Theoretically speaking: the why and how of efficient computation

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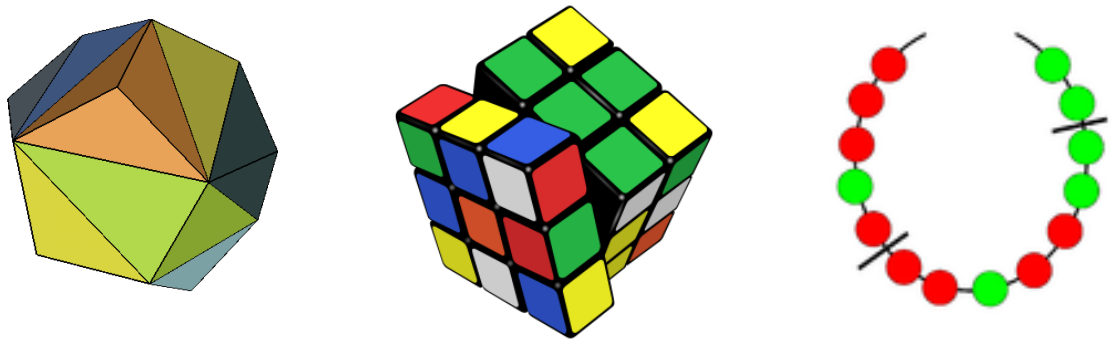
Theoretically speaking ...



Through simple puzzles let us try to understand some aspects of Discrete Mathematics and Design of Algorithms.

Puzzle 1

Puzzle 1: Akbar, Birbal and closed rooms

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The Story

A poor farmer comes to Akbar asking for a loan.

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But you must follow some rules!"

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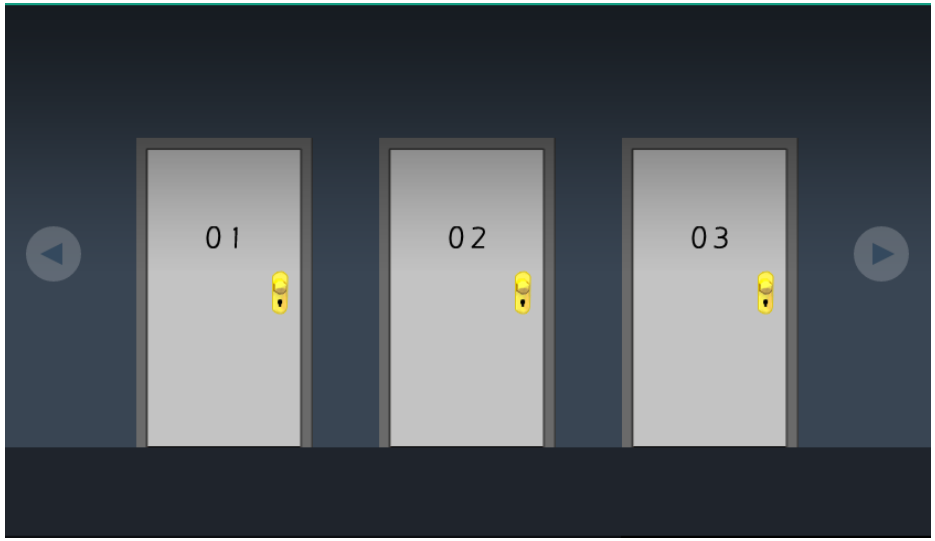

The rules

There are 100 rooms numbered 1 to 100.

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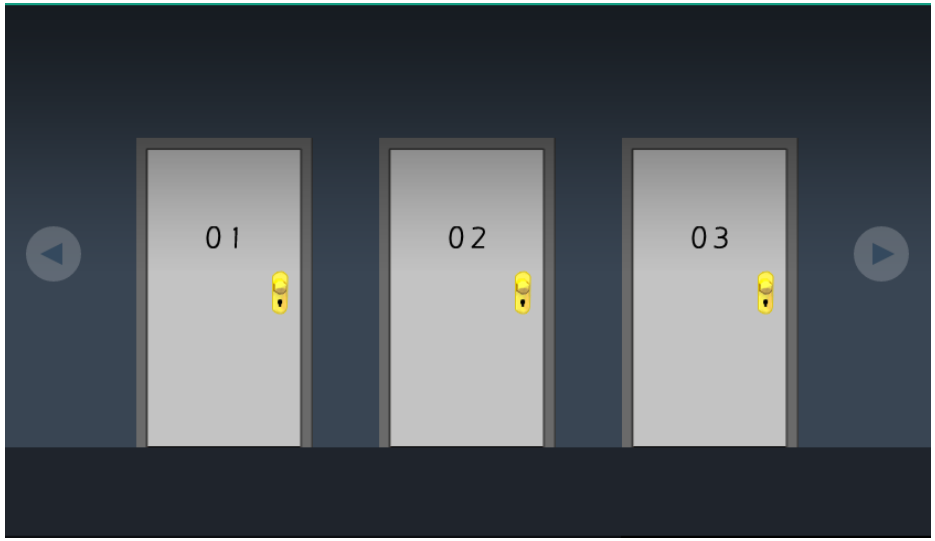

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There is a room with 100 gold coins in it, say room numbered x.

