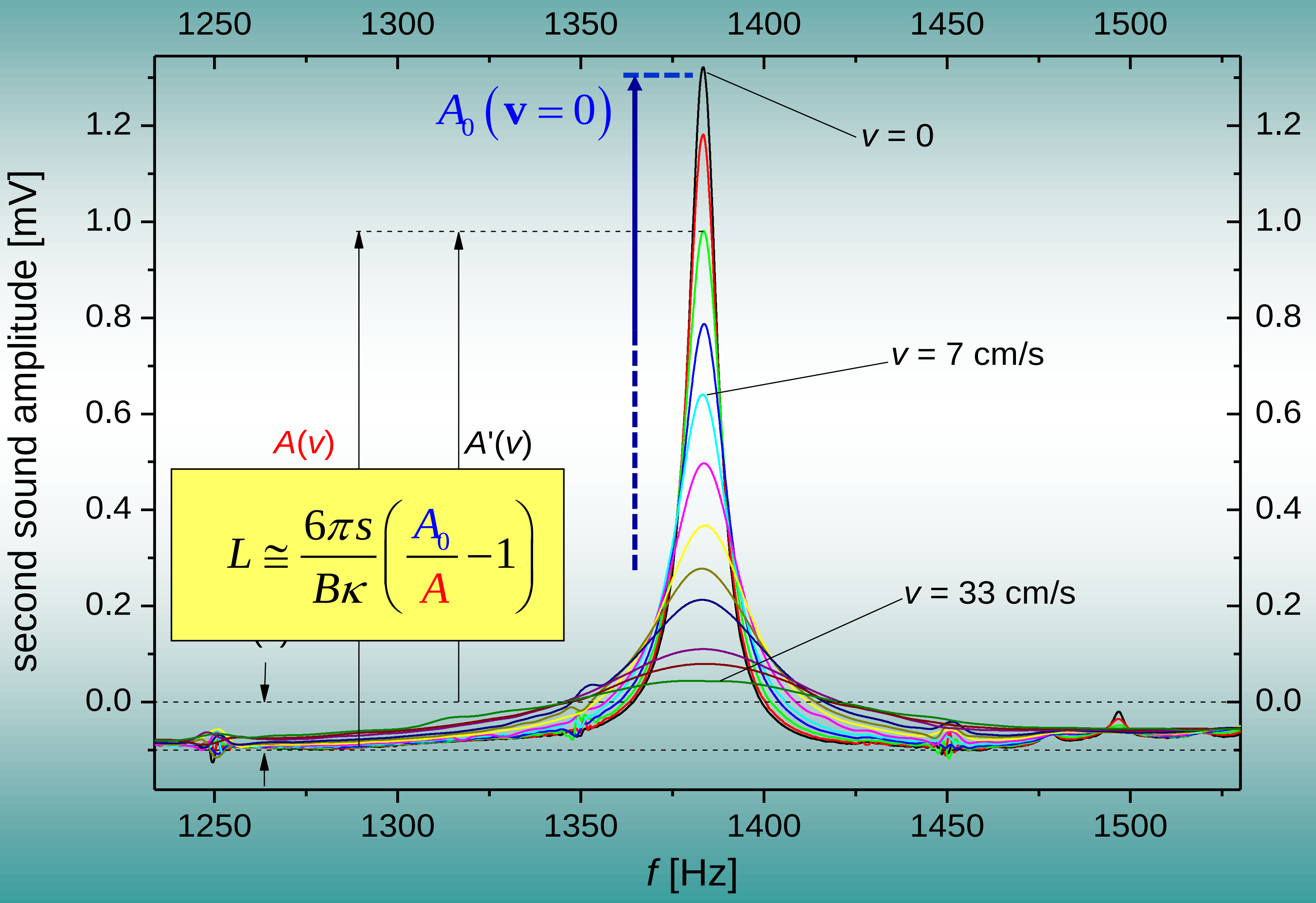


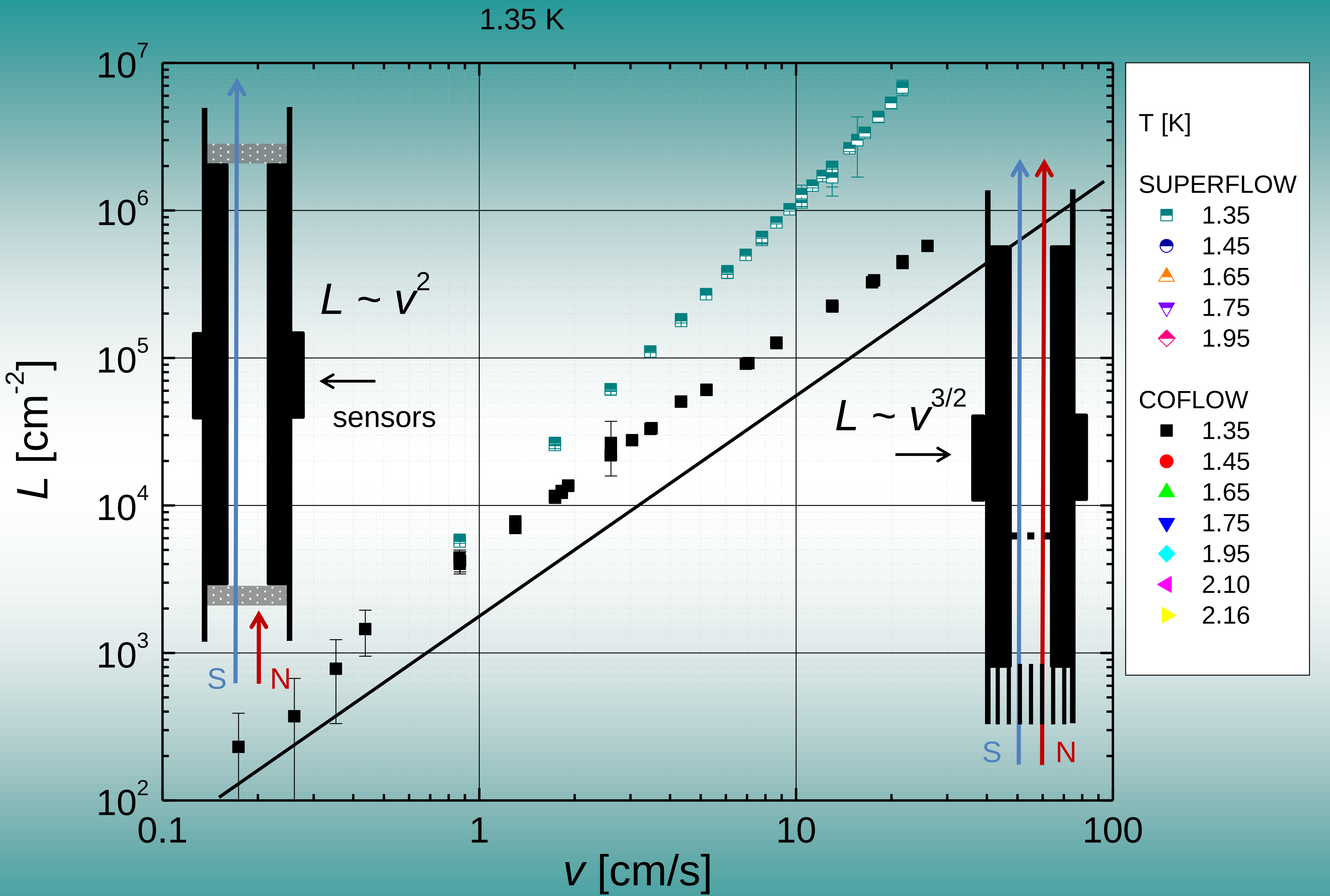
Main observables:

1. Mean superflow velocity, ***v*** → from rate of change of calibrated bellows volume (3% accuracy)
2. Length of quantized vortex lines per unit volume, ***L*** → from attenuation of second sound

