Dynamo Theory
and
Its Application to the Sun

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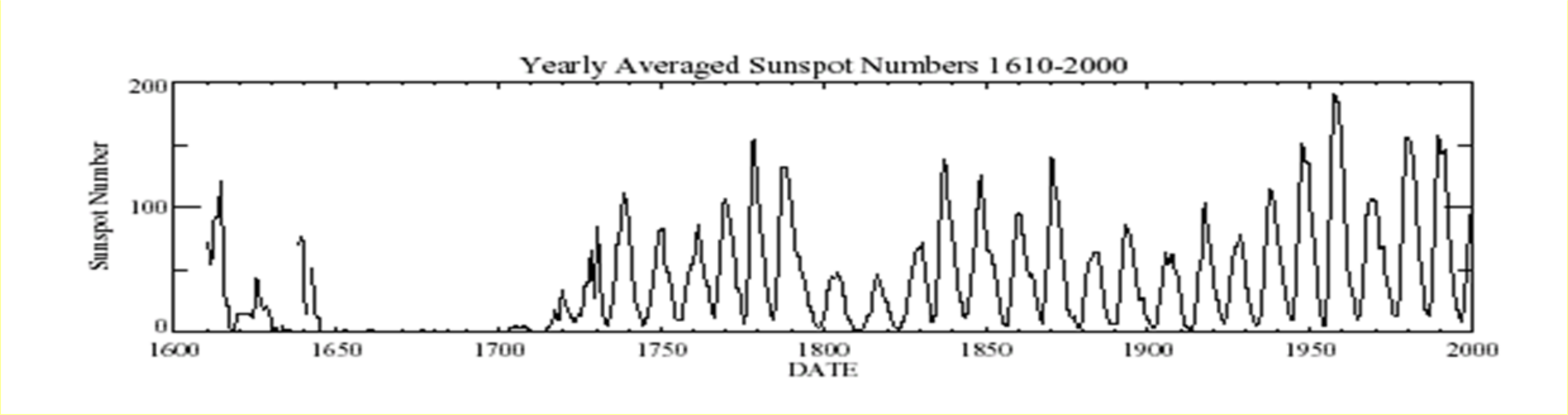
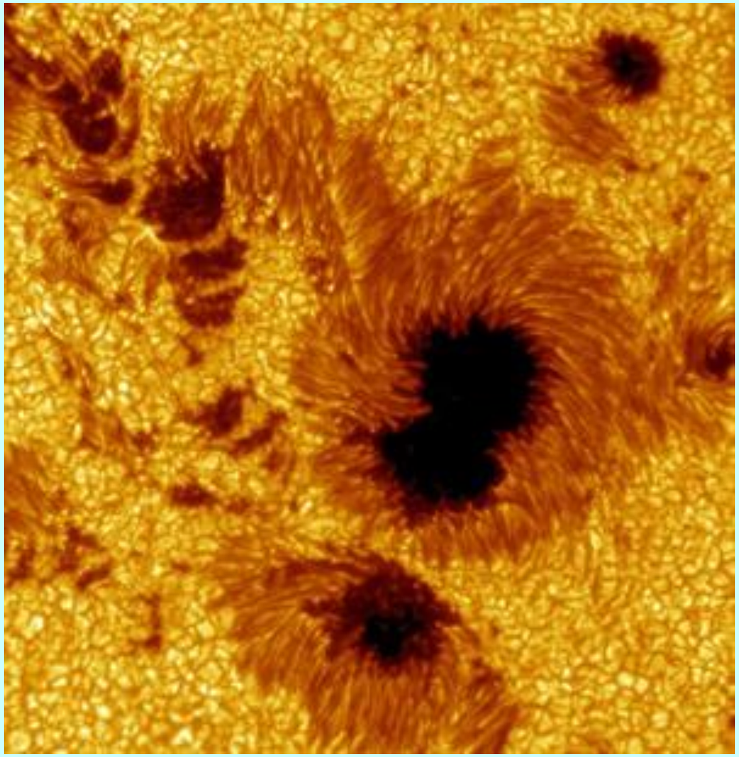
- **1600** - William Gilbert proposes that the Earth is a gigantic magnet
- **1908** – Hale discovered strong magnetic field in sunspots (B about 0.3 tesla)
- **1955** – Parker formulated turbulent dynamo theory for the origin of astronomical magnetic fields

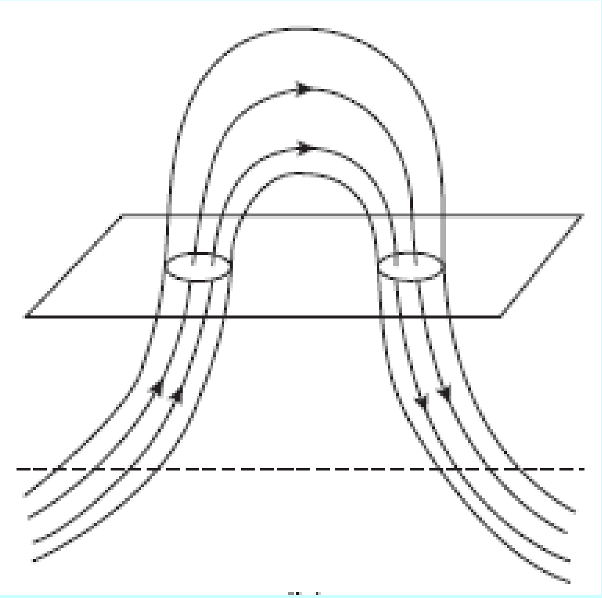


Sun's magnetic field is periodic => Sunspot cycle
Earth's magnetic field is approximately static, except occasional random reversals (last reversal ~ 0.78 million years ago)

