Nuclear Physics in the Era of Lattice QCD

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Mumbai 11/25/2010

Outline

- Looking forward
- Basic technology
- Baryon-Baryon and nuclei
- Conclusion

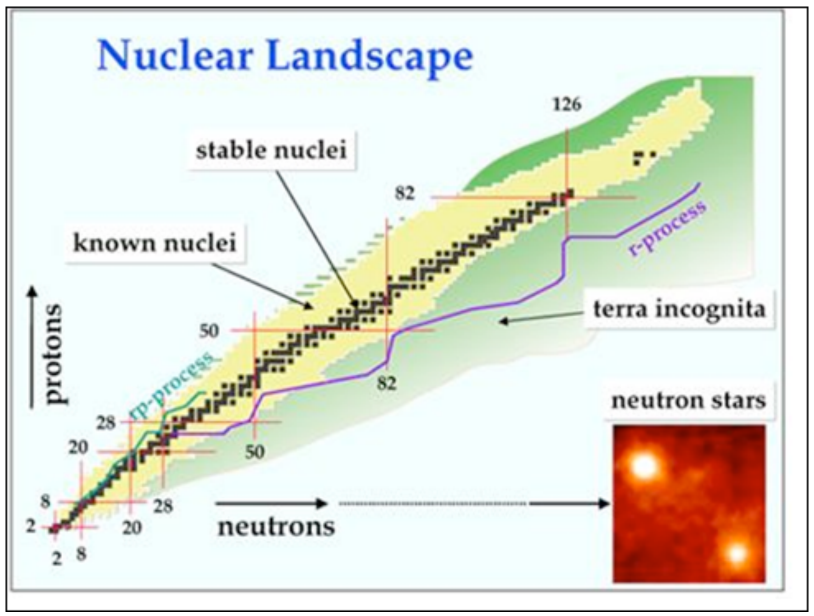
Presently lattice QCD is the only known method for defining QCD outside of perturbation theory and for making quantitative predictions for hadronic quantities with fully controlled uncertainties.