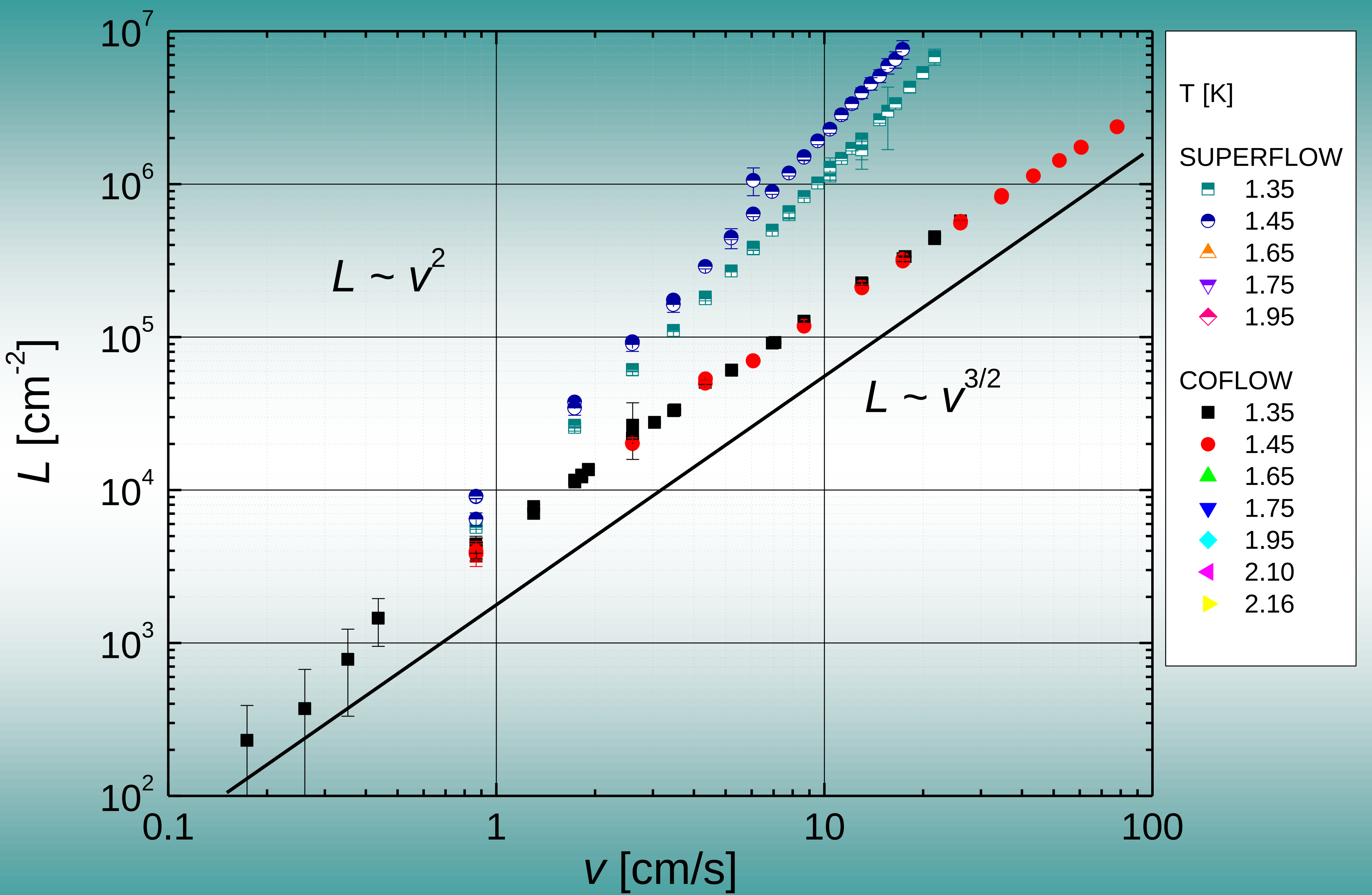

18
7
5
second-sound
sensors
9
grid
flow from bellows
grid
reflections

Coflow with grid, T = 1.65 K

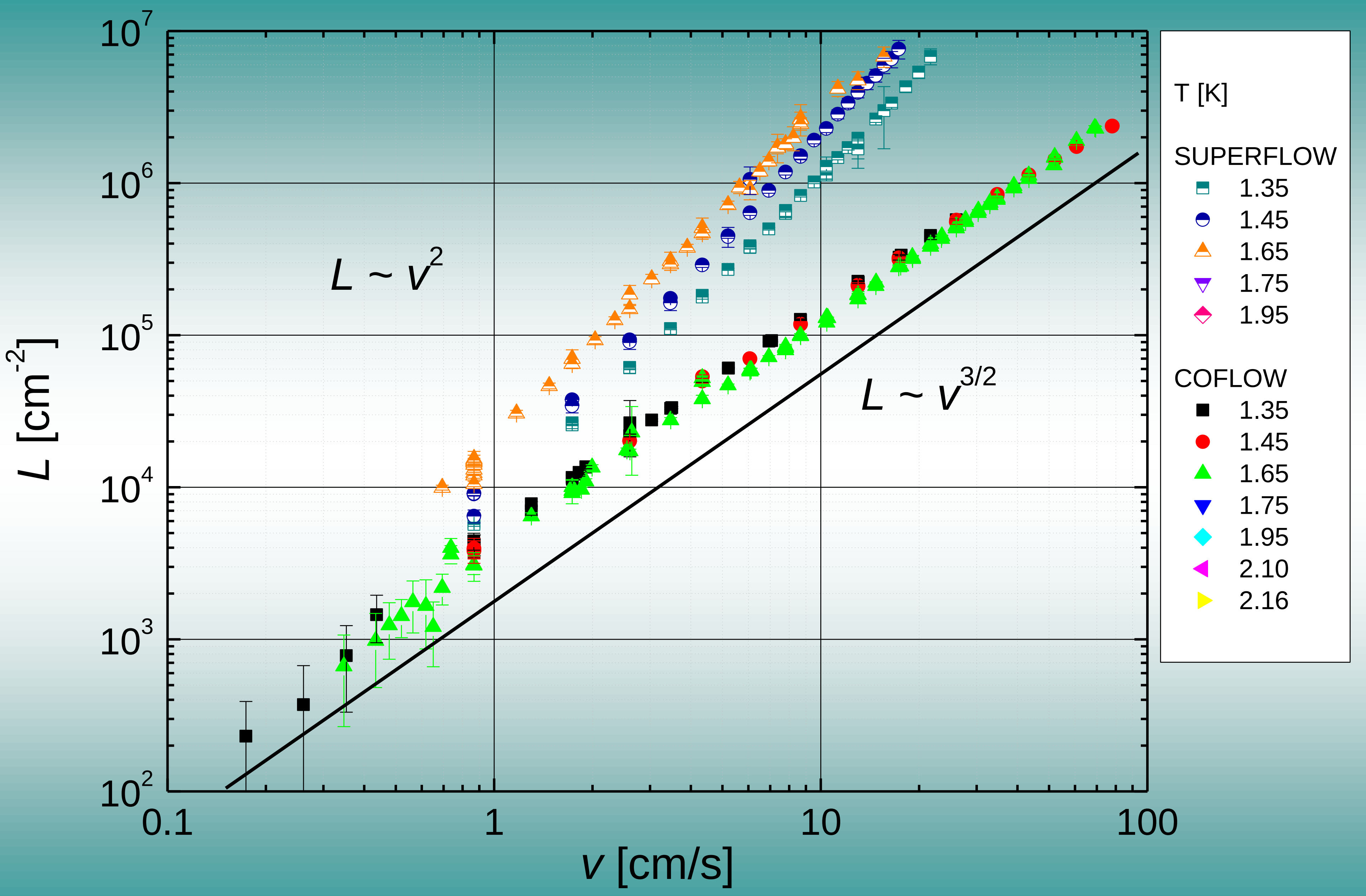




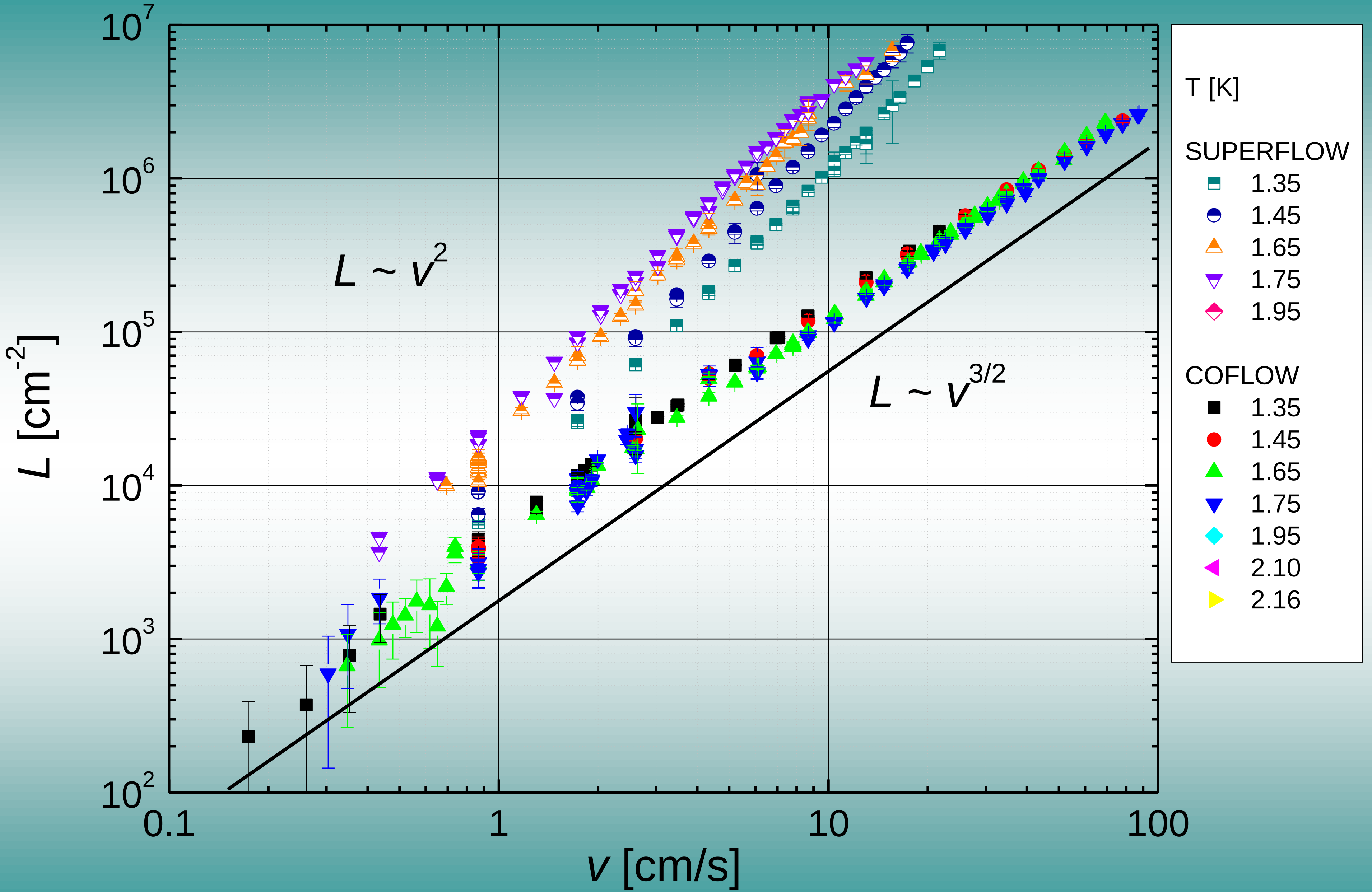
1.35 K
L [cm^-2]
v [cm/s]
0.1
1
10
100
10^2
10^3
10^4
10^5
10^6
10^7
L ~ v^2
sensors
L ~ v^3/2
S
N
S
N
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



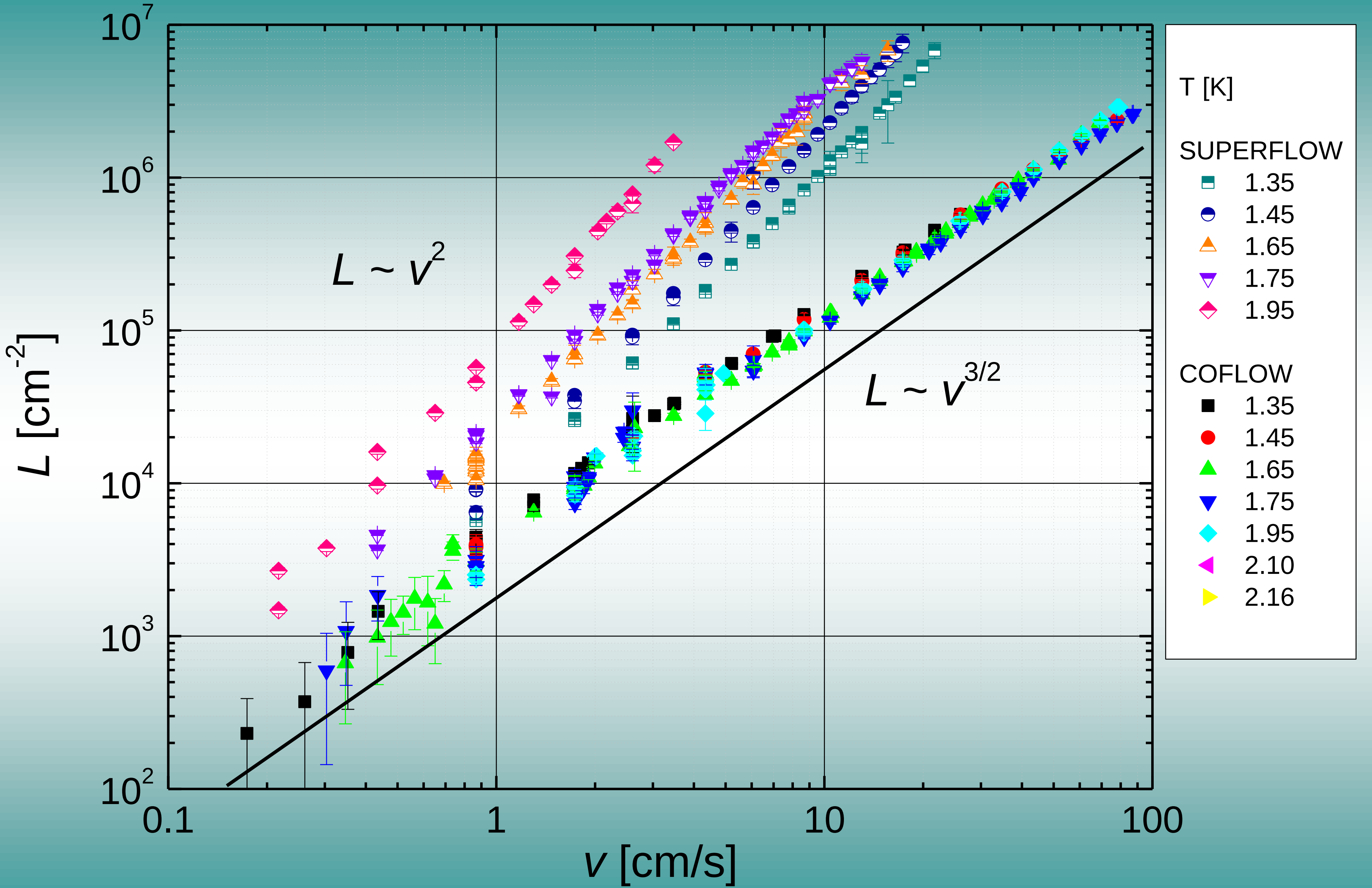
L [cm-2]
v [cm/s]
L ~ v2
L ~ v3/2
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