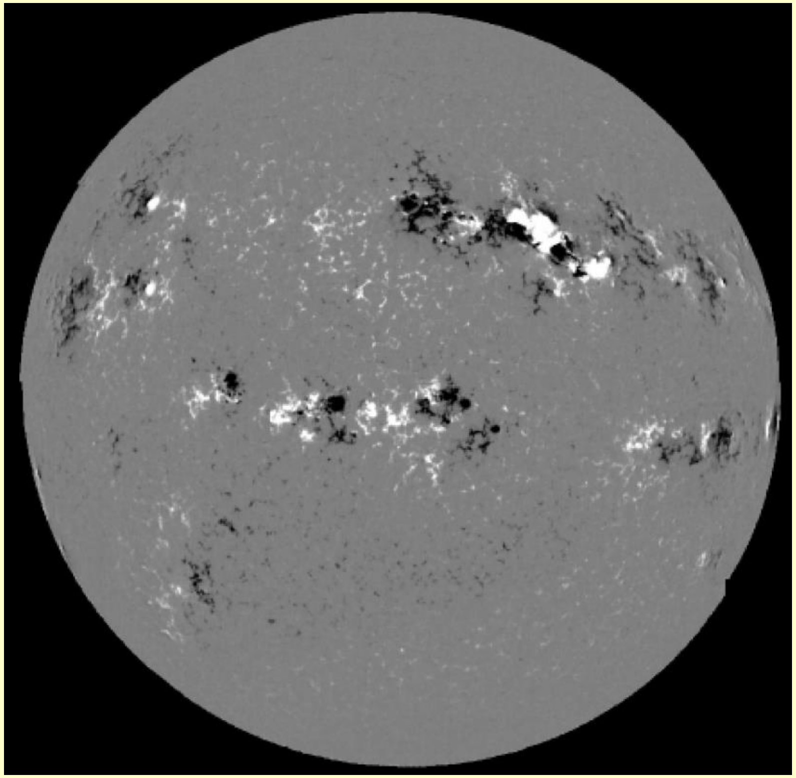
Hale et al. (1919) – Often two large sunspots are seen side by side with opposite polarities

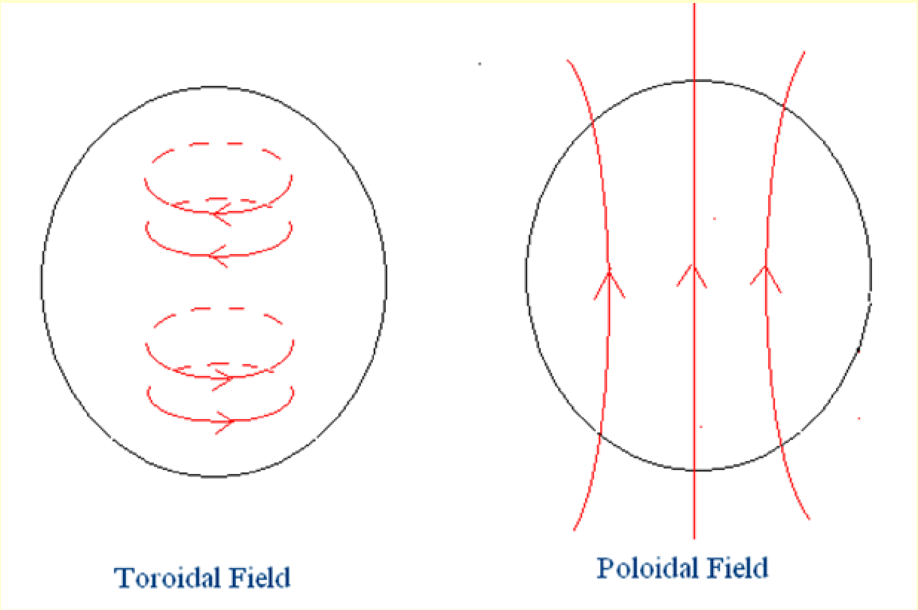


A strand of magnetic flux has come through the surface!



