# Swimming in a sea of data

Or

How I learned to stop worrying and love the data

Or

What is the difference between ‘data science’ and ‘data assimilation’

with sincere apologies for so many cliched phrases - I promise not to continue the trend...

Amit Apte
International Centre for Theoretical Sciences, Tata Institute of Fundamental Research (ICTS-TIFR), Bangalore, India

Summer School for Women in Mathematics and Statistics, ICTS-TIFR
22 May 2019

# Let us start with some data - from IC ‘Theoretical’ S!

## A weather station at ICTS

Quiz: Where on the ICTS campus is this located? (Hint: it is accessible only to authorised persons.)

Measurements taken: temperature, humidity, pressure, wind (speed and direction), rain, ... every 15 minutes, since March-2019 (not a very big dataset)

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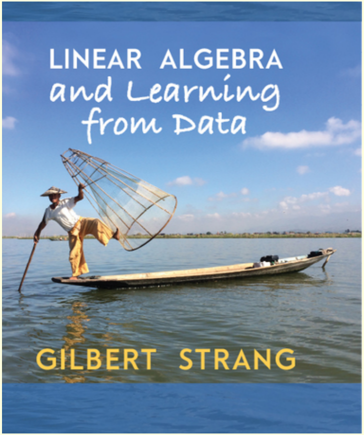
## A weather station at ICTS

So what do these “data” look like?

We will just plot a few things...

# We can plot data, but now what?

A little bit linear algebra helps to go a long way...



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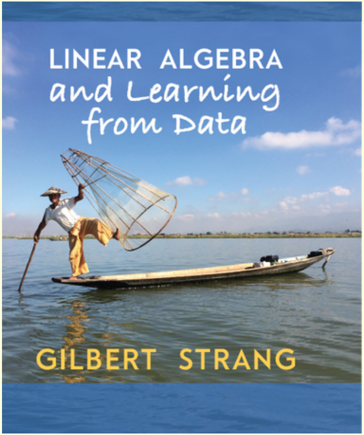

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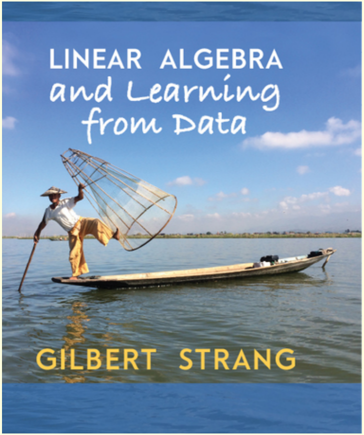

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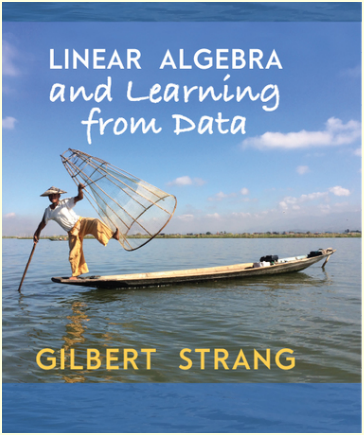

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- each column: measurements at one time instant
- each row: one quantity (temperature, pressure, etc.) at all times

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Recall the $n \times d$ "data matrix" $A =$

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Recall our notes: a change in temperature is somehow related to a change in humidity.

Can we quantify this using linear algebra?

# A little bit of linear algebra

keyword: 'Singular value decomposition'

- (column space) When $A$ is $n \times d$ matrix with columns $A = [a_1, a_2, \ldots, a_d]$, and $v = [v_1, v_2, \ldots, v_d]^t \in \mathbb{R}^d$ is an $d$-dimensional vector, then

**the vector $u = Av$ is a linear combination of columns of $A$:**

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- But the equation $Av = \sigma u$ can make sense for $u \in$ column space!
- That introduces new concepts (to be defined precisely very soon):
  - $v$ as the right singular vectors and $u$ as the left singular vectors, replacing the eigenvectors
  - $\sigma$ as singular values replacing the eigenvalues

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- maximize the ratio $\|Ax\|/\|x\|$ but for those $x \in \mathbb{R}^d$ which are perpendicular to $v_1, \ldots, v_{k-1}$ (prove: they form a subspace!) and denote the answer by $\sigma_k$ and $v_k$ and $u_k$.