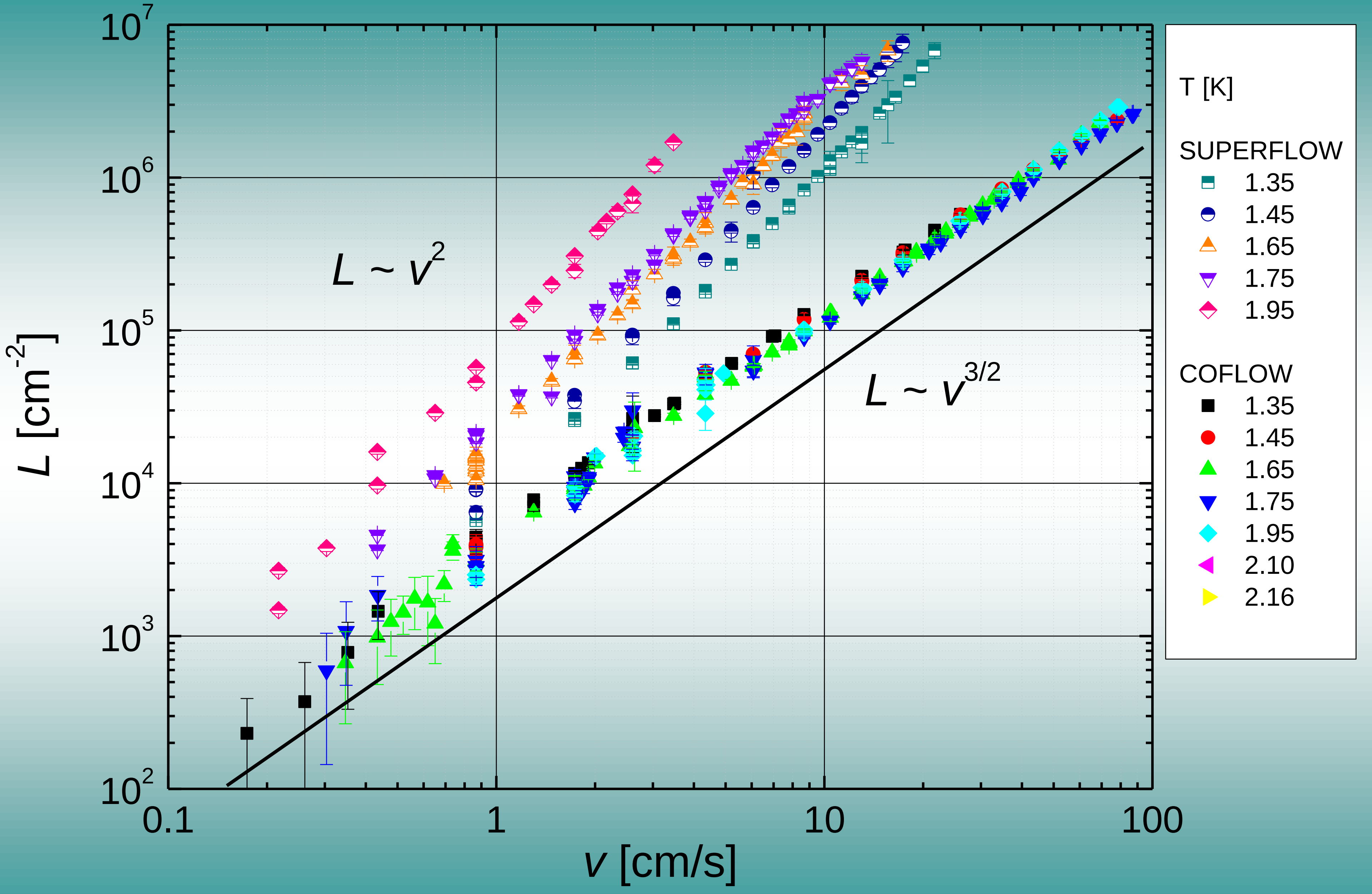
$L \sim v^2$
$L \sim v^{3/2}$
L [cm^{-2}]
v [cm/s]
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



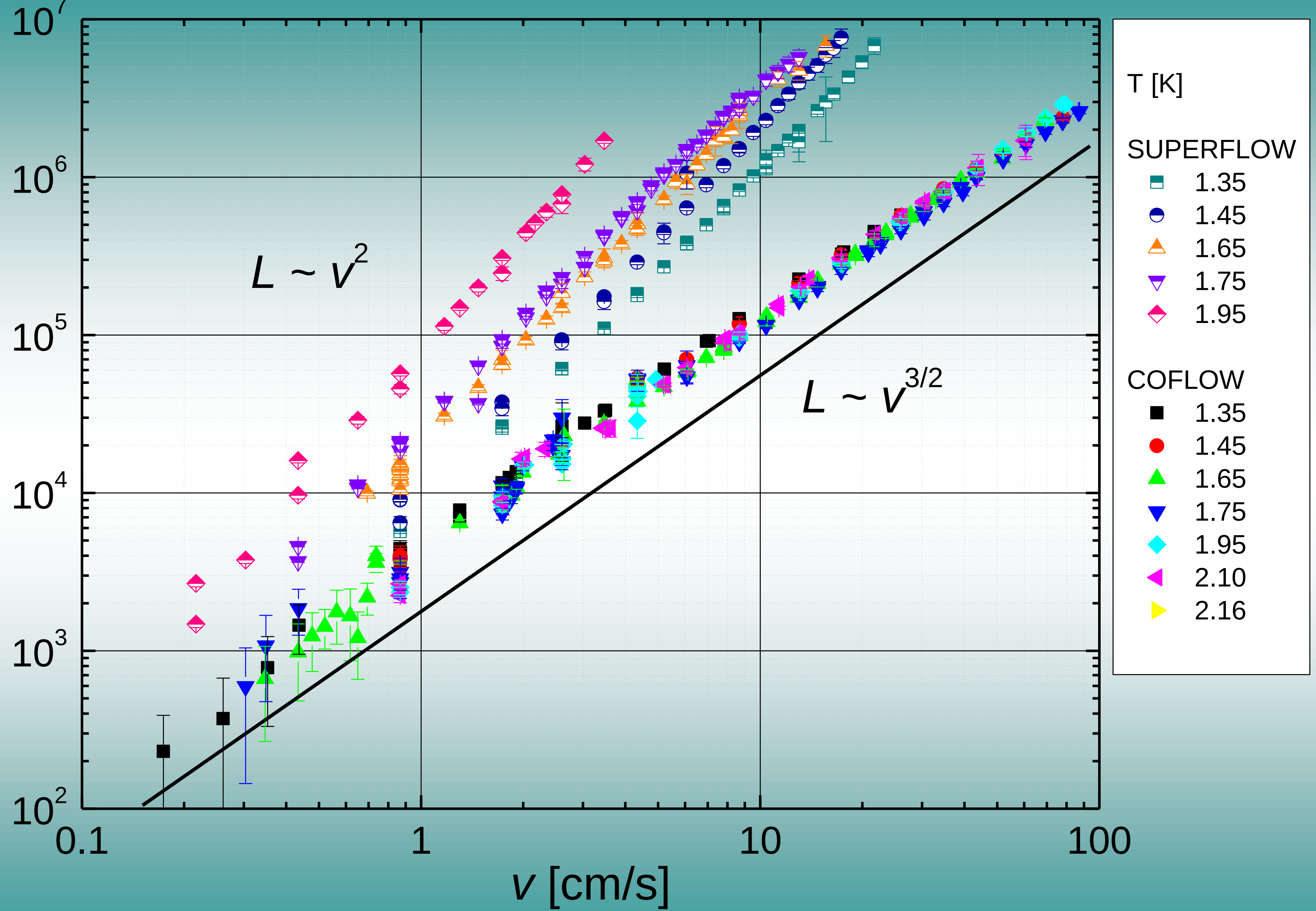
L [cm^-2]
v [cm/s]
L ~ v^2
L ~ v^3/2
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



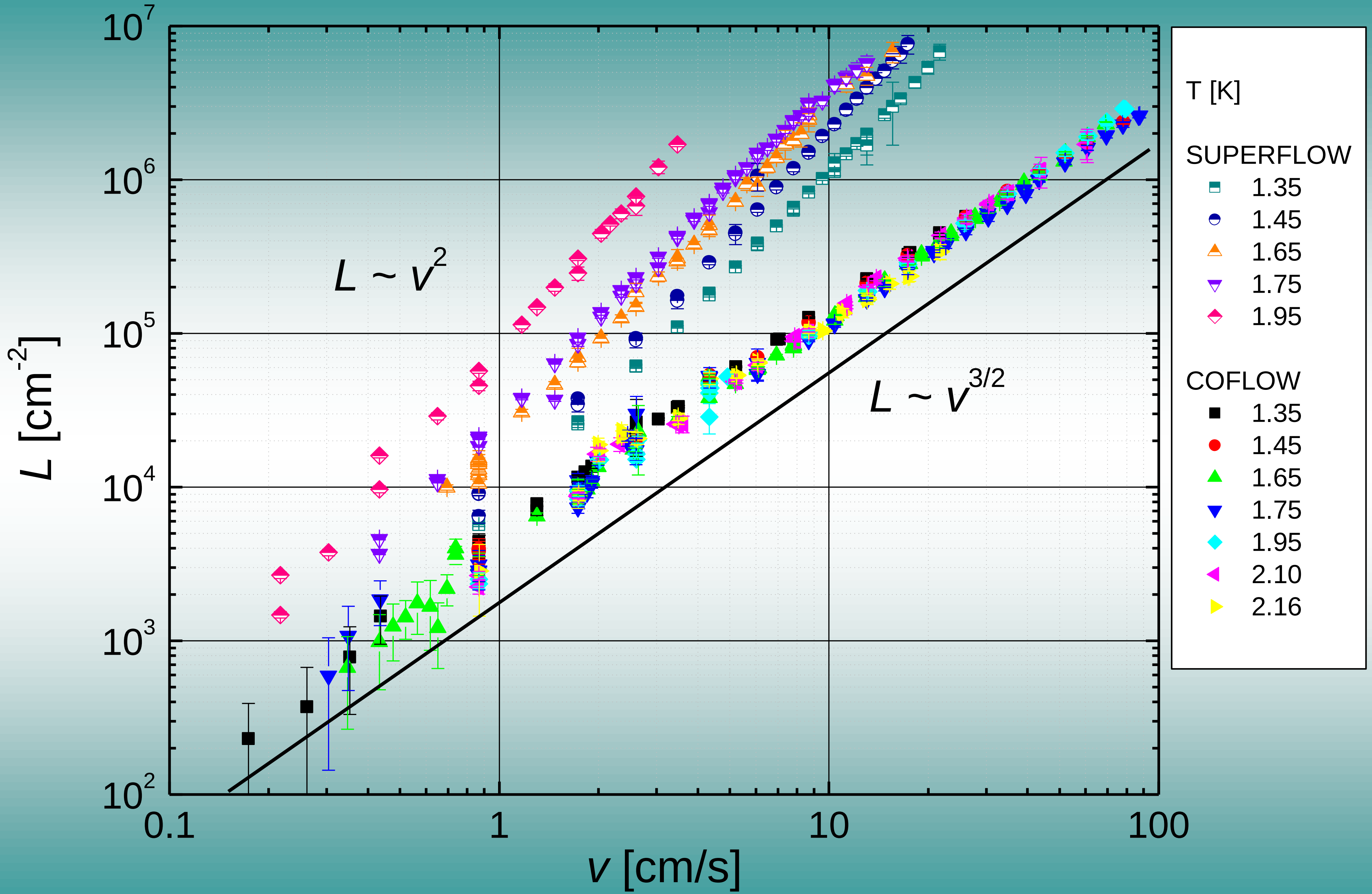
L [cm-2]
v [cm/s]
L ~ v2
L ~ v3/2
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



10^7
10^6
10^5
10^4
10^3
10^2
0.1
1
10
100
L [cm^-2]
v [cm/s]
L ~ v^2
L ~ v^3/2
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



L ~ v^2
L ~ v^3/2
L [cm^-2]
v [cm/s]
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16