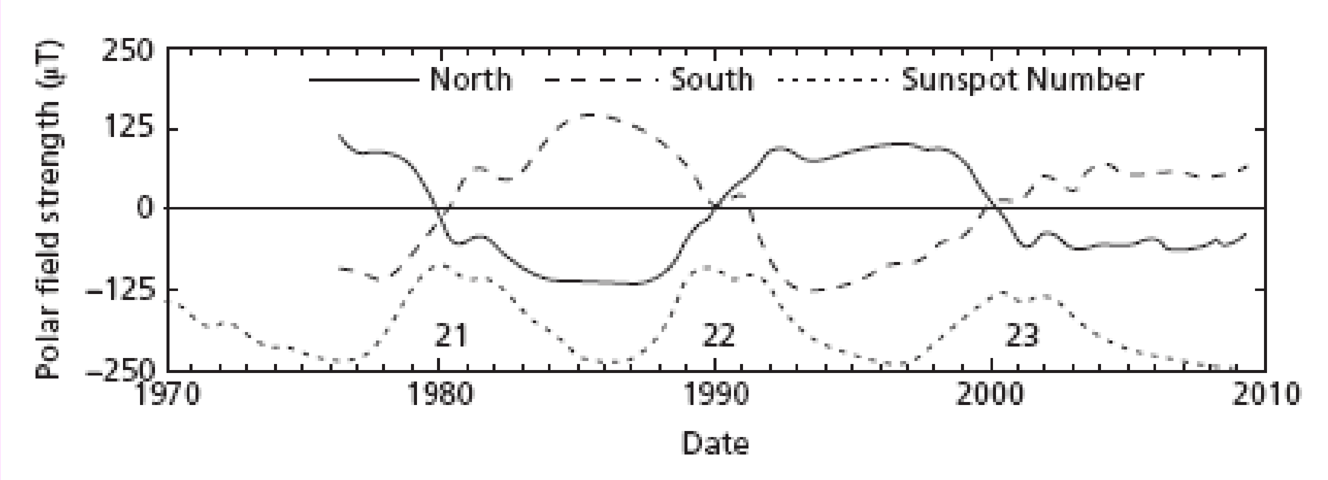
Magnetogram map (white +ve, black –ve)
Polarity is opposite (i) between hemispheres; (ii) from one 11-yr cycle to next >> 22-yr period





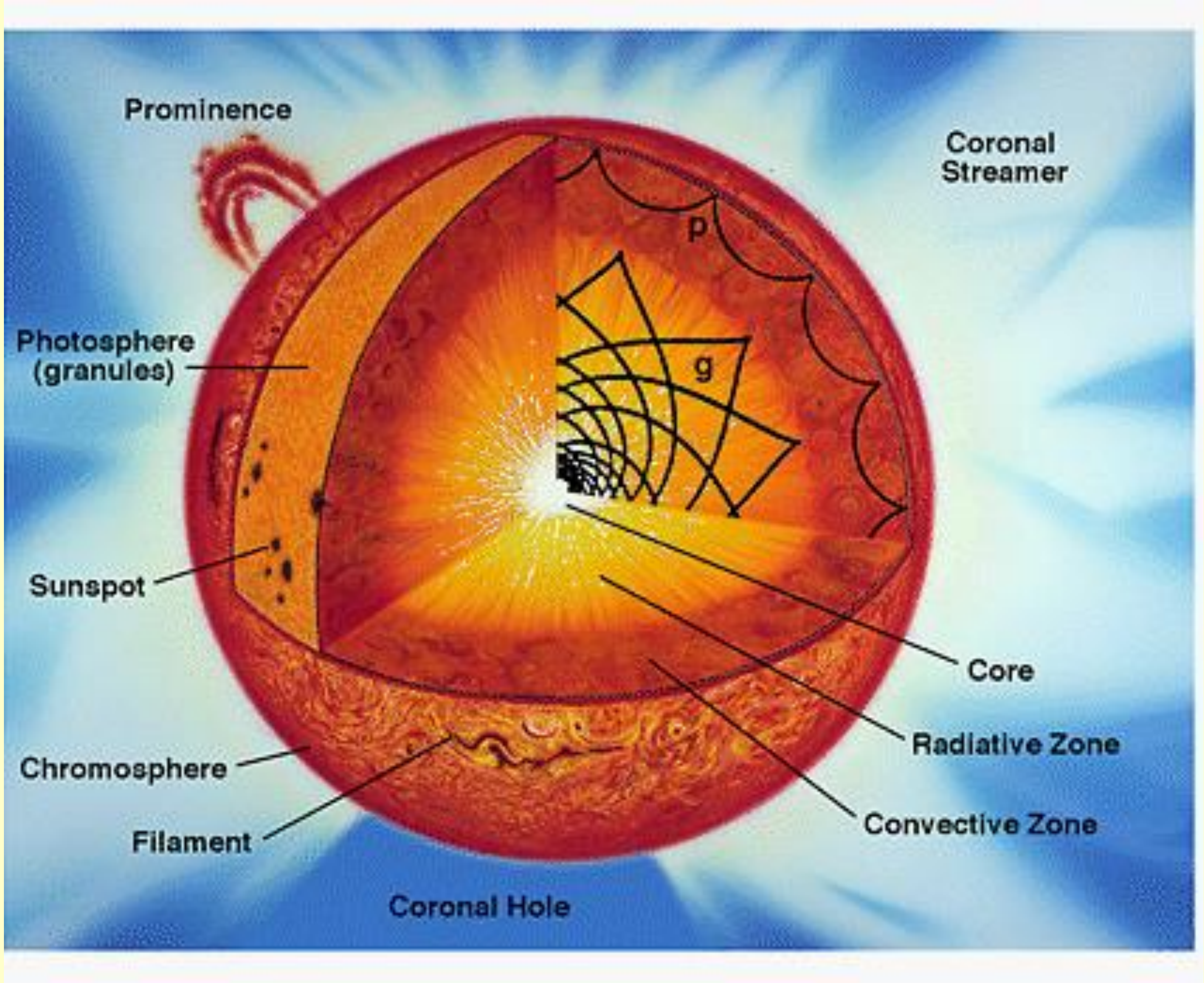
Parker (1955) suggested oscillation between the toroidal and poloidal fields.

