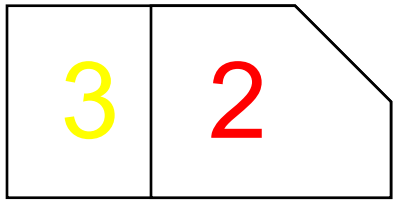
Math: Is it abstract?

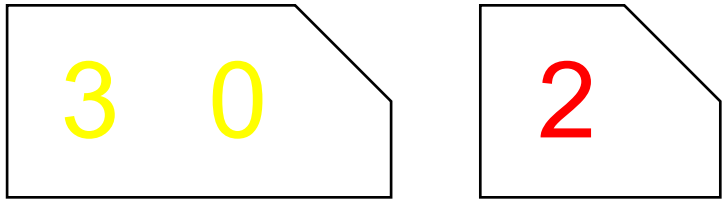
When? Where? How?

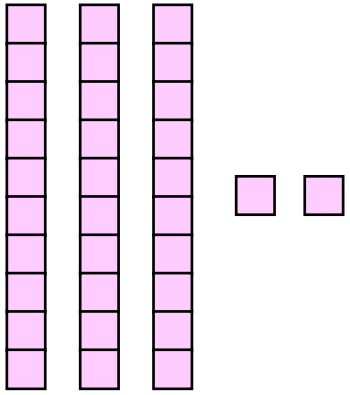
~~Whole numbers~~

Place value

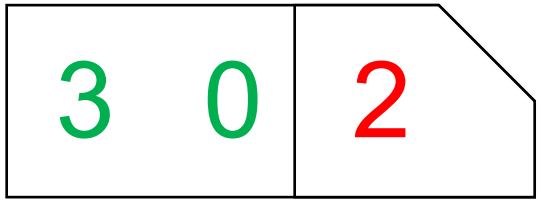
32

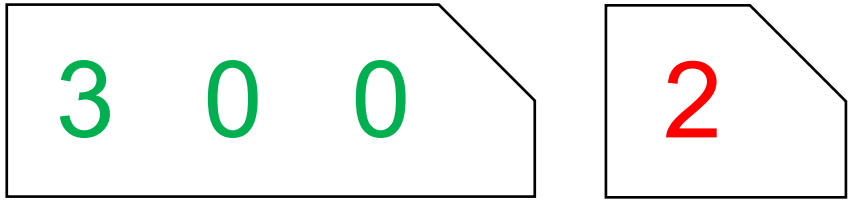


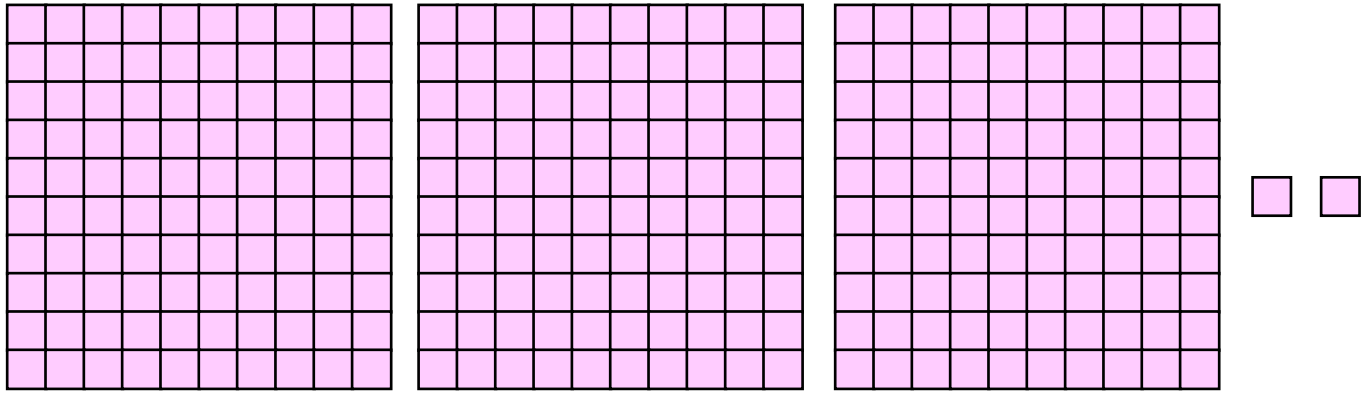




302







Associativity of addition

For whole numbers

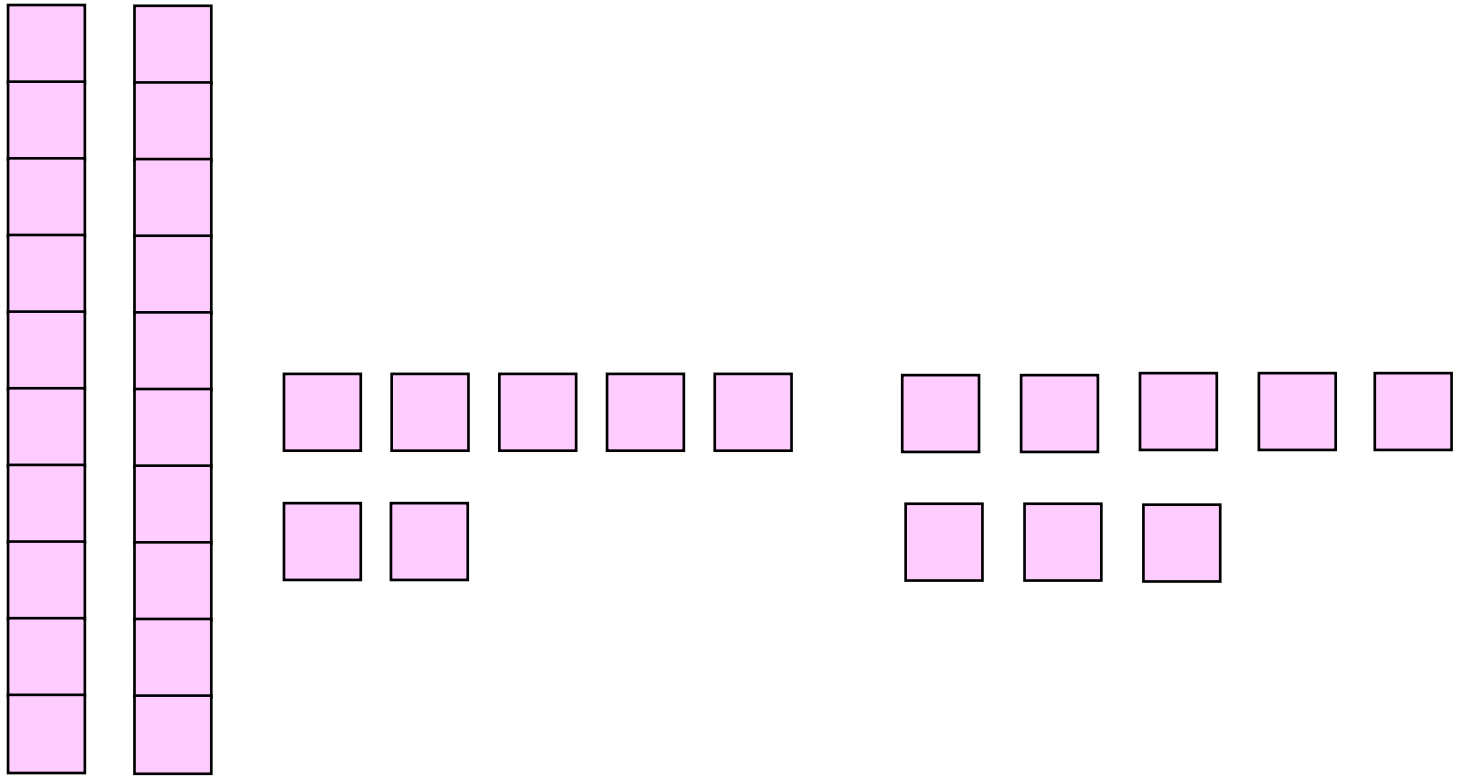
2	7
+	8

27 + 8

= (20 + 7) + 8

= 20 + (7 + 8)