L ~ v^2
L ~ v^3/2
L [cm^-2]
v [cm/s]
0.1
1
10
100
10^2
10^3
10^4
10^5
10^6
10^7
T [K]
SUPERFLOW
1.35
1.45
1.65
1.75
1.95
COFLOW
1.35
1.45
1.65
1.75
1.95
2.10
2.16

Two (steady-state) distinctly diffrent types of scaling:

$$L^{1/2} = \gamma(T)(V_{CF} - V_C)$$

Counterflow, pure superflow

$V_{CF} = \langle |V_n - V_s| \rangle$ mean counterflow velocity

$\chi_1 \, ; \chi_2$ temperature dependent parameter

B mutual friction constant

Vinen equation

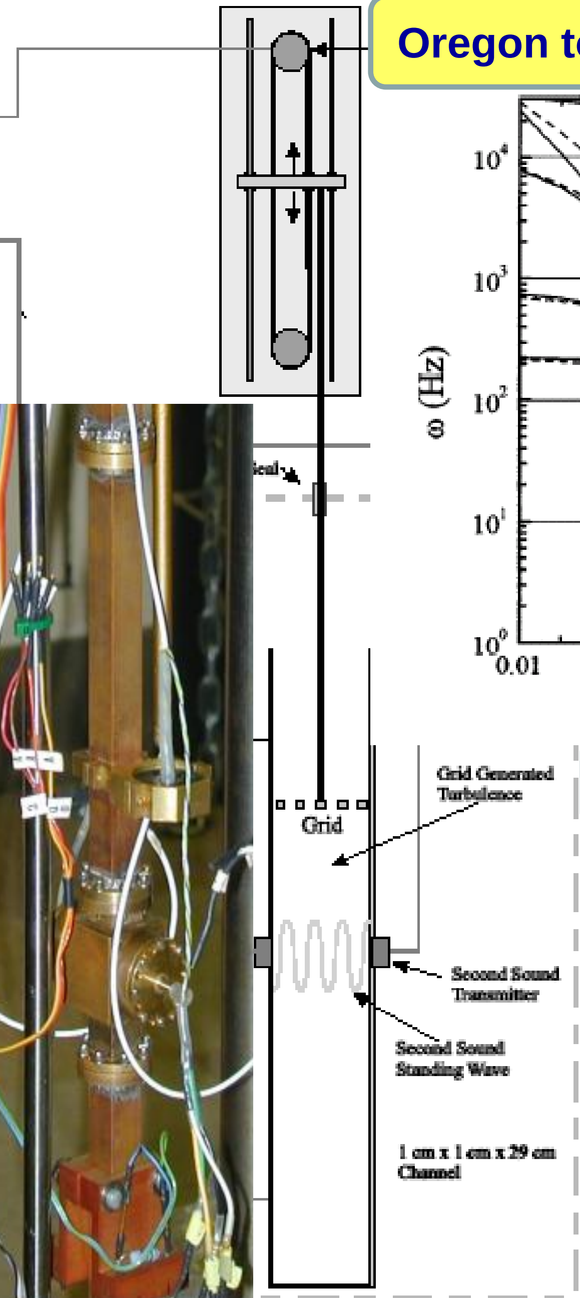
$$\frac{dL}{dt} = \chi_1 \frac{\rho_n B}{2\rho} V_{CF} L^{3/2} - \chi_2 \frac{\kappa}{2\pi} L^2$$

n

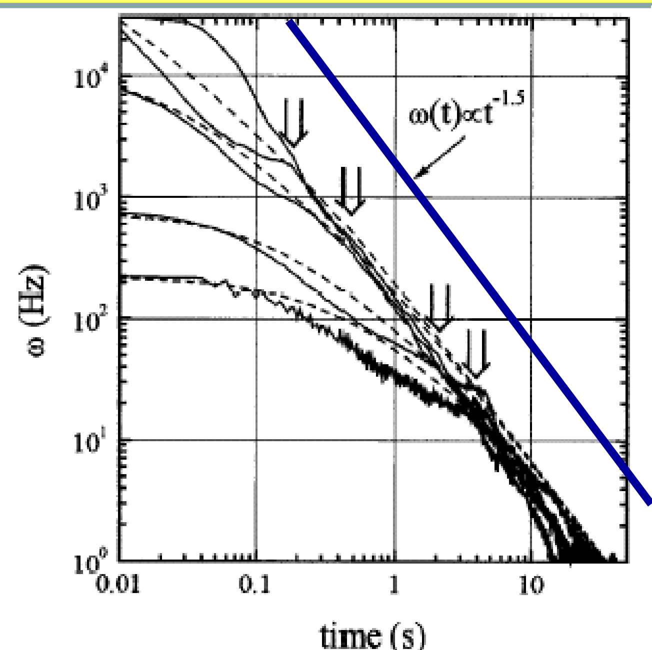
$$L \propto V^{3/2}$$

Coflow – classical scaling

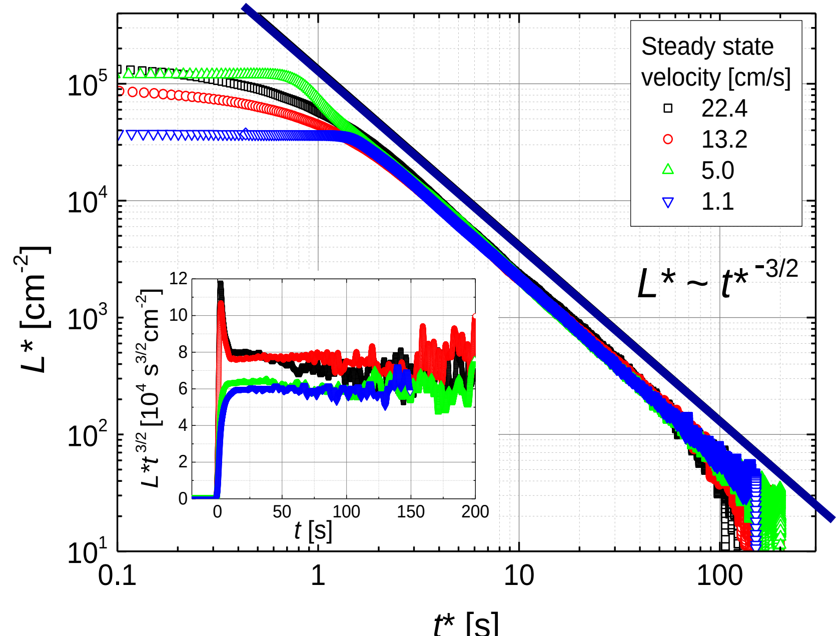
Decay of (two-fluid) quantum turbulence in He II



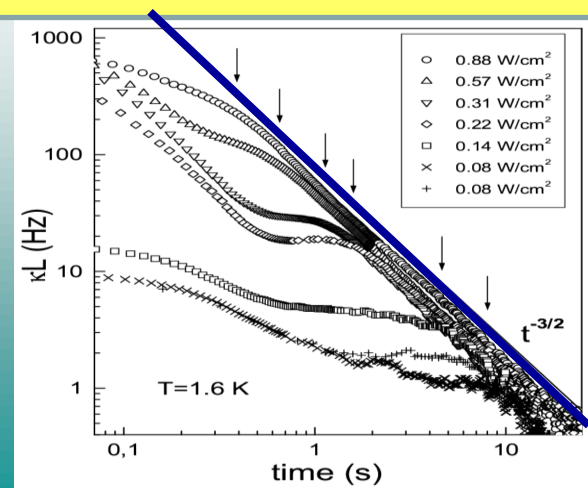
Oregon towed grid experiments



Decay of coflow - Prague



Decay of counterflow – Prague (Tallahassee)



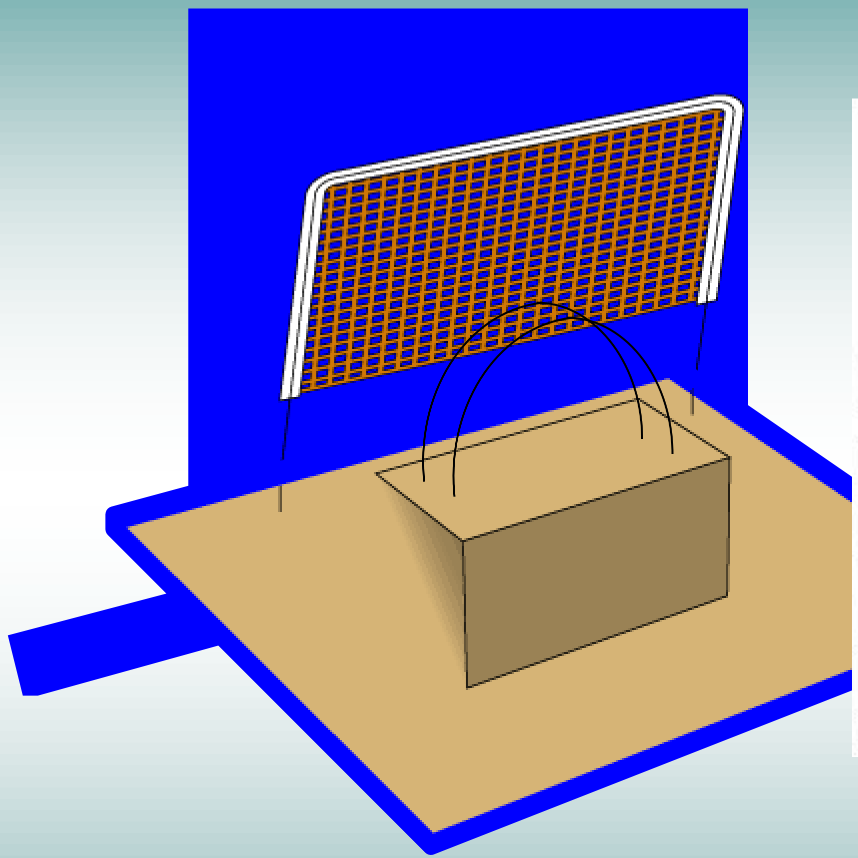
Late decay of vortex line density displays robust (quasi)classical -3/2 power law

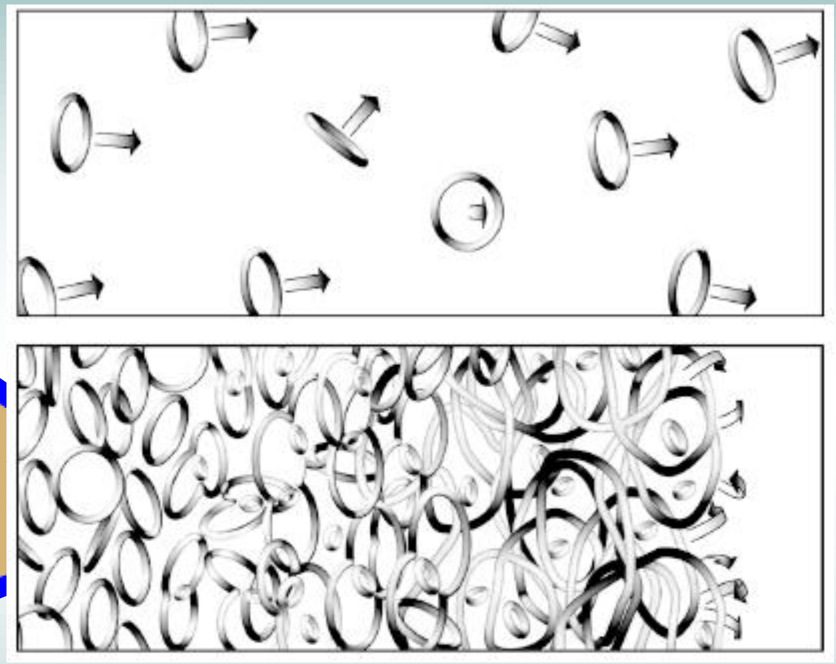
$$L(t) = \frac{D(3C)^{3/2}}{2\pi\kappa\sqrt{\nu_{eff}}}(t+t^*)^{-3/2}$$