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Forming matrices $V = [v_1, v_2, \ldots]$ and $U = [u_1, u_2, \ldots]$ of all these vectors, we arrive at ... (next slide)

# Matrix form of singular value decomposition

- A summary of previous slide (with a bit new terminology): Maximizing $\|Ax\|/\|x\|$ in successively "smaller" sub-spaces leads to three matrices:
  right singular vectors $V = [v_1, v_2, \dots]$
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  - and so is $U$, i.e., $U^t U = I$ (homework: find dimension of this identity matrix too!) and
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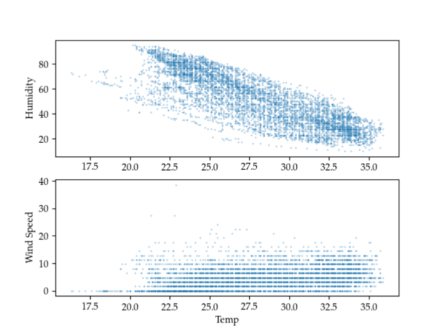
- and finally we get

### Singular value decomposition of $A$

$AV = U\Sigma$ which is equivalent to $A = U\Sigma V^t$.

# What do all these long calculations buy us?

Recall our notes: a change in temperature is somehow related to a change in humidity.



The SVD finds the line closest to the data points: The direction of the first singular vector $u_1$ is the direction of such a line!

## Eckart-Young theorem

If $B$ has rank $k$ then $\|A - A_k\| \leq \|A - B\|$ where $A_k = \sigma_1 u_1 v_1^t + \dots \sigma_k u_k v_k^t$.

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- A new name for an old subject called 'statistics'

  from wikipedia:

  - "Statistics is a branch of mathematics dealing with the collection, analysis, interpretation, presentation, and organization of data..."
  - "Data science is an interdisciplinary field that uses scientific methods, processes, algorithms and systems to extract knowledge and insights from data..."

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Finally: what do I work on? I did not talk about the "time evolution of the data"!! that leads me to data assimilation (not data science!)

# A few questions that data can help us answer!

- When will be the next total solar eclipse visible from Bangalore?
- What will be the closest approach of Halley's comet in next 200 years?
- How many times in the next minute will a double pendulum reach the lowest point? What will be the angle of a double pendulum after 5 min., 10 min., ...?
- What will be the total and regional monsoon rainfall in India 2019?
- When and how strong will be the next El Niño?
- What will be the extent of the Arctic sea-ice over next 50 years?
- What will be the three major U.S. stock indices over the coming week? What will be the next 5 coin tosses (Simpler?)?
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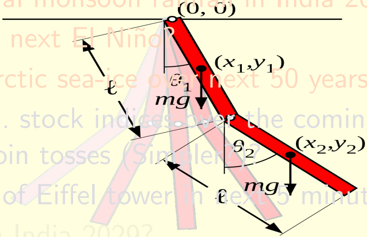

Videos from https://youtu.be/wtnA6ouIu0U

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- H rks w ted in a certain collision at the Large ron Colloider near Geneva?
- and many others ...

**stochastic, answers in probabilistic sense**

**"a million coin tosses!"**

# A few questions that data can help us answer!

- When will be th e visible from Bangalore? e closest approach of Halley's comet in next 200 years?

**deterministic, periodic, predictable**

- How many times in th l a double pendulum reach the e the angle of a double pendulum after 5 min., 10 min., ...?

**deterministic, chaotic, unpredictable**

- What will be the total nsoon rainfall in India 2019?
- What will be the extent of th ice over next 50 years?

**deterministic(?), complex, multi-scale**

**"millions of double pendula!"**

- What will be the three major U.S. stock indices over the coming week? What will be the next 5 coin to Simpler?)?
- How many cars will pass i xt 5 minutes?
- Who will be th
- H rks w ted in a certain collision at the Large on Colloider nea Geneva?