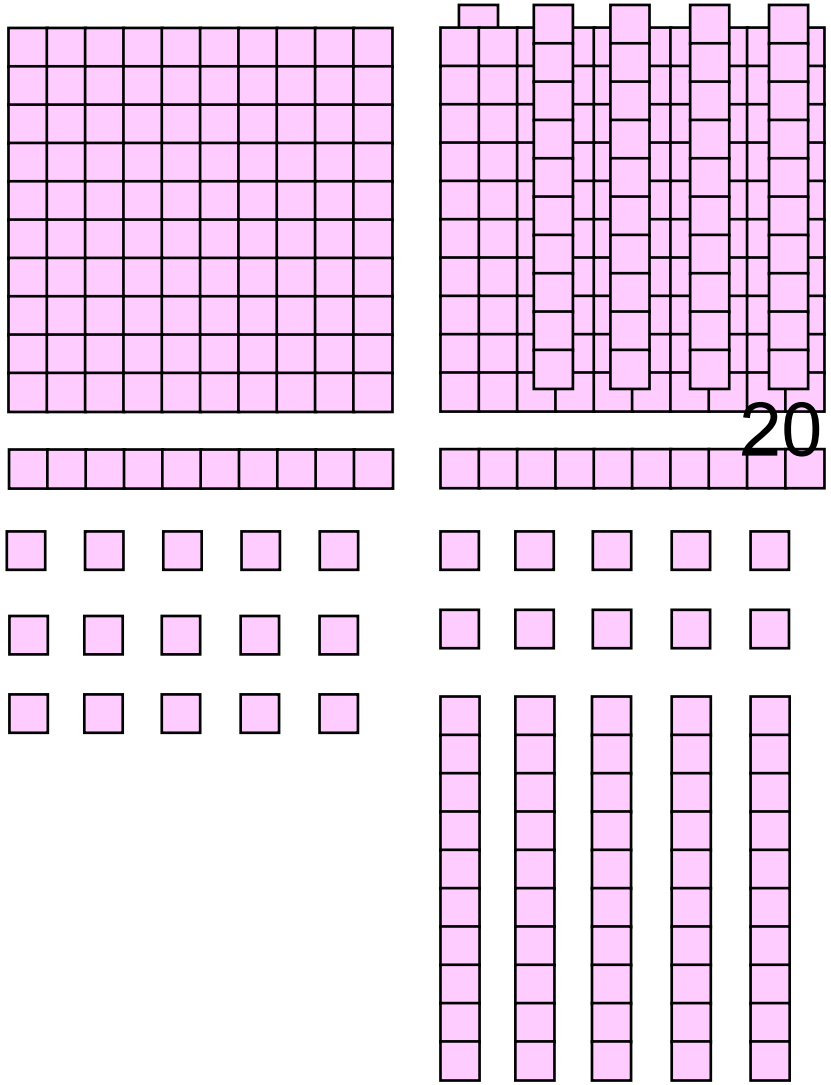
= (7 + 8) + 20



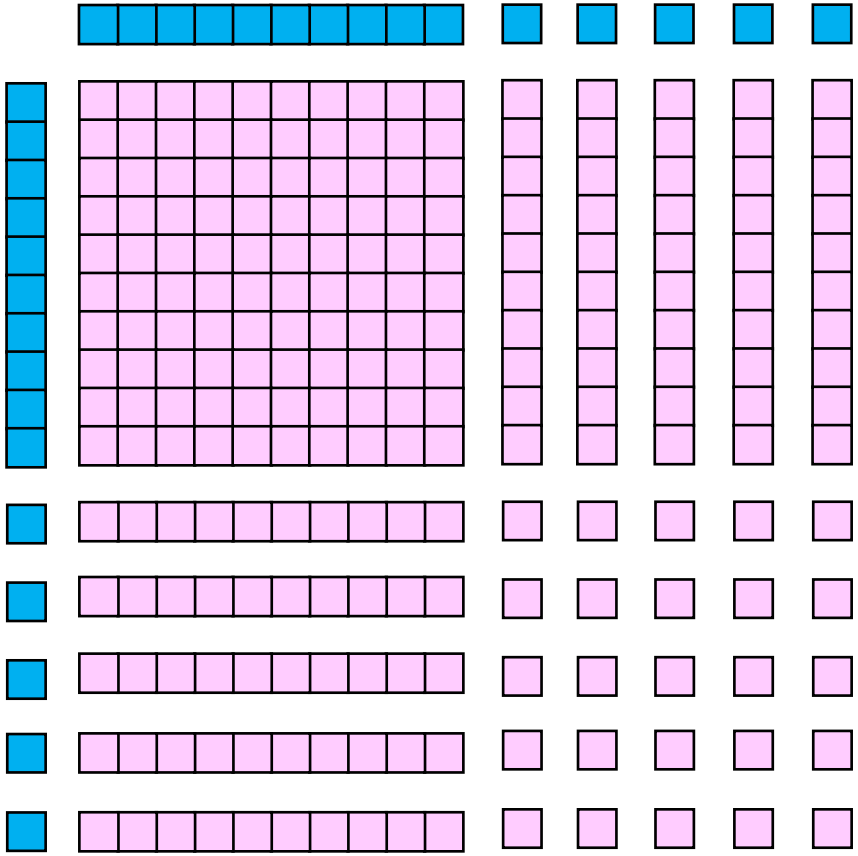
Square root

Division algorithm

225



$$
\begin{array}{r}
1\ \ 5 \\
\hline
225 \\
-\ 100 \\
\hline
20 \div 5 \times 5 \\
-\ 125 \\
\hline
0
\end{array}
\qquad
\begin{array}{)l}
25 \\
125 \\
125
\end{array}
$$



1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th

Multiplying negative numbers?