Effective kinematic viscosity

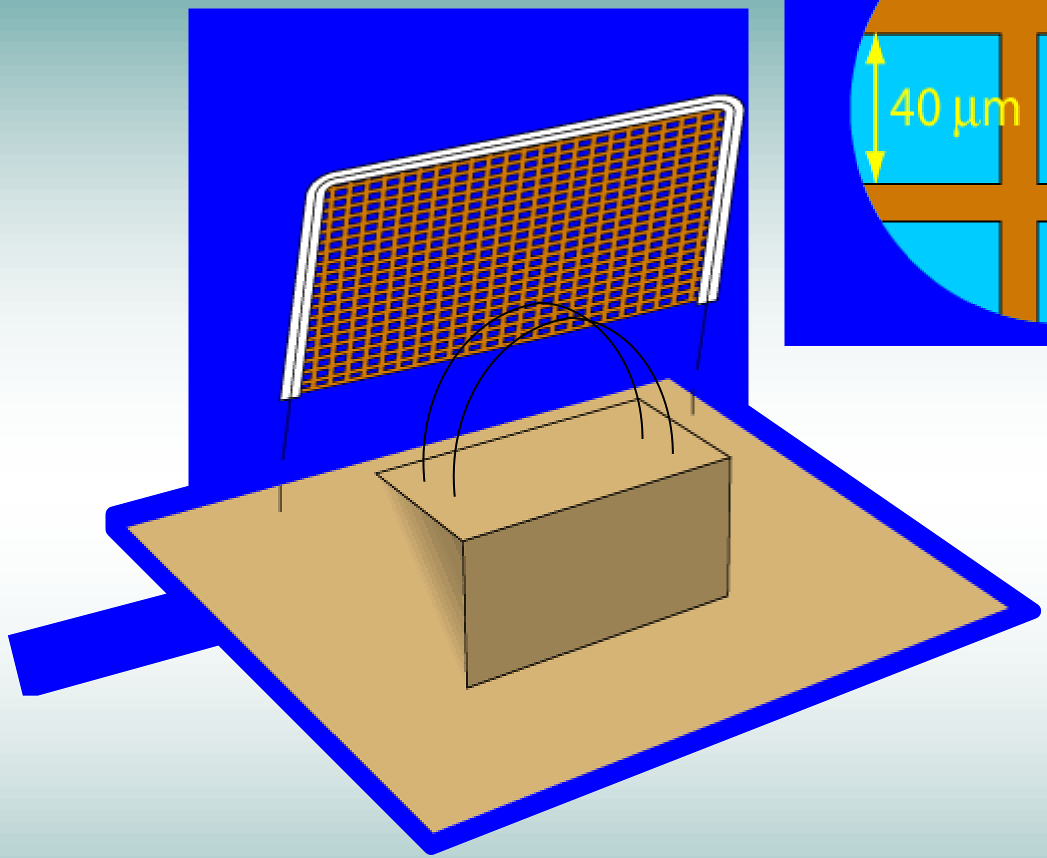
Vibrating grid in 3He B at low temperature - courtesy of S. Fisher

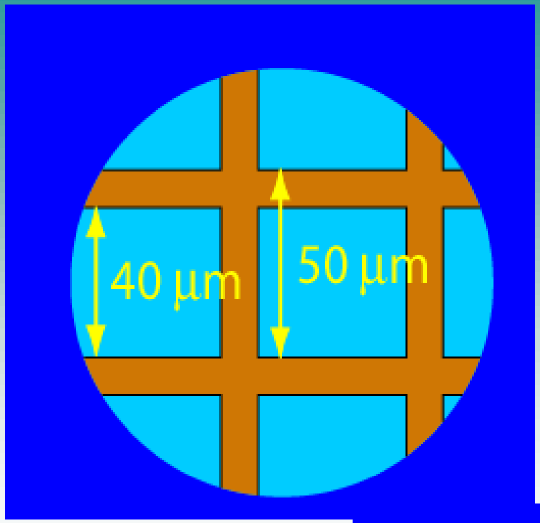
Lancaster group

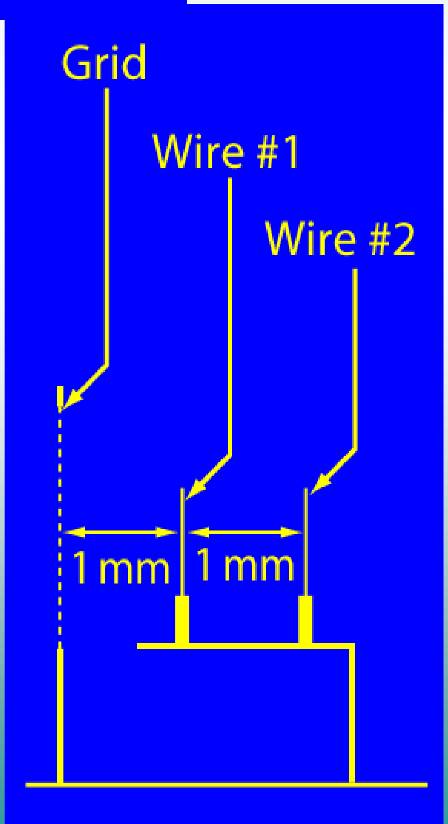




Temperature below 0.2 mK !!!



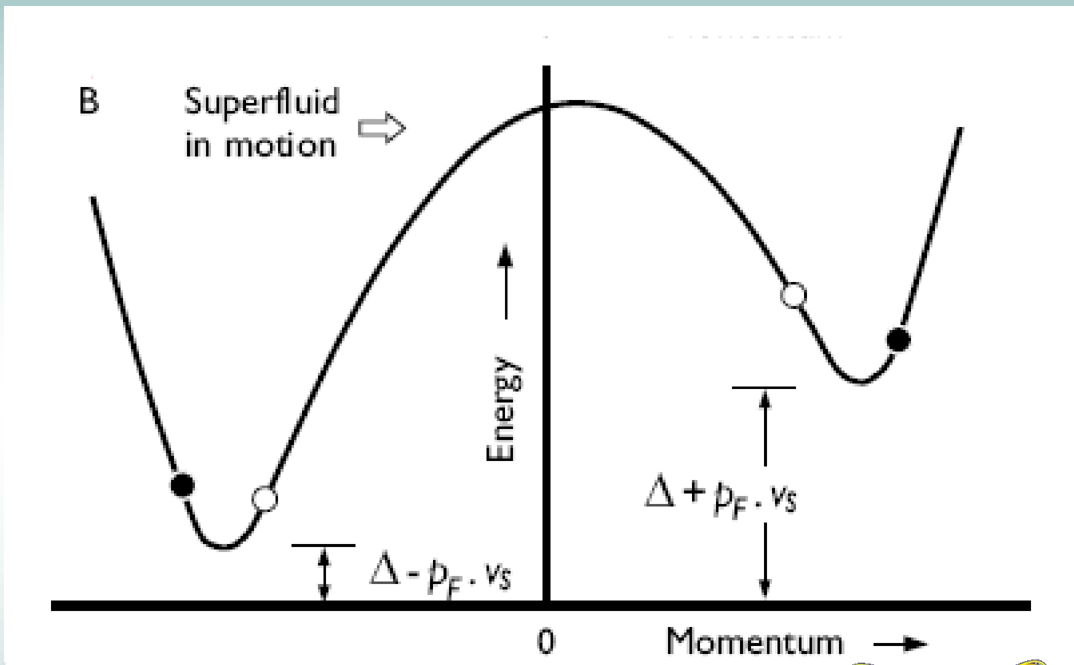


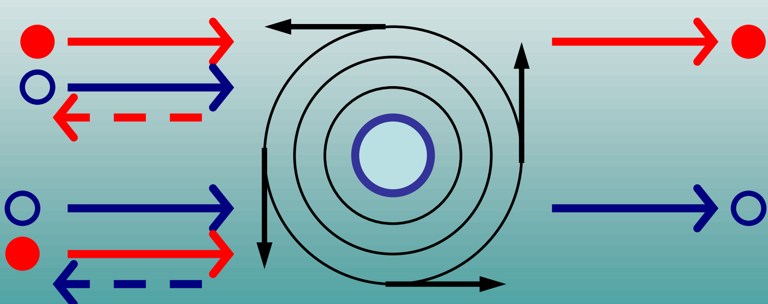


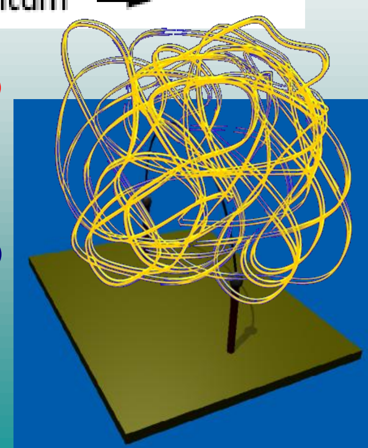
Temperature below 0.2 mK !!!

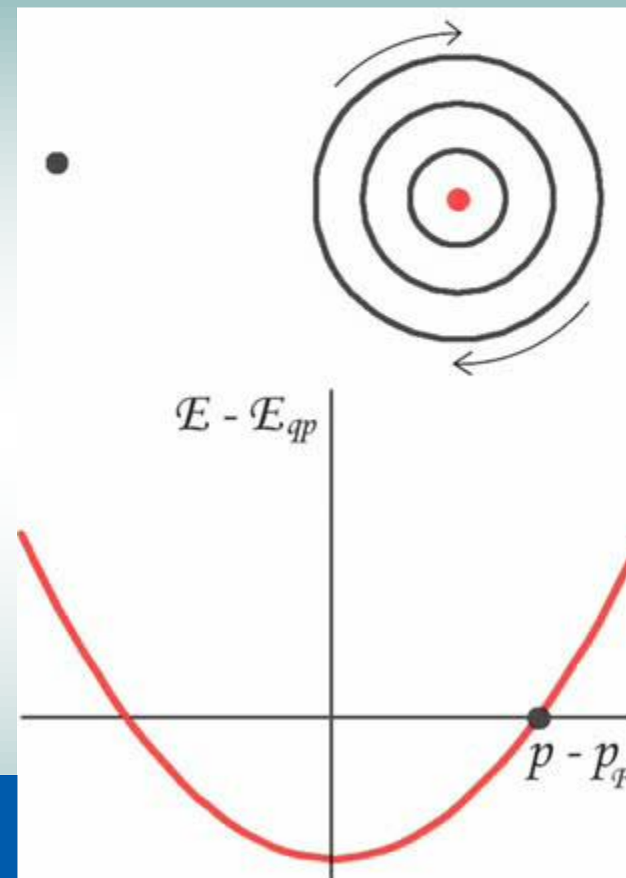
Andreev reflection in ^{3}He-B

Non-classical scattering of thermal excitations - quasiparticles, quasiholes.