Babcock and Babcock (1955) detected the weak poloidal field (~ 0.001 tesla)



The polar fields and the sunspot number as functions of time

There is indeed an oscillation between toroidal magnetic field (indicated by sunspot number) and poloidal magnetic field (indicated by polar field), as envisaged by Parker (1955)



Sunspots are magnetic field concentrations in turbulent plasma

Solar equator rotates faster than the solar pole

Basic equations of Magnetohydrodynamics (MHD)

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\nabla)\mathbf{v}\right]=-\nabla\left(P+\frac{B^2}{2\mu_0}\right)+\frac{(\mathbf{B}.\nabla)\mathbf{B}}{\mu_0}+\rho\mathbf{g}+\nu\nabla^2\mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B})+\frac{1}{\mu_0\sigma}\nabla^2\mathbf{B},$$

Induction equation

Magnetic Reynolds number

$$\mathcal{R}_{\mathrm{M}}=\frac{VB/L}{B/\mu_0\sigma L^2}=\mu_0\sigma VL$$

Laboratory: $\mathcal{R}_{\mathrm{M}}$ <<1

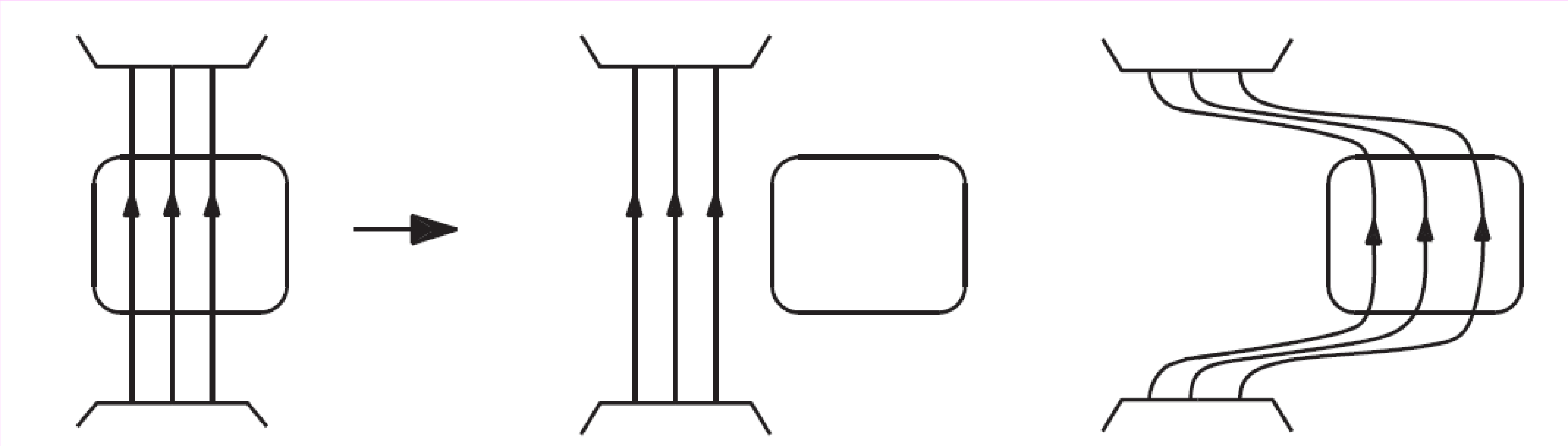
$$\frac{\partial \mathbf{B}}{\partial t}\approx\frac{1}{\mu_0\sigma}\nabla^2\mathbf{B}$$

Astrophysics: $\mathcal{R}_{\mathrm{M}}$ >> 1

$$\frac{\partial \mathbf{B}}{\partial t}\approx\nabla\times(\mathbf{v}\times\mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}).$$

=> Alfven's theorem of flux freezing (Alfven 1942)



$\mathcal{R}_M$ << 1 $\mathcal{R}_M$ >> 1

Can we have time-independent solution of induction equation?

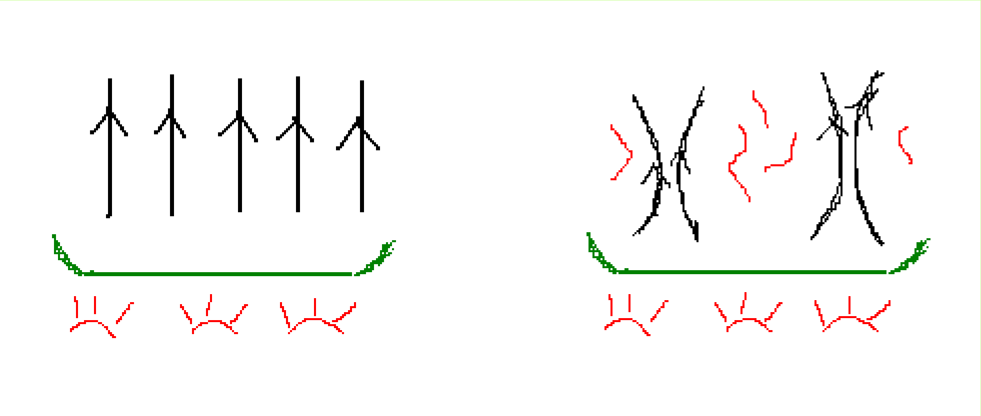
$$\nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} = 0$$