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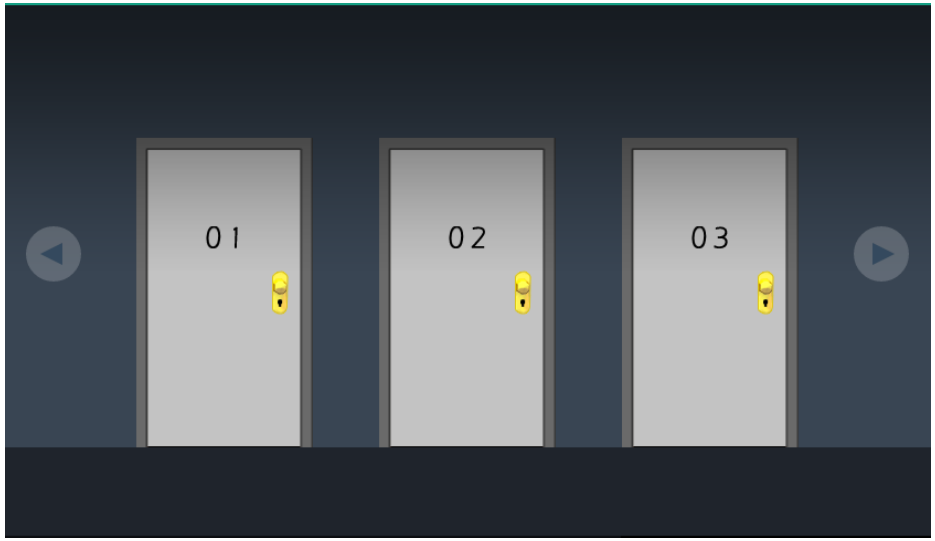

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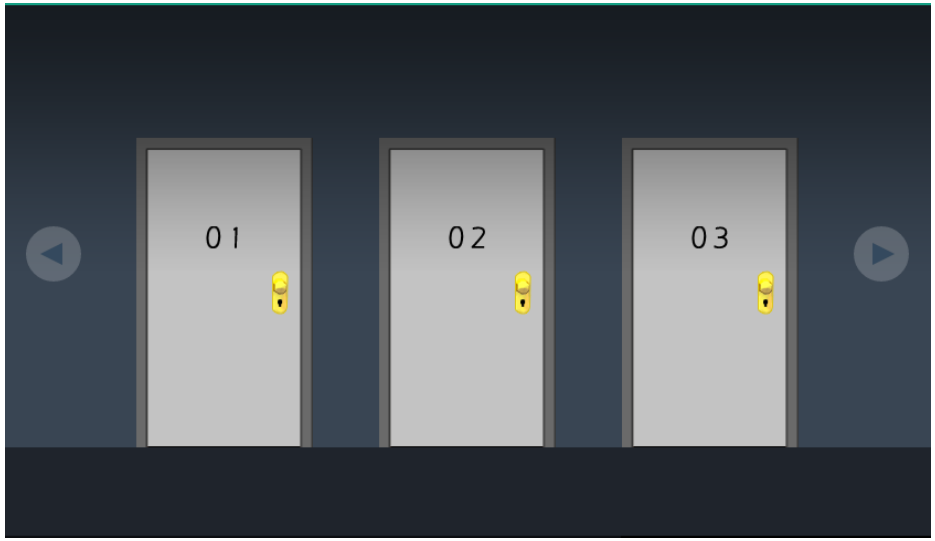

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If room y such that $y < x$ is opened, then you pay 1 gold coin.

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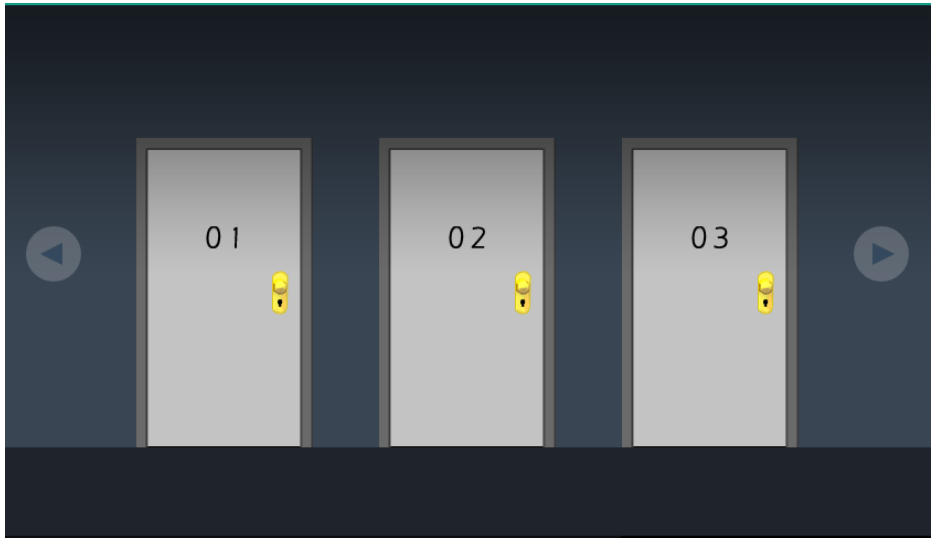