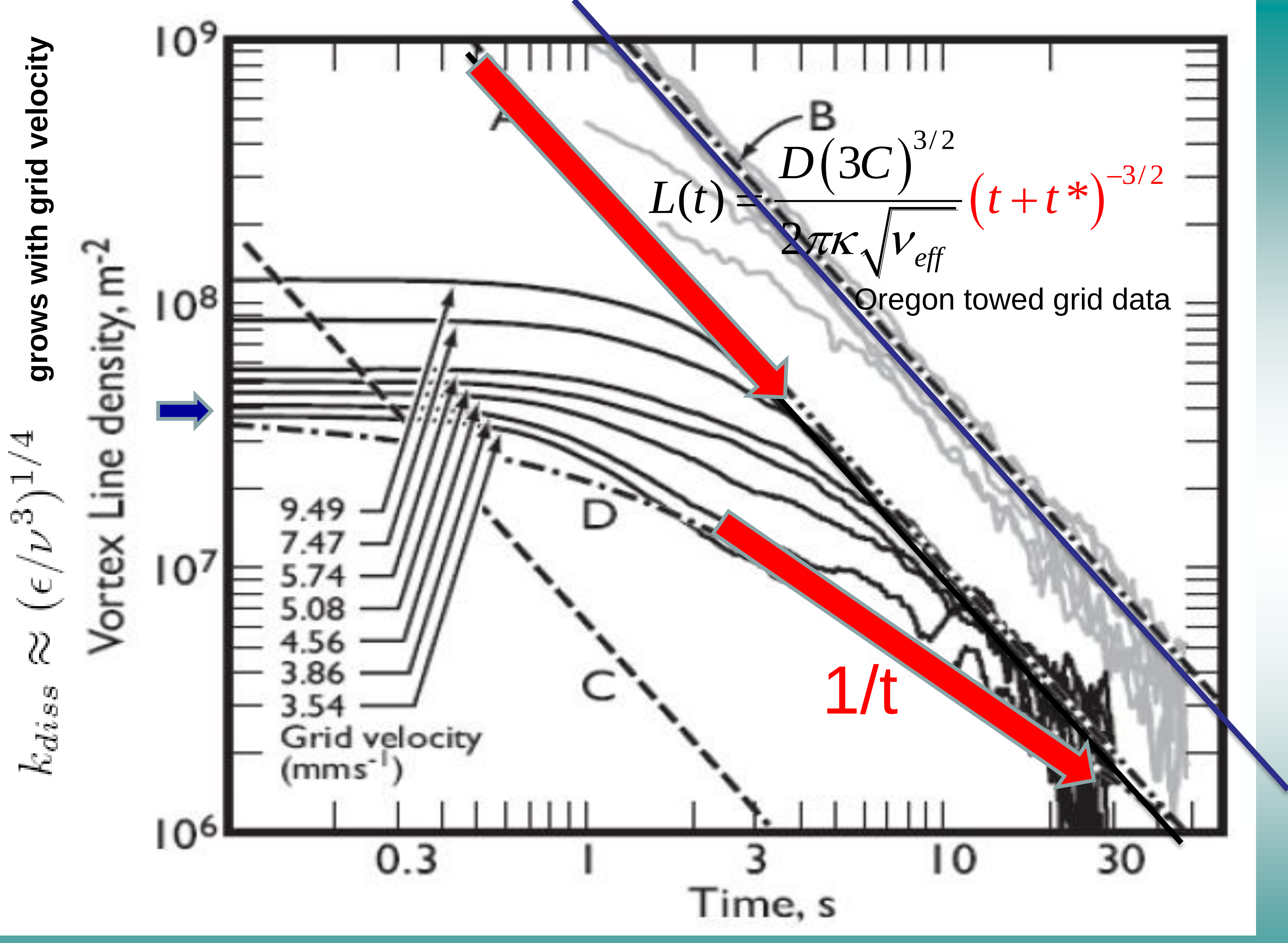






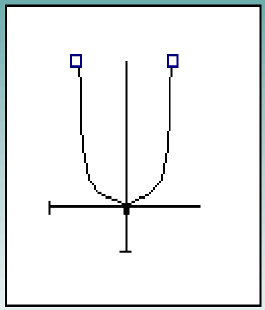


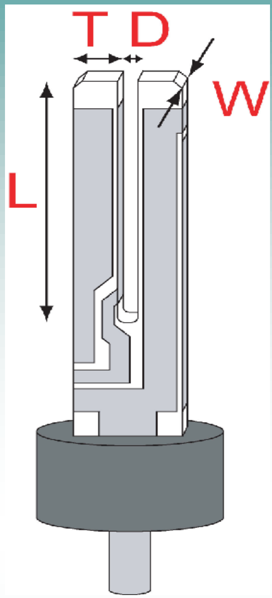
Quasiparticle damping reduced
by surrounding vortices



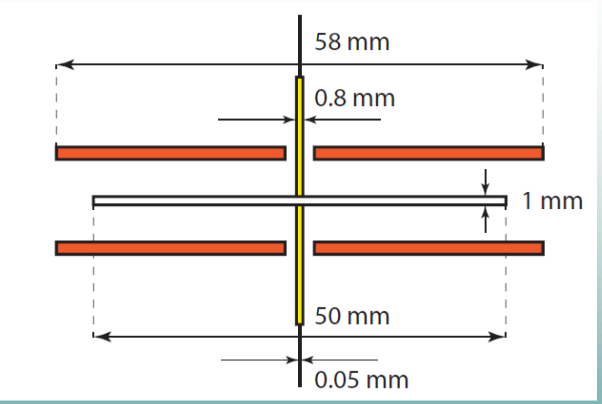
grows with grid velocity
$k_{diss} \approx (\epsilon/\nu^3)^{1/4}$
Vortex Line density, m^{-2}
10^9
10^8
10^7
10^6
A
B
C
D
$L(t) = \frac{D(3C)^{3/2}}{2\pi\kappa\sqrt{\nu_{eff}}}(t+t^*)^{-3/2}$
Oregon towed grid data
1/t
9.49
7.47
5.74
5.08
4.56
3.86
3.54
Grid velocity
(mms^{-1})
0.3
1
3
10
30
Time, s

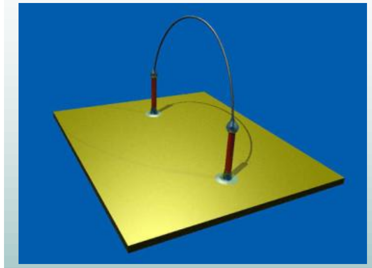
Experiments with oscillating objects

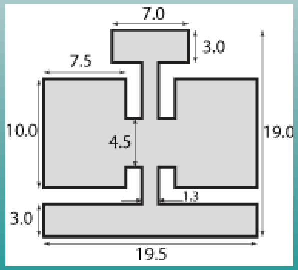


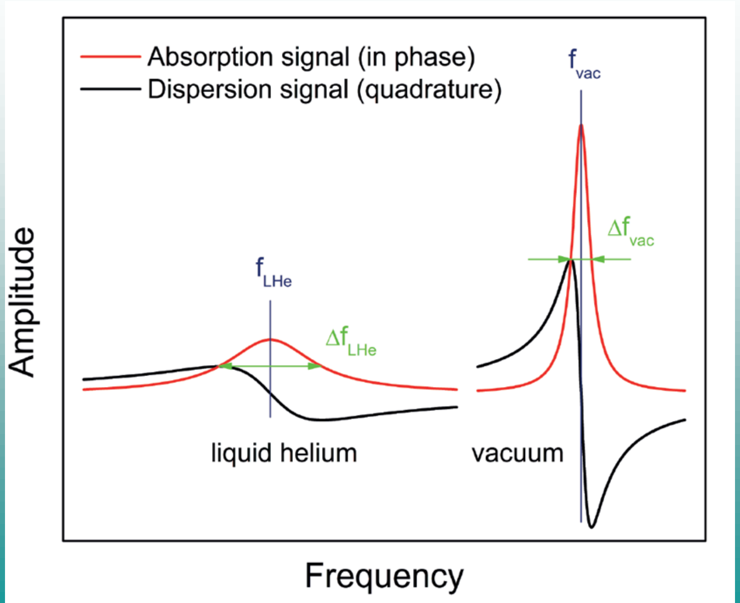


- commercial quartz tuning fork (32 kHz)
  - L = 3.65 mm, T = 680 μm, W = 460 μm, D = 180 μm; surface ~ 5 - 10 μm
- custom-made tuning forks, Lancaster (quartz + $LiNbO_3$)
  - L = 3.5 mm, T = 90 μm, W = 75 μm, D = 90 μm; surface ~ 1 μm, f = 6.5 kHz (41 kHz)
  - L = 6 mm, T = 1.1 mm, W = 1 mm, D = 0.6 mm; surface ~ 1 μm, f = 24.9 kHz; $LiNbO_3$
- vibrating NbTi wire (d = 40 μm, D = 2 mm)
- double-paddle, J. Luzuriaga (wing: 10 x 7.5 mm^2; thickness 0.2 mm)
- large torsionally oscillating disc (D = 40 mm, h = 1 mm)

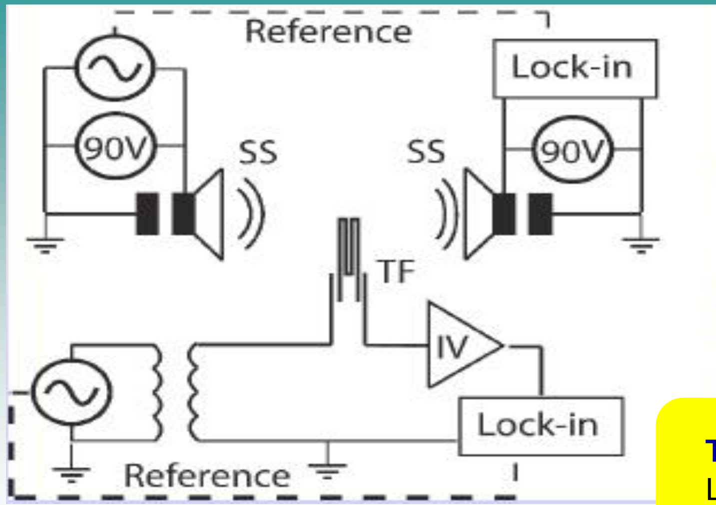








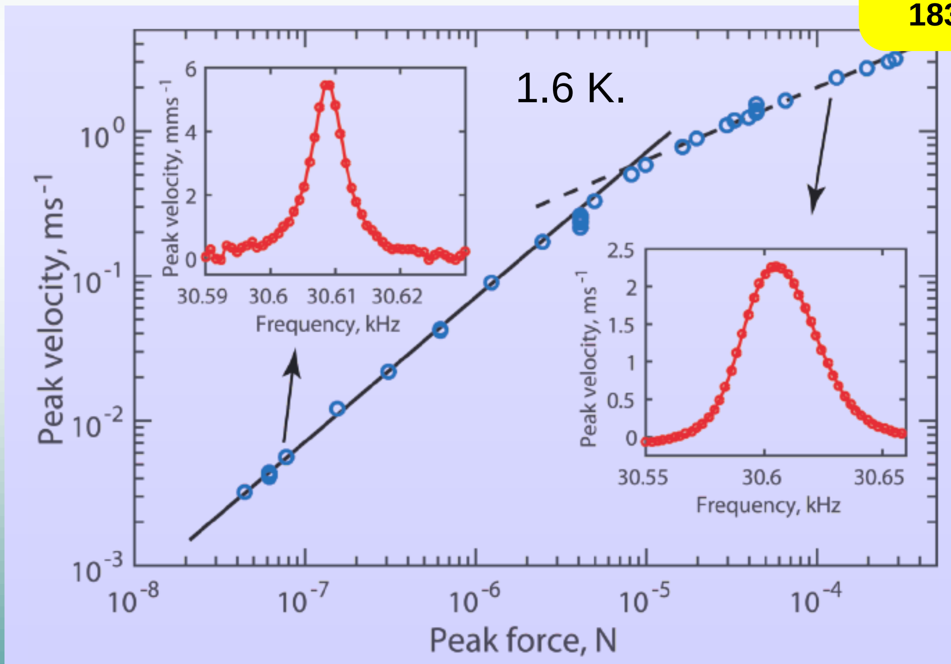
Does the oscillating tuning fork produce quantized vortices ?

