STELLAR CONVECTION

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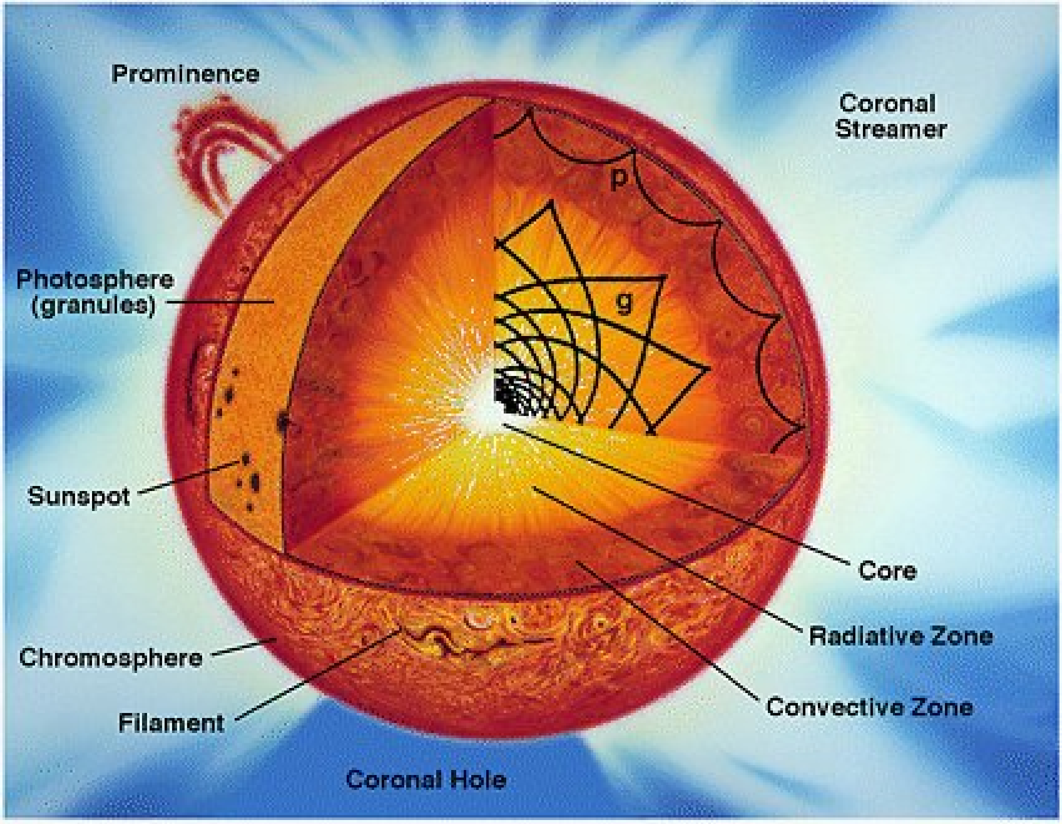
- The Sun being our nearest star, provides us with a unique opportunity to study stellar physics. The Sun has also played a key role in our understanding of physics, e.g., general relativity and neutrino physics.

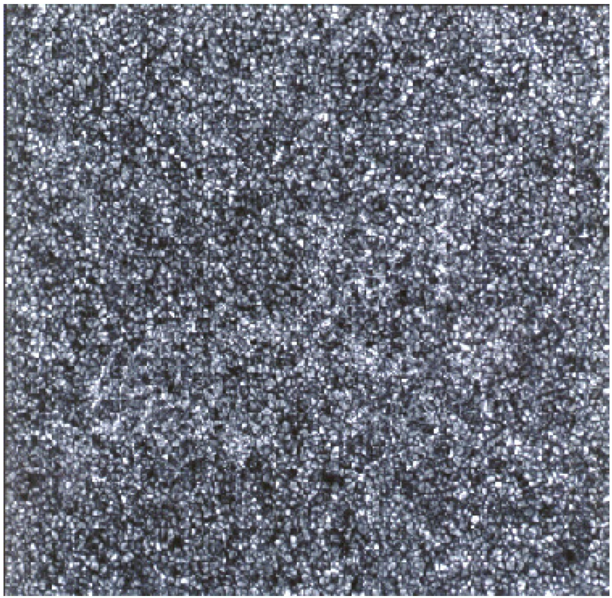
- A major source of uncertainty in current solar models and in our understanding of solar interior is the treatment of convection. The solar convection zone is highly turbulent and a proper treatment of all length scales of turbulence is beyond the scope of computers in foreseeable future.

- Convection plays a central role in maintaining the differential rotation profile as well as the meridional flow and solar dynamo.

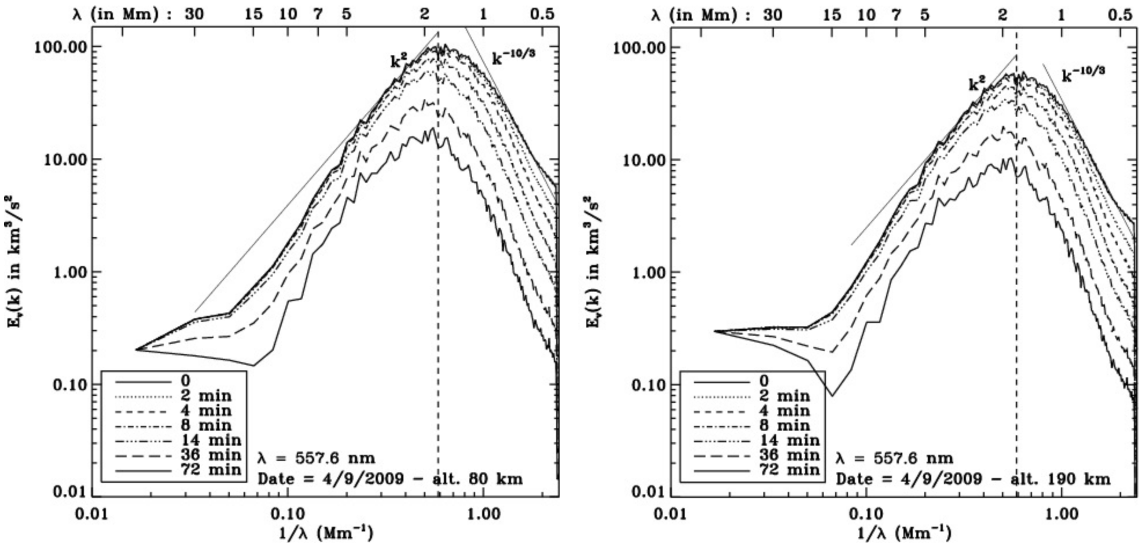


Prominence
Coronal
Streamer
p
g
Photosphere
(granules)
Sunspot
Core
Radiative Zone
Chromosphere
Convective Zone
Filament
Coronal Hole