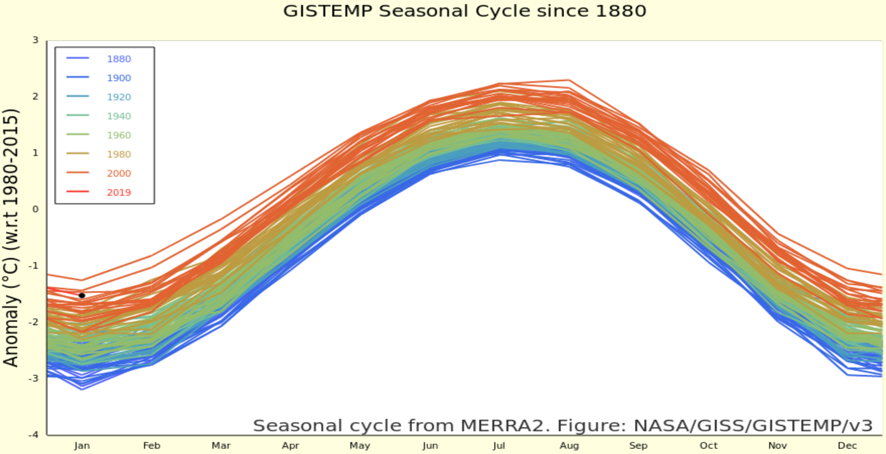
**stochastic, answers in probabilistic sense**

**"a million coin tosses!"**

- and many others ...

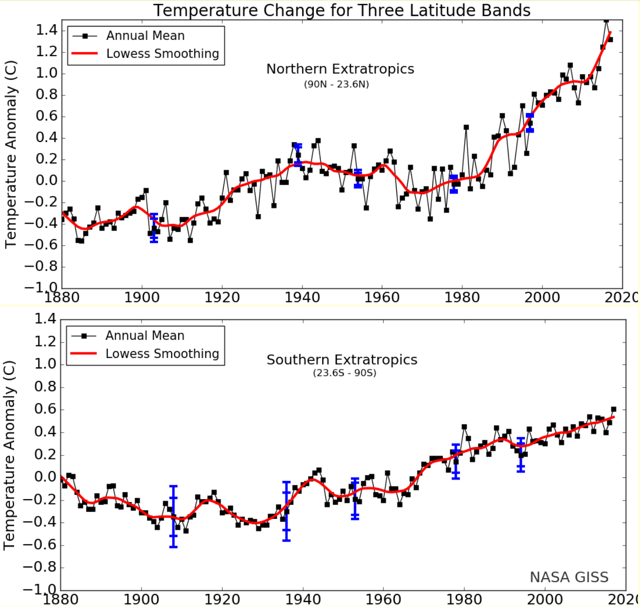
# Climate change is an important problem...

So what about changes in the climate? Or global warming? Here is what we know (temperature of the earth from 1880-2017)



`https://data.giss.nasa.gov/gistemp/graphs/`

# Climate change is an important problem...



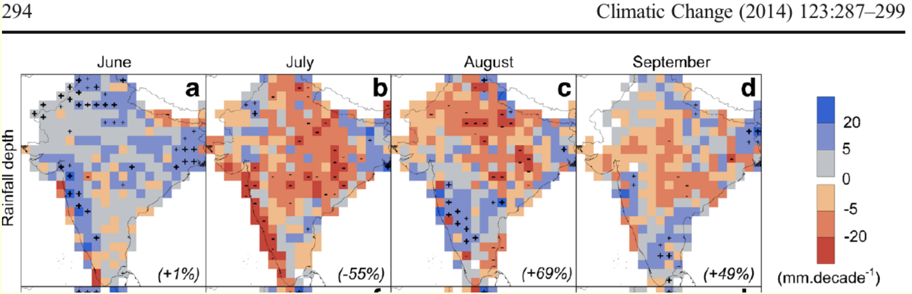
But, the global changes are not uniform: Northern extra-tropics have warmed more than the southern.