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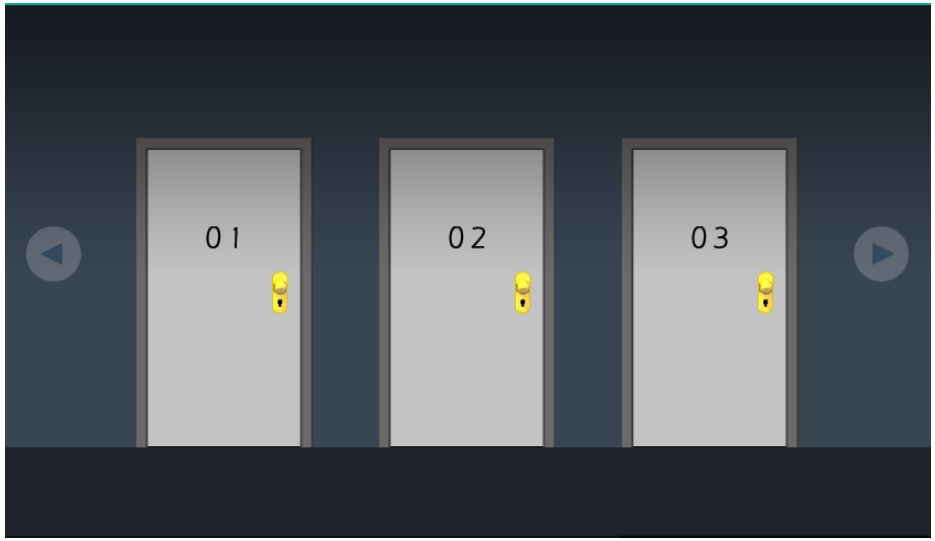

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Farmer does not know x.

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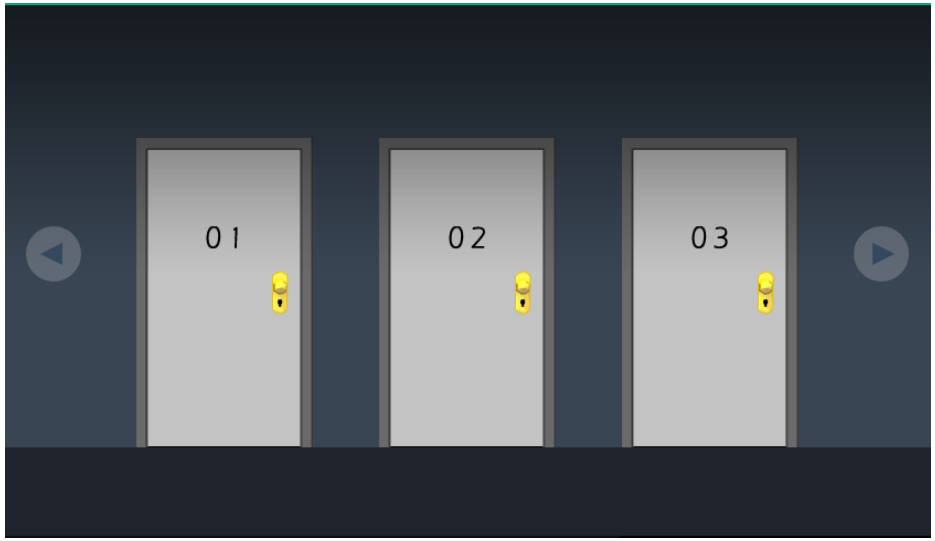

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Farmer does not know x.

What is the maximum number of coins he can take home?

With Birbal's help!

At least one gold coin

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Open all doors sequentially starting from 1.

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In the worst case $x = 100$: Farmer pays 99 and earns 100. Profit of 1.

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Open the 50th door. If it has 100 coins then done.

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Open the 50th door. If it has 100 coins then done.

If charged 1 coin, open all doors sequentially starting from 51.

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Open the 50th door. If it has 100 coins then done.

If charged 1 coin, open all doors sequentially starting from 51.

If charged 40 coins, open all doors sequentially starting from 1.

In the worst case farmer pays $40 + 48$ and earns 100. Profit of 12.

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Let $i = 1$. Open $10i$th door. If charged 1 coin, let $i \leftarrow i + 1$.

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In the worst case, $x = 89$. Farmer pays 8 coins for opening doors $10, 20, \ldots, 80$; pays 40 to open door 90

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A more realistic situation

Detecting defective memory cells

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In an array of n memory cells, there is a cell k such that all cells from $k+1$ to n are defective.

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Detecting defective memory cells

In an array of n memory cells, there is a cell k such that all cells from $k+1$ to n are defective.

The processor does not know k.

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Find k with at most 2 system crashes.

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Analysis of our strategy.

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Some questions and take back message

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Puzzle 2

Akbar, Birbal and some food for thought

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The citizens who have that dish come forward.

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Try to serve as many dishes to Akbar as possible.

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But x_2 does not have d_2, however, x_1 had d_2 as well.

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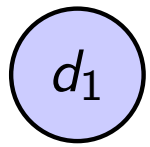
Birbal's predicament and Graph Theory

Model the puzzle as a graph

$$G = (X = \{x_1, x_2\} \cup \{d_1, d_2\}, E = \{(d_1, x_1), (d_1, x_2), (d_2, x_1)\})$$



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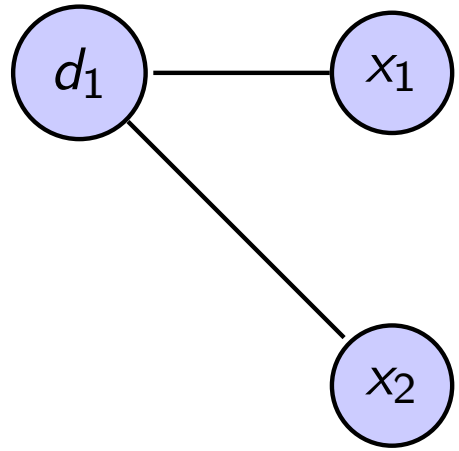