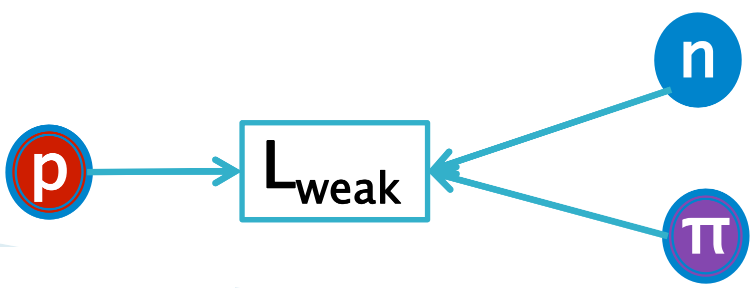
nuclear structure

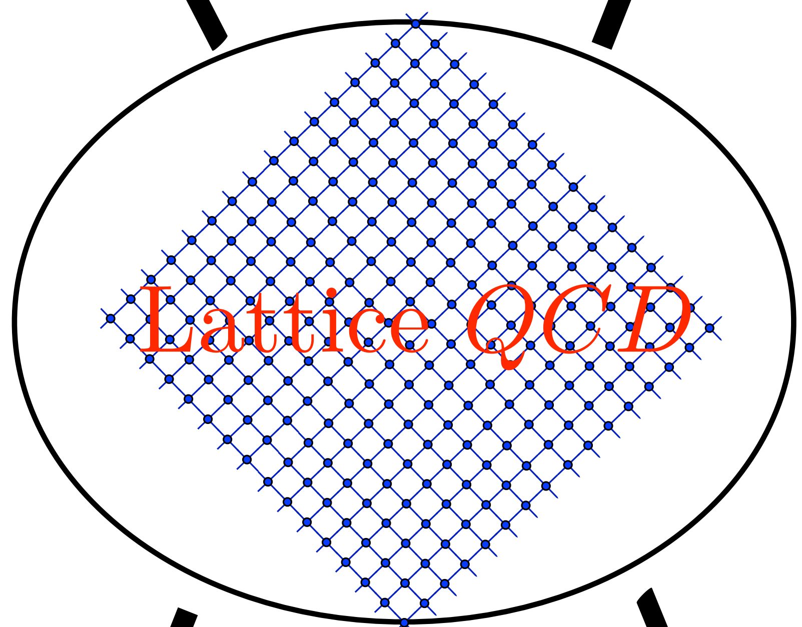
nucleon structure

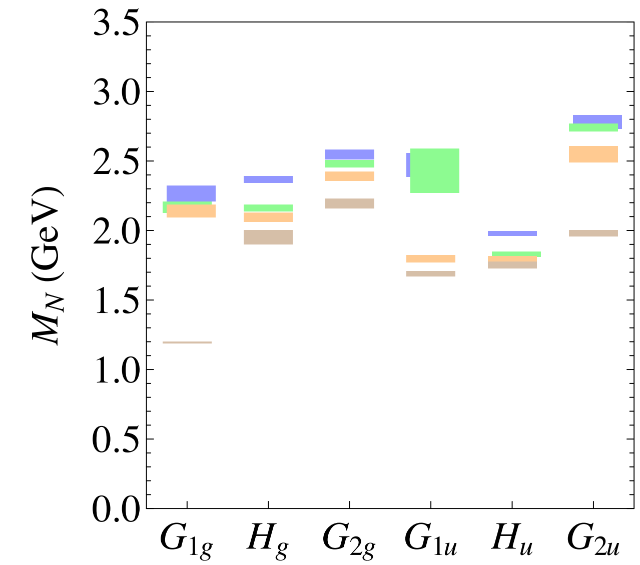
$e.g.\ nnn$

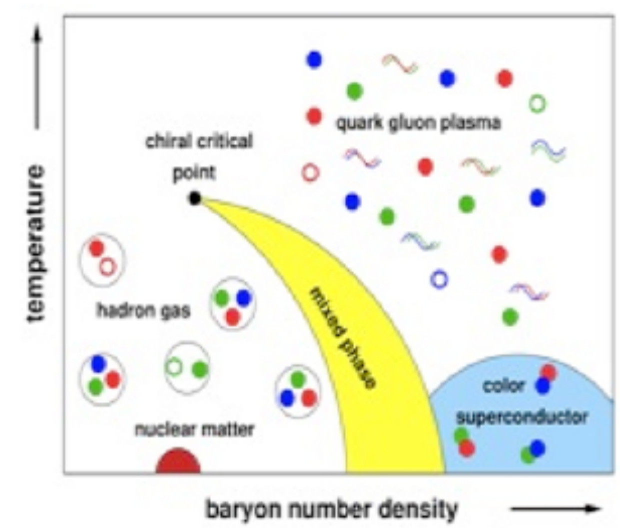
$e.g.$ excited baryons











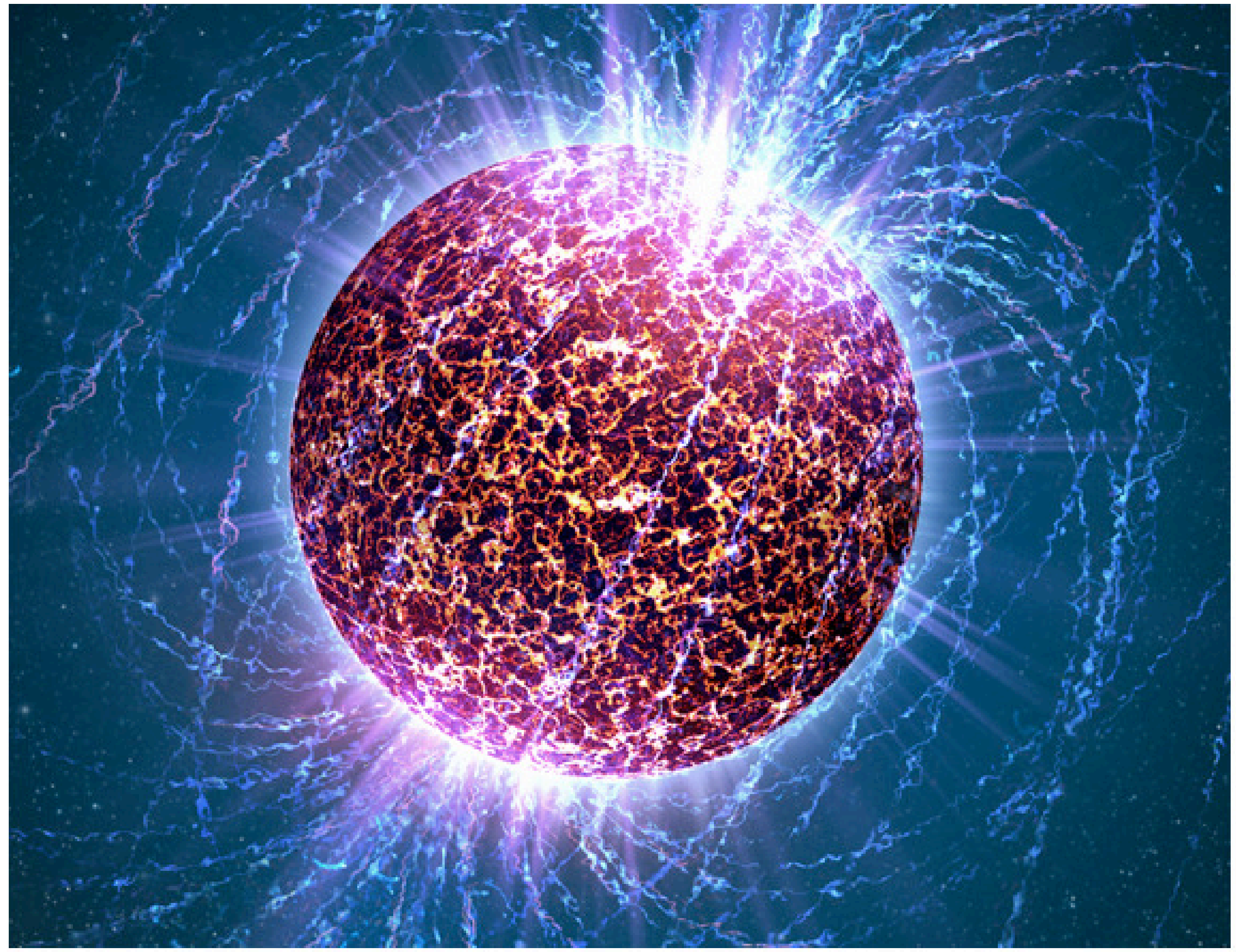
$e.g.\ h_{\pi NN}$

$e.g.$ critical point

precision electroweak

equation of state

NEOS and the fate of dense astrophysical objects

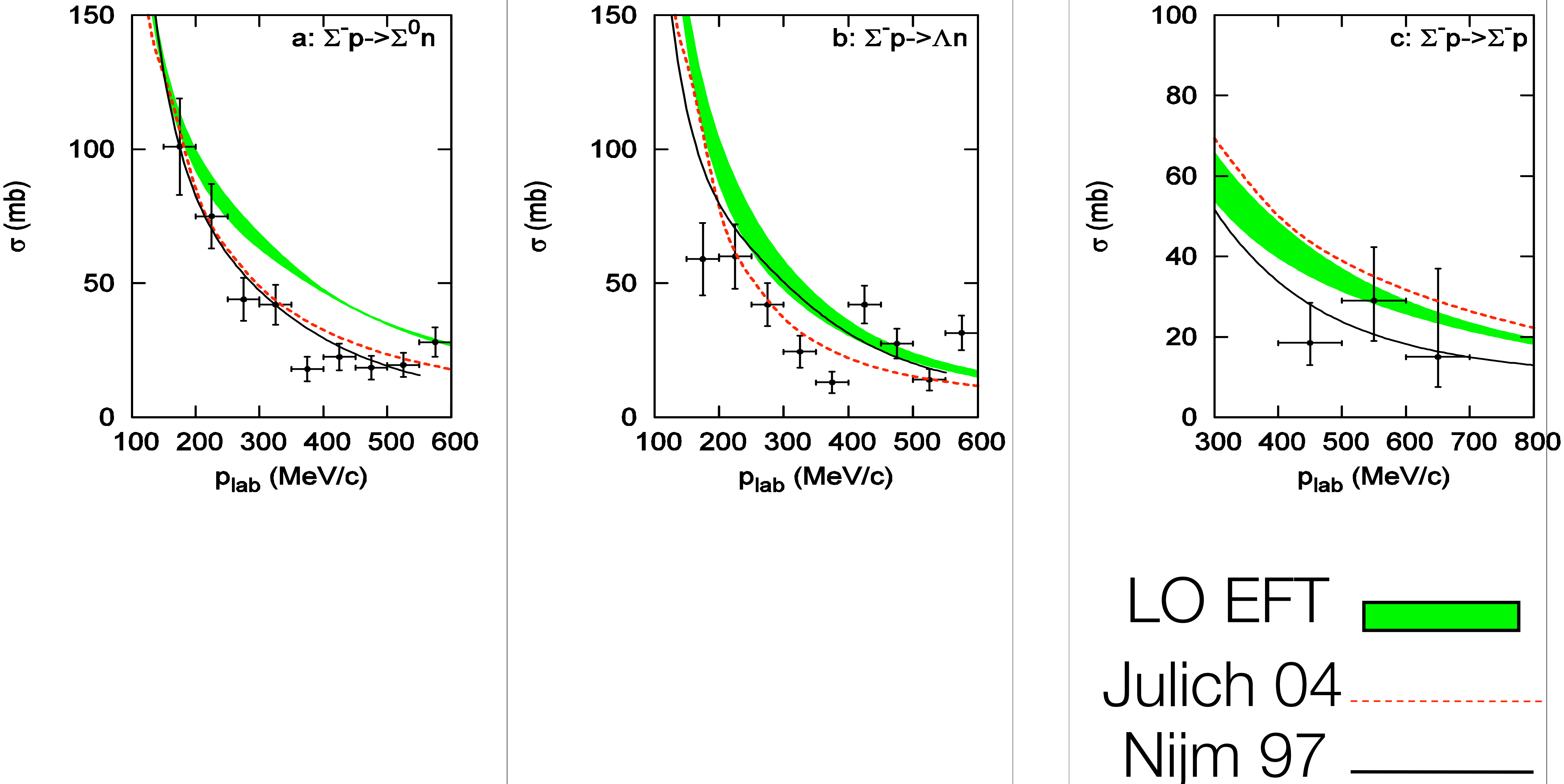


Extreme conditions $\longrightarrow$ No experiments

$$\text{Example}: p\Sigma^- \text{ poorly known}$$

What are the hyperon-nucleon scattering parameters?

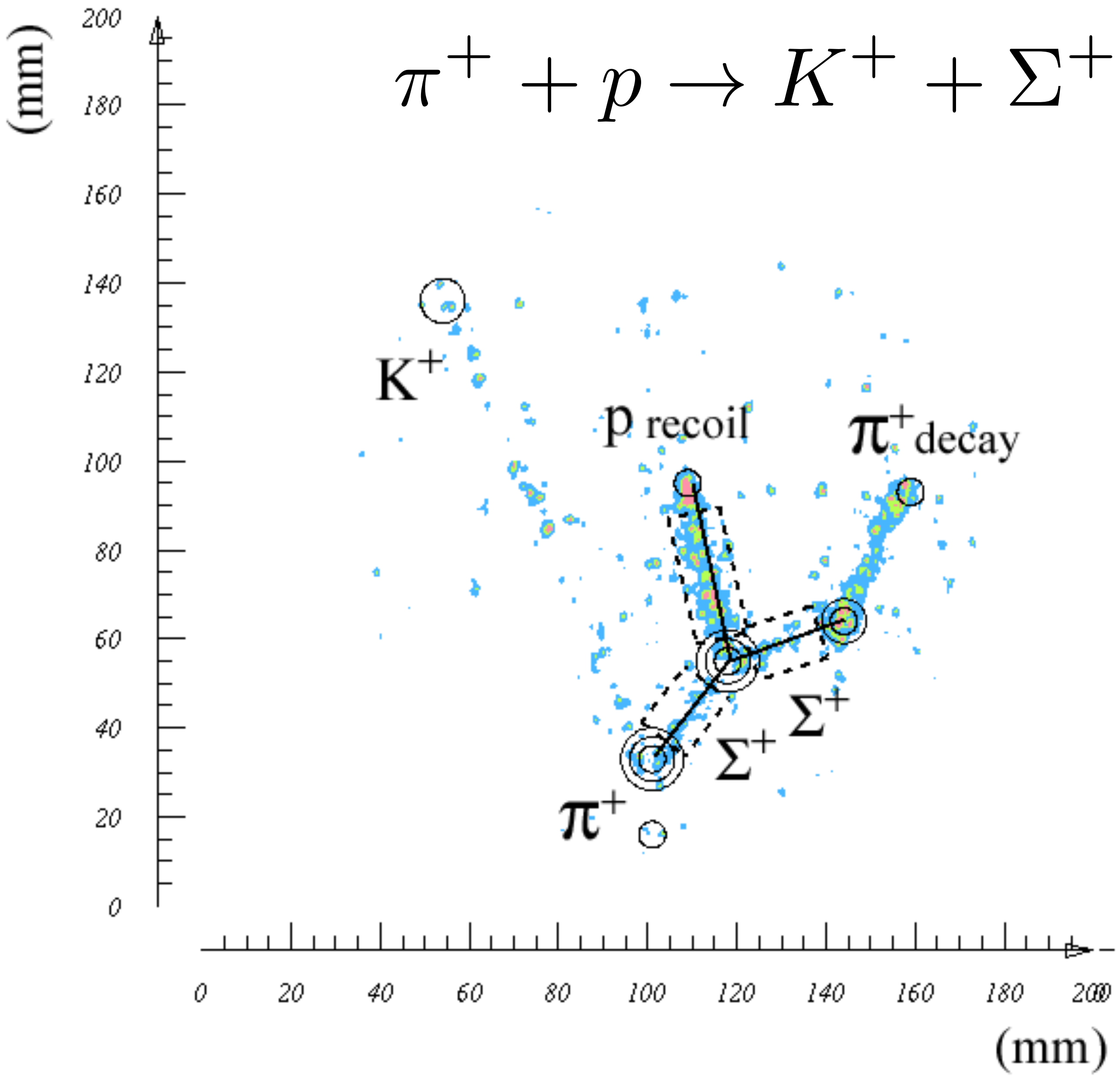
State-of-the-art

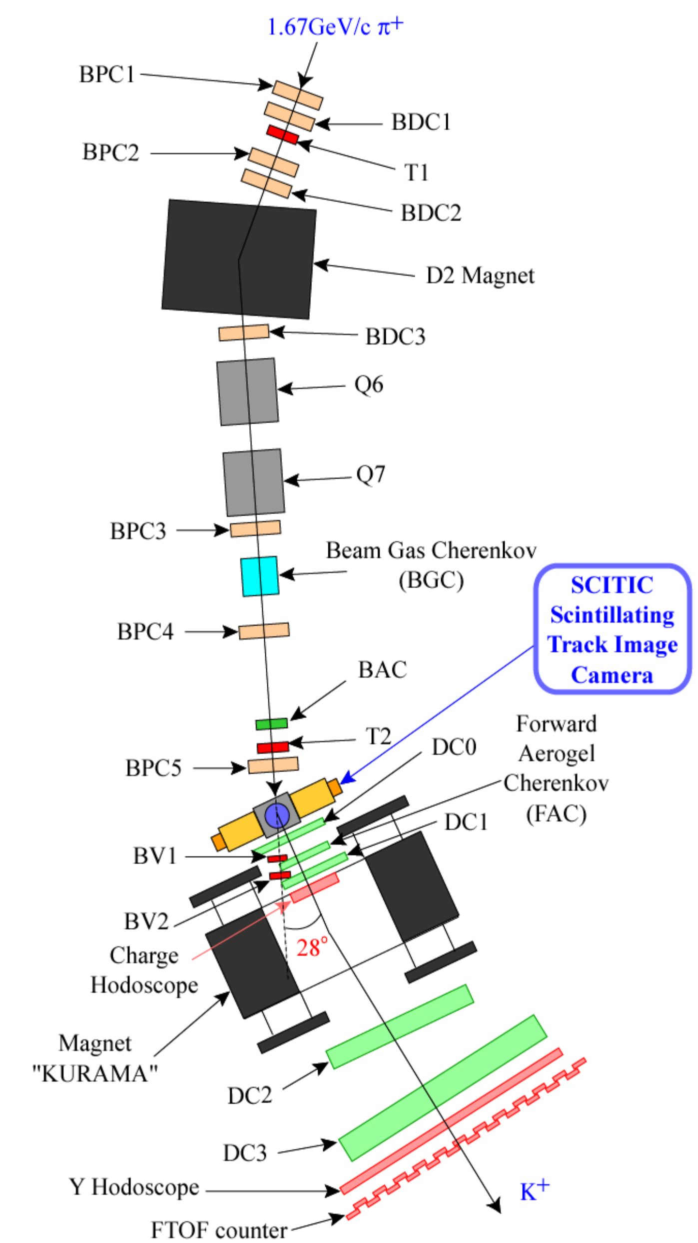


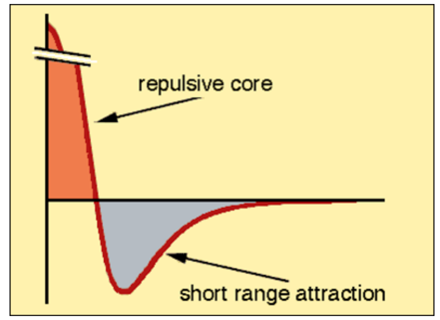
YN interactions from experiment?