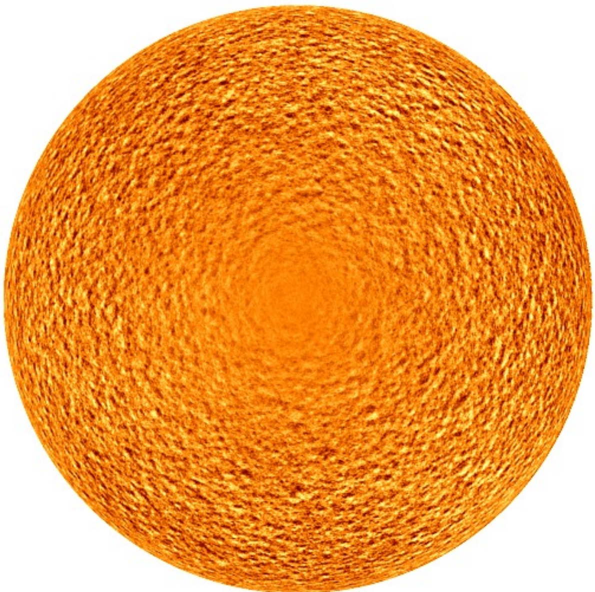
- A visible manifestation of solar convection is the granulation pattern on the solar surface first seen by Huggins (1866). Granules have a length scale of about 1000–2000 km and a life-time of 5–10 minutes. The velocity is about 1–2 km s^{-1}.



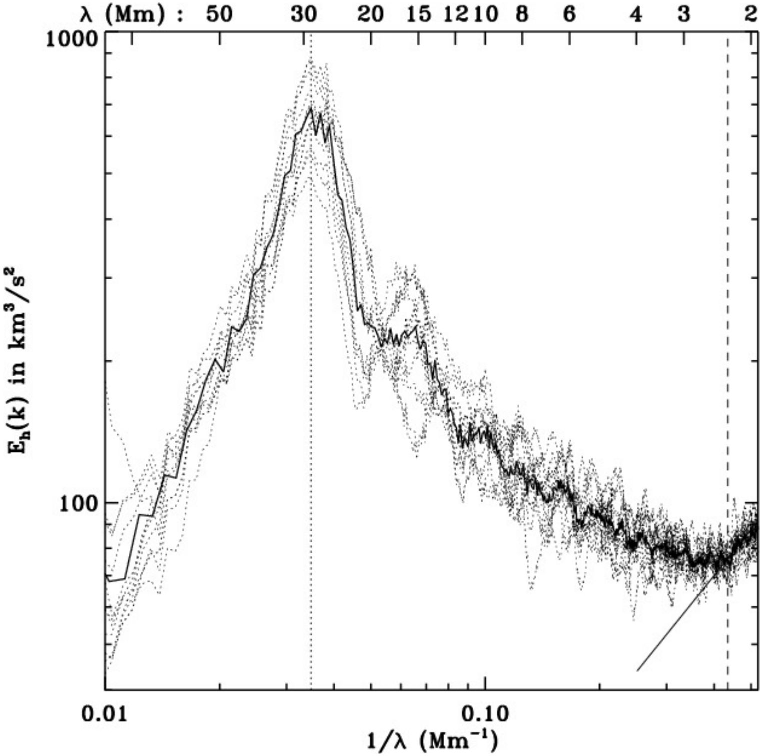
Power spectrum of vertical velocity (Rieutord et al. 2010)



- While studying solar rotation, Hart (1954) discovered the existence of systematic horizontal velocity pattern, which is now called supergranulation. With a length scale of about 30000 km and a time scale of about 1 day and horizontal velocity of about 500 m s^{-1}.



Power spectrum of horizontal velocity (Rieutord et al. 2010)



- There are some claims of mesogranulation with length and time scale between granulation and supergranulation.

- Similarly, there are also claims of giant cells comparable to solar radius, but these are not established.

- While the scale of granulation can be understood theoretically as the dominant scale of convection near the solar surface, there is no accepted explanation for occurrence of multiple scales.

- Supergranulation could be due to helium ionization layer below the solar surface, but this is not established.

SOLAR CONVECTION ZONE

- In the outer $\approx 30\%$ of the solar radius, energy is predominantly transmitted by convection. There is no theory to calculate the convective flux in highly turbulent compressible fluid. This is a major source of uncertainty in solar models. Convection also controls other phenomenon, like differential rotation, meridional flow, dynamo, etc.

- In the upper layers of the solar convection zone, radiative transfer is also important and it adds to the complication.

- The stellar models treat convection using the mixing-length theory

$$F_{\mathrm{conv}} \sim \rho W C_p L \left(\left| \frac{dT}{dr} \right| - \left| \frac{dT}{dr} \right|_{\mathrm{adia}} \right)$$

- The mixing-length theory has been very successful in constructing stellar models. The main deviation occurs close to the surface where the convection is not very efficient. The reason for this is that in most of the stellar interior, the convection is very efficient and the temperature gradient is close to the adiabatic value.

- Only rigorous method to compute convective flux is the direct numerical simulation, i.e., solving the equations of fluid dynamics along with radiative transfer in the context of the convection zone.

- A measure of turbulence is provided by the Reynolds number

$$Re = \frac{VL}{\nu}$$

where V and L are the characteristic velocity and lengths, respectively and ν is the kinematic viscosity.

- In the solar convection zone typically $Re = 10^{15}$

- The convective eddies have a wide range of length and time-scales and numerical simulations have to resolve all these scales.

- Further, the solar convection zone has strong stratification with density varying by more than 10^6 over its depth, which means that compressibility has to be included.

- From theory of turbulence the ratio of typical length scale to the smallest length scale of turbulence, which gives the typical No. of mesh points required to resolve all length scales is $Re^{3/4}$. Thus the number of grid points is $Re^{9/4}$. In order to solve the equations the time step will be limited by the Courant condition which gives similar value for the No. of time steps. Thus the total effort required is $Re^3 \sim 10^{45}$.

- This will only cover the dynamical time scale. If we wish to evolve the fluid over thermal time scale we need to add a few orders of magnitude to make it $\sim 10^{50}$.

- Even if all existing computers in the world are employed to do the calculation for the entire age of the Universe, they will fall short by many orders of magnitude.

- If we assume the Moore's law to be valid for the rest of the 21st century, we will have computers capable of about 10^{35} FLOPS by 2100, which will still need more than hundred million years to complete the computation.

- Thus the simulation of the solar convection zone is beyond the capabilities of computers in foreseeable future.

- In actual practice the maximum achievable mesh on currently available supercomputers is 4096^3, which can effectively deal with $Re \lesssim 10^6$, which is far from the solar value.

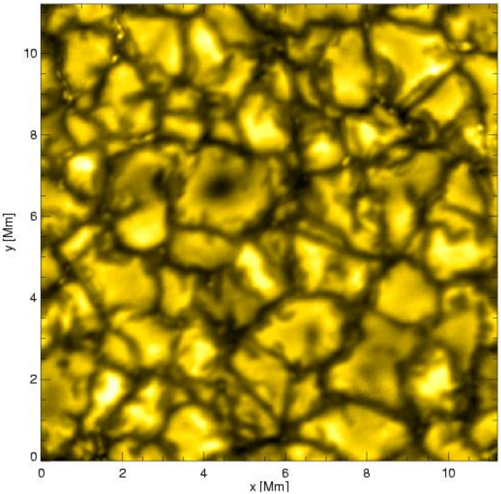
- One technique to deal with large Re is the Large Eddy Simulations, where the effect of unresolved eddies is approximated by using some treatment of turbulence. This is not very effective in solar convection zone because of extreme stratification.