If multiplication table is written as

$1 \times 5 = 5$

$2 \times 5 = 10$

$3 \times 5 = 15$

⋮

Then

$3 \times (-5) = (-5) + (-5) + (-5) = -15$

But

$(-3) \times 5 = ?$

Commutativity is yet to be explored

$(-3) \times (-5) = ?$

And

$(3/5) \times (-7/8) = ?$

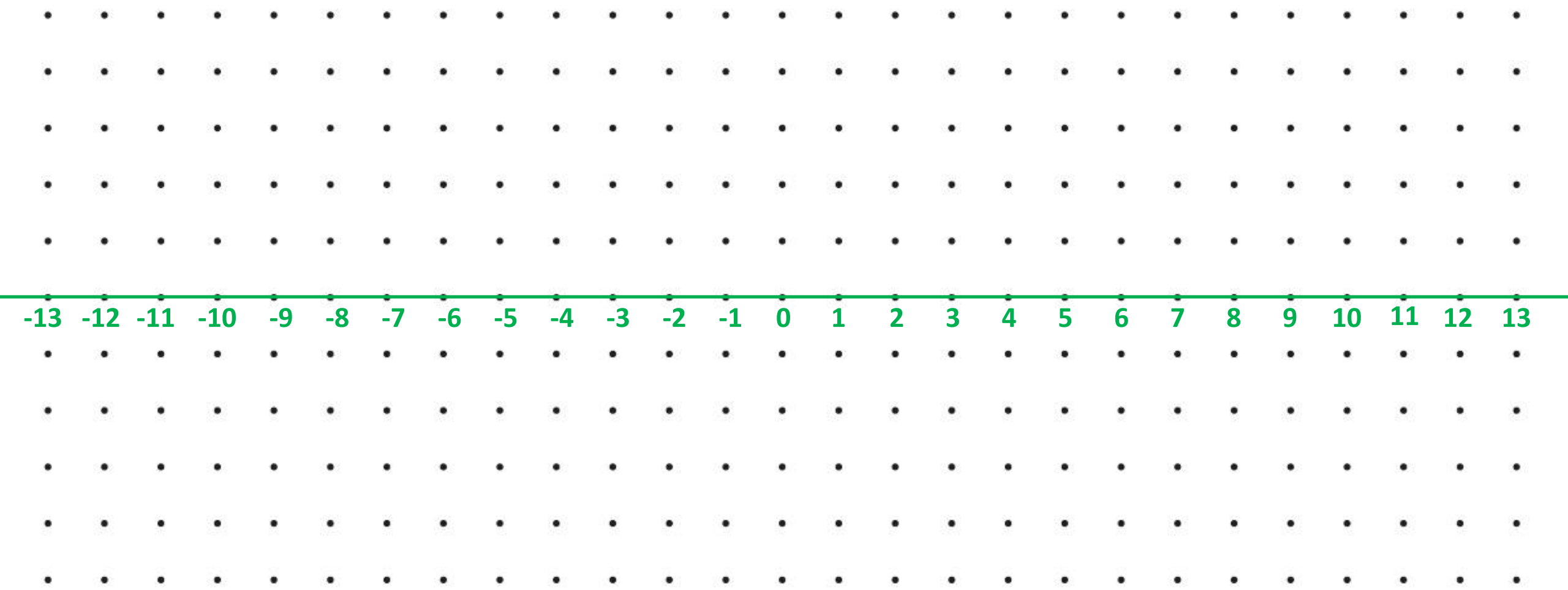
$(-1/3) \times (4/7) = ?$

$(-2/5) \times (-3/7) = ?$

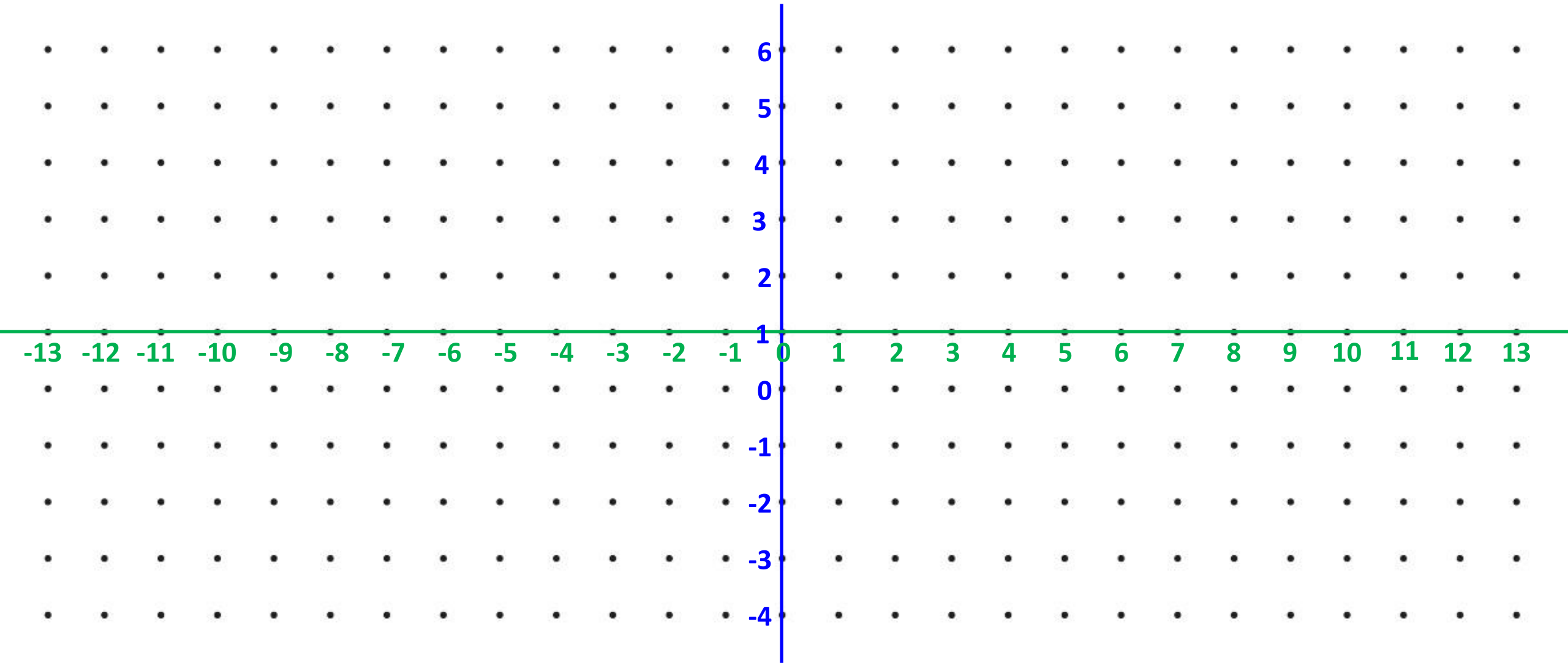