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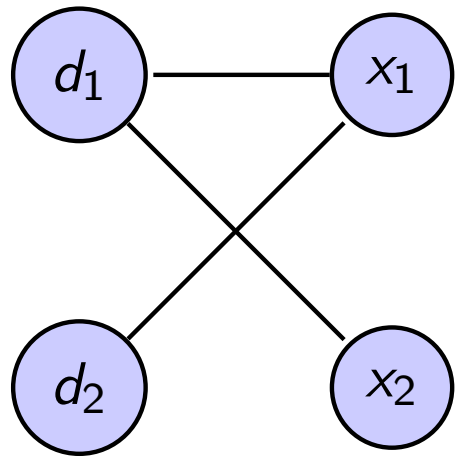

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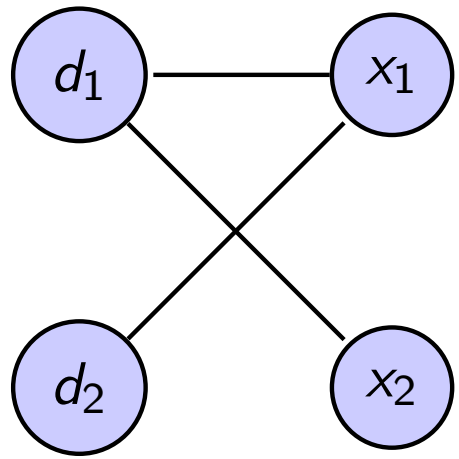

Maximum matching in the graph

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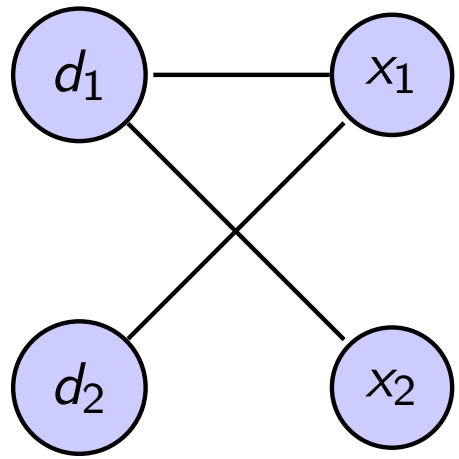

Maximum matching in the graph

Birbal needs to find the maximum sized set, say M^*, of edges such that no two edges in M^* share an end point.

$M^* = \{(d_1, x_2), (d_2, x_1)\}$.

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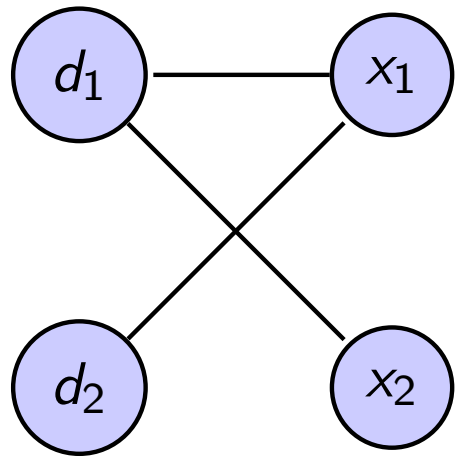

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Modeling the puzzle: maximum matching in graphs

Problem (Maximum Matching)

Given: *Given a vertex set $X = U \cup V$ and collection of edges $e_1, e_2, \ldots, e_m$ s.t. each e_i has one of its vertex in U and the other in V.*

Compute: *the size of the maximum matching in the graph.*

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Compute: *the size of the maximum matching in the graph.*

A possible approach

$M \leftarrow \emptyset$;
while *there exists* $e_i = (u_{i_1}, u_{i_2})$, *an input edge* **do**
  if $\forall e = (u, v) \in M$: $(u \neq u_{i_1} \neq u_{i_2})$ *and* $(v \neq u_{i_1} \neq u_{i_2})$ **then**
    $M \leftarrow M \cup \{e_i\}$;
  end
end
Output $|M|$

Modeling the puzzle: maximum matching in graphs

Problem (Maximum Matching)

Given: *Given a vertex set $X = U \cup V$ and collection of edges $e_1, e_2, \ldots, e_m$ s.t. each e_i has one of its vertex in U and the other in V.*

Compute: *the size of the maximum matching in the graph.*

A possible approach

$M \leftarrow \emptyset$;
while *there exists* $e_i = (u_{i_1}, u_{i_2})$, *an input edge* **do**
  if $\forall e = (u, v) \in M$: $(u \neq u_{i_1} \neq u_{i_2})$ *and* $(v \neq u_{i_1} \neq u_{i_2})$ **then**
    $M \leftarrow M \cup \{e_i\}$;
  end
end
Output $|M|$

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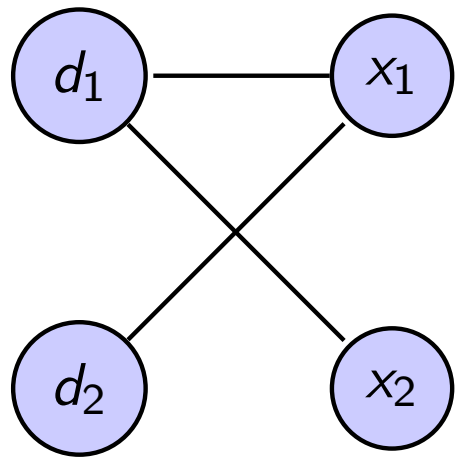
Analysis of the approach

Does it find the maximum matching?