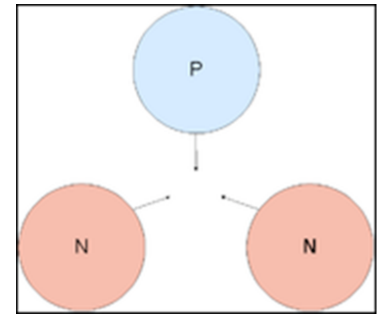
Kozi Nakai (KEK)

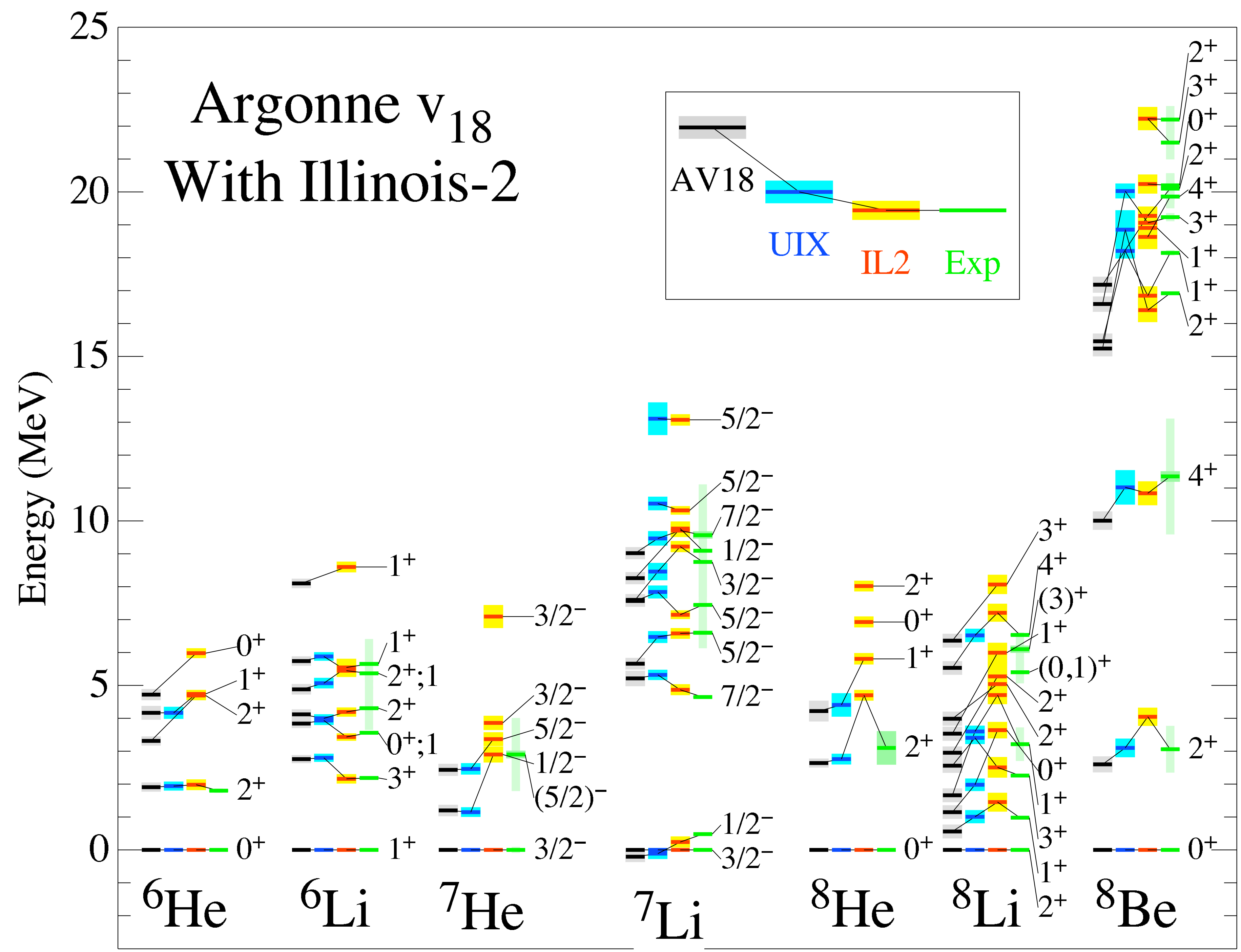
$$\pi^+ + p \to K^+ + \Sigma^+$$



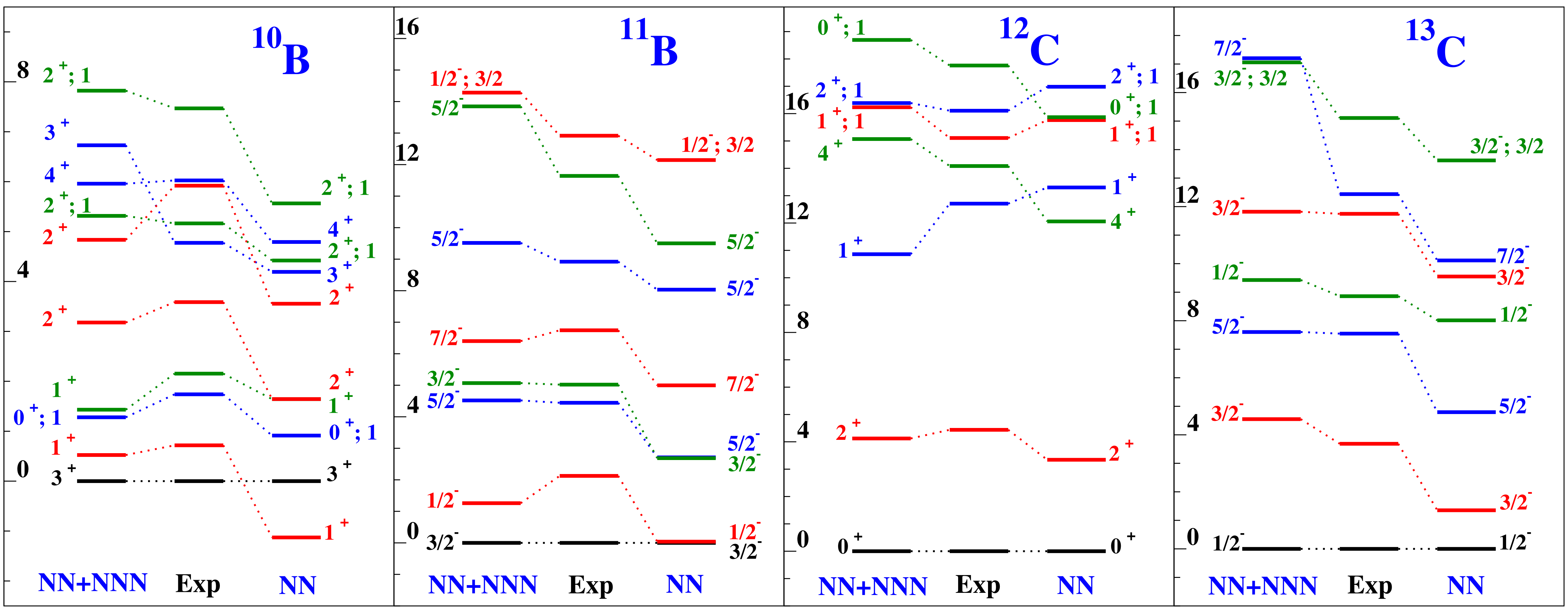



repulsive core
short range attraction


P
N
N

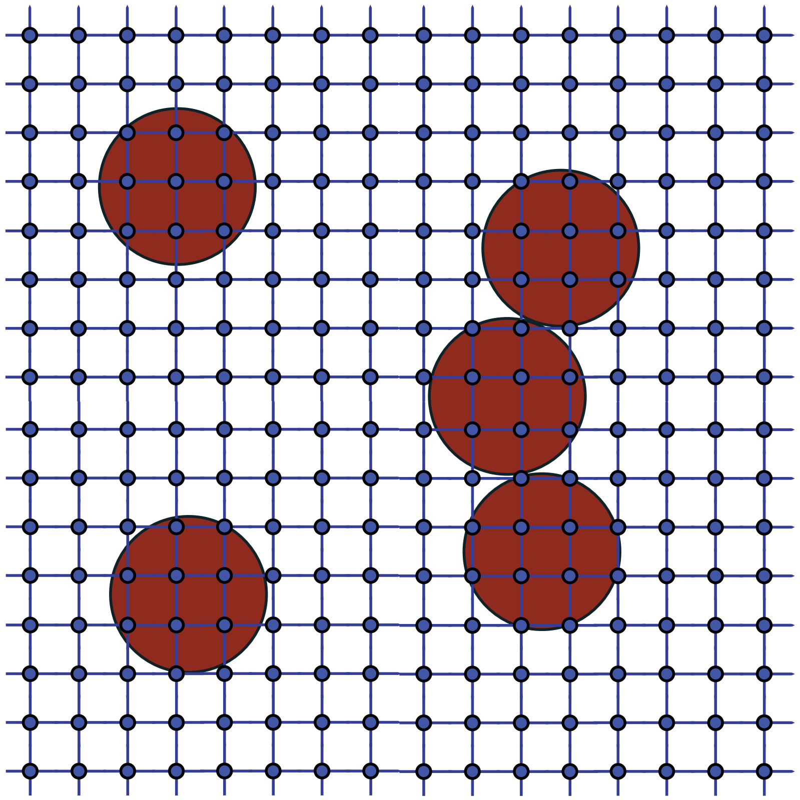

Argonne v_{18}
With Illinois-2
Energy (MeV)
25
20
15
10
5
0
AV18
UIX
IL2
Exp
^{6}He
^{6}Li
^{7}He
^{7}Li
^{8}He
^{8}Li
^{8}Be

NCSM



^{10}B
8
4
0
2+; 1
3+
4+
2+; 1
2+
2+
1+
0+; 1
1+
3+
2+; 1
4+
2+; 1
3+
2+
2+
1+
0+; 1
3+
1+
NN+NNN
Exp
NN
^{11}B
16
12
8
4
0
1/2-; 3/2
5/2-
5/2-
7/2-
3/2-
5/2-
1/2-
3/2-
1/2-; 3/2
5/2-
5/2-
7/2-
5/2-
3/2-
1/2-
3/2-
NN+NNN
Exp
NN
^{12}C
16
12
8
4
0
0+; 1
2+; 1
1+; 1
4+
1+
2+
0+
2+; 1
0+; 1
1+; 1
1+
4+
2+
0+
NN+NNN
Exp
NN
^{13}C
16
12
8
4
0
7/2-
3/2-; 3/2
3/2-
1/2-
5/2-
3/2-
1/2-
3/2-; 3/2
7/2-
3/2-
1/2-
5/2-
3/2-
1/2-
NN+NNN
Exp
NN

Lattice QCD: Multi-baryon interactions



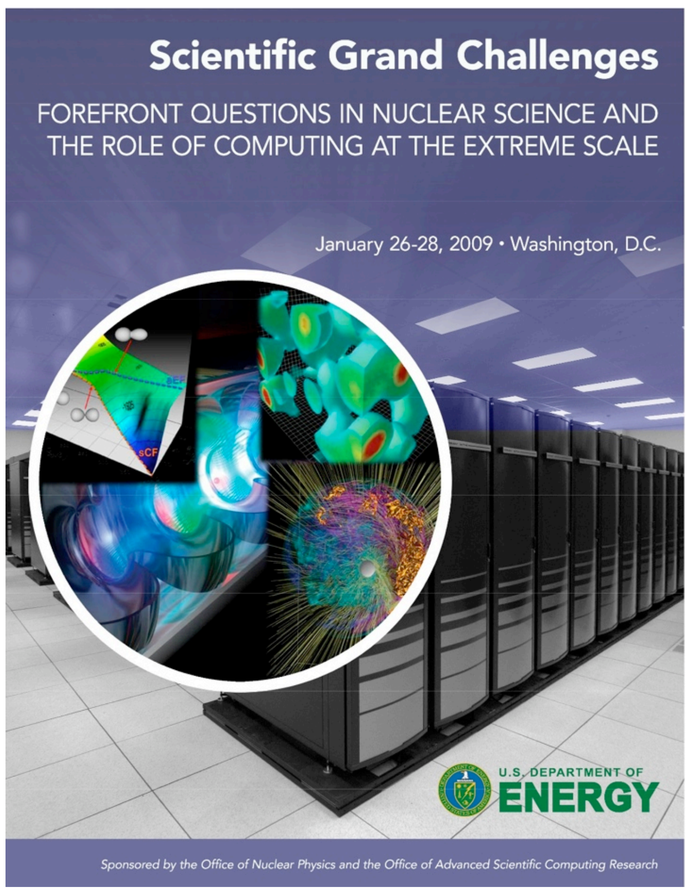
Three- and higher-body interactions are poorly known and yet dramatically impact the properties of nuclei

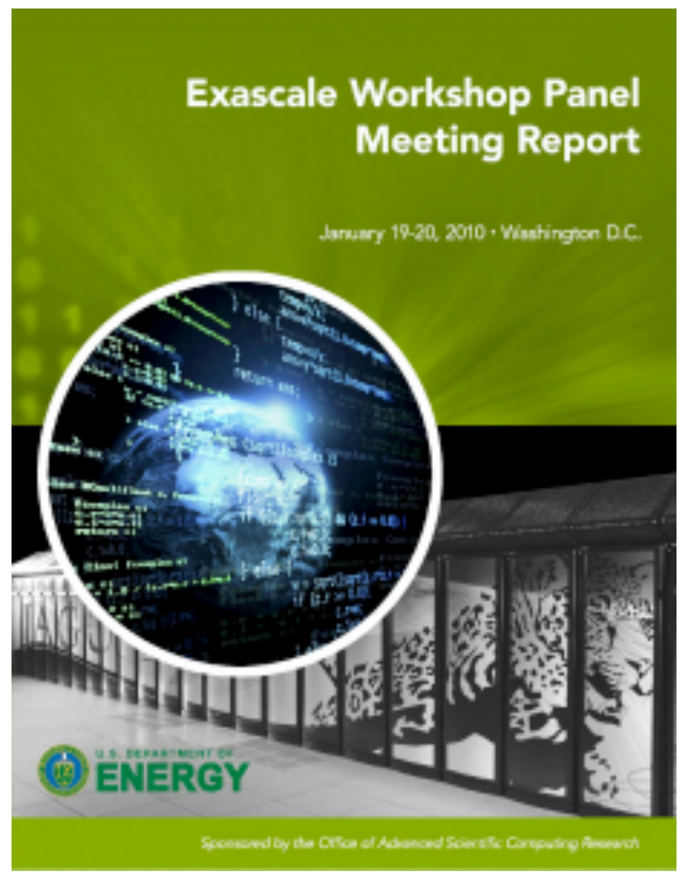
E.g., significant role in/effect on:

- "spin-orbit" properties of the nucleus
- stability of borromean nuclei (e.g. ^{6,8}He, ^{9}Be, ^{8}Li)
- scattering processes, *etc.*

How do nuclei emerge from QCD?