`https://data.giss.nasa.gov/`

`gistemp/graphs/`

# Climate change is an important problem...

Even more locally, changes are non-uniform:



294 Climatic Change (2014) 123:287–299

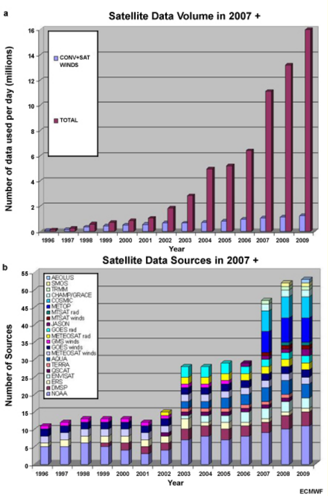
Rate of change of rainfall is more in one part than in other.
(Even the sign of change is different).

Lacombe G, McCartney M (2014) Uncovering consistencies in Indian rainfall trends observed over the last half century. Clim. Change 123(2): 287-299. http://dx.doi.org/10.1007/s10584-013-1036-5

# Data are the key to unravelling these complex mysteries

A remarkable change in the last 20-30 years: the amount of data is increasing "exponentially." Example from weather prediction:

- The first attempt at weather prediction used around 50-100 data points (Richardson 1920s)
- Next attempts: von Neumann, Charney, 1950s: a few KB (kilo=1000) of data
- 1970s - 1980s: a few 100 KB / a few MB (mega=1000 KB)
- Currently: 100s of MB / a few GB (giga=1000 MB)
- 2015-2020: a few TB (tera=1000GB)



ECMWF

# The other key is: scientific computations

Computing power has increased at the same rate as the availability of data.

- The first attempt at weather prediction needed a few weeks of calculations by hand!! (Richardson 1920s)
- Next attempts: von Neumann, Charney, 1950s: a few Kilo-flop/s
- 1970s - 1980s: a few Mega-flop/s
- Currently: Peta-flop/s
- 2015-2020: Exa-flop/s

"32 x 2000 = 64000 computers [humans!!] would be needed to race the weather for the whole globe. That is a staggering figure." (Richardson 1922, p.219); image (c) Stephen Conlin 1986

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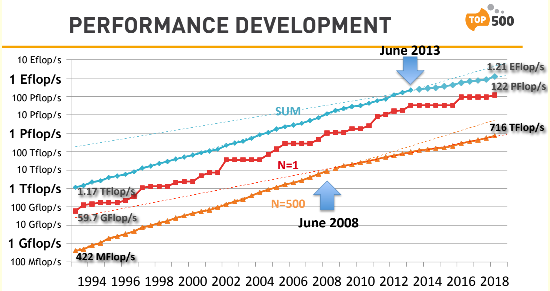
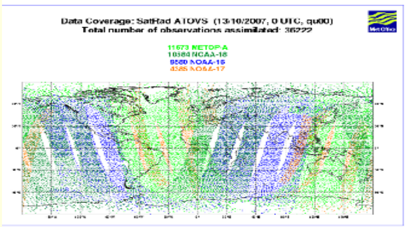


from https:

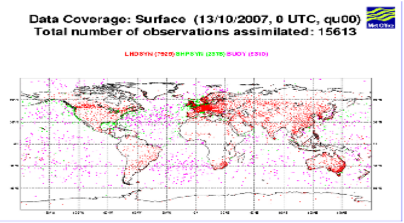
//www.top500.org/static/media/uploads/top500_ppt_201806.pdf

# Two specific areas that aim to combine data with models

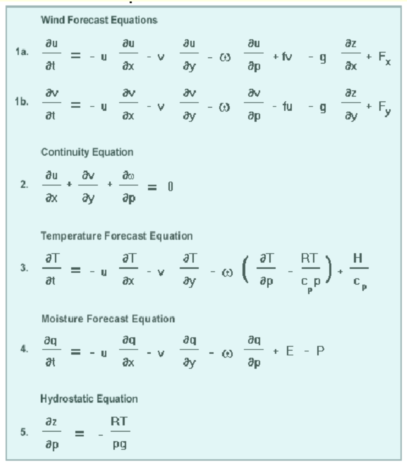
- Data assimilation: how do we use the observations, e.g. each dot on the left panel and more, with numerical models, e.g. equations shown on right?