$$G = (X = \{x_1, x_2\} \cup \{d_1, d_2\}, E = \{(d_1, x_1), (d_1, x_2), (d_2, x_1)\})$$



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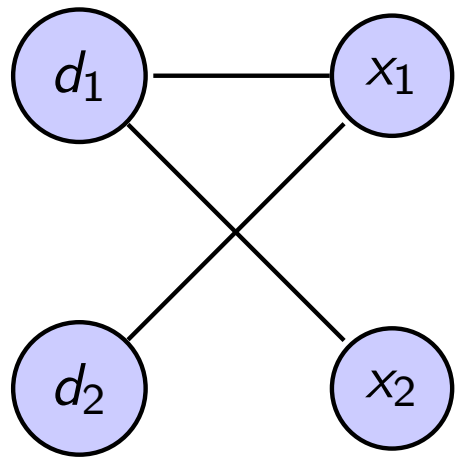

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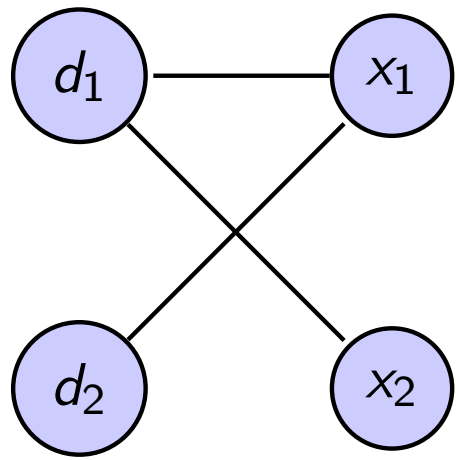

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Can it always satisfy half of Akbar's requests?

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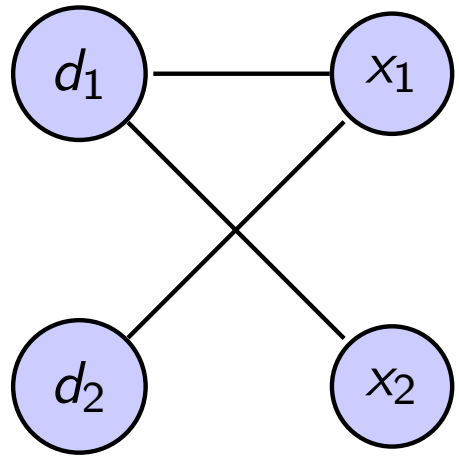

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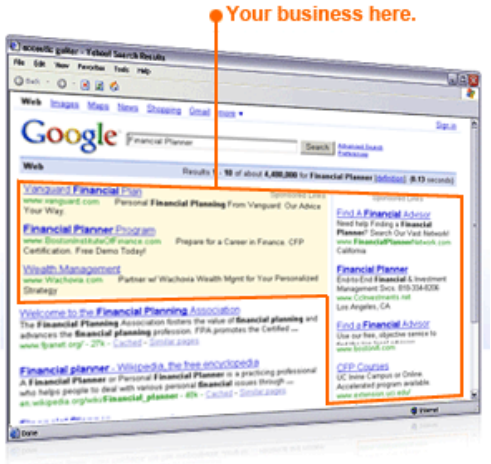
Can it always satisfy half of Akbar's requests?

Every edge added in M can leave out at most two edges of the maximum matching.

Indeed it can!

A more realistic situation

Google AdWords



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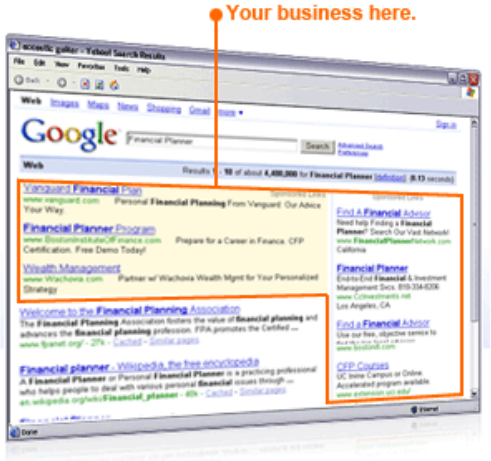

Bidders $\{b_1, b_2, \ldots, b_m\}$ and query phrases $\{q_1, q_2, \ldots, q_n\}$.

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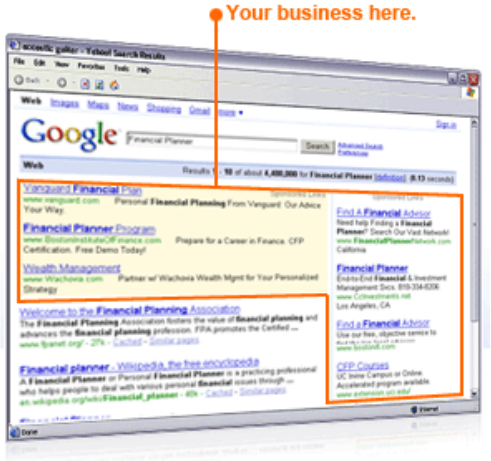

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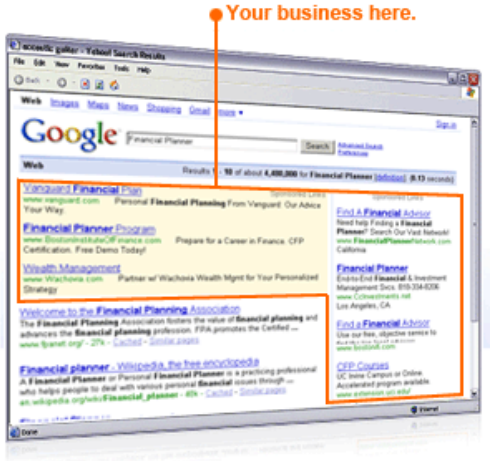