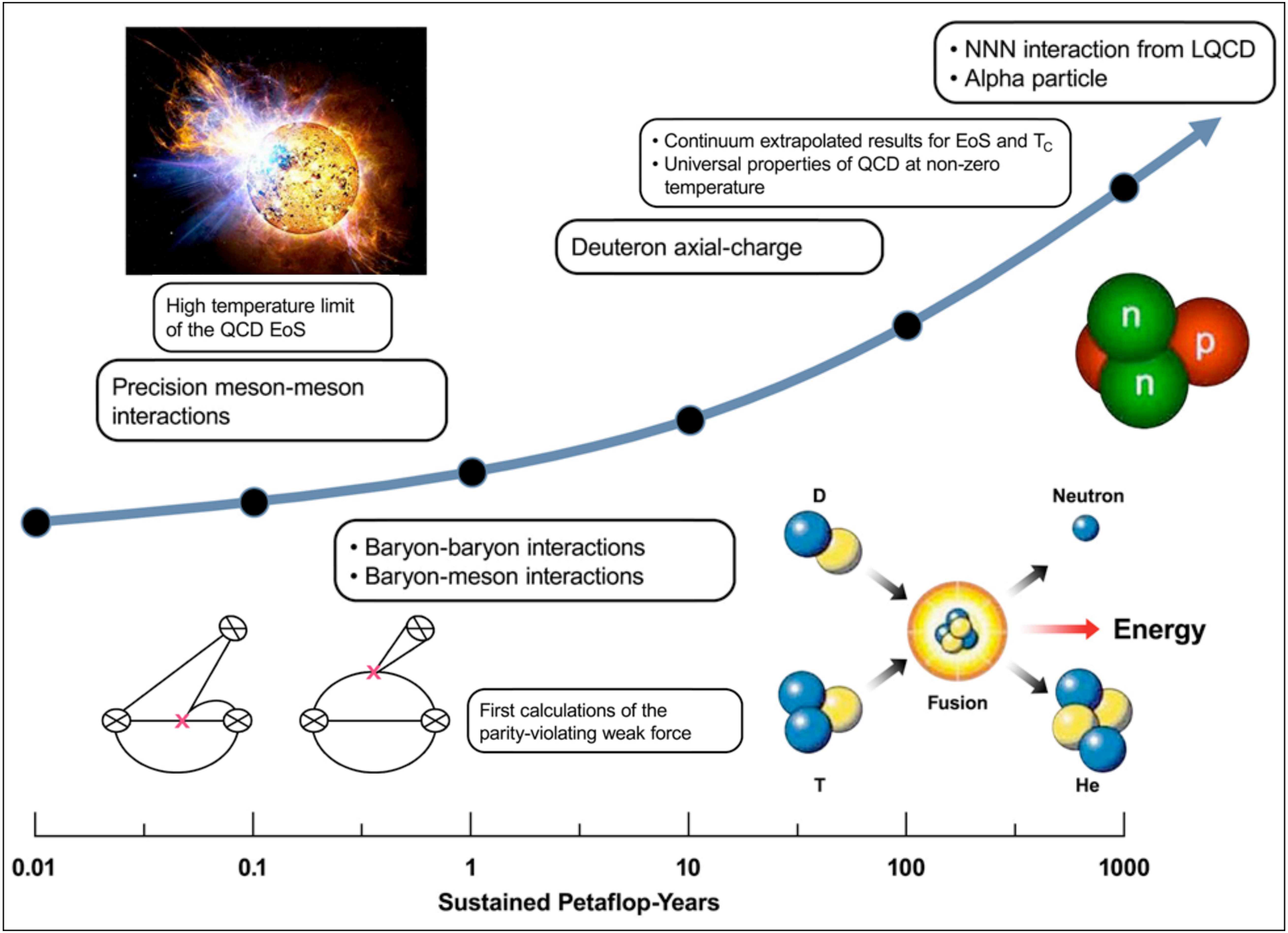
$$1 \text{ Exaflop} = 10^3 \text{Petaflops} = 10^6 \text{Teraflops} = 10^9 \text{Gigaflops}$$





(Trivelpiece committee)

- Architectures and Technology
- Biology
- Basic Energy Sciences
- Climate Science
- Cross-Cutting Workshop
- Fusion Energy
- High Energy Physics
- National Security
- Nuclear Energy
- Nuclear Physics



• NNN interaction from LQCD
• Alpha particle
• Continuum extrapolated results for EoS and T_C
• Universal properties of QCD at non-zero temperature
Deuteron axial-charge
High temperature limit of the QCD EoS
Precision meson-meson interactions
n
p
n
• Baryon-baryon interactions
• Baryon-meson interactions
First calculations of the parity-violating weak force
D
T
Fusion
Neutron
Energy
He
0.01
0.1
1
10
100
1000
Sustained Petaflop-Years

How do we extract s-wave scattering information (phase shifts and binding energies) from a lattice calculation?

Finite Volume

$$p \cot\delta(p) = \frac{1}{\pi L}\,\mathcal{S}_3\left(\frac{pL}{2\pi}\right) \qquad \mathcal{S}_3(\eta) \equiv \sum_{\mathbf{n}}^{\Lambda_n} \frac{1}{\mathbf{n}^2 - \eta^2} - 4\pi\Lambda_n$$

$$+\,\mathcal{O}\left(e^{-M_\pi L}\right)$$

Weak coupling expansion:

$$\Delta E_0(2,L) = \frac{4\pi\, a_{\pi\pi}}{m_\pi L^3}\left\{1 - \left(\frac{a_{\pi\pi}}{\pi L}\right)\mathcal{I} + \left(\frac{a_{\pi\pi}}{\pi L}\right)^2\left[\mathcal{I}^2 - \mathcal{J}\right] + \left(\frac{a_{\pi\pi}}{\pi L}\right)^3\left[-\mathcal{I}^3 + 3\mathcal{I}\mathcal{J} - \mathcal{K}\right]\right\} + \frac{8\pi^2 a_{\pi\pi}{}^3}{m_\pi L^6}\, r_{\pi\pi} + \mathcal{O}\left(L^{-7}\right)$$

Calculated on the lattice!

phase shift

$$\mathcal{I} = \lim_{\Lambda_j\to\infty} \sum_{\mathbf{i}\neq\mathbf{0}}^{|\mathbf{i}|\leq\Lambda_j} \frac{1}{|\mathbf{i}|^2} - 4\pi\Lambda_j = -8.91363291781$$

$$\mathcal{J} = \sum_{\mathbf{i}\neq\mathbf{0}} \frac{1}{|\mathbf{i}|^4} = 16.532315959$$

$$\mathcal{K} = \sum_{\mathbf{i}\neq\mathbf{0}} \frac{1}{|\mathbf{i}|^6} = 8.401923974433$$

What about bound states?

$$\boxed{\mathcal{A}_2(p) = \frac{8\pi}{M}\frac{1}{p\cot\delta(p) - ip} \longrightarrow \cot\delta(i\gamma) = i}$$

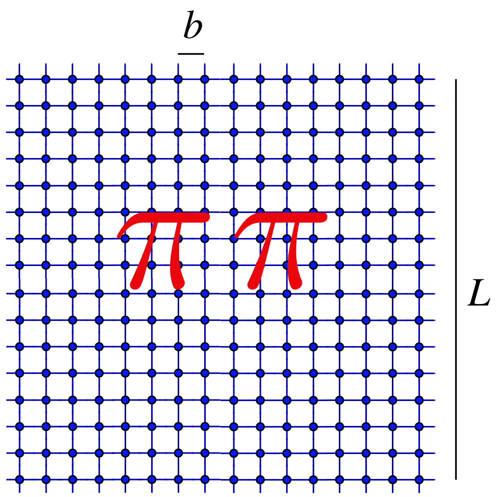
Finite-V:

$$\cot\delta(i\kappa) = i \; - \; i\sum_{\mathbf{m}\neq 0}\frac{e^{-|\mathbf{m}|\kappa L}}{|\mathbf{m}|\kappa L}$$

$$\boxed{\kappa = \gamma \; + \; \frac{6}{L}\frac{e^{-\gamma L}}{1 \; - \; \gamma r_3} \; + \; \mathcal{O}(e^{-\sqrt{2}\gamma L})}$$

Need several volumes!