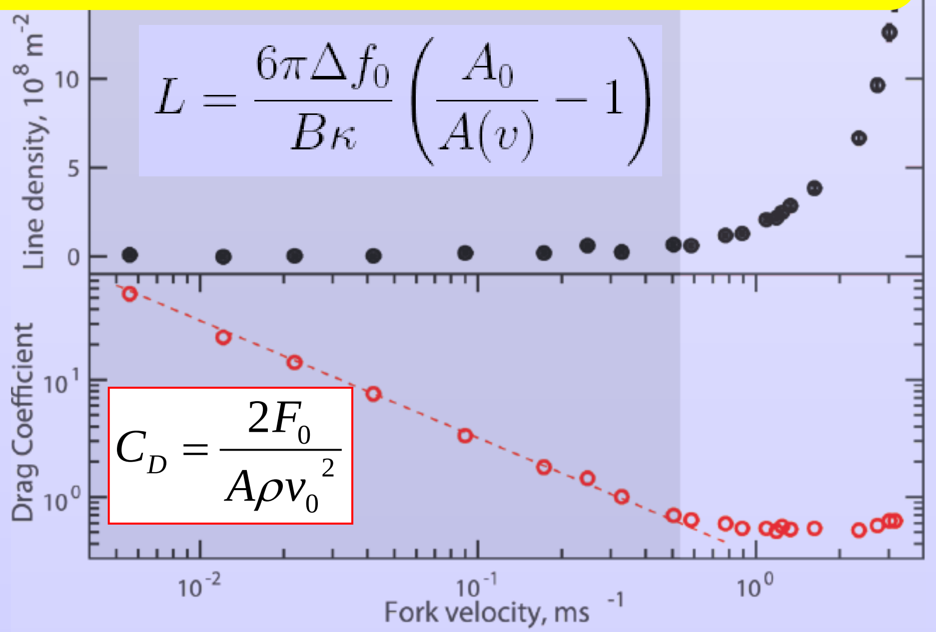


cylindrical second sound resonator

Lancaster University

Measurements of Vortex Line Density Generated by a Quartz Tuning Fork in Superfluid 4He D. Schmoranzer, M.J. Jackson, LS, O. Kolosov, V. Tsepelin, A.J. Woods, J. Low Temp. Physics, **183,** 208-214 (2016)





$$L = \frac{6\pi\Delta f_0}{B\kappa}\left(\frac{A_0}{A(v)} - 1\right)$$

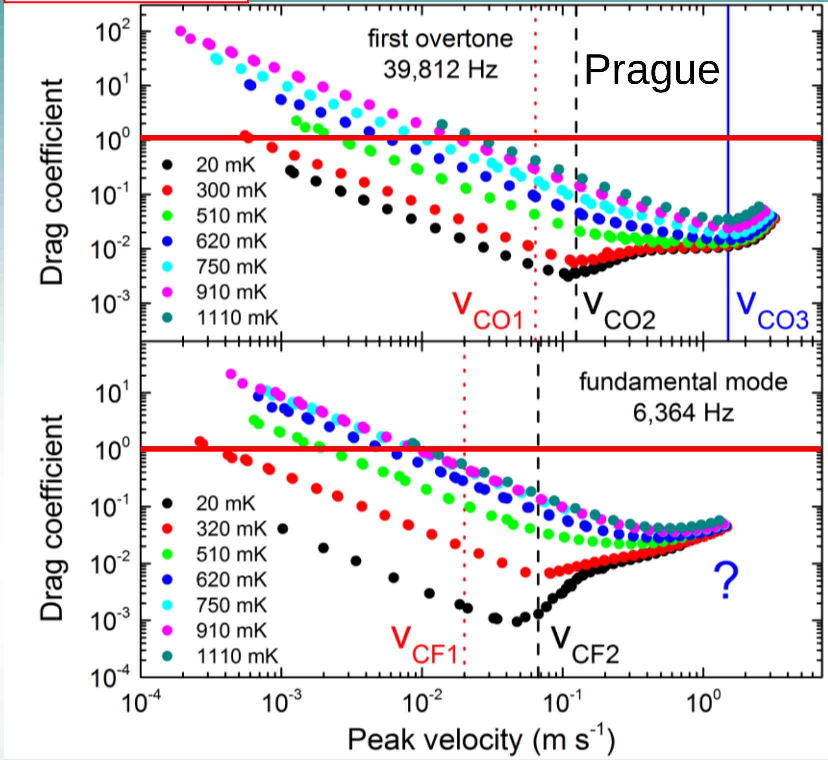
$$C_D = \frac{2F_0}{A\rho v_0^2}$$

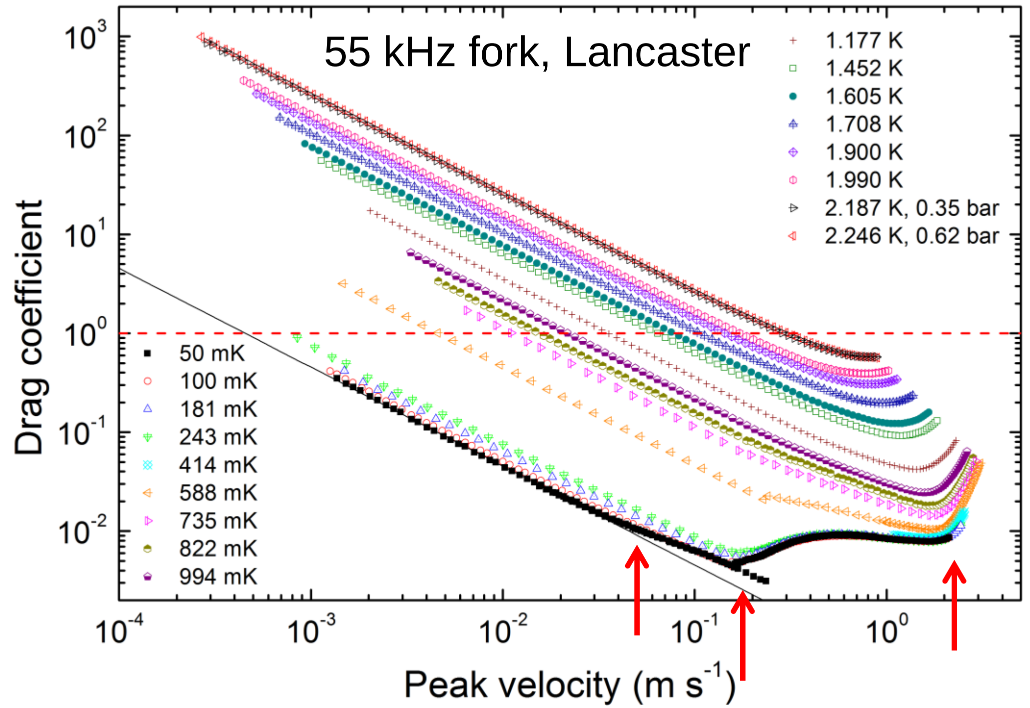
Directly tested by second sound attenuation:

Production of quantized vortices is directly related to the onset of excess damping

$$C_D = \frac{2F_0}{A\rho v_0^2}$$

Drag coefficient displays three critical velocities (zero T limit)



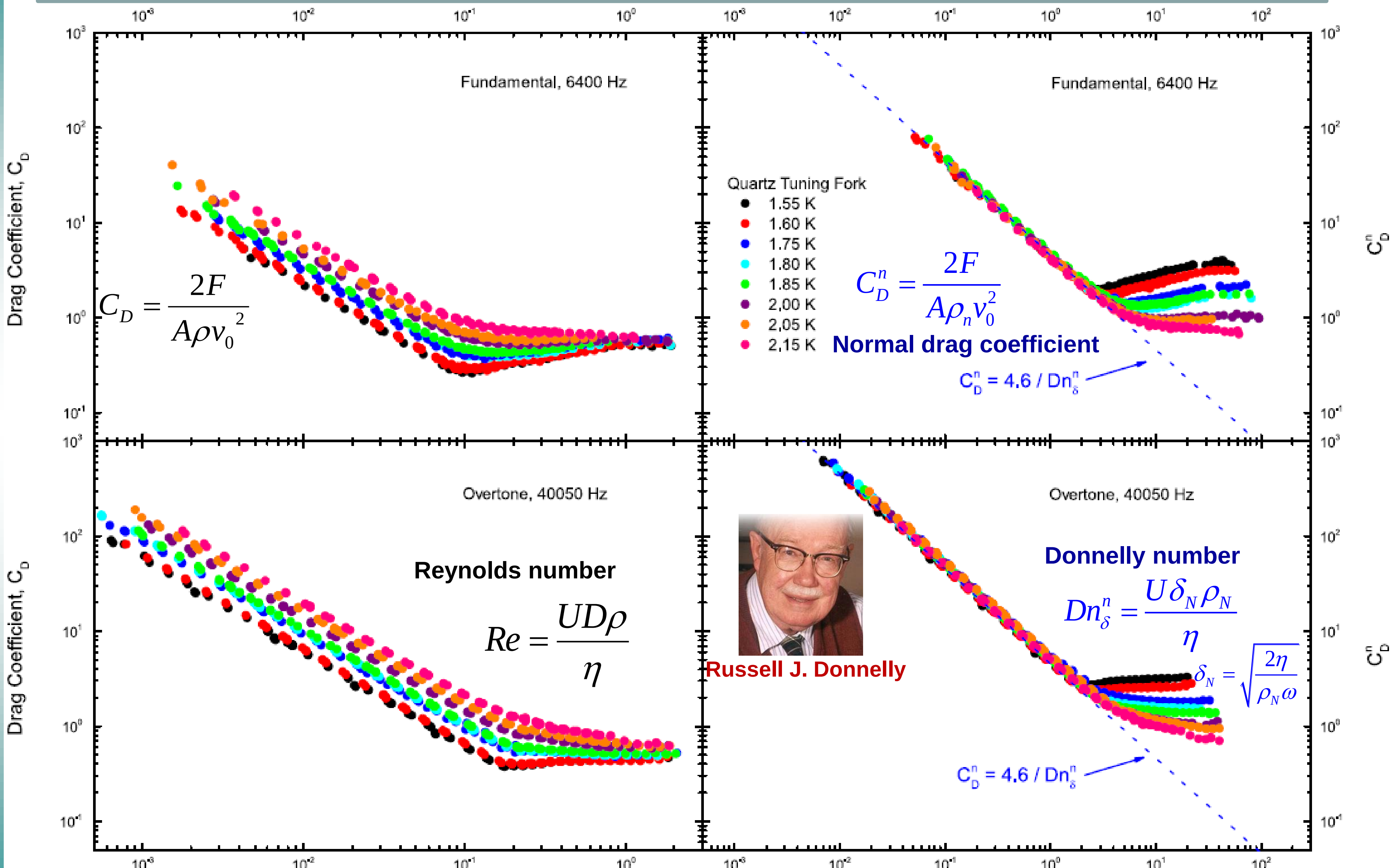


- **First critical velocity** – changes in frequency, little effect on drag force
  - effective mass rises due to vortices pinned on the oscillator surface
  - **need not lead to increased drag (mostly potential flow)**
- **Second critical velocity** – non-linear dissipation sets in
  - vortices spread into the bulk, carrying energy & momentum away
  - if $C_D \ll 1$ – the "wake" past the oscillator is not classical-like, no large structures in the flow, building up the vortex tangle
- **Third critical velocity** – large structures start to develop in the tangle
  - drag rises towards classical value (full pressure drag, developed wake)

For details, see: D. Schmoranzer, M. J. Jackson, V. Tsepelin, M. Poole, A. J. Woods, M. Clovecko and LS: **Multiple Critical Velocities in Oscillatory Flow of Superfluid 4He due to Quartz Tuning Forks** Phys. Rev. B **94**, 214503 (2016)

