Hats off to theoretical computer science

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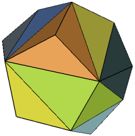
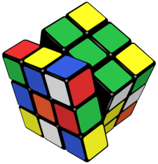
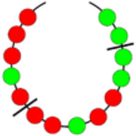
May 17, 2018

Theoretically speaking ...

Through simple puzzles let us try to understand some aspects of Logic, Computation of Boolean Functions, and Coding Theory.

Puzzle 1

Three girls and hats

The story:
Three girls sitting in a room.

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Three girls sitting in a room.
Each wears a red colored or white colored hat.

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Teacher: there is alteast one red hat.

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A: I don't know the color of my hat.
B: Even I don't know the color of my hat.

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Go figure!

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Due to teacher's claim, not all hats white.

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Then *B* would know her hat color
But even *B* does not know her hat color!!

Discussion

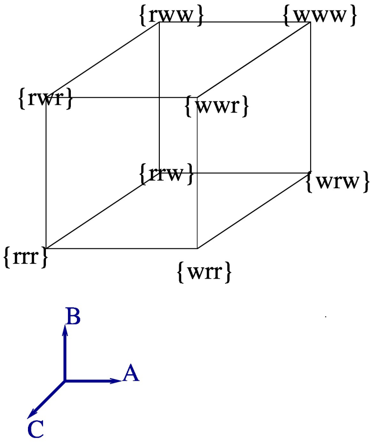
Due to teacher's claim, not all hats white.

A doesn't know the color of her hat, therefore:
Both B's and C's hats cannot be white.

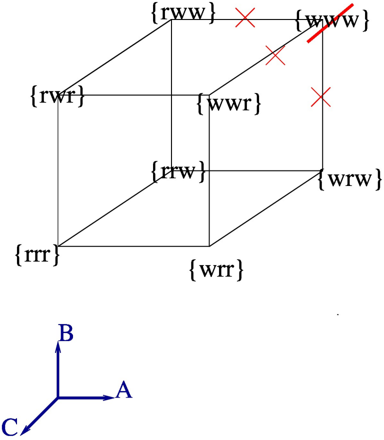
Say, C's hat is white and A's red:
Then B would know her hat color
But even B does not know her hat color!!
Hence C can deduce . . . everything!

What happened?

Initial information set:



After Teacher's announcement



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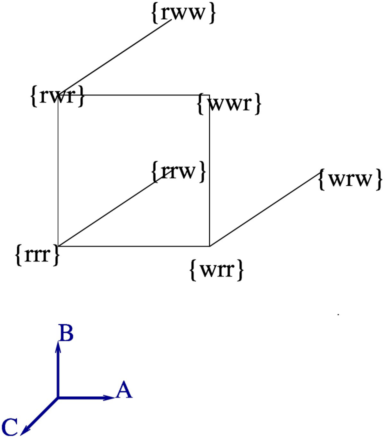
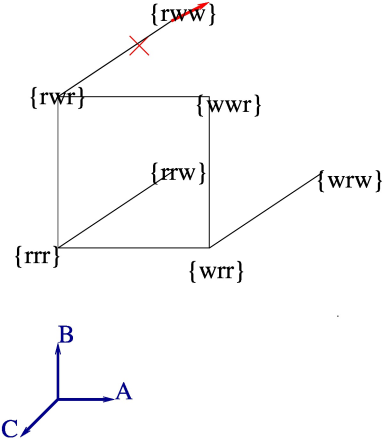
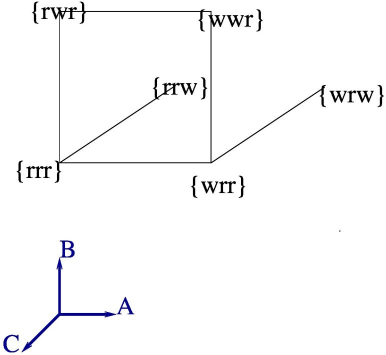


new information set

After *A*'s announcement

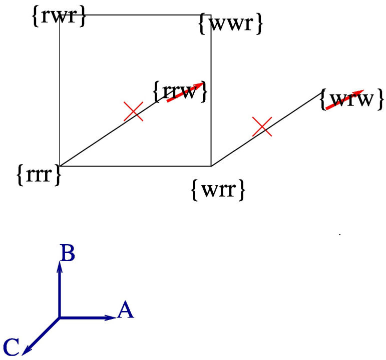


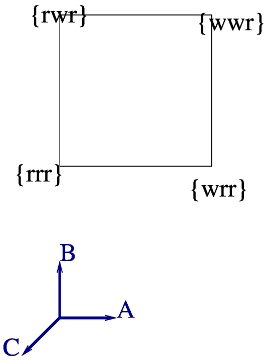
{*rww*} deleted



new information set

After B's announcement

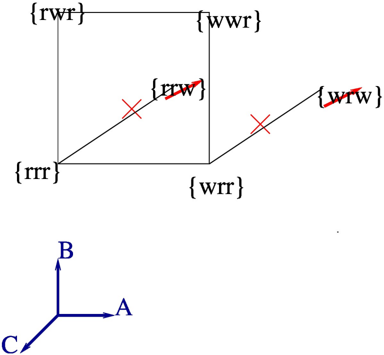


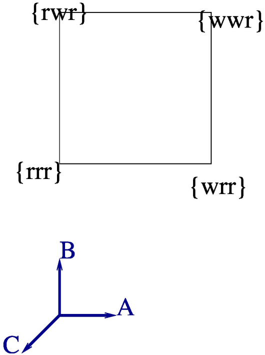


$\{rrw\}$ and $\{wrw\}$ deleted

new information set

After B's announcement





$\{rrw\}$ and $\{wrw\}$ deleted — new information set

Observe, C's color can only be red!