- Most calculations adopt an adhoc approach, like artificially increasing viscosity, etc.

RESULTS FROM NUMERICAL SIMULATIONS

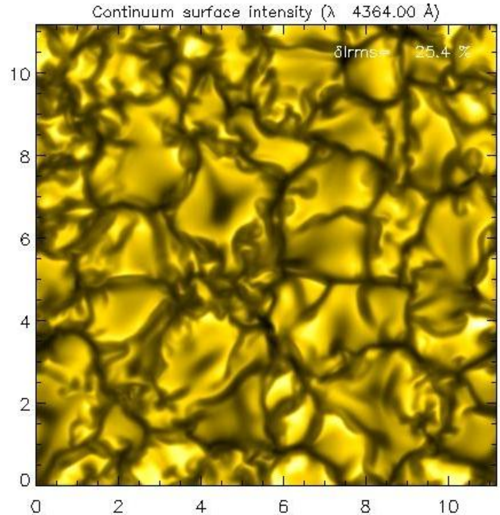
- Numerical simulations of the solar convection zone typically consider a box of $6000 \times 6000 \times 4000$ km, representing a very small part of outer solar convection zone.

- Observations show that the dominant size of convective cells in the Sun is about 2000 km. This is reproduced by these simulations.



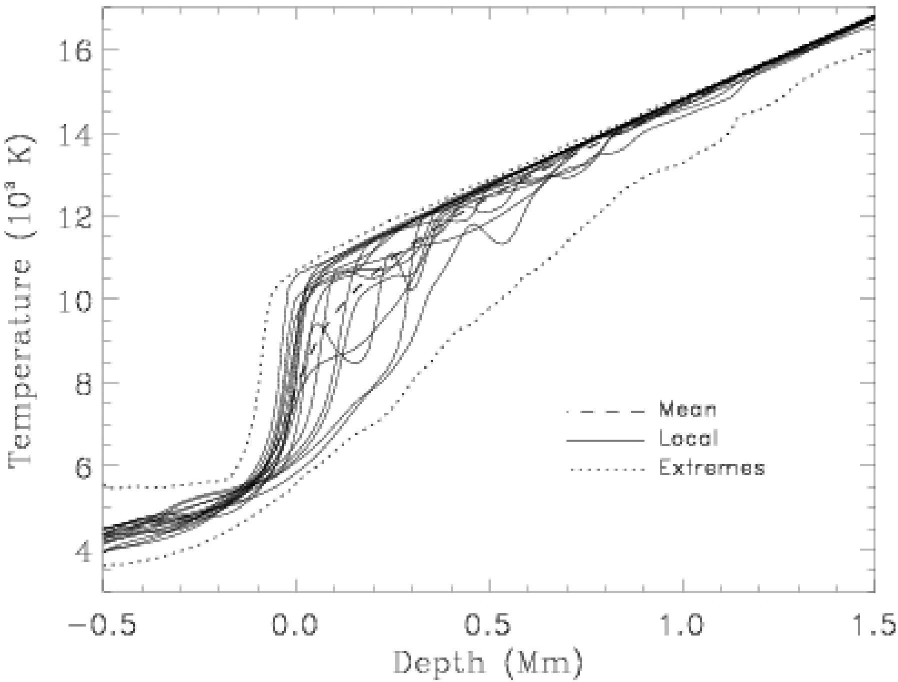
Solar Granulation

Swedish Solar Telescope on La Palma by Carlsson et al.