$\pi\pi$ scattering in lattice QCD



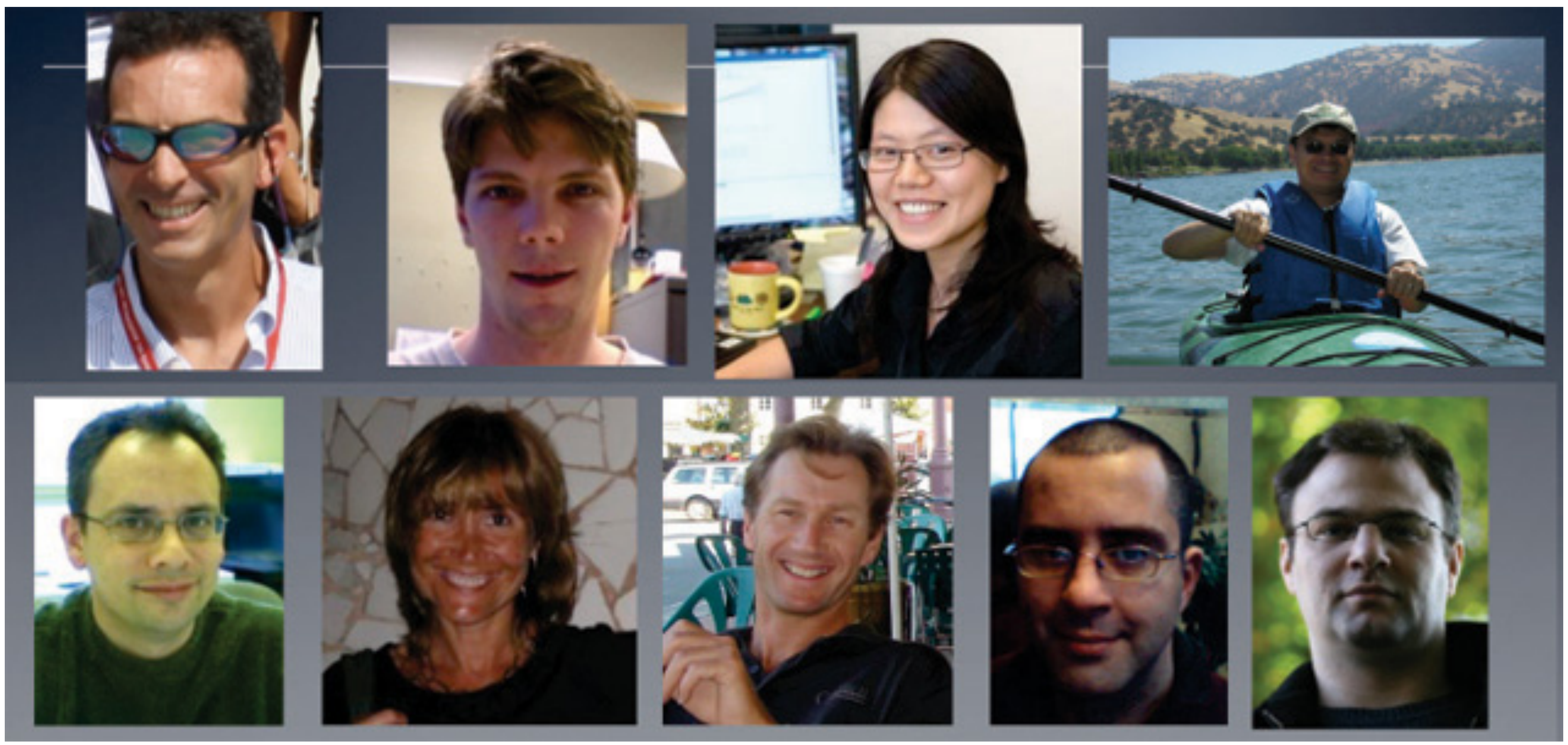
$$\mathcal{O}_{\pi^+}(t,\vec{x}) \;=\; \overline{u}(t,\vec{x})\gamma_5 d(t,\vec{x})$$

$$C_{\pi^+\pi^+}(p,t) \quad = \quad \langle 0| \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{O}_{\pi^-}(t,\mathbf{x})\ \mathcal{O}_{\pi^-}(t,\mathbf{y})\ \mathcal{O}_{\pi^+}(0,\mathbf{0})\ \mathcal{O}_{\pi^+}(0,\mathbf{0})|0\rangle$$

$$\boxed{\frac{C_{\pi^+\pi^+}(p,t)}{C_{\pi^+}(t)C_{\pi^+}(t)} \quad \rightarrow \sum_{n=0}^{\infty} \mathcal{A}_n\ e^{-\Delta E_n(2,L)\ t}}$$

$$\Delta E_n(2,L) \;\equiv\; 2\sqrt{\ \vec{p}_n^{\,2} \;+\; m_\pi^2\ } \;-\; 2m_\pi$$

NPLQCD Collaboration



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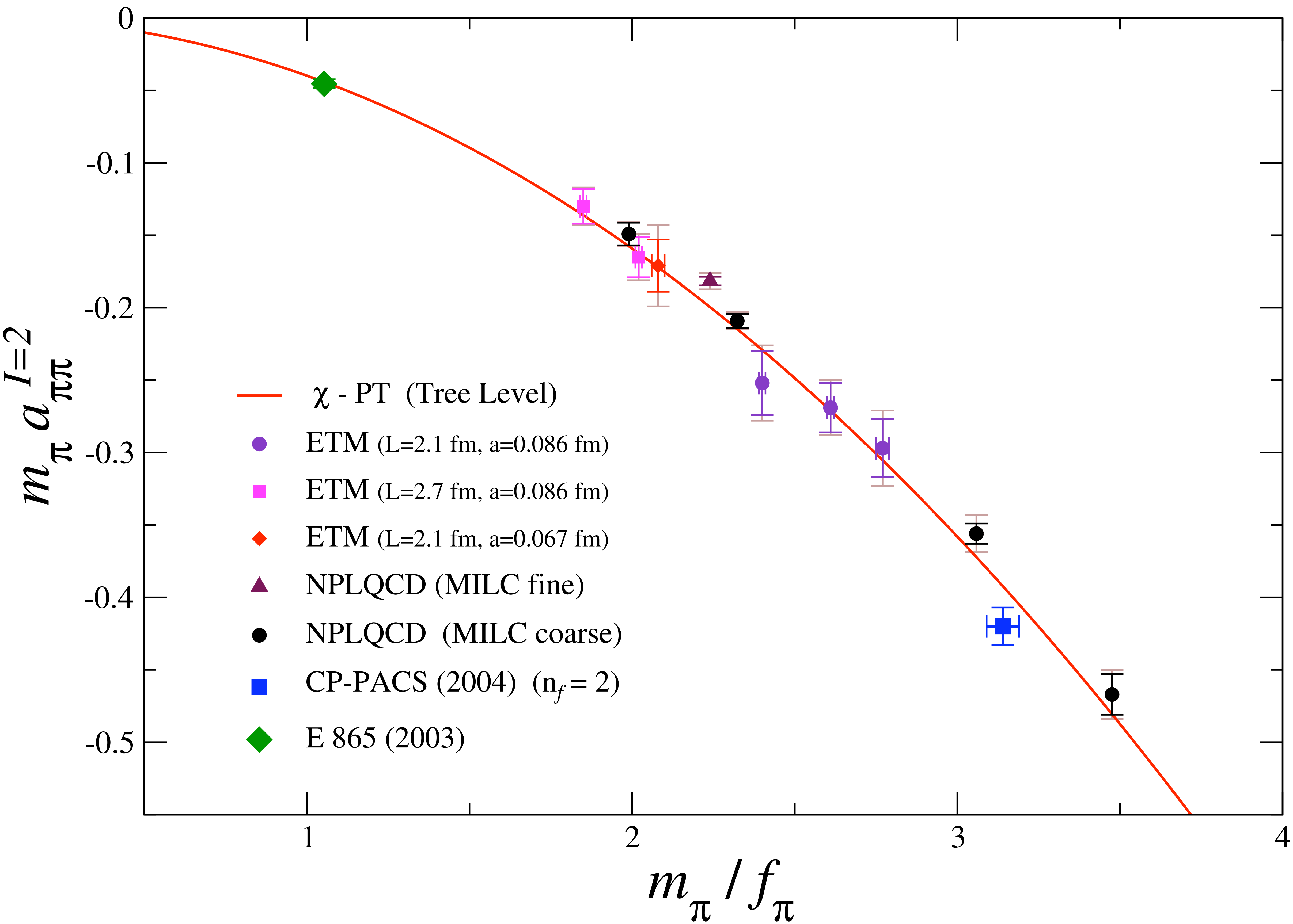
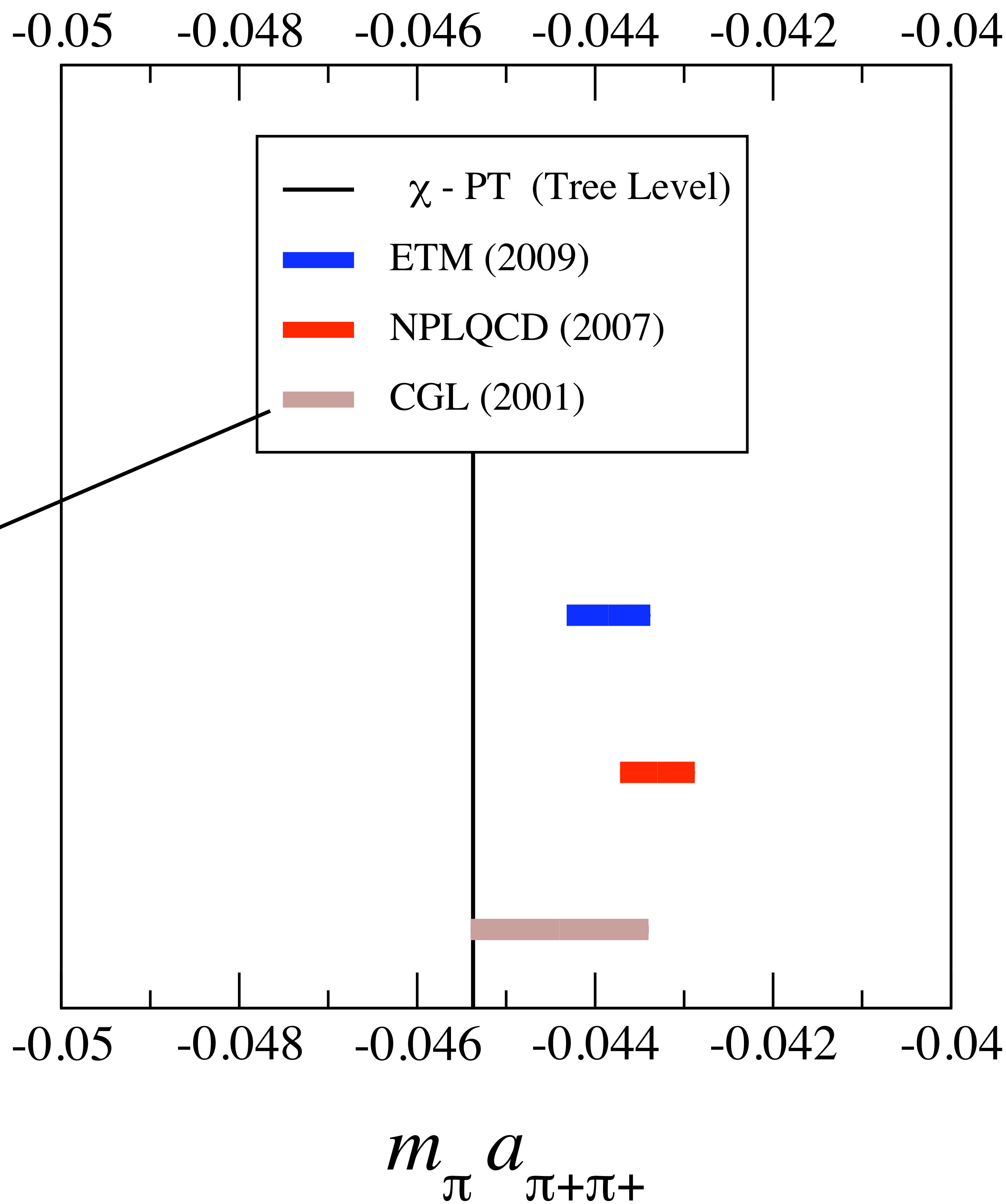
Benchmarking: $\pi^+\pi^+$ $(I=2)$