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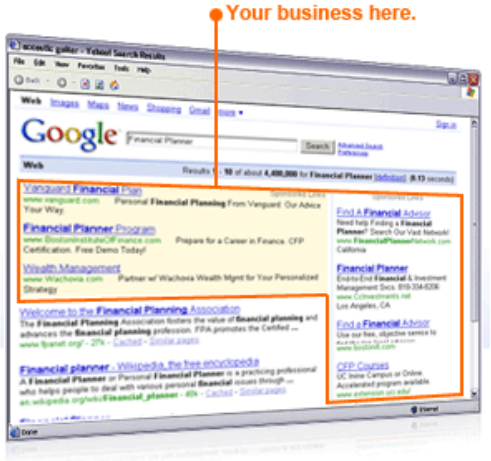

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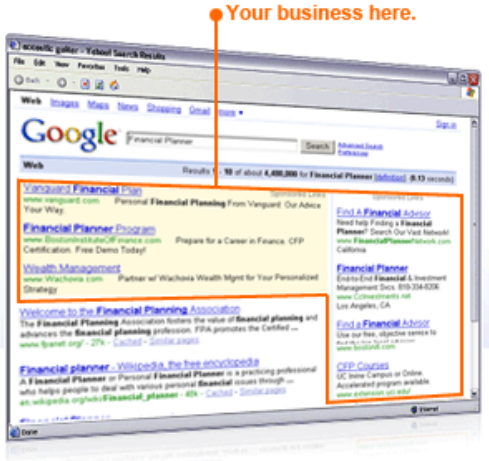

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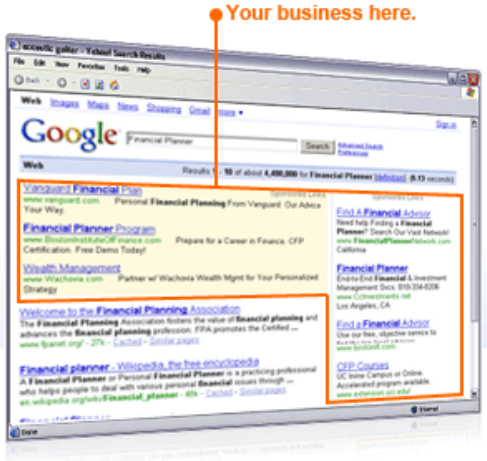

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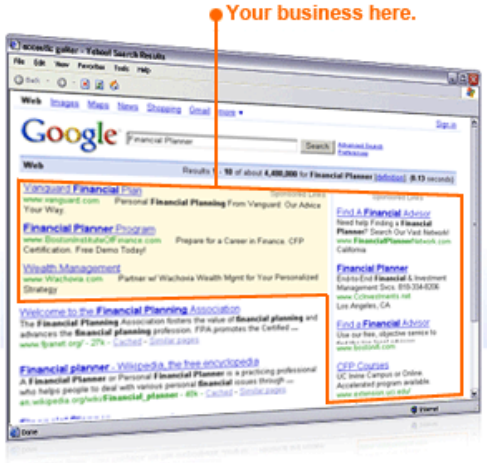

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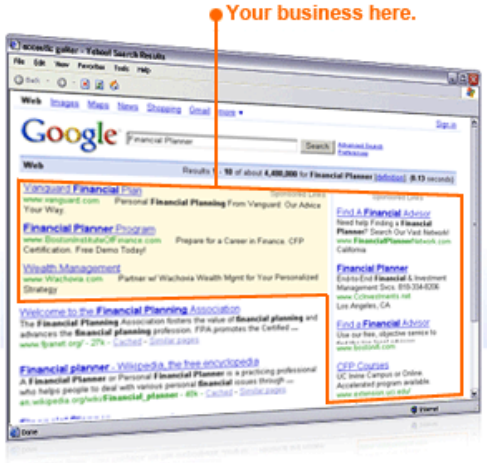

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Some questions and take back message

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Can Birbal serve more than 1/2 franction of all the dishes requested by Akbar?

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Beyond what fraction of requests can never be served?

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Take back message

To create a company like Google, know your graph theory!

Puzzle 3

Akbar, Birbal and the farmer ... story continues

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The story

The farmer: "Your highness, 40 gold coins are too few for my big family."

Akbar, Birbal and the farmer ... story continues

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The farmer: "Your highness, 40 gold coins are too few for my big family."

Akbar says yet again: " Take what you can ...

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There are 100 doors arranged as shown.

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As soon as you find a door such that all its neighboring doors have numbers smaller than it I will give you 100 gold coins.

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What is the maximum number of gold coins farmer can win?

How much can Birbal help?

Opens each door row-wise from left to right

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The farmer may end up with 0 coins.

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9	10	11	12
5	6	7	8
1	2	3	4

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A simpler situation

Doors arranged in a row (instead of on a grid)

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To find a local maxima

Open the middle door and two doors adjacent to it.

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$100 - \log_2(100) \sim 93$

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Doors arranged on a grid

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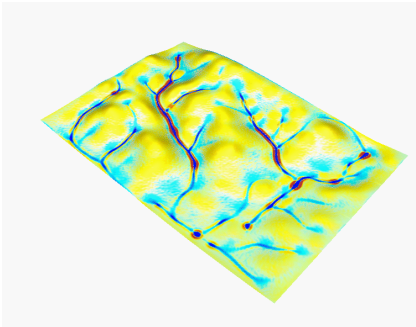
If n^2 doors are arranged on an $n \times n$ grid, then as a function of n, how many coins can the farmer win?

