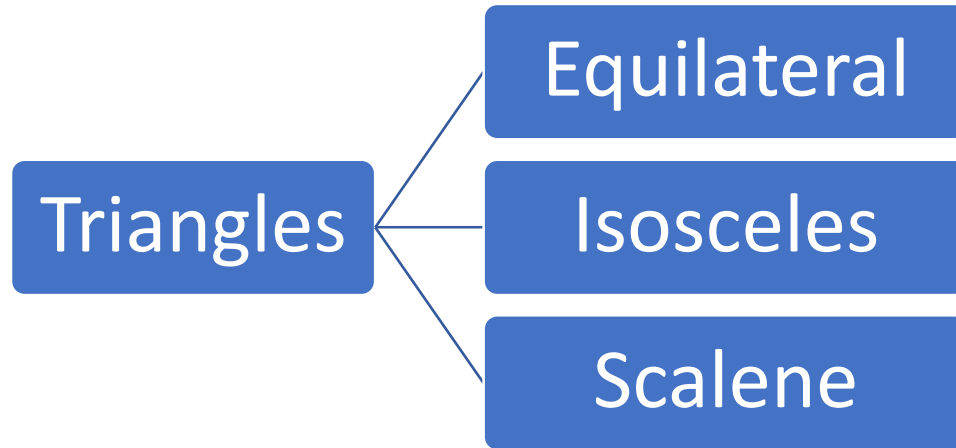


# Unknown Identities

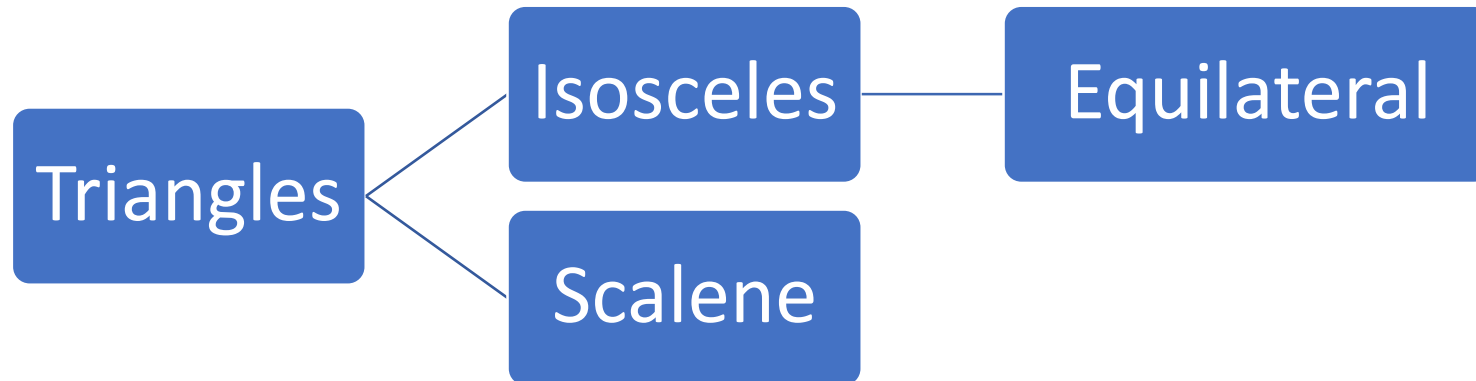
Of Known Shapes

# Triangles: side-wise

- Old



- New



- Which is better? Why?

# Triangles: another look

- Which triangles can be folded in half?
- Is there any triangle which is special? How?

Take any triangle. Join midpoints of all sides to form 4 congruent smaller triangles, each similar to the original triangle. Fold to form a tetrahedron

- Which triangles can be folded to form a tetrahedron?
- Which can't be?
- Any other case?

# Triangles

## Side-wise

- Isosceles  
= triangles with (line) symmetry
- Equilateral  
= triangles with rotational symmetry  
= triangles with multiple line symmetries
- Scalene  
= triangles without symmetry

## Angle-wise

- Acute = can be folded to form a tetrahedron
- Right = tetrahedron flattens out
- Obtuse = can't be folded to form a tetrahedron

# Triangles

	<b>Rotational symmetry</b>	<b>No rotational symmetry</b>
<b>Line symmetry</b>	Equilateral	Isosceles
<b>No line symmetry</b>		Scalene

# Quadrilaterals

- Equilateral quad?
- Equiangular quad?
  
- Circumcircles?
- Incircles?
  
- Symmetries?

# Quadrilaterals with incircles

- Quad with incircle  $\Rightarrow$  sums of opposite sides are equal
- Quad with equal sums of opposite sides  $\Rightarrow$  incircle?