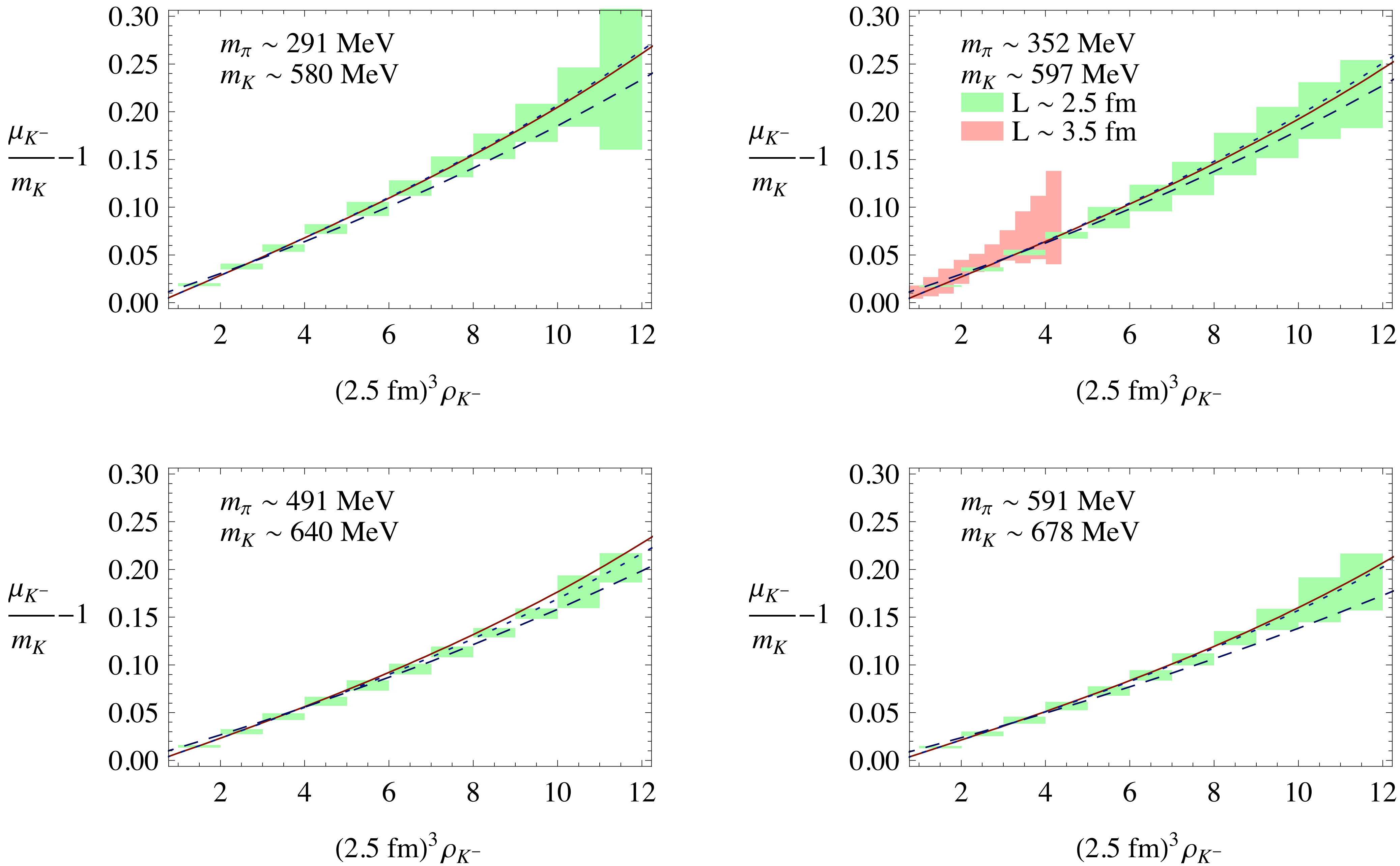


Roy equations

+

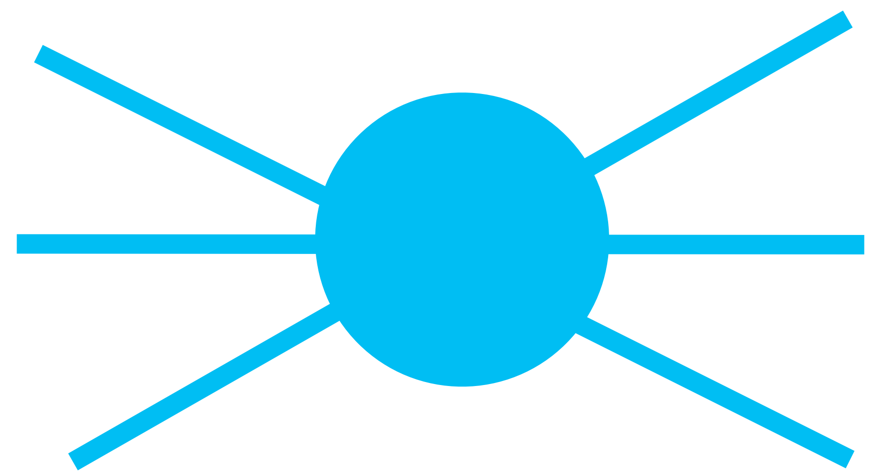
chiral symmetry

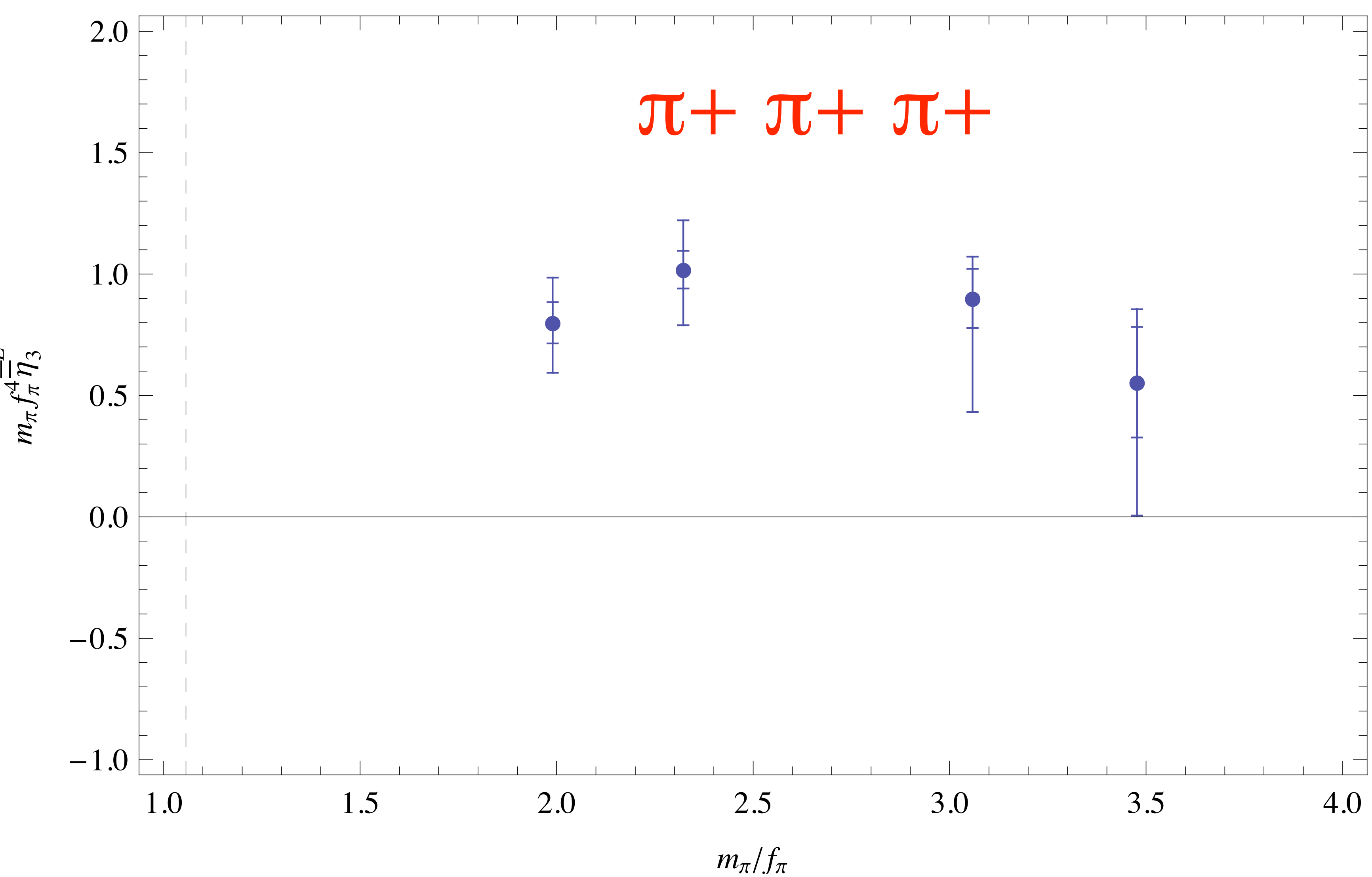
Many-Meson Physics



Bose-Einstein condensates of mesons!

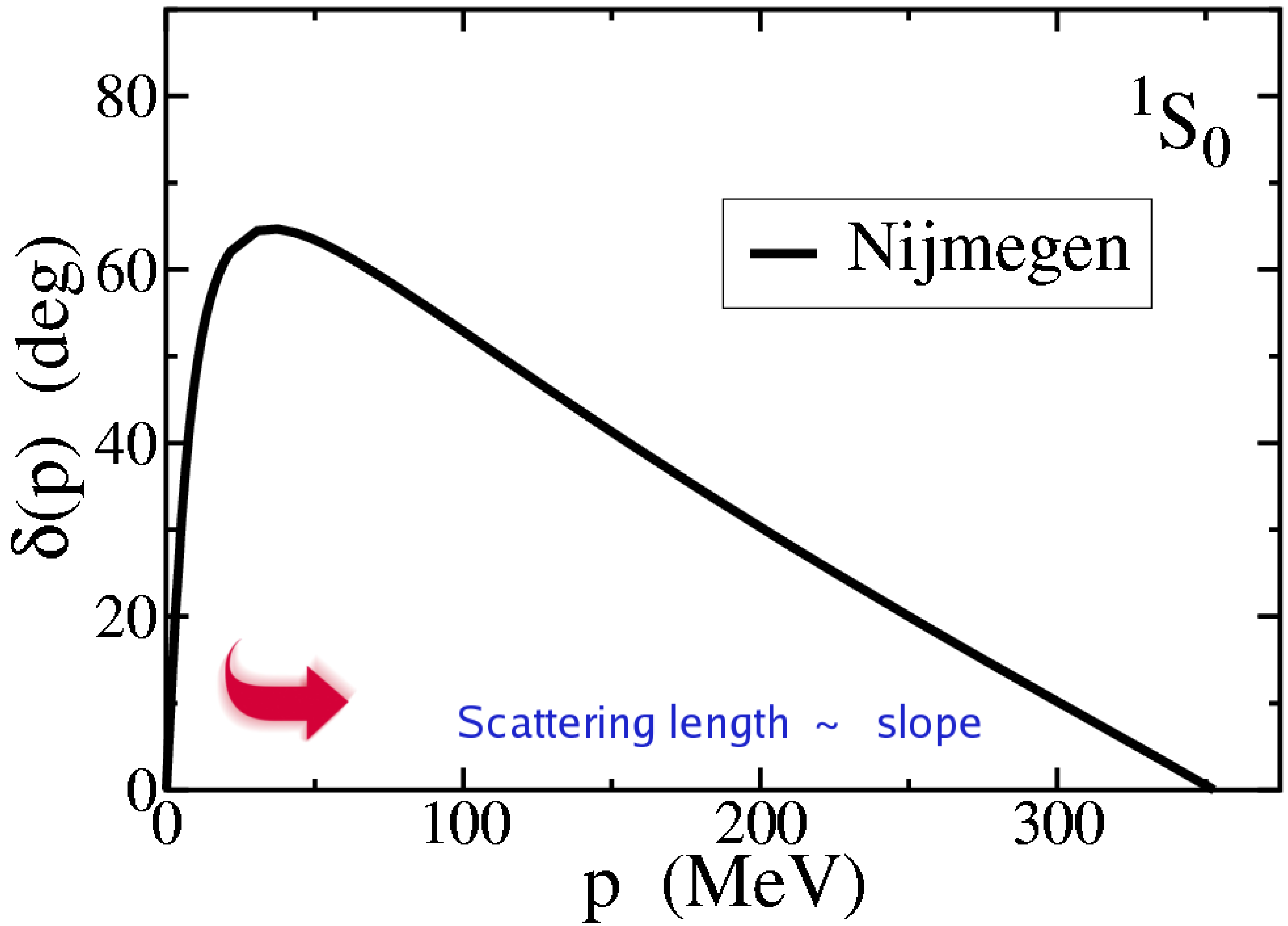
Pion 3-Body Interaction





Why is nuclear physics special?