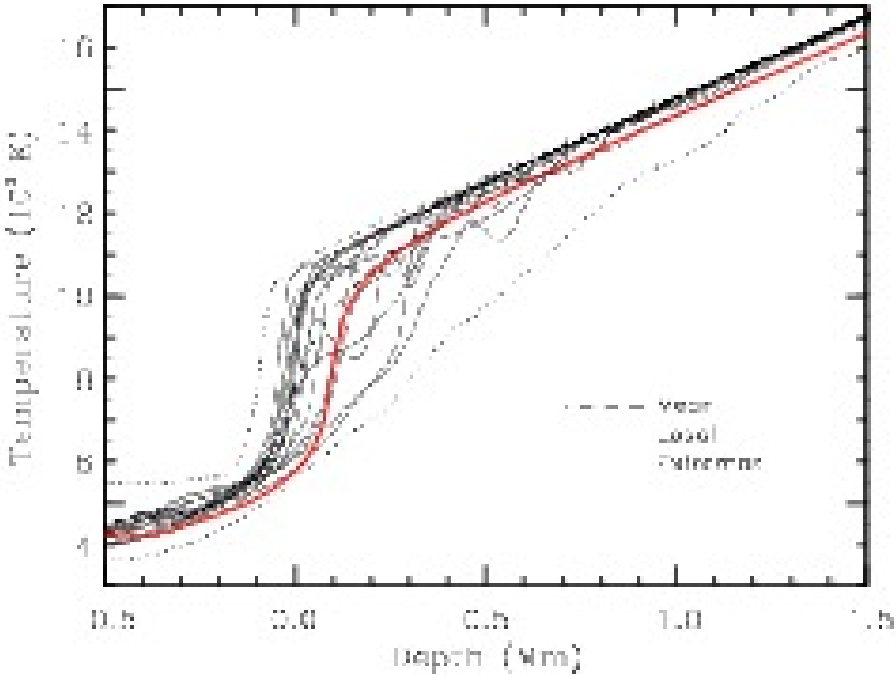


Numerical Simulation

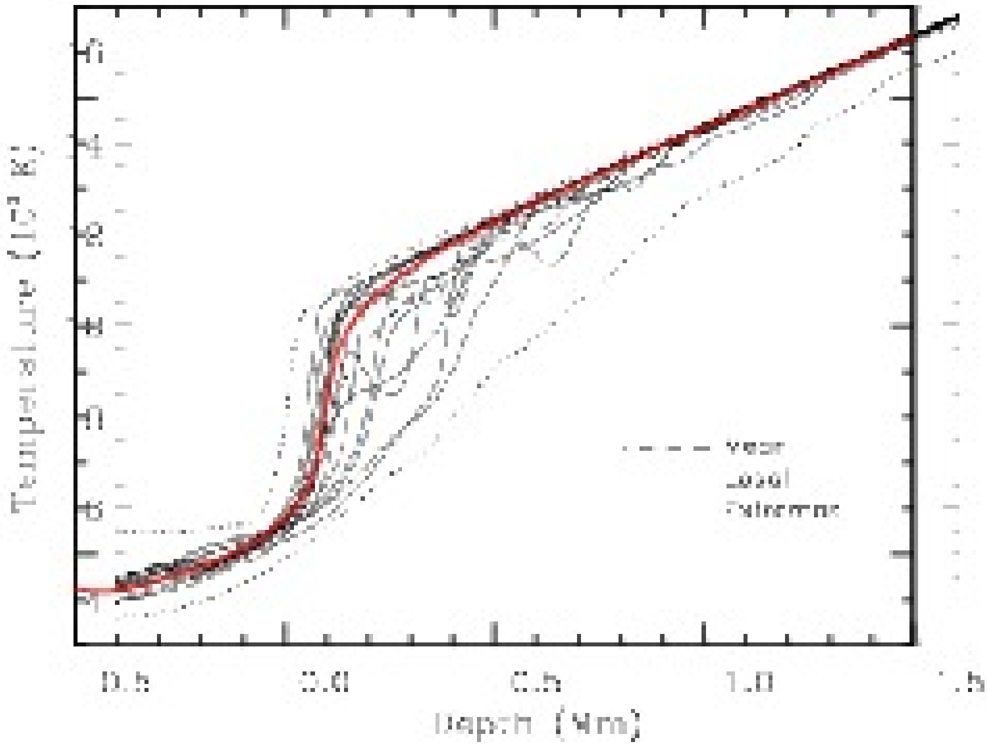
CO^{5}BOLD by Matthias Steffen and Bernd Freytag



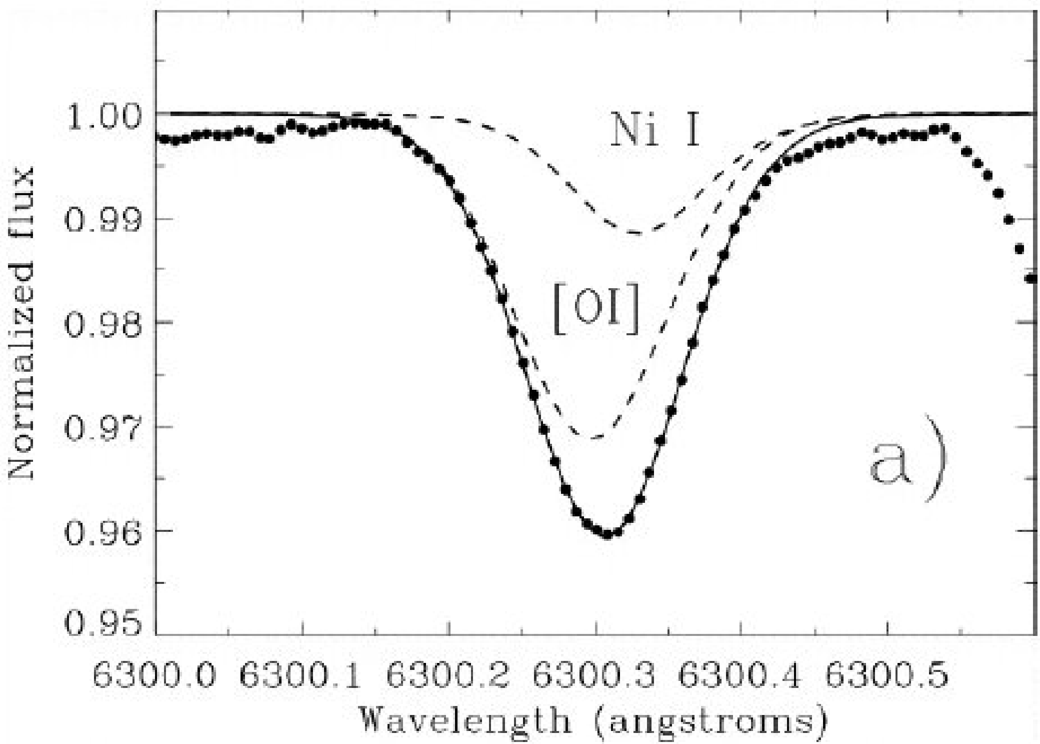
Temperature profile from Stein & Nordlund (1998)



Depth (Mm)
0.0
0.5
1.0



Temperature (10³ K)
Mean
Local
Extremes
-0.5
0.0
0.5
1.0
1.5
Depth (Mm)



[OI] line profile at 6300 Å from Allende Prieto et al. (2001)

- Due to correlation between temperature and velocity the line profiles are asymmetric and shifted towards the blue side. The line bisector is an important diagnostic of convection.
- Near the centre of solar disk the net blue-shift is about 500 m s^{-1}, which largely cancels the gravitational red-shift of 633 m s^{-1}. Convective blue-shift varies with position on the solar disk because of projection effect.
- Because of this gravitational red-shift was not confirmed in solar spectral lines, until numerical simulations of solar convection zone were done in 1980s.
- Before that exotic models were proposed to explain the observed line-shift, including nonzero photon mass or finite lifetime (Crawford 1979, Nature 277, 633).

letters

Photon decay in curved space-time

RECENT astrophysical observations[1,2] have raised doubts as to whether the large redshifts of distant galaxies (the Hubble redshift) are due entirely to cosmological expansion. The strongest argument[3] in favour of cosmological expansion is that there is no known hypothesis consistent with the laws of physics (other than the Doppler shift hypothesis) that can explain the observed redshifts. An alternative explanation—a gradual energy loss of photons due to their interaction with curved space-time—is considered here. The basic premise is that because photons have a finite spread they are subject to tidal stresses and that this provides a mechanism for the transfer of momentum from the photon to the mass producing curved space-time. Any transfer of momentum without an equivalent transfer of energy will destroy the concept of the photon as a single elementary particle. It is therefore postulated that the interaction of the photon with curved space-time causes it to lose energy in the form of very low energy secondary photons. As well as providing an explanation for the Hubble redshift this hypothesis can also explain the solar limb effect, that is, the increasing redshift of solar spectral lines as the viewpoint approaches the limb of the Sun.

Most texts that consider the passage of light in curved space-time equate the trajectory of a photon to that of a light ray, thereby neglecting any lateral spread the photon may have. If we exclude the possibility of infinite energy density then in some sense not yet defined a photon must have a finite non-zero cross-sectional area. A better model for the photon is a bundle of light rays. The effect of the tidal stresses on this bundle is shown by their relative deviations from a central reference geodesic. An appropriate coordinate system is an orthonormal Riemann coordinate system that is parallel transported along this central geodesic. Furthermore, every point along the central geodesic is uniquely labelled by the value of an affine parameter τ. As the deviations that are linear in τ given only reversible focusing (or defocusing) effects they will be ignored and we shall consider only the irreversible deviations that are quadratic in τ. These are given by the familiar equation for geodesic deviation.