# Quadrilaterals with line symmetry

Line of symmetry passes through

- A vertex

⇒ kite

- A side

⇒ isosceles trapezium



# Quadrilaterals

	<b>Rotational symmetry</b>	<b>No rotational symmetry</b>
<b>Line symmetry</b>	Rhombus Rectangle	Kite (and arrowhead) Isosceles trapezium
<b>No line symmetry</b>	Parallelogram	Non-isosceles trapezium Concave quads And others

# Food for thought

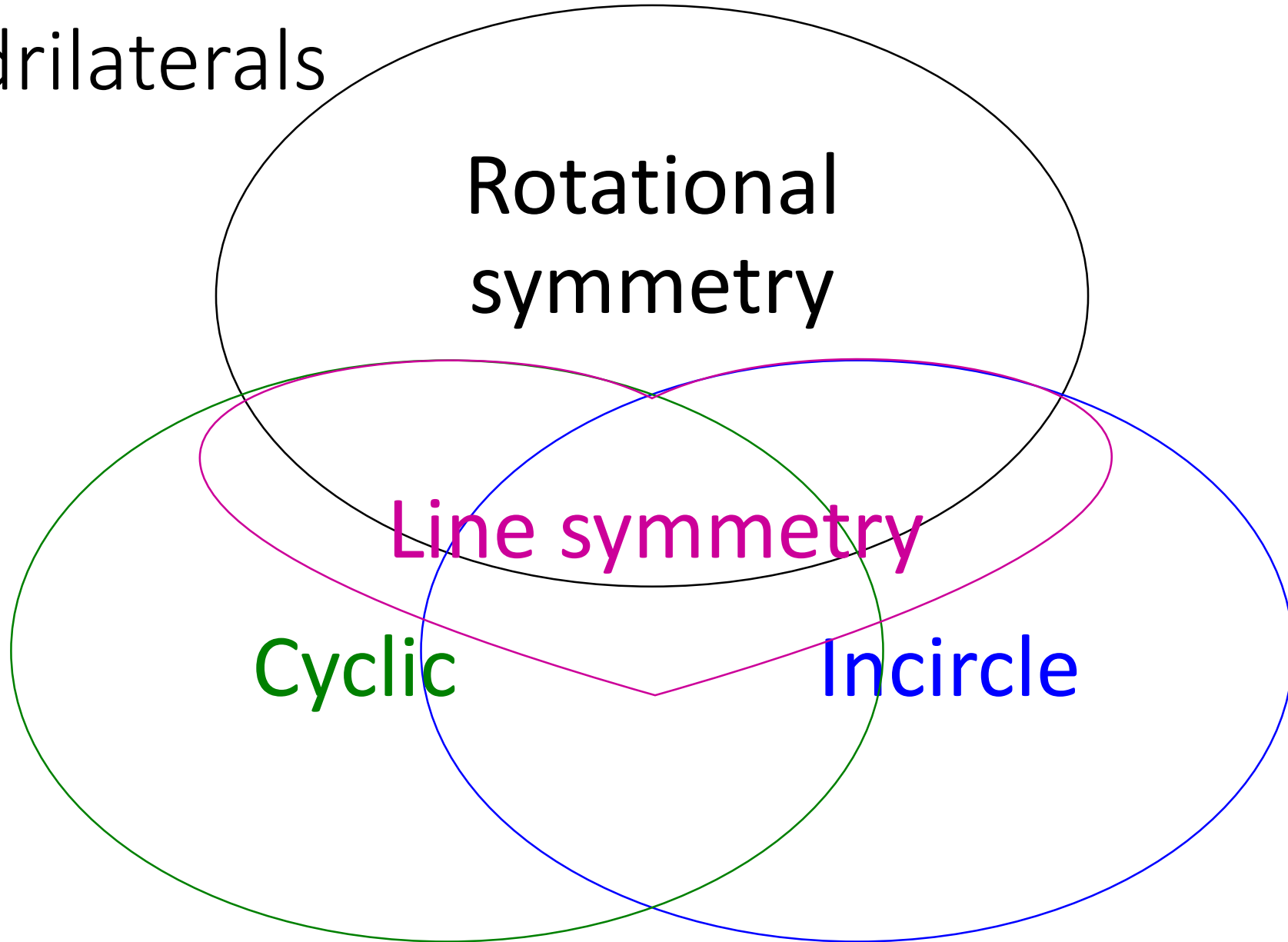
If a shape has both rotational and line symmetries, then can it have only one line of symmetry?

If a shape has more than one line of symmetry, does it also have rotational symmetry?

# Quadrilateral – duality

Rhombus: equilateral quad	Rectangle: equiangular quad
Kite: 2 pairs of equal adjacent sides	IT: 2 pairs of equal adjacent angles
Incircle $\Leftrightarrow$ equal sums of opp. sides	Cyclic $\Leftrightarrow$ equal sums of opp. angles
Incenter: intersection of angle bisectors	Circumcenter: intersection of (perpendicular) bisector of sides

Quadrilaterals



Rotational  
symmetry

Line symmetry

Cyclic

Incircle

# Isosceles trapezium – definition

- Trapezium  $\Rightarrow$  one pair of parallel sides
- Old: the non-parallel sides are equal
  - Need for non-parallel: otherwise it may mean one pair of equal and parallel sides  $\Rightarrow$  parallelogram
  - Problem: rules out rectangles
- New: the other pair of opposite sides are equal

# Are these really needed?

- Who cares about topology? 3Blue1Brown

<https://www.youtube.com/watch?v=AmgkSdhK4K8>

- “... mathematics is an art” – Lockhart Lament

[https://www.maa.org/external\\_archive/devlin/LockhartsLament.pdf](https://www.maa.org/external_archive/devlin/LockhartsLament.pdf)

Thank you! 😊

Questions/comments?

Contact: [mathspace@apu.edu.in](mailto:mathspace@apu.edu.in)

7406333305