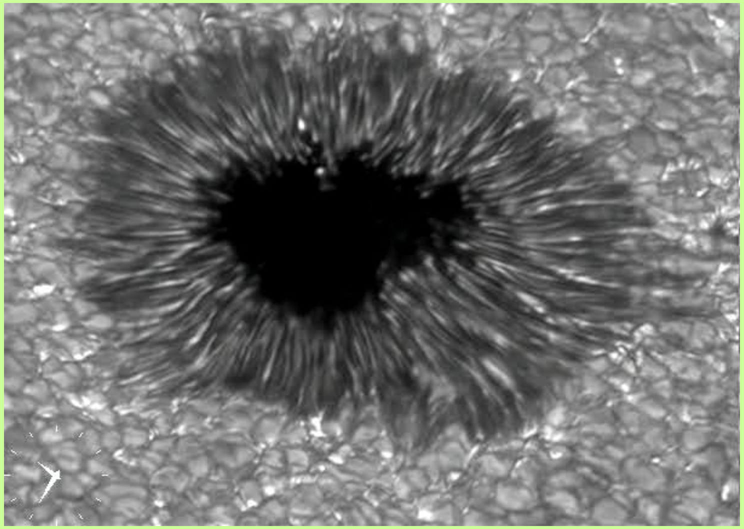
Cowling's anti-dynamo theorem: Axisymmetric solution not possible (Cowling 1933)

Magnetoconvection

Linear theory – Chandrasekhar 1952

Nonlinear evolution – Weiss 1981; . . .





Sunspots are magnetic field concentrations with suppressed convection

Magnetic field probably exists as flux tubes within the solar convection zone

Axisymmetric magnetic field

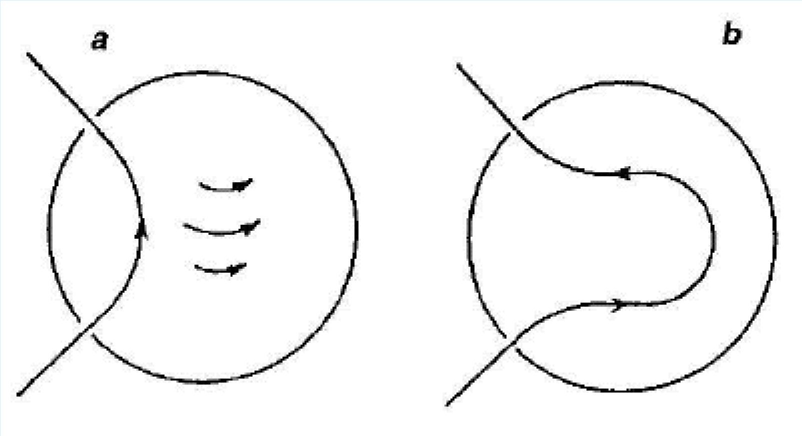
$$\mathbf{B} = B_r\mathbf{e}_r + B_\theta\mathbf{e}_\theta + B_\phi\mathbf{e}_\phi$$

Poloidal Component

Responsible for weak fields

Toroidal Component

Gives rise to sunspots



Differential rotation – produces toroidal field from poloidal field

Bottom of the convection zone – strong differential rotation, generation of toroidal magnetic field

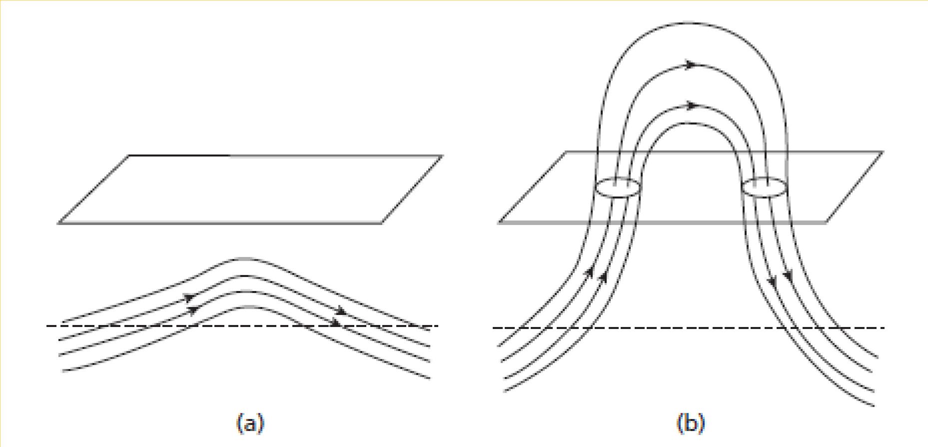
Flux Tube

$$p_{\text{out}} = p_{\text{in}} + \frac{B^2}{2\mu} \longrightarrow p_{\text{in}} \le p_{\text{out}}$$