The propagation of light in curved space-time is identical to propagation in a medium with a heterogeneous refractive index[4]. As the solution of Maxwell's equations is much easier for the latter case this approach is adopted here. The refractive index that gives the same geometric behaviour of light rays as the deviation of geodesics is radially symmetric about the axis of propagation and has the form

$$n = (1 - \varepsilon^2((x^2)^2 + (x^3)^2))^{1/2} \qquad (1)$$

where ε is a small constant and the central geodesic is along the x^1 axis. Then to order ε propagating wave solutions can be found with a gaussian amplitude distribution in the radial direction and requiring the dispersion relationship

$$\omega_0^2 = c^2 k_0^2 + 2\varepsilon\omega_0 c \qquad (2)$$

where ω_0 is the circular frequency and k_0 is the wavenumber. This gives a group velocity of $v = (1 - \varepsilon/k_0)c$, where c is the velocity of light along the reference geodesic.

Equation (2) can be interpreted as giving the wave packet an effective mass of $m = (2\varepsilon/k_0)^{1/2}(E_0/c^2)$ where $E_0 = \hbar\omega_0$ is the energy of the wave packet. The basic hypothesis here is that the same result applies to a photon in curved space-time and that this effective rest mass is just the energy of two secondary photons that are being emitted. Two secondaries are required to preserve spin. Each secondary has a wavenumber $k = (\frac{1}{2}\varepsilon k_0)^{1/2}$.

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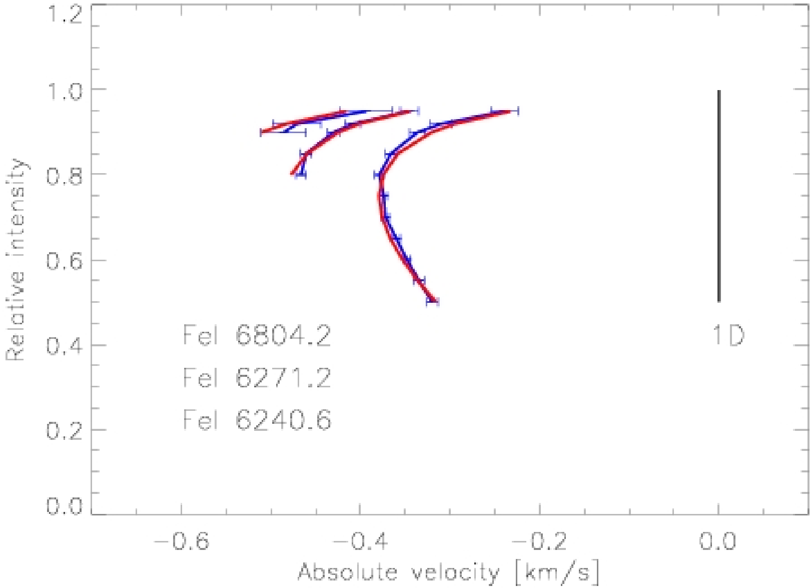
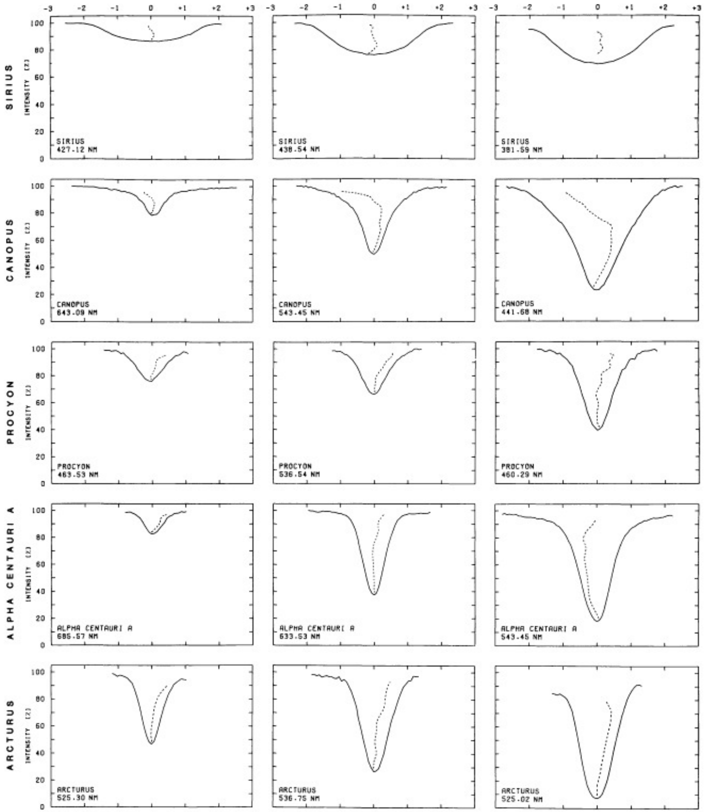


Comparison of predicted and observed line bisectors for a Few FeI lines from Asplund et al. (2000)

- The asymmetry in line-profiles and the shape of line bisector can be used to study convection on other stars where the surface cannot be resolved to study granulation directly.

- Dravins (1987, A&A 172, 211) and others have modelled the bisector in many stars.


SIRIUS
CANOPUS
PROCYON
ALPHA CENTAURI A
ARCTURUS
INTENSITY (%)
SIRIUS 427.12 NM
SIRIUS 438.54 NM
SIRIUS 381.59 NM
CANOPUS 643.09 NM
CANOPUS 543.45 NM
CANOPUS 441.68 NM
PROCYON 463.53 NM
PROCYON 536.54 NM
PROCYON 460.29 NM
ALPHA CENTAURI A 686.57 NM
ALPHA CENTAURI A 633.53 NM
ALPHA CENTAURI A 543.45 NM
ARCTURUS 525.30 NM
ARCTURUS 536.75 NM
ARCTURUS 525.02 NM

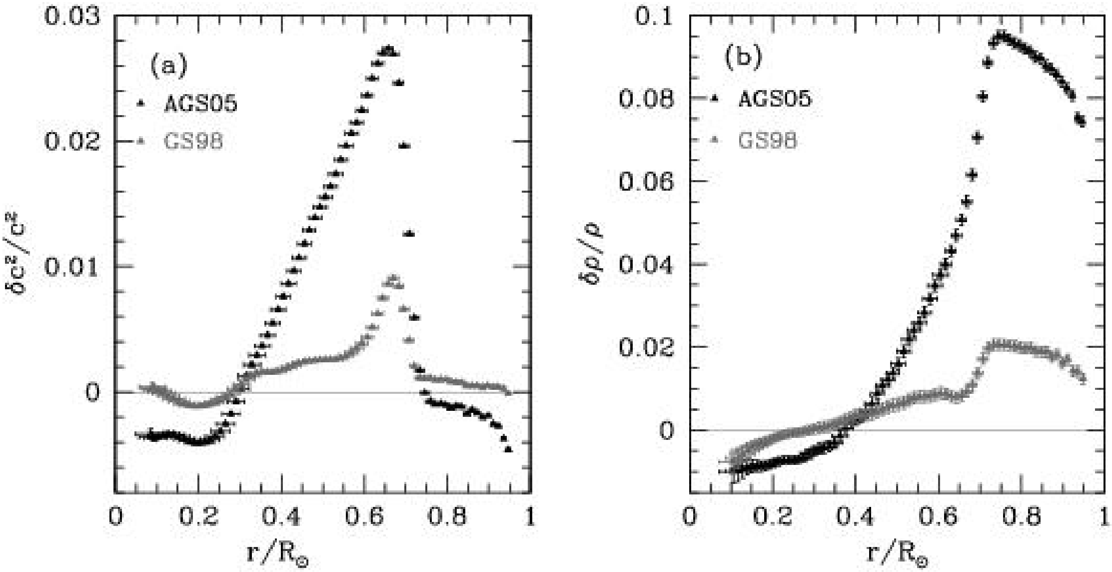
- Spectral line profiles can also be used to calculated abundances of elements in the Sun.