Consider neutron-proton scattering in the $^1S_0$ channel



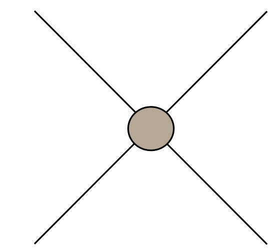
$$a_s^{^1S_0} \simeq -23 \text{ fm} \simeq \frac{1}{8 \text{ MeV}}$$

Phase shift varies over $\Delta p \sim 8$ MeV: NO Taylor expansion in $\frac{p}{m_\pi}$!

Benchmarking: NN

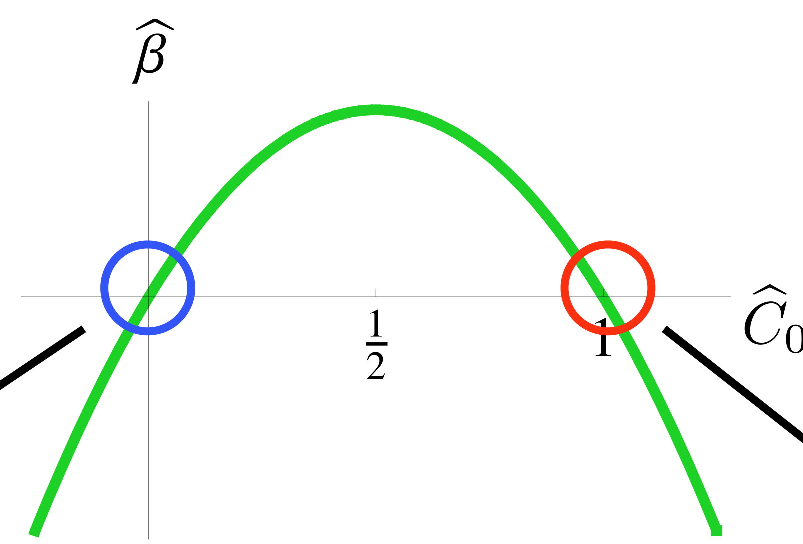
Experiment:

$$a_s^{^1S_0} = -23.714 \text{ fm} \qquad r_s^{^1S_0} = 2.73 \text{ fm}$$
$$a_s^{^3S_1} = 5.425 \text{ fm} \qquad r_s^{^3S_1} = 1.749 \text{ fm}$$



$$a_s \gg \Lambda_{QCD}^{-1}$$

$$\hat{\beta}_0 = \mu \frac{d}{d\mu} \hat{C}_0 = -\hat{C}_0(\hat{C}_0 - 1)$$

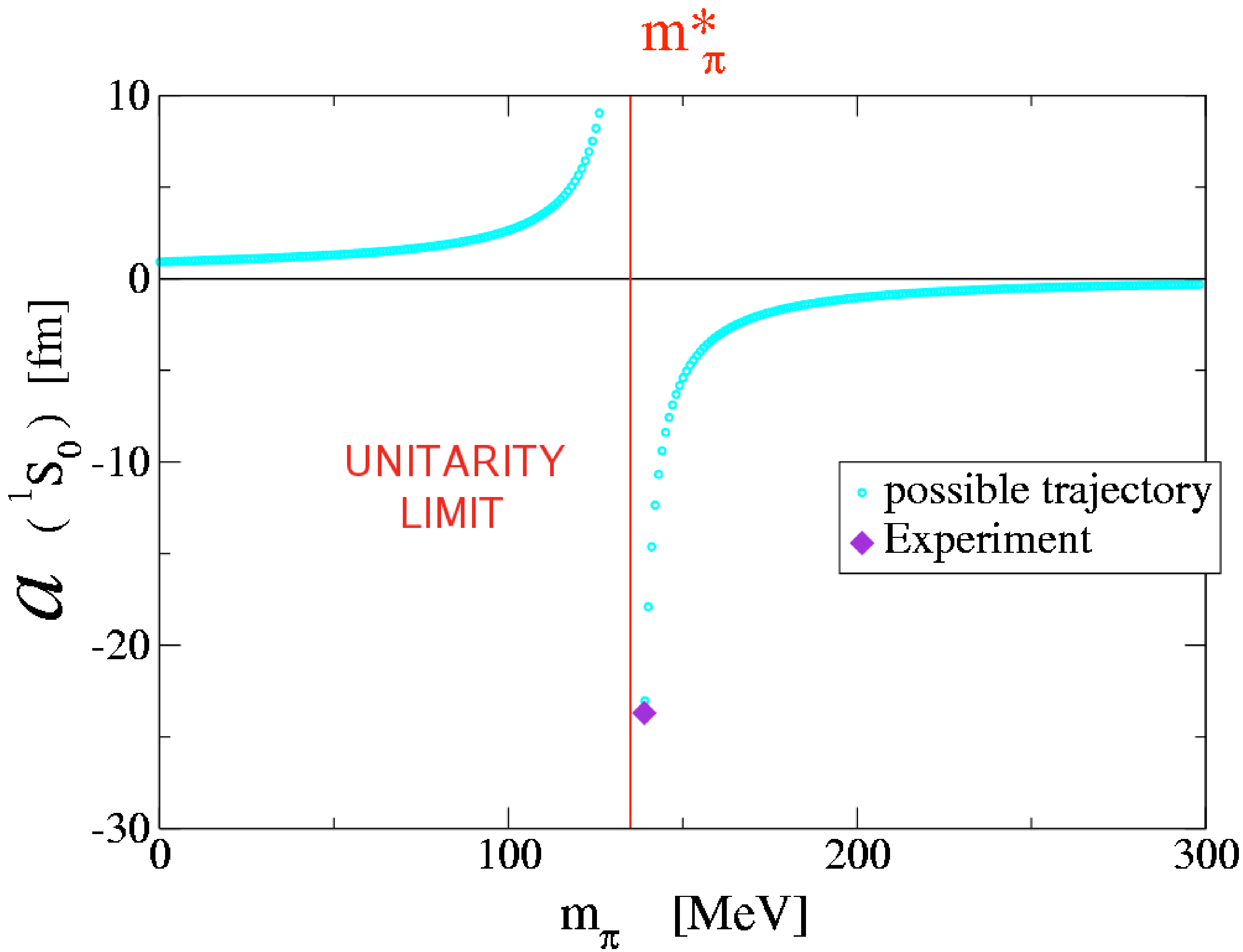


Trivial IR fixed point: "natural case"

Nontrivial UV fixed point: "unnatural case"

"Unitarity"

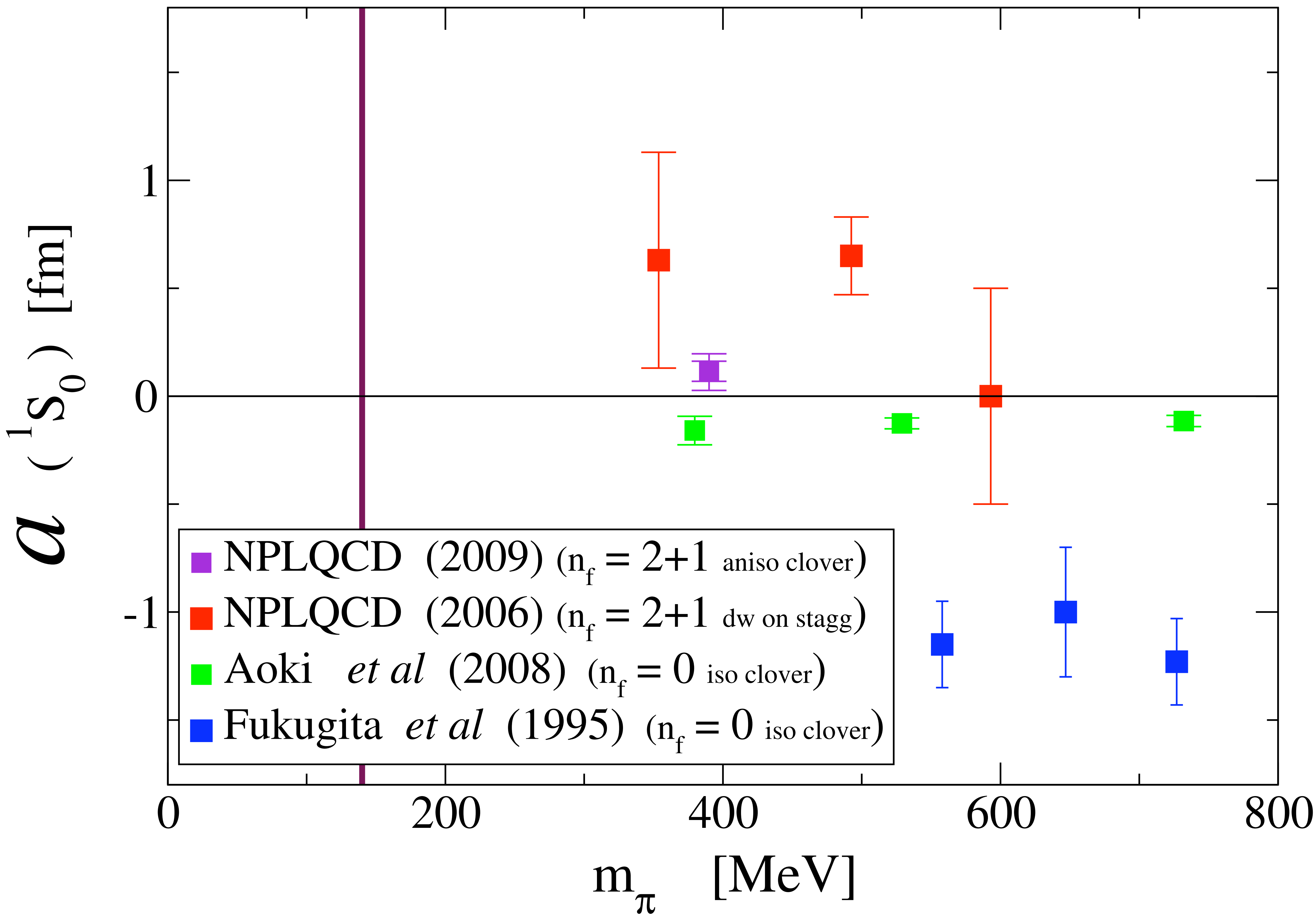
Why is nuclear physics near this UV fixed point??