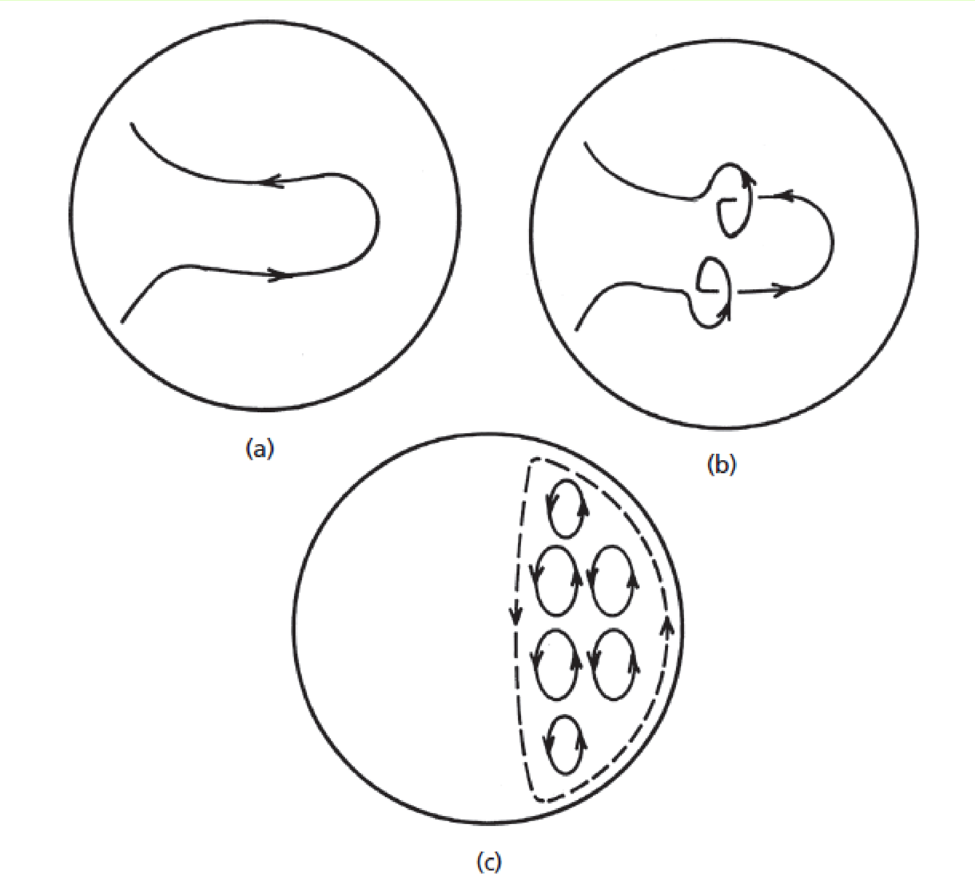
Usually the inside is under-dense

Magnetic buoyancy (Parker 1955)

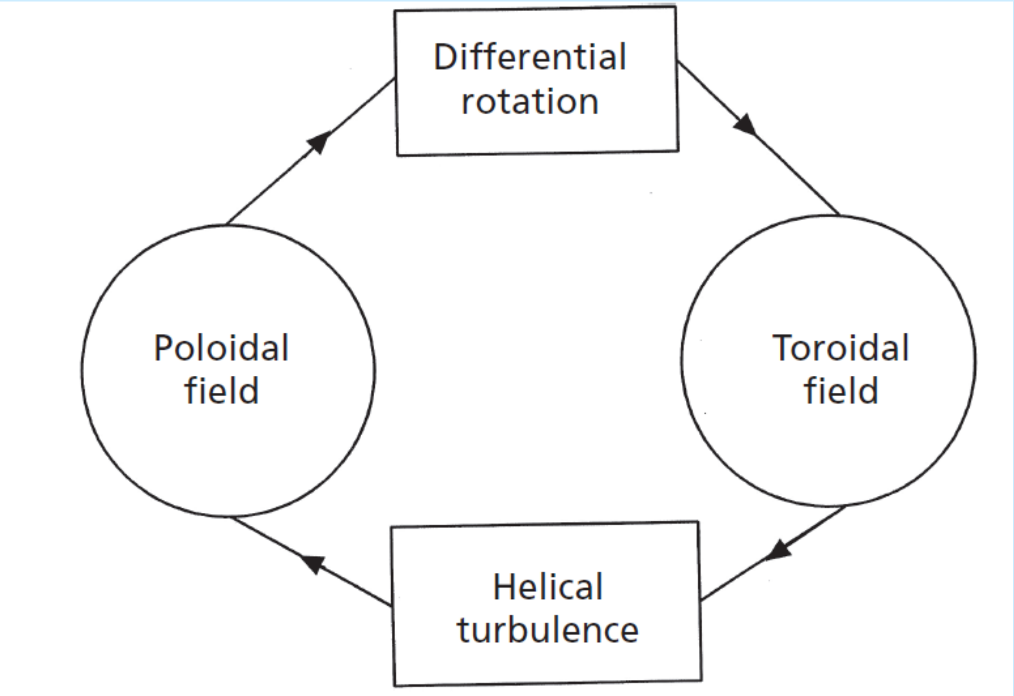
Very destabilizing within the convection zone, but much suppressed below its bottom



Generation of the poloidal field from the toroidal field by helical turbulence (Parker 1955; Steenbeck, Krause & Radler 1966)



α-effect: toroidal field energy density should be less than kinetic energy density of turbulence



The dynamo cycle – classical model of solar dynamo proposed by Parker (1955)

Detailed magnetic buoyancy calculations suggested very strong toroidal field (D'Silva & Choudhuri 1993)

Classical solar dynamo has to be modified => flux transport dynamo model (Choudhuri, Schussler & Dikpati 1995)