- Before 3D simulations were possible, abundances were calculated from 1D semi-empirical solar atmosphere models, where some parameters characterising turbulence were adjusted to match observed spectrum.

- When these abundances were used to construct solar models, the structure matched the sound speed and density profiles inferred from helioseismology.

- These abundances were revised downwards when 3D simulations were used (Asplund et al. 2004). Oxygen abundance was reduced by 45%. The abundance of other heavy elements was also reduced, thus reducing Z/X from 0.023 (Grevesse & Sauval 1998) to 0.0165, thus reducing the opacity.

- These revised abundances are inconsistent with helioseismology and all efforts to resolve the discrepancy have failed. Seismic estimates of abundances yield higher values consistent with old abundances.



Basu & Antia (2008)

- Because of reduction in opacity, the depth of the convection zone in a solar model is reduced (Bahcall & Pinnsoneault 2004).

- Opacities near the base of the convection zone need to be increased by 11–25% (Basu & Antia 2004; Bahcall et al. 2004, 2005).

- Difference between the OP and OPAL opacity calculations is only 2% near the base of the convection zone (Badnell et al. 2004).

- Recent determination of O abundances have yielded higher values, e.g.,

  Centeno & Socas-Navarro (2008) using Ni/O ratio in blend found O abundance of 8.86 ± 0.07 close to GS98 value (8.83 ± 0.06).

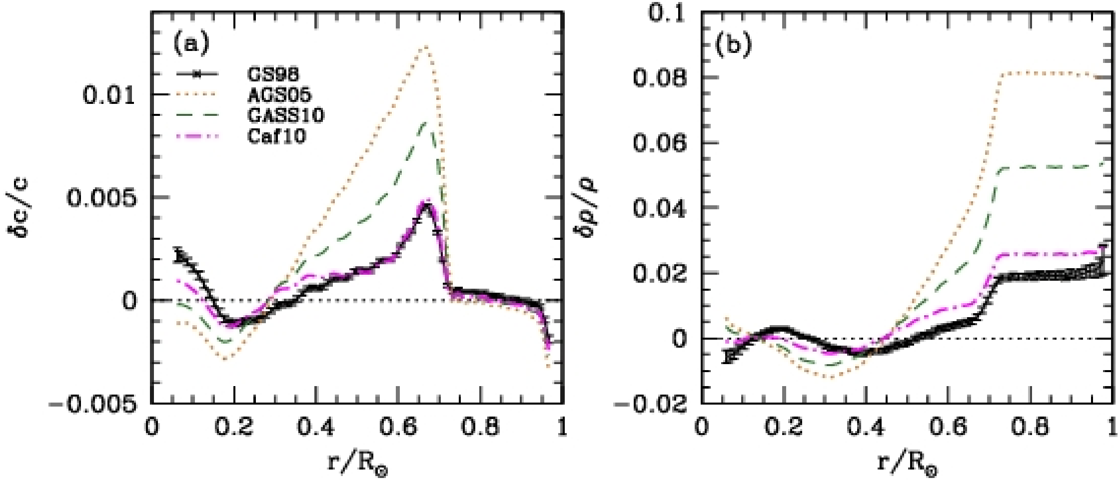
  Caffau et al. (2008) using an independent 3D atmospheric model found O abundance of 8.76 ± 0.07 between the GS98 and AGS05 values.

  Asplund et al. (2009) have revised their estimates slightly upwards to get $Z/X = 0.0181$

  Lodders (2010) has given table of abundances with $Z/X = 0.019$.

  Caffau et al. (2011) combined revised abundance of all elements to obtain $Z/X = 0.0209$.

  Bailey et al. (2015) using the Sandia Z facility measured opacity due to Iron at 2×10^6 K to find higher value



Difference in sound speed and density between solar models and the Sun (Antia & Basu 2010)

- Numerical simulations showed large overshoot below the solar convection zone (e.g., Hurlburt et al. 1986).

- Helioseismic results and observed Lithium and Beryllium abundance at solar surface put limits on overshoot.

- Improved numerical simulations have found penetration depth decreases with some measure of turbulence. Though it is not possible to do simulations with realistic measures.

- The main problem with all simulations is that they cannot resolve all relevant scales of turbulence.

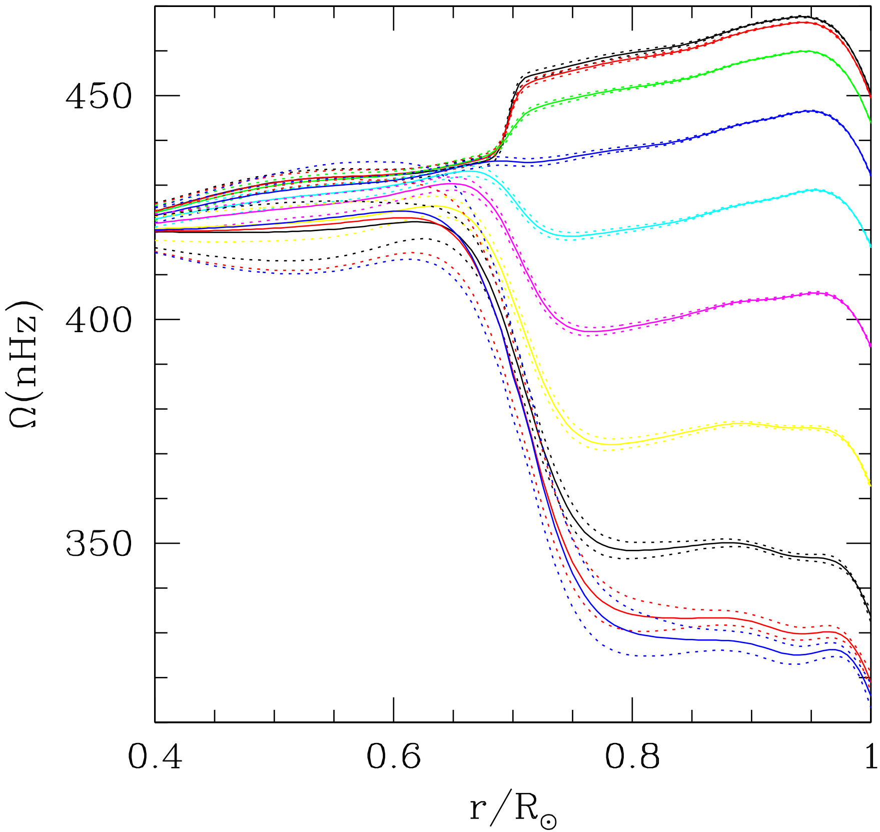
SOLAR DIFFERENTIAL ROTATION

- Observations of sunspot and other features on solar surface show that the Sun is rotating differentially, with equator rotating faster than the poles.