More realistic situations

Given a terrain locating water bodies



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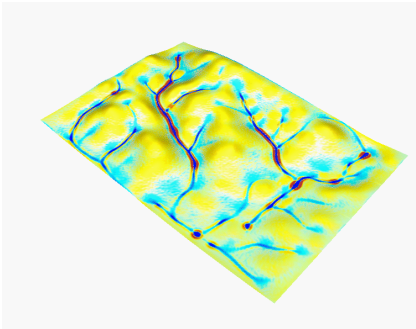

The heights denote the numbers in the grid.

Very likely to find water in local minima.

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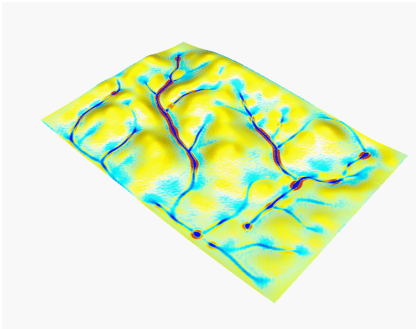

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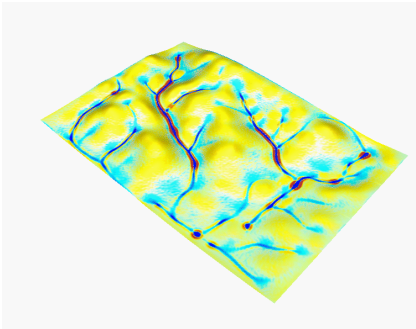

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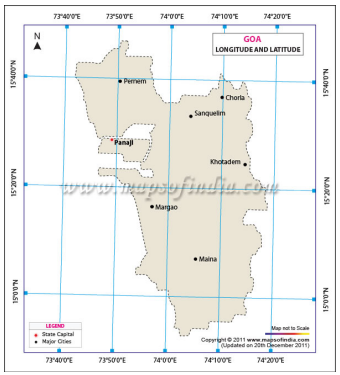

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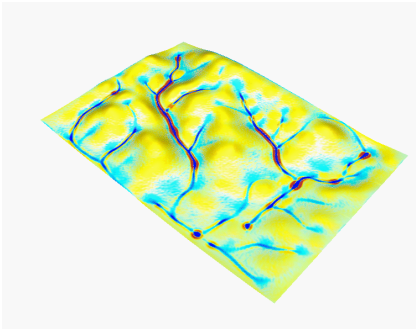

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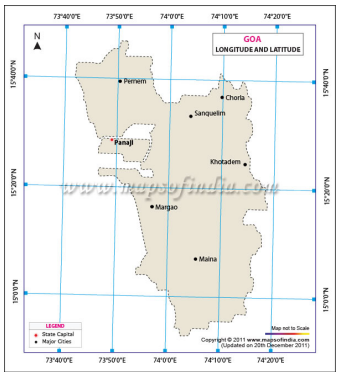

Heat maps of Goa and where to set up your ice cream stall today?

Some take back message

Want to help create Indian Google?