Mean field MHD (Steenbeck, Krause & Radler 1966)

Substitute $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}', \ \mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'$

in induction equation $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B},$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} + \frac{\partial \mathbf{B}'}{\partial t} = \nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}} + \overline{\mathbf{v}} \times \mathbf{B}' + \mathbf{v}' \times \overline{\mathbf{B}} + \mathbf{v}' \times \mathbf{B}') + \frac{1}{\mu_0 \sigma} \nabla^2 (\overline{\mathbf{B}} + \mathbf{B}')$$

Average this equation term by term:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}}) + \nabla \times \mathcal{E} + \frac{1}{\mu_0 \sigma} \nabla^2 \overline{\mathbf{B}},$$

where $\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'}$

This can be non-zero if $\mathbf{v}'$ and $\mathbf{B}'$ are correlated

First order smoothing approximation (FOSA):

$$\mathcal{E} = \alpha\overline{\mathbf{B}} - \beta\nabla\times\overline{\mathbf{B}}$$

where $\alpha = -\frac{1}{3}\overline{\mathbf{v}'.(\nabla\times\mathbf{v}')}\,\tau$ and $\beta = \frac{1}{3}\overline{\mathbf{v}'.\mathbf{v}'}\,\tau.$

This gives us

$$\frac{\partial\overline{\mathbf{B}}}{\partial t} = \nabla\times(\overline{\mathbf{v}}\times\overline{\mathbf{B}}) + \nabla\times(\alpha\overline{\mathbf{B}}) + \left[\frac{1}{\mu_0\sigma} + \beta\right]\nabla^2\overline{\mathbf{B}}$$

α is dynamo source term and β is turbulent diffusion

Mean field dynamo equation:

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla\times(\alpha\mathbf{B}) + \lambda_T\nabla^2\mathbf{B}$$

=> Kinematic mean field dynamo models

Dynamo Equation in spherical geometry

Magnetic field $\mathbf{B} = B(r,\theta)\mathbf{e}_\phi + \nabla \times [A(r,\theta)\mathbf{e}_\phi]$

Velocity field $\mathbf{v} + r\sin\theta\Omega(r,\theta)\mathbf{e}_\phi$

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\mathbf{v}.\nabla)(sA) = \lambda_T\left(\nabla^2 - \frac{1}{s^2}\right)A + \alpha B,$$

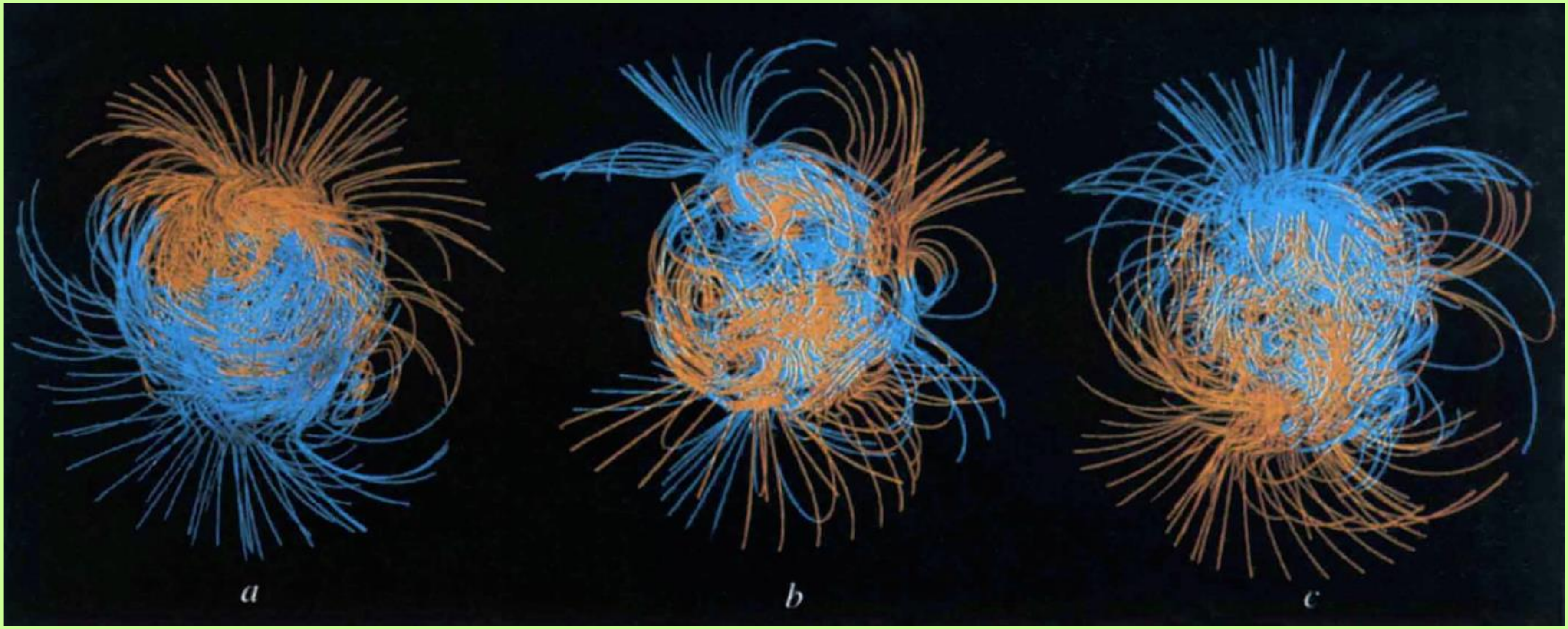
$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right] = \lambda_T\left(\nabla^2 - \frac{1}{s^2}\right)B + \boxed{s(\mathbf{B}_p.\nabla)\Omega} + \boxed{[\nabla\times(\alpha\mathbf{B}_p)]_\phi}$$