- Numerical simulations of solar convection zone (Glatzmaier & Gilman 1982), showed that rotation rate should be constant on cylinders, which can yield differential rotation at the surface.

- When helioseismic data was used to infer solar rotation in the interior, the result was totally different.

- Since then improved simulations with lot of tuning have yielded results closer to observations



450
400
350
Ω(nHz)
0.4
0.6
0.8
1
r/R⊙

- Helioseismic results show that differential rotation is confined to the convection zone, while below that rotation is essentially constant. The transition region is known as tachocline and its origin is also not understood.

- Lot of simulations with magnetic field have been done to explain the formation of tachocline, but they do not fully explain tachocline.

MERIDIONAL FLOW

- The North-South component of velocity has been observed at the solar surface with solar plasma moving from equator towards the poles in both hemispheres with an amplitude of 10–20 m s^{-1}. Attempts have been made to infer meridional flow in the interior using helioseismology.

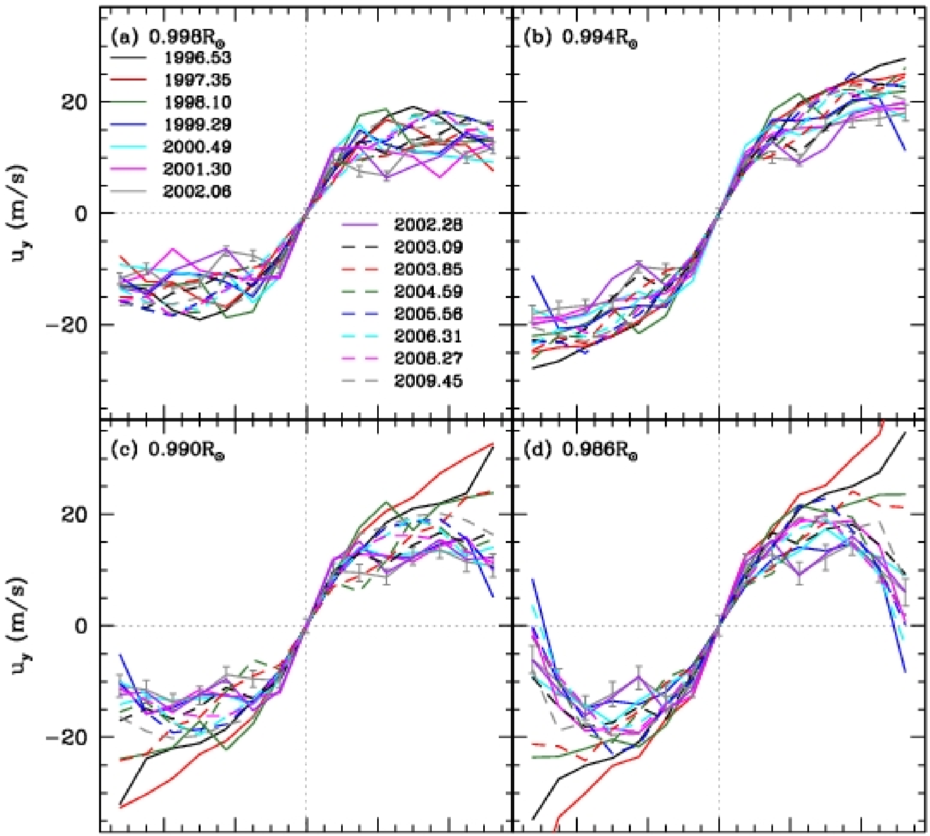
- The depth at which the flow reverses is important for solar dynamo theories.

- To first order the contribution of meridional flow to frequencies of global modes vanishes and hence these cannot be used to infer meridional flow. Quasi-degenerate perturbation theory can be used to estimate the second order contributions due to mode coupling.

- Local helioseismic techniques, time-distance or ring diagram techniques have been used to study meridional flows

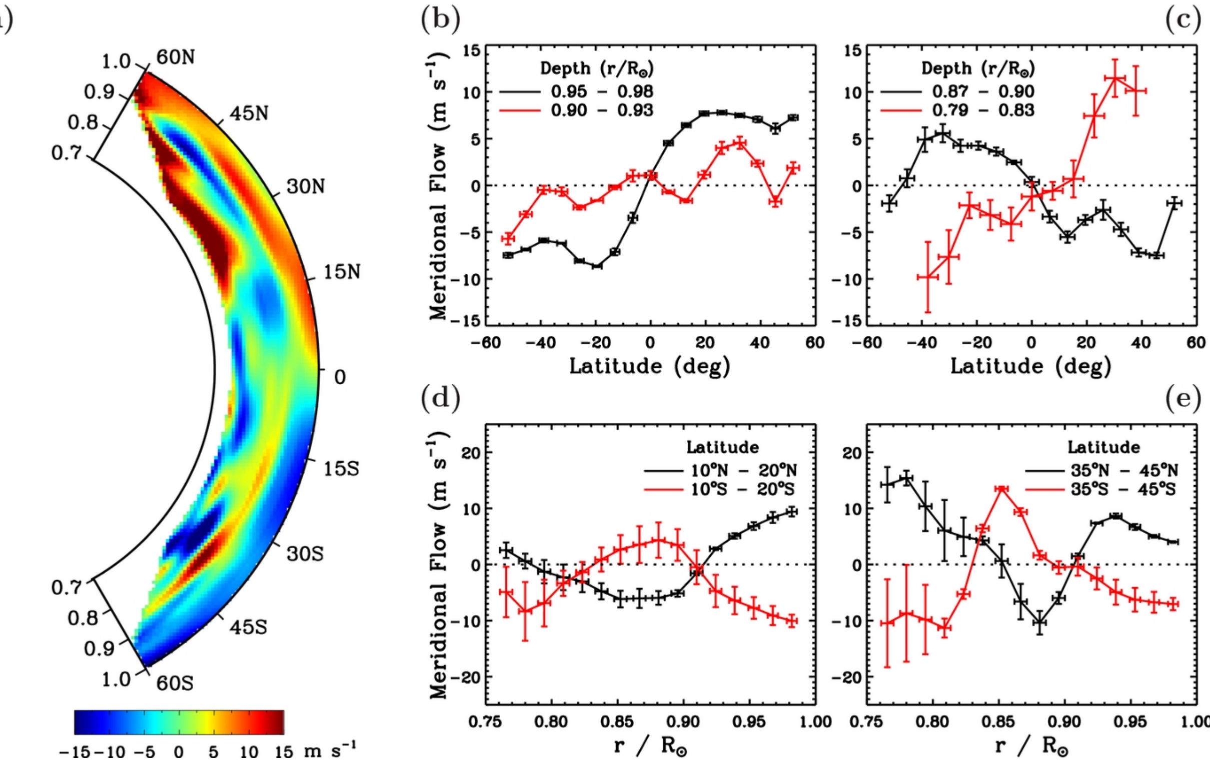
- These techniques can give meridional velocity as a function of depth, latitude and longitude. Averaging over the longitude range gives the velocity as a function of latitude and depth.