Epistemic Logic

Logic that reasons about knowledge.

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Many applications in computer science, economics.

Examples are robotics, network security, study of social interactions etc.

Puzzle 2

Only we can save us

The story:
100 prisoners standing in a line.
Each can see everyone ahead of him but none behind him.
Each wears a red or a white hat.

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Lots of people may die

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If each choses either red or white arbitrarily; all may die.

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If each choses either red or white arbitrarily; all may die.

All but one may die.

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All may die.
If each choses either red or white arbitrarily; all may die.
All but one may die.
If atleast one red (white) hat last prisoner says red (white, respectively), others copy. This will save atleast one person. In worst case exactly one person will survive.

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Can atleast 50 (half) survive?

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Idea: Majority

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Everyone else sticks to that answer.

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Not necessarily save oneself.

Save almost all

Can we save 99?

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Idea: Notice each prisoner computes a function.

Solution	strategy	function
1	none	constant function
2	last computes; others copy	$\vee$ (OR)
3	last computes; others copy	majority

Majority is an involved function (provavbly harder to compute) as compared to the constant function or the OR function.
Leads to saving more people.

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Leads to saving more people.
Increase the complexity of functions each prisoner computes–to save more people.

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Parity:

Say red= 1 and white= 0.

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Say red$=1$ and white$=0$.

Last prisoner says red (white) if he sees odd (even,respectively) number of red hats ahead of him.

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$$\oplus(x_1, x_2, \ldots, x_n) = \begin{cases} 1 & \text{if } \left(\sum_{i=1}^{n} x_i\right) \equiv 1 \ (mod 2) \\ 0 & \text{otherwise} \end{cases}$$

Which of these functions are hard to compute?

Proposition

If we can compute Majority, we can compute Th^{α} *for any integer* α *such that* $0 \leq \alpha \leq n$.

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Easy to see that the proof generalises for any α.

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$$\oplus(x_1, x_2, \ldots, x_n) = \bigvee_{i=0}^{\lfloor \frac{n}{2} \rfloor} [Th^{2i+1}(x_1, x_2, \ldots, x_n) \wedge Th^{2i}(x_1, x_2, \ldots, x_n)]$$

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Majority is harder than *Th* which is harder than $\oplus$.

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- vaguely, one needs to say **all** possible such combinations fail
- evidence: decades of efforts has yielded only a few results
- pessimism: some techniques have been classified as useless for proving such results
- optimism: because so less is known, there is a lot to discover!

Puzzle 3

Farther the better.

The story:

Seven students sitting in a circle. Each wearing either a red or a white hat.

Each can see all but his own hat.

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The story:
Seven students sitting in a circle. Each wearing either a red or a white hat.
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What is the probability that the students win?

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What is the probability that the students win? Uniform distribution on all the possible configurations of the hats.

Can they win with probability half?

One student says ‘red’. All others say pass.

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Half the times no student is wrong.

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0 1 0	p 1 p	1
0 1 1	0 p p	1
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Strategy: If equal number of red and white then 'pass'

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1 1 0	p p 0	1
1 1 1	0 0 0	0

Strategy: If equal number of red and white then 'pass'
else invert the value.

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Observe: Students are correct with probability $\frac{3}{4}$

Strategies

Strategy: If equal number of red and white then 'pass' else invert the value.

As stated this does not generalise.

Is there a better way of stating it, which will generalise?

Codes

Codes are functions such that $f : \{0,1\}^k \to \{0,1\}^n$ with $k < n$ (injective, with size of the range $= 2^k$).

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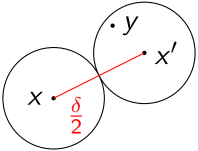

Figure: x and x' are δ apart. y corrected to its nearest neighbor x'

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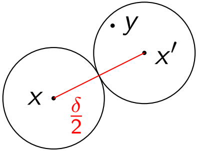

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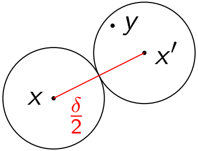

Figure: x and x' are δ apart. y corrected to its nearest neighbor x'

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Hence, with distance 2 or 3 one error can be corrected.

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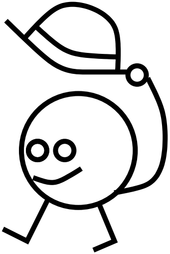

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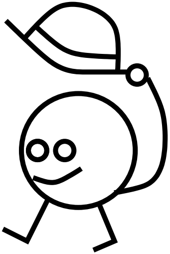

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Thank you!