Lancaster University

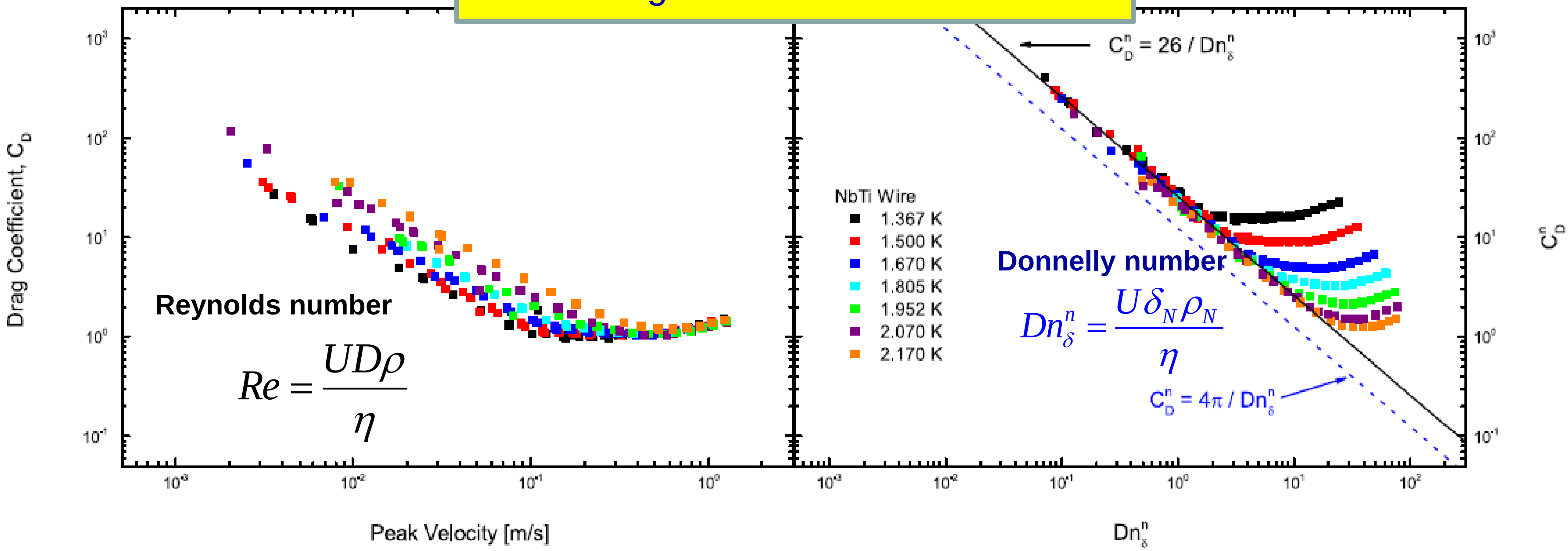
Commercial quartz tuning fork -- drag force scaling in the two-fluid regime



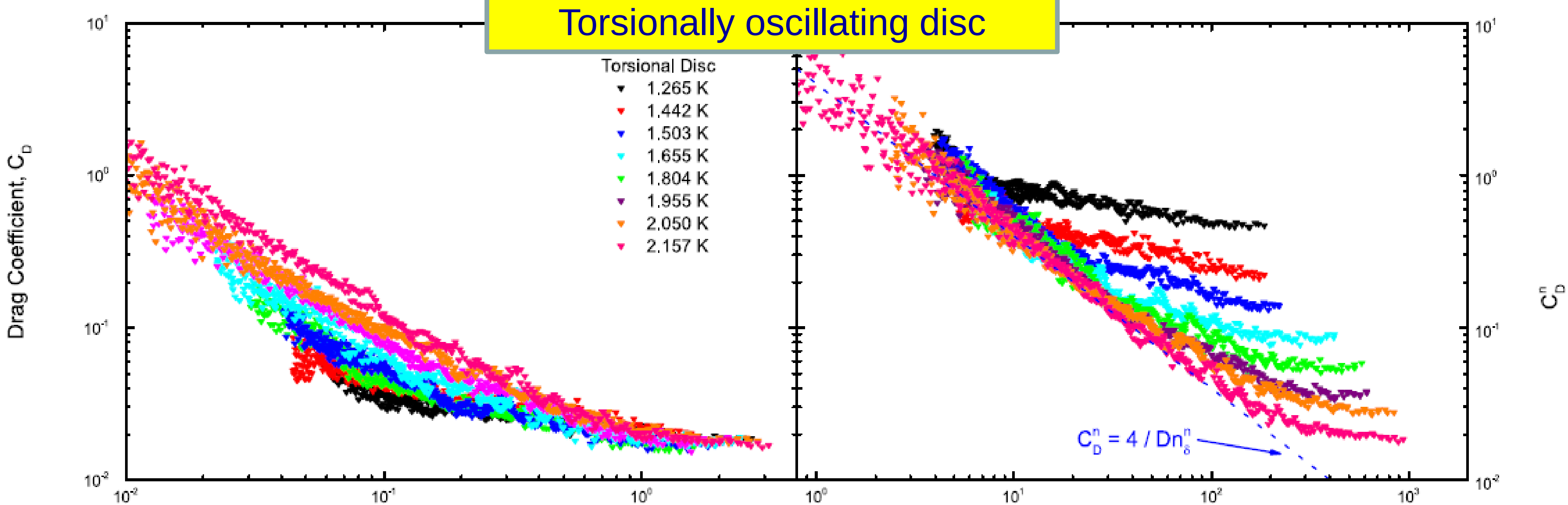
Peak velocity [m/s]

Donnelly number

Vibrating NbTi wire 40 micron dia



Torsionally oscillating disc

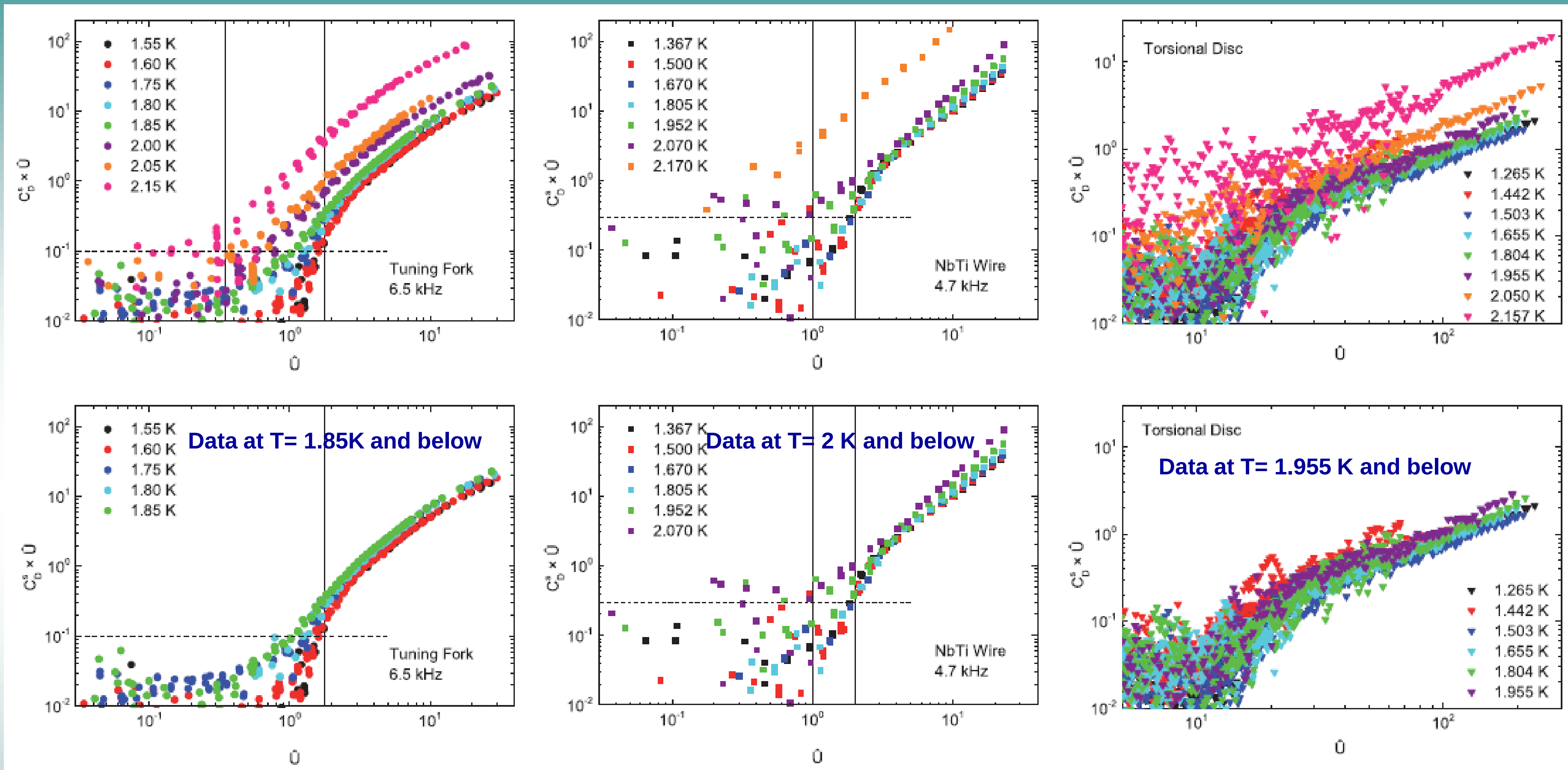


angular velocity [rad/s]

Donnelly number

Plots of non-linear drag versus nondimensionalized velocity

$$U' = \frac{U}{\sqrt{\kappa\omega}}$$



•The non-linear drag contribution is *T*- independent belows ome T showing that the superfluid component undergoes the transition alone, while the normal component remains mostly laminar.

•This is due to the kinematic viscosity of the normal component rapidly increasing as temperature is decreased.

In He II, normal component flow may be laminar even if turbulence exists in the superfluid component, like in 3He-B

Summary

•All forms of 4He - cryogenic helium gas, normal liquid He I and superfluid He II – as well as superfluid 3He-B - serve as outstanding working fluids for **cryogenic fluid dynamics** and **quantum turbulence**

•Extremely high Re and Ra flows can be studied under controlled laboratory conditions

•**Quantum turbulence** has been investigated over 50 years -**a lot is known about it, but it is still only partly understood**

•In the zero temperature limit QT represents the simplest prototype of turbulence

•At finite temperature, in the two-fluid regime, QT is more complex than classical turbulence, combining classical turbulencein the normal fluid with the dynamics of the vortex tangle in the superfluid, coupled by the mutual friction force

Does 4He, together with other quantum fluids, hold the key to unlocking the underlying physics of fluid turbulence?

Plenty of interesting physics to play with…

Introduction to quantum turbulence

Carlo F. Barenghi[a,1], Ladislav Skrbek[b], and Katepalli R. Sreenivasan[c]

Charles University in Prague
Universitas Carolina

Visualization of two-fluid flows of superfluid helium-4

Wei Guo[a,b], Marco La Mantia[c], Daniel P. Lathrop[d], and Steven W. Van Sciver[a,b,1]

Andreev reflection, a tool to investigate vortex dynamics and quantum turbulence in ^{3}He-B

Shaun Neil Fisher[a], Martin James Jackson[b], Yuri A. Sergeev[c,d], and Viktor Tsepelin[a,1]

Quantum turbulence generated by oscillating structures

William F. Vinen[a,1] and Ladislav Skrbek[b]

Direct observation of Kelvin waves excited by quantized vortex reconnection

Enrico Fonda[a,b,c], David P. Meichle[a,d], Nicholas T. Ouellette[a,e], Sahand Hormoz[f], and Daniel P. Lathrop[a,d,1]

Dynamics of quantum turbulence of different spectra

Paul Walmsley[a,1], Dmitry Zmeev[a,b], Fatemeh Pakpour[a], and Andrei Golov[a]

Quantum turbulence in superfluids with wall-clamped normal component

Vladimir Eltsov[1], Risto Hänninen, and Matti Krusius

Modeling quantum fluid dynamics at nonzero temperatures

Natalia G. Berloff[a,b,1], Marc Brachet[c], and Nick P. Proukakis[d]

Experimental, numerical, and analytical velocity spectra in turbulent quantum fluid

Carlo F. Barenghi[a], Victor S. L'vov[b], and Philippe-E. Roche[c,d,1]

Vortex filament method as a tool for computational visualization of quantum turbulence

Risto Hänninen[a,1] and Andrew W. Baggaley[b]

Wave turbulence in quantum fluids

German V. Kolmakov[a], Peter Vaughan Elsmere McClintock[b,1], and Sergey V. Nazarenko[c]

Vortices and turbulence in trapped atomic condensates

Angela C. White[a,1,2], Brian P. Anderson[b], and Vanderlei S. Bagnato[c]