Data Coverage: SatRad ATOVS (13/10/2007, 0 UTC, qu00)
Total number of observations assimilated: 36222



Data Coverage: Surface (13/10/2007, 0 UTC, qu00)
Total number of observations assimilated: 15613



Wind Forecast Equations

1a. $$\frac{\partial u}{\partial t} = - u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \omega \frac{\partial u}{\partial p} + fv - g \frac{\partial z}{\partial x} + F_x$$

1b. $$\frac{\partial v}{\partial t} = - u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - \omega \frac{\partial v}{\partial p} - fu - g \frac{\partial z}{\partial y} + F_y$$

Continuity Equation

2. $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

Temperature Forecast Equation

3. $$\frac{\partial T}{\partial t} = - u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} - \omega \left( \frac{\partial T}{\partial p} - \frac{RT}{c_p p} \right) + \frac{H}{c_p}$$

Moisture Forecast Equation

4. $$\frac{\partial q}{\partial t} = - u \frac{\partial q}{\partial x} - v \frac{\partial q}{\partial y} - \omega \frac{\partial q}{\partial p} + E - P$$

Hydrostatic Equation

5. $$\frac{\partial z}{\partial p} = - \frac{RT}{pg}$$

# Two specific areas that aim to combine data with models

- Markov random field model: can we find dominant patterns in Indian summer monsoon rainfall over last 100 years?

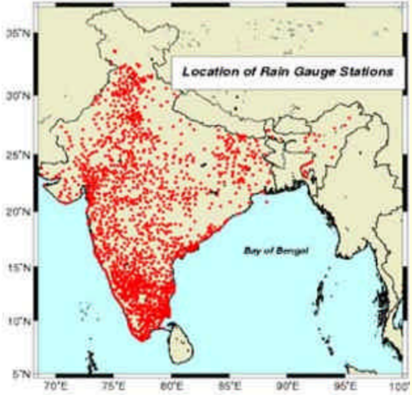


**Figure 1.** Location of 1803 rain gauge stations.

CURRENT SCIENCE, VOL. 91, NO. 3, 10 AUGUST 2006

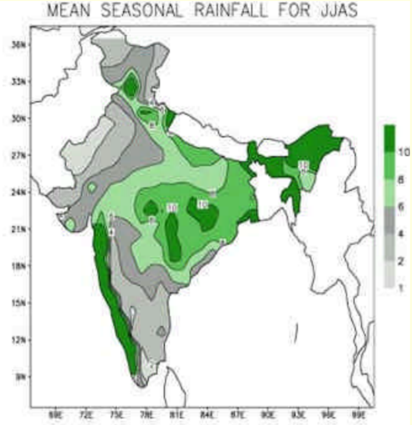


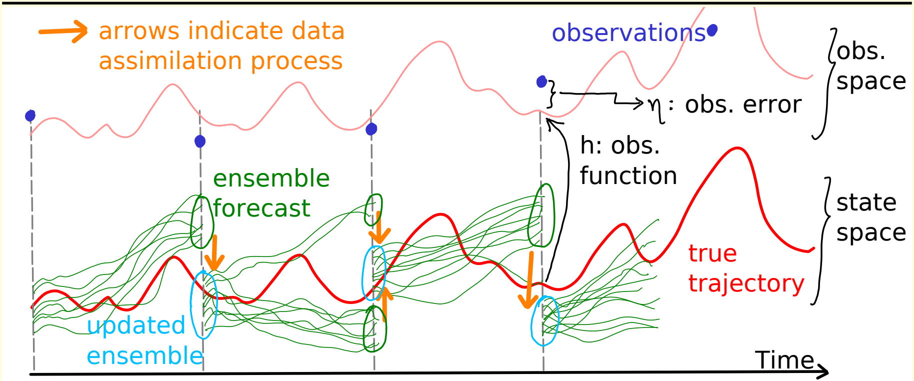
**Figure 3.** Spatial pattern of southwest monsoon seasonal (June t September) mean rainfall (mm/day).