Scaling model

Of multiplication

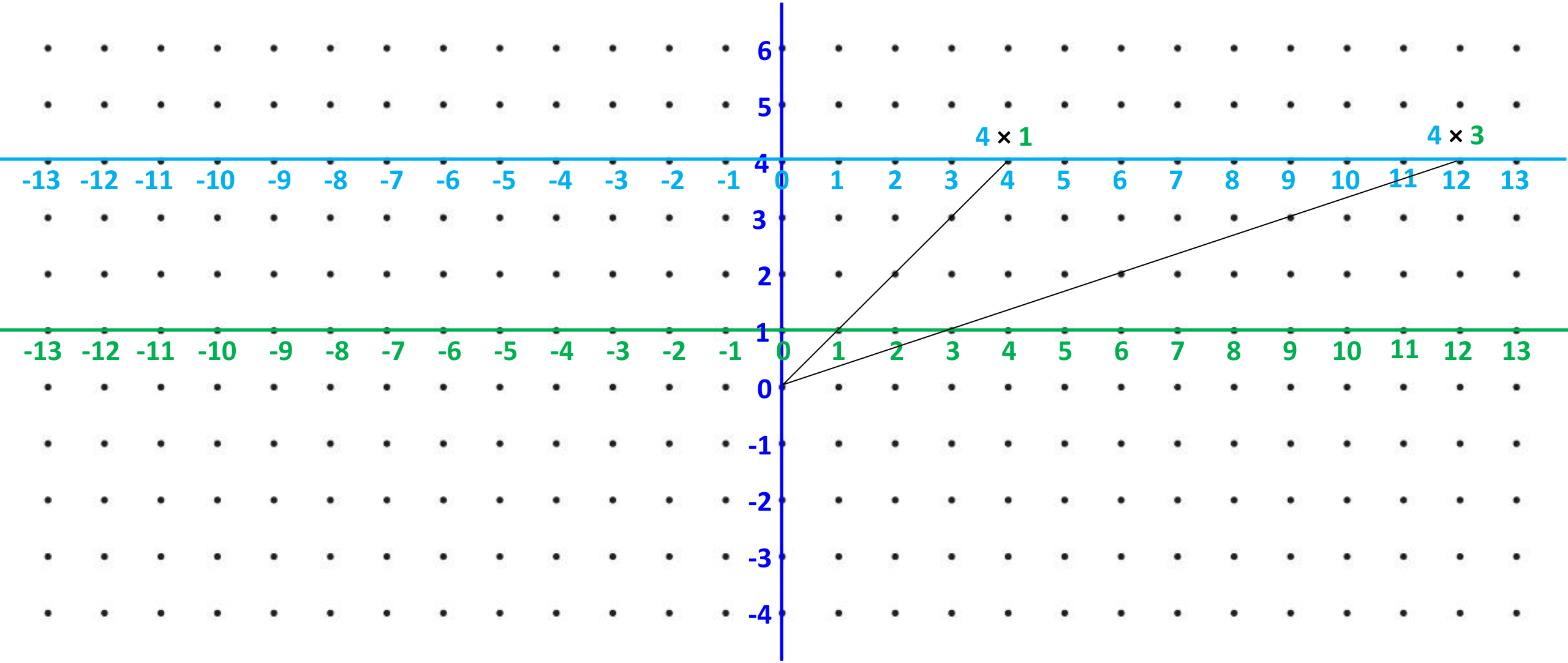
1st number line



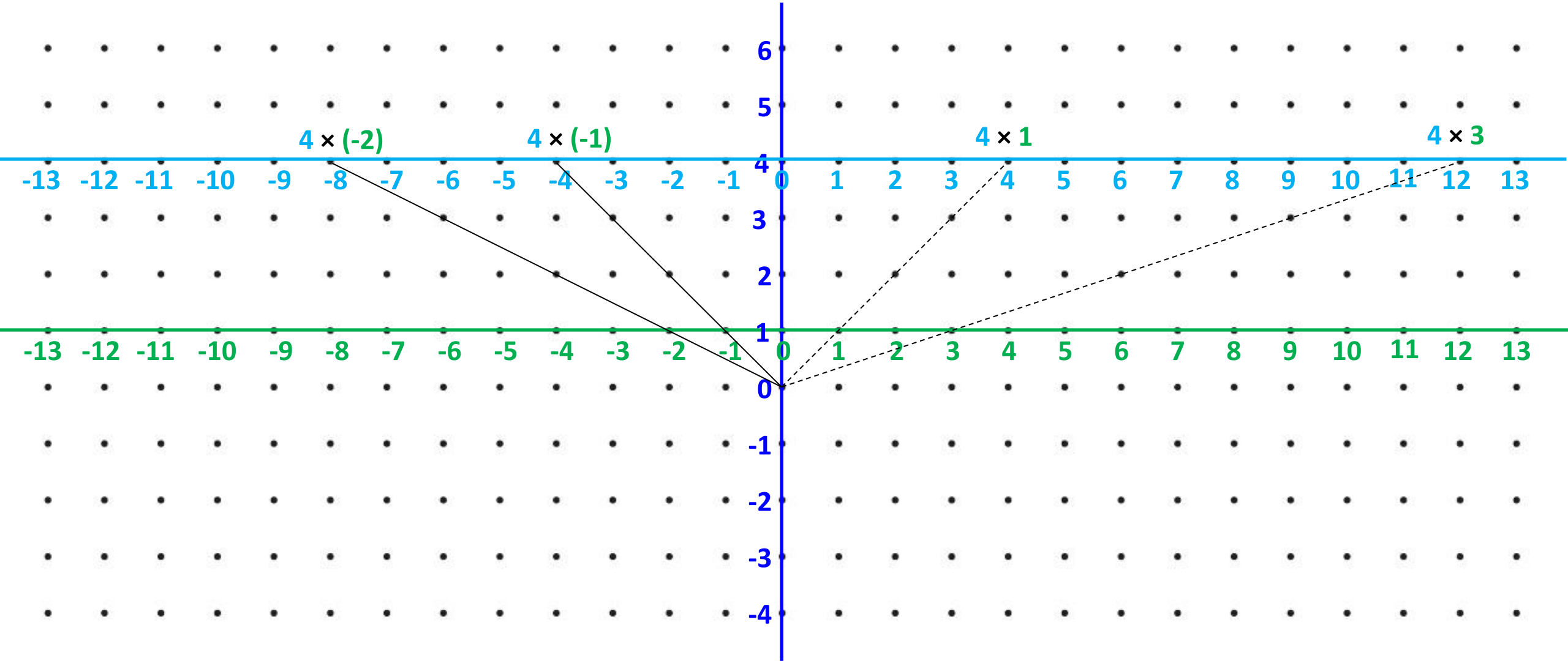
2nd number line



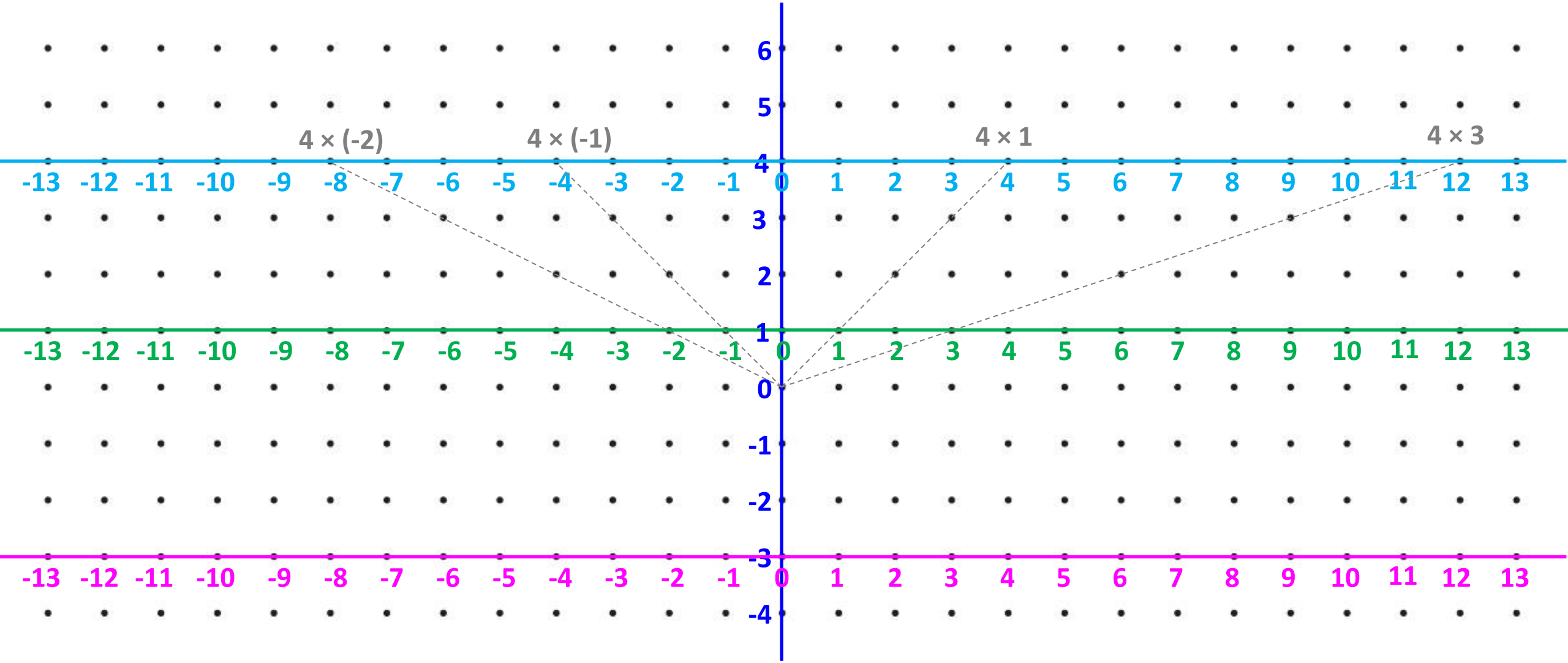