- The ring diagram technique studies solar oscillations in a small region on the solar surface. This can study modes with small length scales which are confined to outer regions of the Sun.



(a) 0.998R$_\odot$
(b) 0.994R$_\odot$
(c) 0.990R$_\odot$
(d) 0.986R$_\odot$
1996.53
1997.35
1998.10
1999.29
2000.49
2001.30
2002.06
2002.28
2003.09
2003.85
2004.59
2005.56
2006.31
2008.27
2009.45
u_y (m/s)
20
0
−20

- Time-distance analysis considers the wave travel-time between two points on the solar surface. If there is no flow the travel-time from point A to B will be same as that from B to A. The difference in this travel time is sensitive to flows and can be used to study meridional flow.

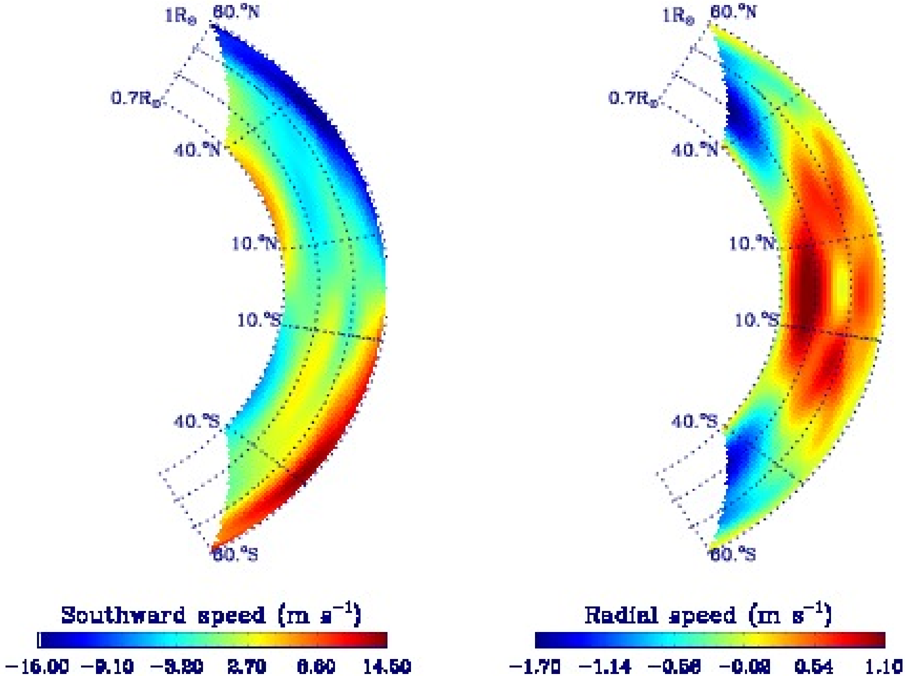


Meridional flow from time-distance analysis (Zhao et al. 2013)

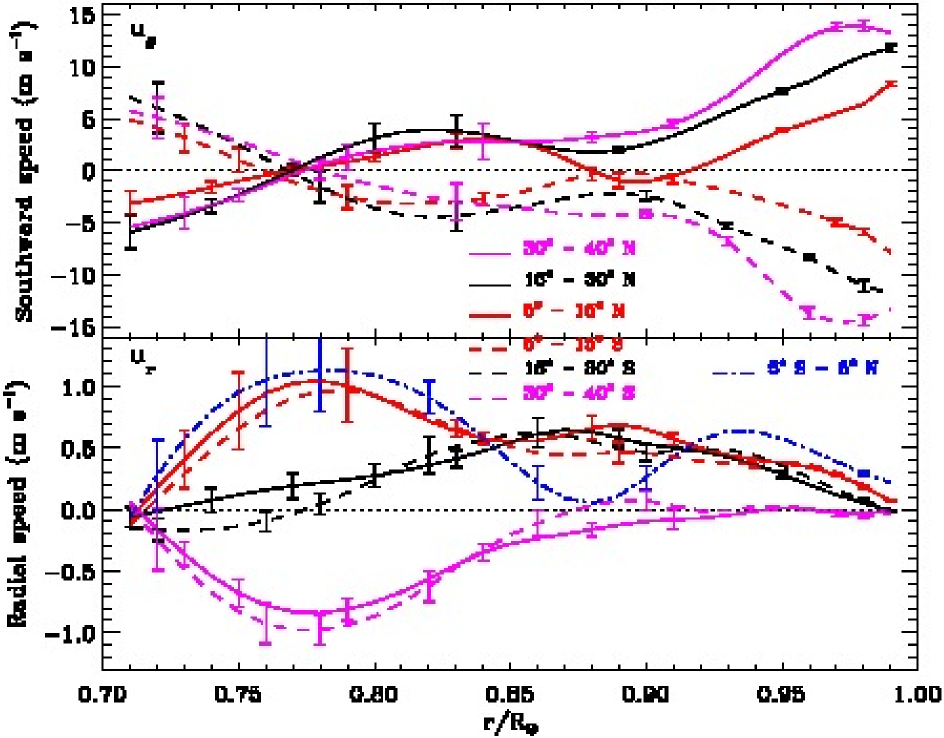
- Rajaguru & Antia (2015) used 4 years of data from HMI
- Used stream function to represent meridional velocity:

$$\rho u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \frac{\cos\theta}{r\sin\theta}\psi$$
$$\rho u_\theta = -\frac{\partial\psi}{\partial r} - \frac{\psi}{r}$$

  which ensures that the solution automatically satisfies the continuity equation and also enables u_r to be determined.
- Inversions are done using the Regularised Least Squares (RLS) technique.
- To get more realistic error estimates on inverted velocity we also perturb the smoothing used in inversion which gives some estimate of systematic errors in inversions.



Rajaguru & Antia (2015)



Rajaguru & Antia (2015)

- Convection is modified by presence of magnetic field and numerical simulations have been used to study the role of magnetic field.

- These studies can be used to study the dynamics of magnetic field in solar convection zone, formation of tachocline, structure of sunspots and possible dynamo action. There is probably no striking success in these areas.

- In the region above the photosphere, density and pressure are low and magnetic field plays a controlling role.

- Further, these regions are not in equilibrium and the structure is not spherically symmetric. All these factors make it more difficult to simulate these layers.

- This region is important for our understanding of coronal heating, solar flares and coronal mass ejections as well as solar wind.

Summary

- Numerical treatment of stellar convection is beyond the scope of computers in near future.
- Numerical simulations have been successful in reproducing solar granulation and absorption line-profiles in stellar spectrum.
- However, using the profiles to calculate solar abundances has considerable uncertainties.
- Simulations have failed to identify multiple scales of solar convection.
- There is only a limited success in simulating solar differential rotation or meridional flow or solar dynamo.