# Data assimilation

## Combining data with observational models

The art of optimally incorporating

- partial and noisy observational data of a
- chaotic, nonlinear, complex dynamical system with an
- imperfect model (of the data and the system dynamics) to get an
- estimate and the associated uncertainty for the system state

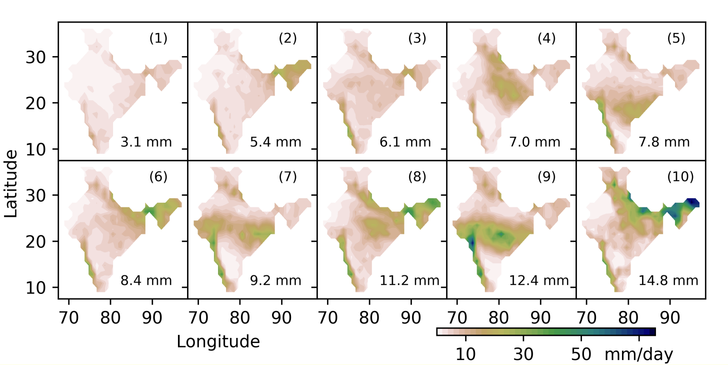


# Markov random fields

## Extracting patterns from data

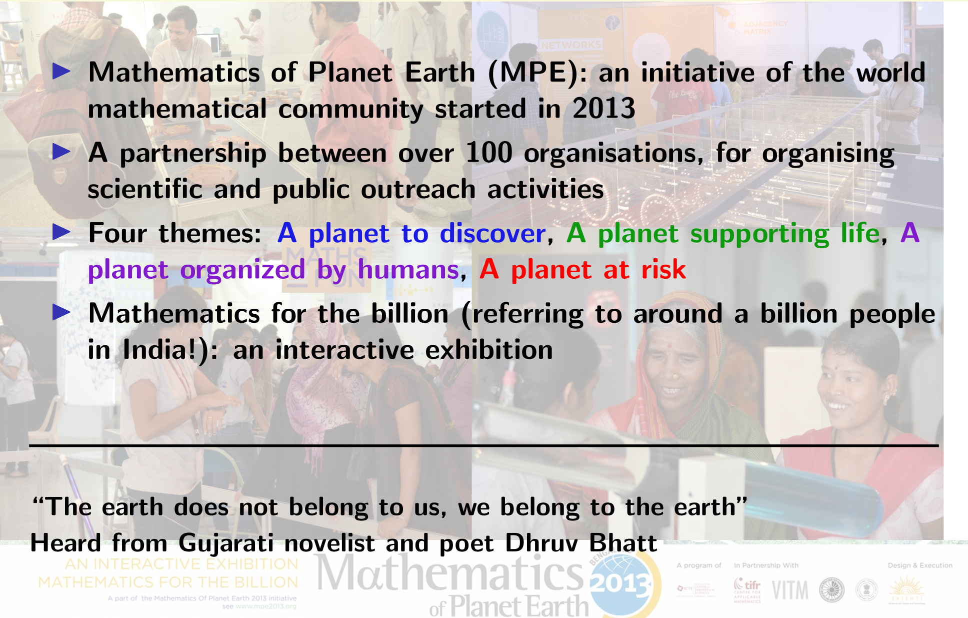
A probabilistic model that

- incorporates "domain knowledge" using probabilities, which are
- conditioned on observed rainfall data, with the aim of
- achieving clustering of locations and of days, and
- identifying dominant patterns in monsoon rainfall data



Common rainfall patterns for Indian summer monsoon (from `https://doi.org/10.1093/climsys/dzy009`)

# Mathematics: for the Planet Earth


NETWORKS
ADJACENCY MATRIX
MATHS FUN
AN INTERACTIVE EXHIBITION
MATHEMATICS FOR THE BILLION
A part of the Mathematics Of Planet Earth 2013 initiative
see www.mpe2013.org
Mathematics of Planet Earth
BENGALURU 2013
A program of
In Partnership With
tifr CENTRE FOR APPLICABLE MATHEMATICS
VITM
Design & Execution
SRISHTI

- **Mathematics of Planet Earth (MPE): an initiative of the world mathematical community started in 2013**
- **A partnership between over 100 organisations, for organising scientific and public outreach activities**
- **Four themes: A planet to discover, A planet supporting life, A planet organized by humans, A planet at risk**
- **Mathematics for the billion (referring to around a billion people in India!): an interactive exhibition**

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**“The earth does not belong to us, we belong to the earth”**
**Heard from Gujarati novelist and poet Dhruv Bhatt**