3rd number line



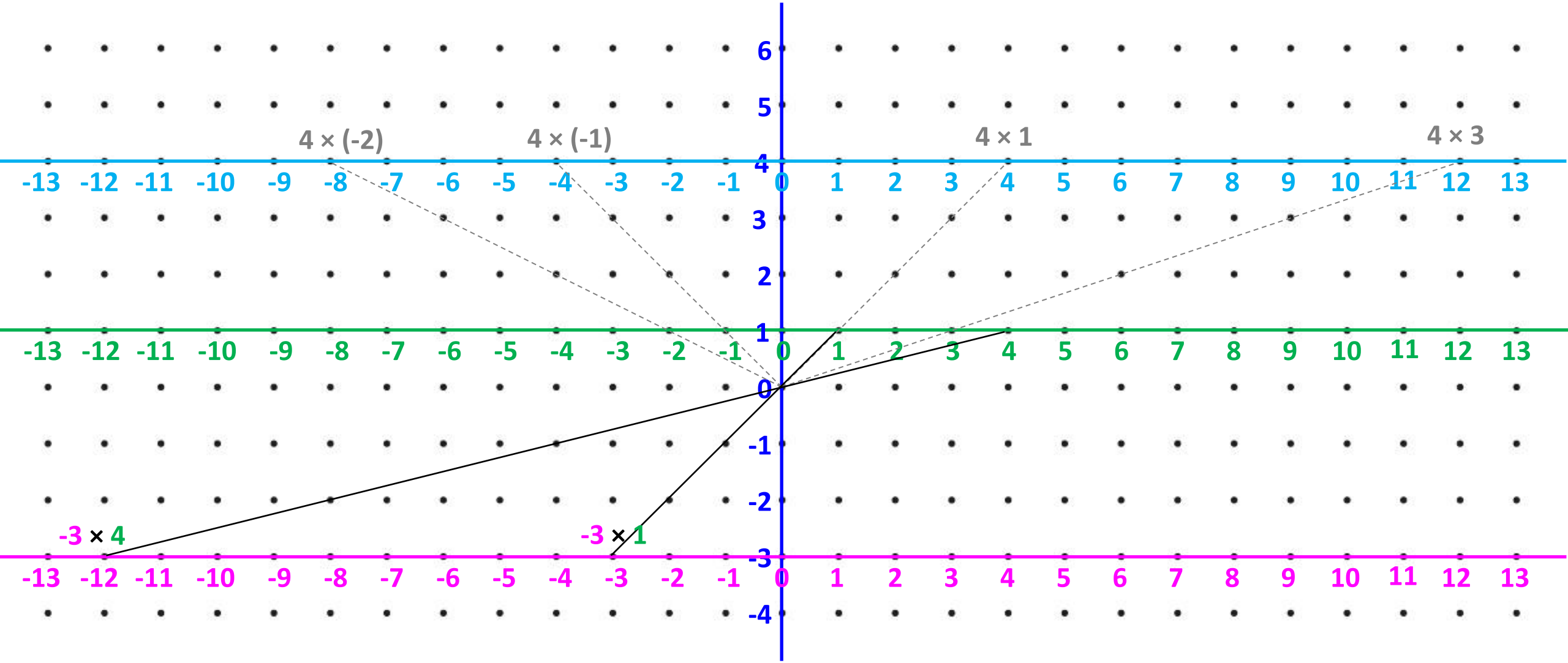
Multiplying negative numbers



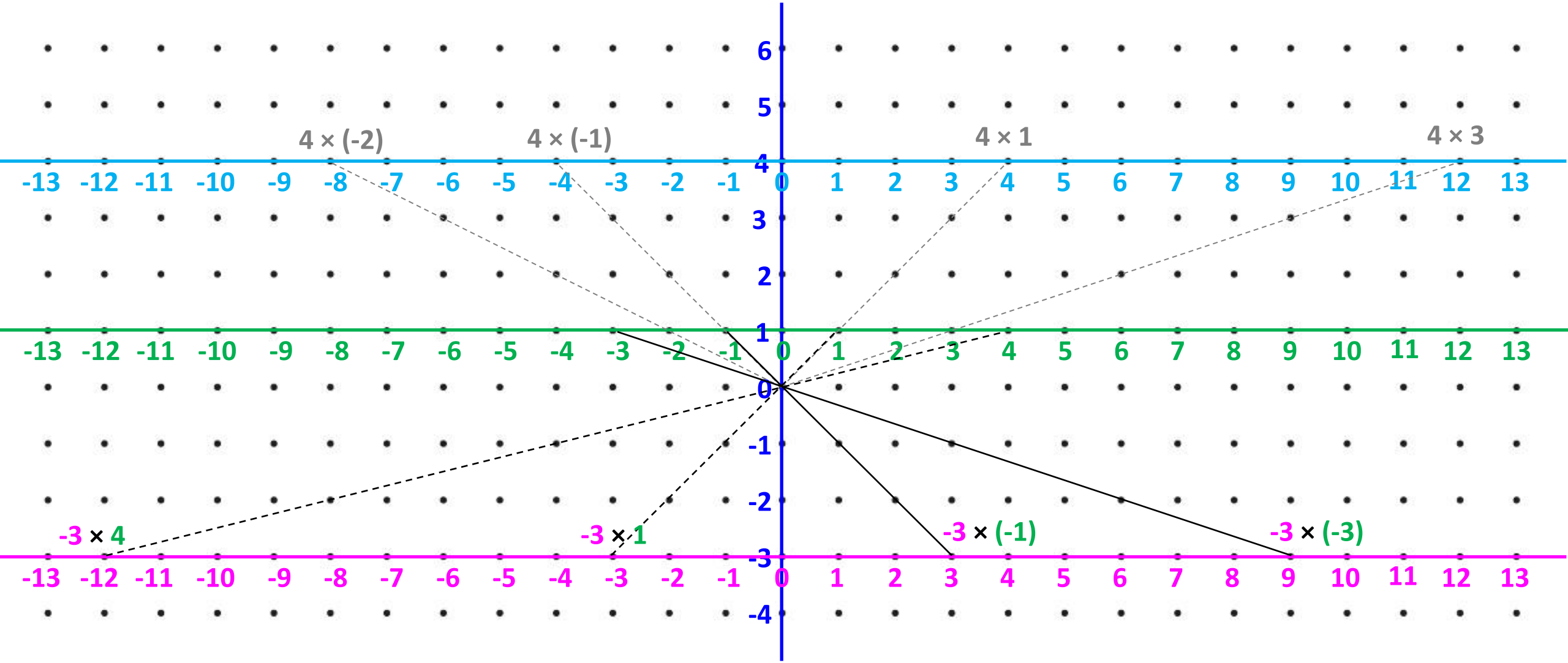
4^{th} number line (through negative scale factor)