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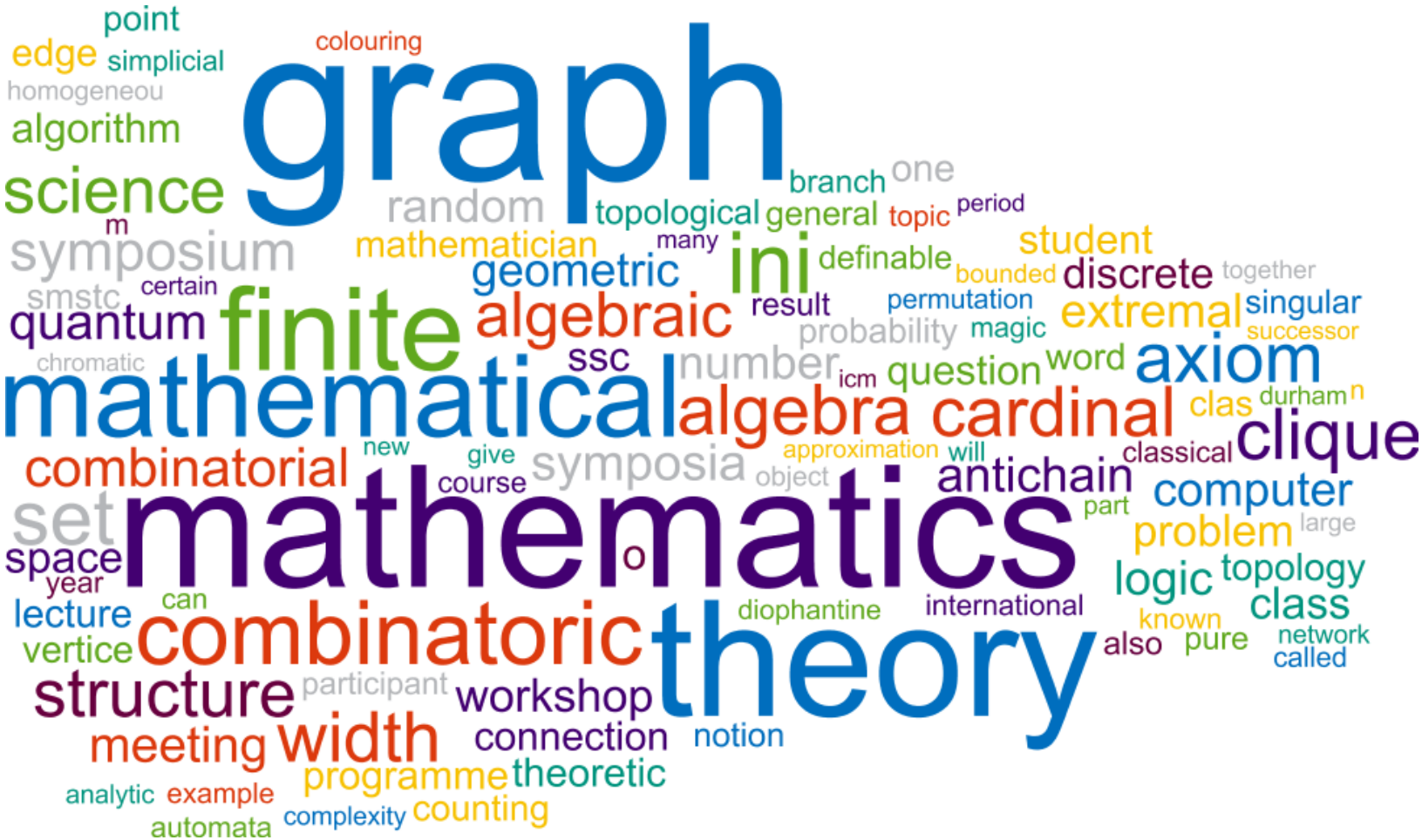
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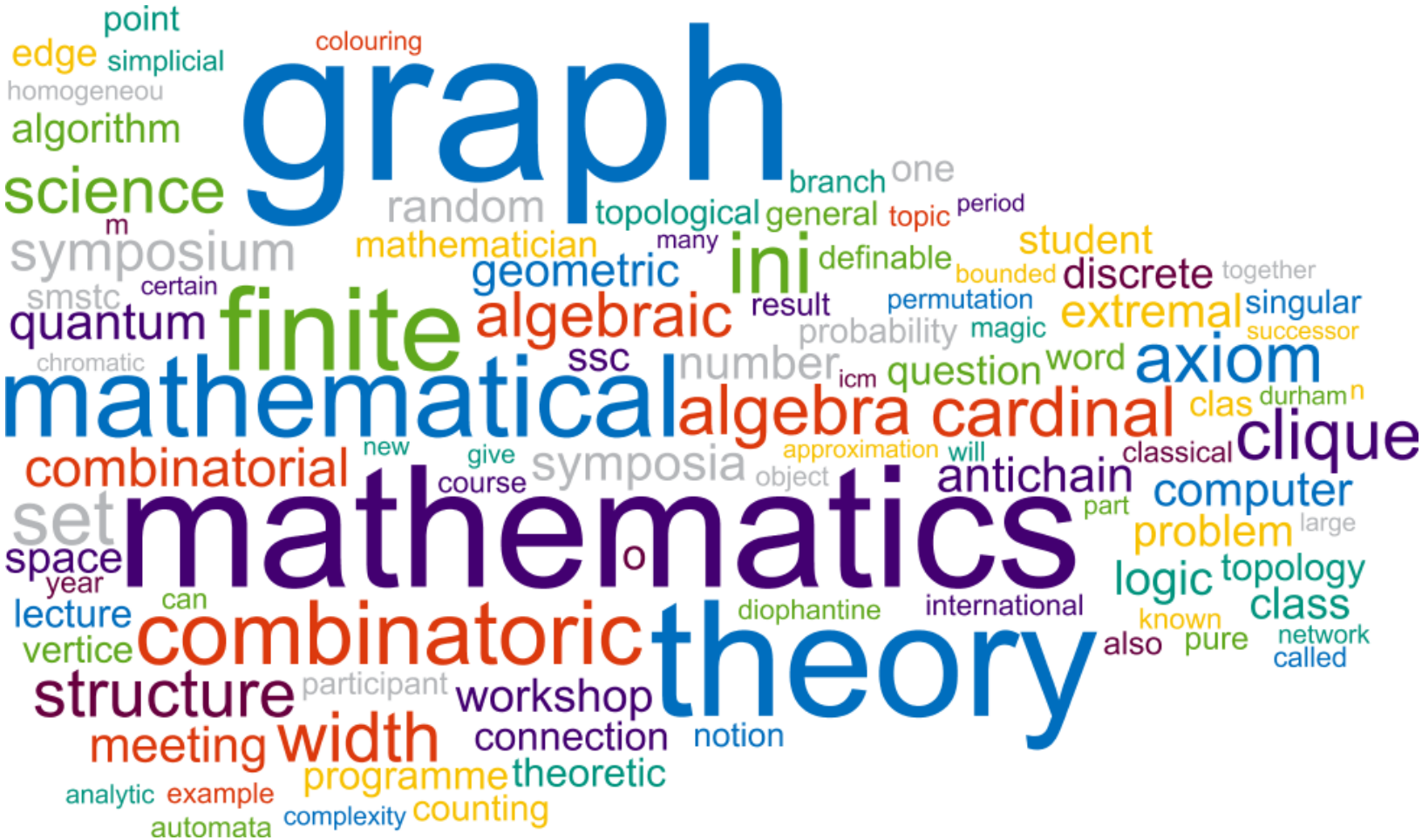
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Thank You!