T1 T2

where $s = r\sin\theta$ and $\mathbf{B}_p$ is poloidal field $\nabla \times [A(r,\theta)\mathbf{e}_\phi]$

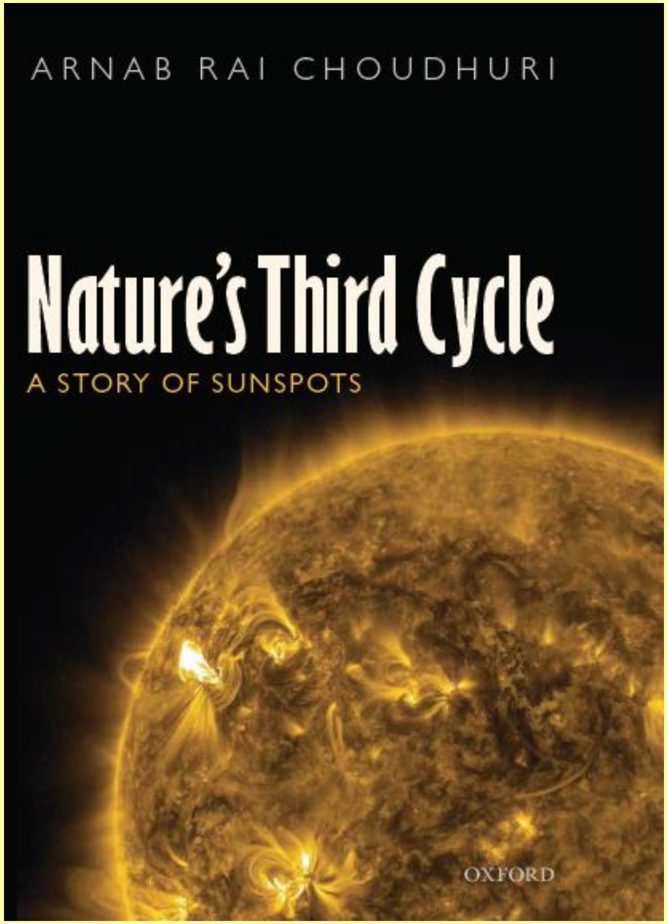
T1 >> T2: αΩ dynamo (solar and stellar dynamos)
T1 << T2: α^2 dynamo (planetary dynamos)

Direct numerical simulations (DNS) – turbulence and dynamo process simulated simultanously

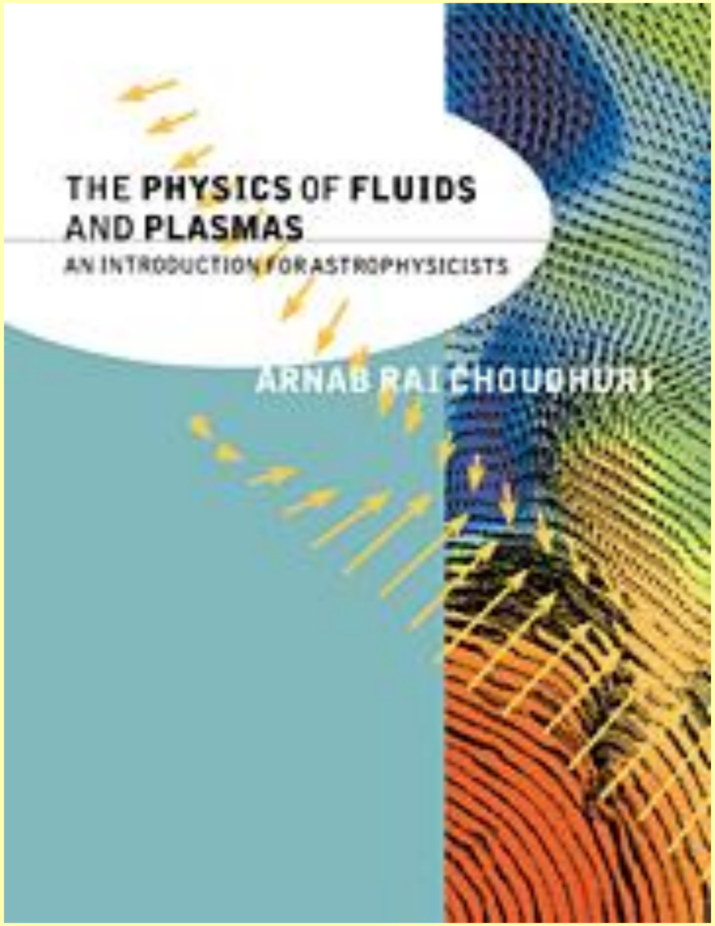


Geodynamo simulation of Glatzmaier and Roberts (1996) showed a geomagnetic reversal

For further studies



Popular science book
(Oxford University Press)



Graduate textbook
(Cambridge University Press)