Multiplying positive numbers



Multiplying negative numbers

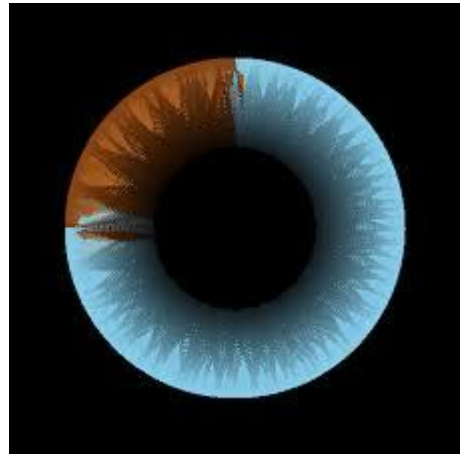


Going higher…

Hyperbolic planes by Diana Tamiana

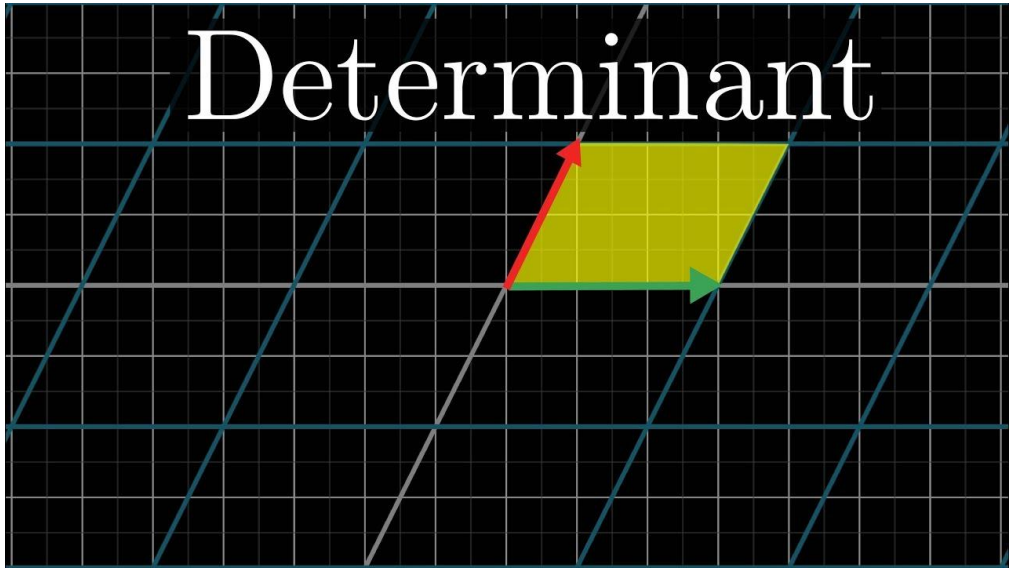
https://www.youtube.com/watch?v=w1TBZhd-sN0 (TedX)

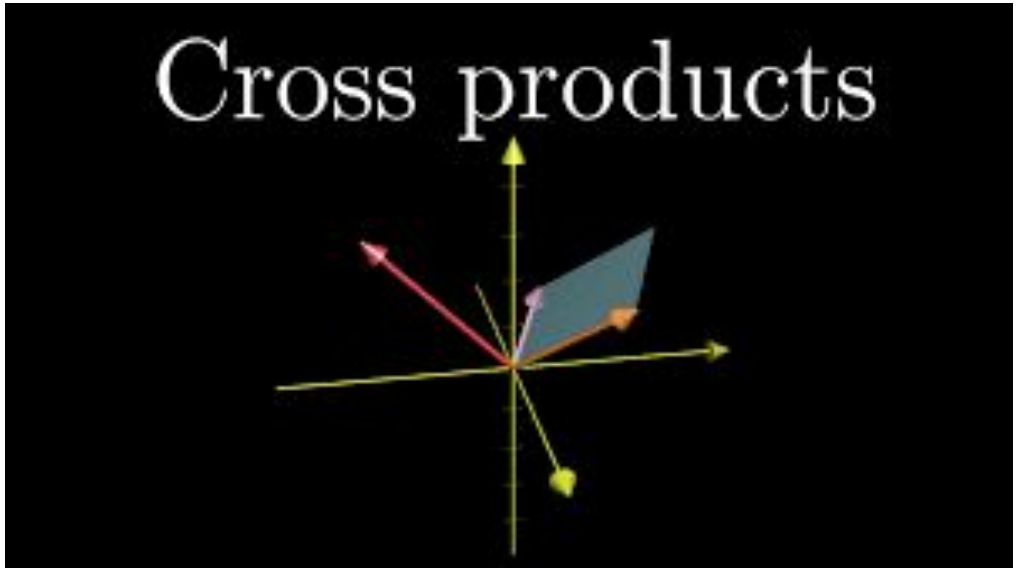
Youtube channel: 3blue 1brown



Linear algebra, vector spaces, matrices …

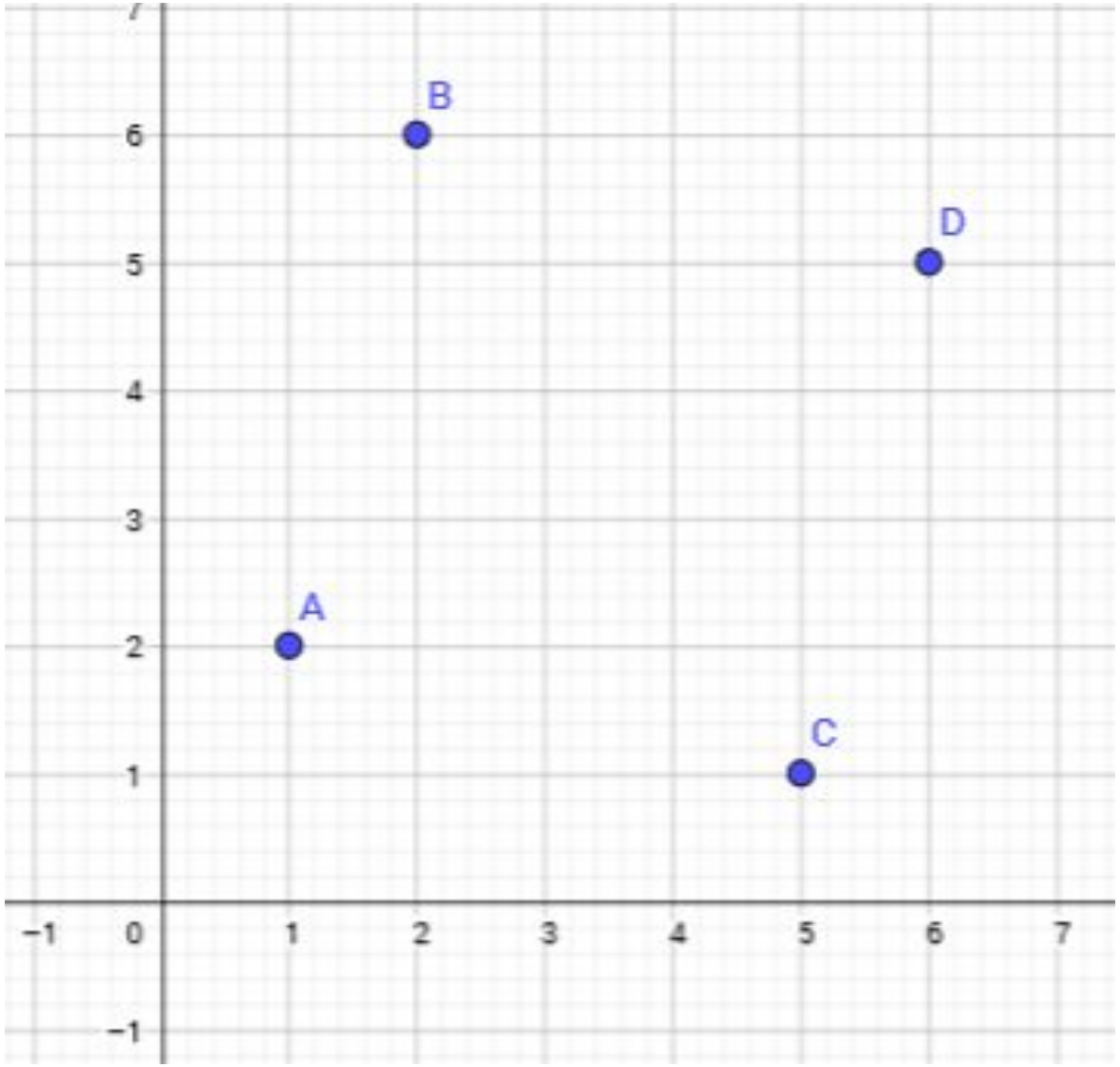
- Linear transformation
- Eigen values and eigen vectors





Is it a square?

- A (1, 2), B (2, 6), C (5, 1) and D (6, 5)
  - ✓ Calculate all distances
  - ✓ Calculate slopes of all line segments
  - ✓ Plot the points
- If these points were in 3D?
  - ✓ Calculate all distances
  - ✓ Calculate all angles – dot products
  - ✓ Plot the points
- If they are in 4D? 5D? n-D?
  - ✓ Calculate all distances
  - ✓ Calculate slopes of all line segments



Points equidistant from each other

- 2 points
  - Line segment ∴ Fits within 1D
  - A = 1 and B = -1
- 3 points
  - Equilateral triangle ∴ Needs minimum 2D
  - A = (1,0) B = (-1,0) and C = (0,√3)
- 4 points
  - Regular tetrahedron ∴ Needs minimum 3D
  - A = (1,0,0) B = (-1,0,0) C = (0, √3,0) and D = $\left(0, \frac{1}{\sqrt{3}}, 2\sqrt{\frac{2}{3}}\right)$

Equidistant points (contd.)

- 5 points?
  - ✓Let E = (x, y, z, w)
  - ✓$BE^2 = AE^2 \Rightarrow x = 0$
  - ✓$CE^2 = AE^2 \Rightarrow y = \frac{1}{\sqrt{3}}$
  - ✓$DE^2 = AE^2 \Rightarrow z = \frac{1}{\sqrt{6}}$
  - ✓$DE^2 = AB^2 \Rightarrow w = \sqrt{\frac{5}{2}}$
- 6 points?
  - ✓$E = \left(0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, \sqrt{\frac{5}{2}}, 0\right)$
  - ✓Let F = (x, y, z, w, v)
  - ✓$BF^2 = AF^2 \Rightarrow x = 0$
  - ✓$CF^2 = AF^2 \Rightarrow y = \frac{1}{\sqrt{3}}$
  - ✓$DF^2 = AF^2 \Rightarrow z = \frac{1}{\sqrt{6}}$
  - ✓$EF^2 = AF^2 \Rightarrow w = \frac{1}{\sqrt{10}}$
  - ✓$EF^2 = AB^2 \Rightarrow v = 2\sqrt{\frac{3}{5}}$

What have we got so far…

A = (1, 0 … 0) B = (-1, 0 … 0) C = (0, $\sqrt{3}$, 0 … 0)

	D	E	F
3D			
4D			
5D			

Going forward to n such points in (n – 1)D

$A_0 = (1, 0 \dots 0)$ $A_1 = (-1, 0 \dots 0)$ $A_2 = (0, \sqrt{3}, 0 \dots 0)$

$$A_{n-1} = \left(0, \frac{1}{\sqrt{3}}, \dots, \frac{1}{\sqrt{T_{n-2}}}, \frac{n}{\sqrt{T_{n-1}}}\right) = \left(0, \frac{1}{\sqrt{3}}, \dots, \sqrt{\frac{2}{(n-2)(n-1)}}, \sqrt{\frac{2n}{n-1}}\right)$$

$$A_{n-2} = \left(0, \frac{1}{\sqrt{3}}, \dots, \frac{1}{\sqrt{T_{n-3}}}, \frac{n-1}{\sqrt{T_{n-2}}}, 0\right) = \left(0, \frac{1}{\sqrt{3}}, \dots, \sqrt{\frac{2}{(n-3)(n-2)}}, \sqrt{\frac{2(n-1)}{n-2}}, 0\right)$$

$\vdots$

$$A_{n-i} = \left(0, \frac{1}{\sqrt{3}}, \dots, \frac{1}{\sqrt{T_{n-i-1}}}, \frac{n-i+1}{\sqrt{T_{n-i}}}, 0 \dots 0\right)$$ with i – 1 coordinates

Thank you!

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7406 3333 05

At Right Angles: ttp://azimpremjiuniversity.edu.in/SitePages/resources-at-right-angles.aspx

Teachers of India: http://teachersofindia.org/en