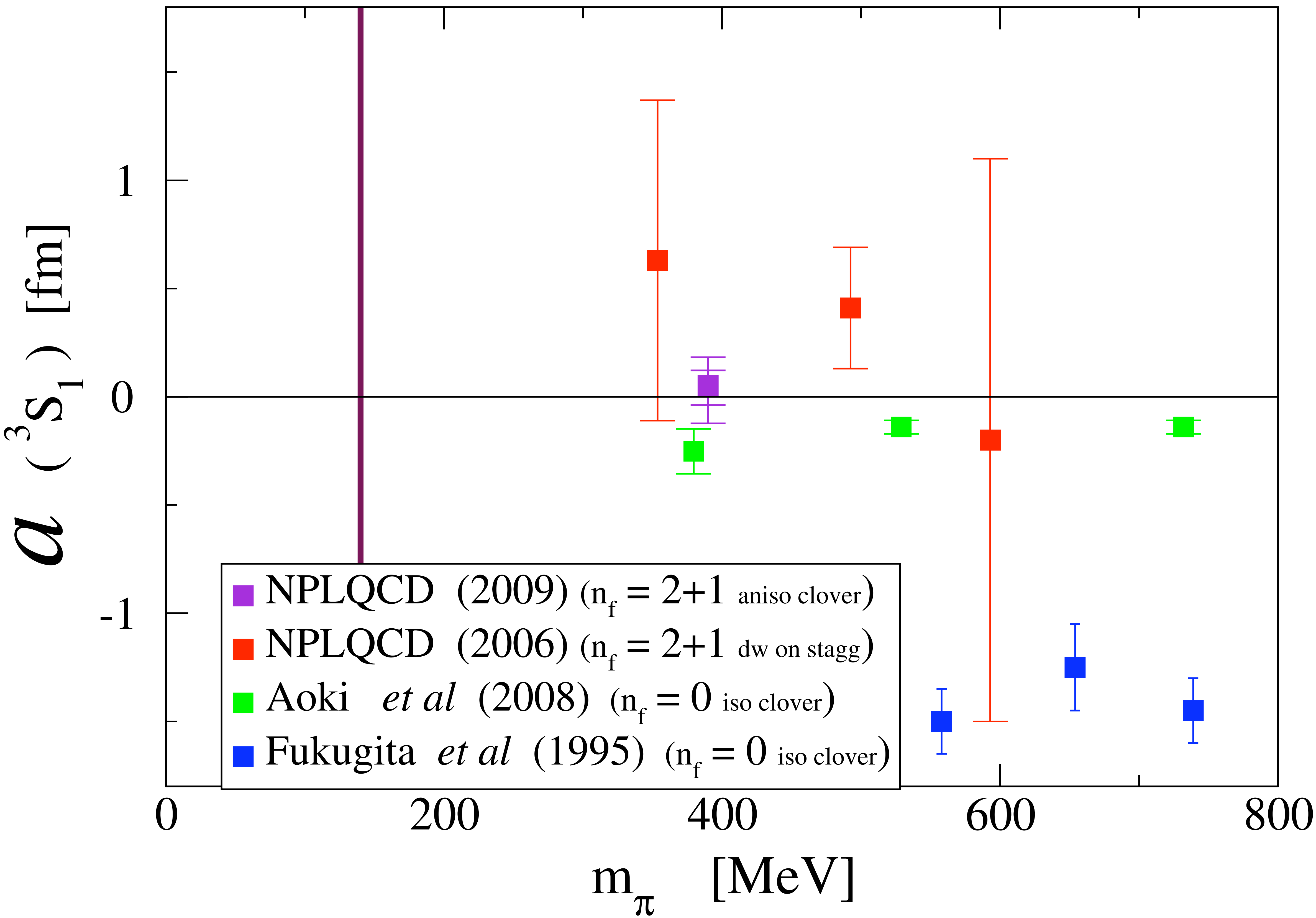


$$a_s^{-1} \sim \frac{m_\pi - m_\pi^*}{m_\pi} \Lambda_{QCD}$$

Lattice QCD will answer this question!

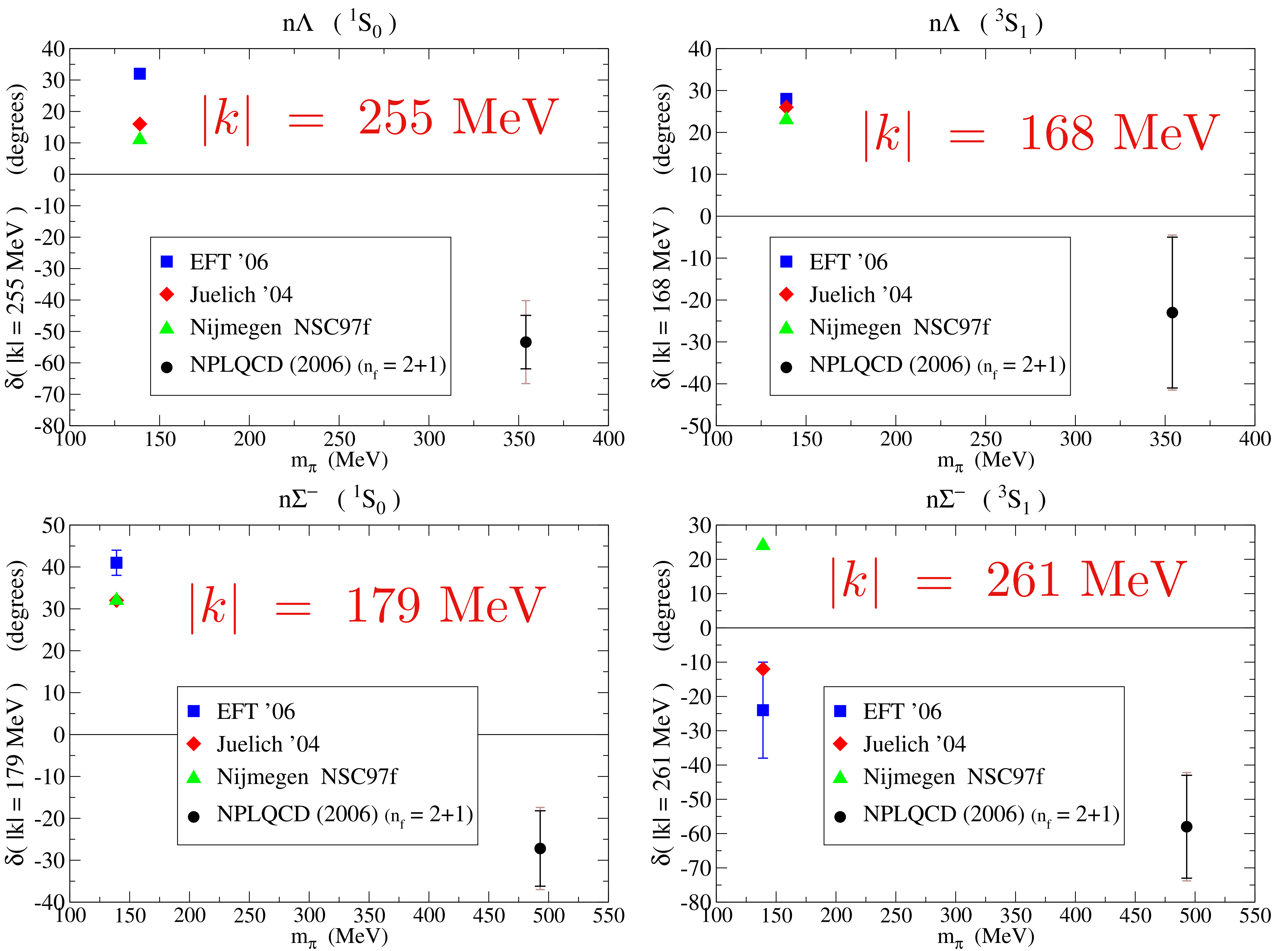
Lattice QCD: NN





NPLQCD (2009) (n_f = 2+1 aniso clover)
NPLQCD (2006) (n_f = 2+1 dw on stagg)
Aoki *et al* (2008) (n_f = 0 iso clover)
Fukugita *et al* (1995) (n_f = 0 iso clover)
$a\ (^3S_1)$ [fm]
m_π [MeV]
1
0
-1
0
200
400
600
800

YN interactions



Does signal/noise decay exponentially?

Does signal/noise decay exponentially?

Yes!

For a system of A *nucleons:*

$$\boxed{\frac{\text{noise}}{\text{signal}} \xrightarrow[t\to\infty]{} \frac{1}{\sqrt{N}}\, e^{A\left(m_p - \frac{3}{2} m_\pi\right)t}}$$

Does signal/noise decay exponentially?

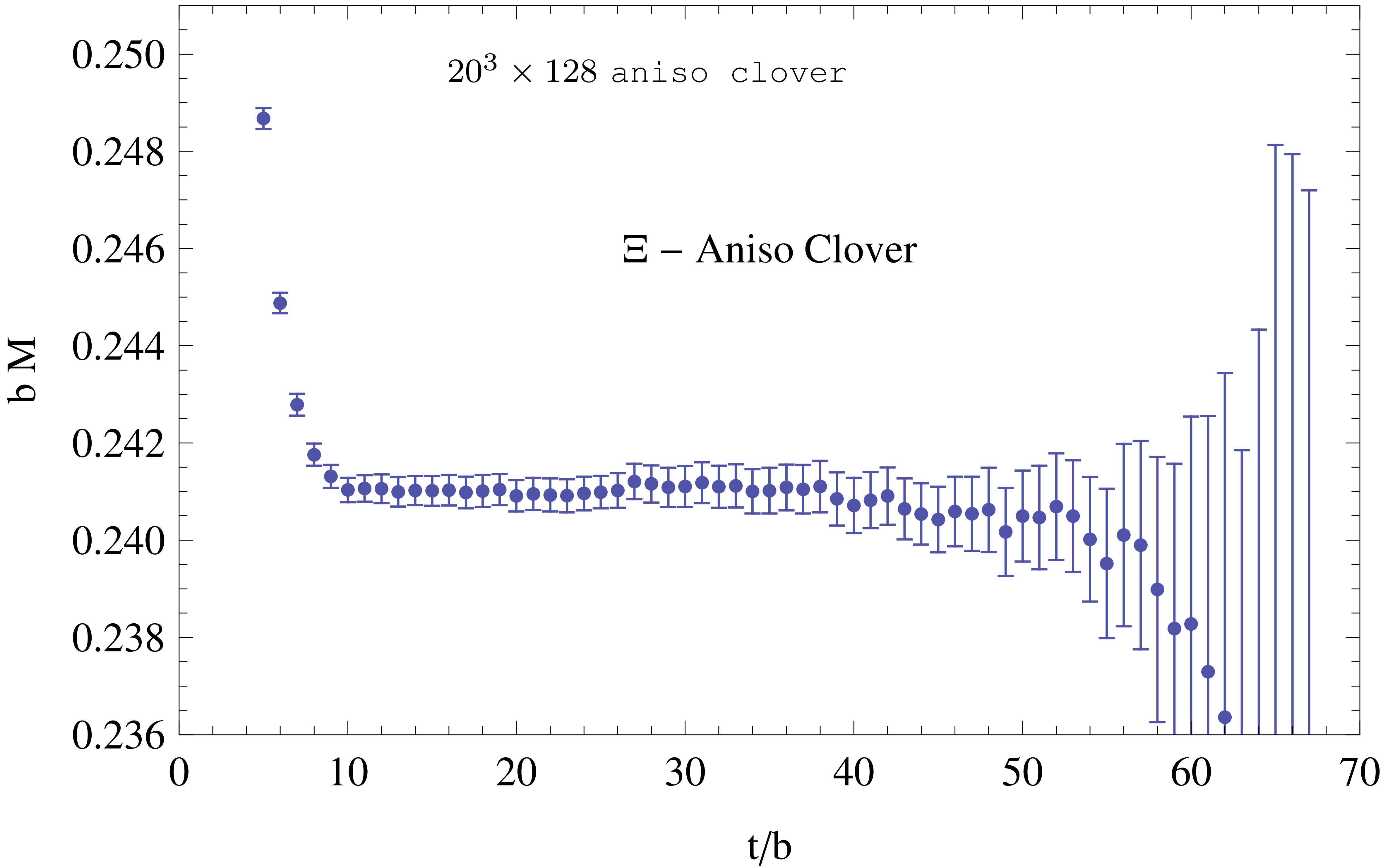
Yes!

For a system of A *nucleons:*

$$\boxed{\frac{\text{noise}}{\text{signal}} \xrightarrow[t \to \infty]{} \frac{1}{\sqrt{N}}\, e^{A\left(m_p - \frac{3}{2} m_\pi\right)t}}$$

However, only *asymptotically*!

Anisotropic clover lattices *with high statistics* NPLQCD (2009)

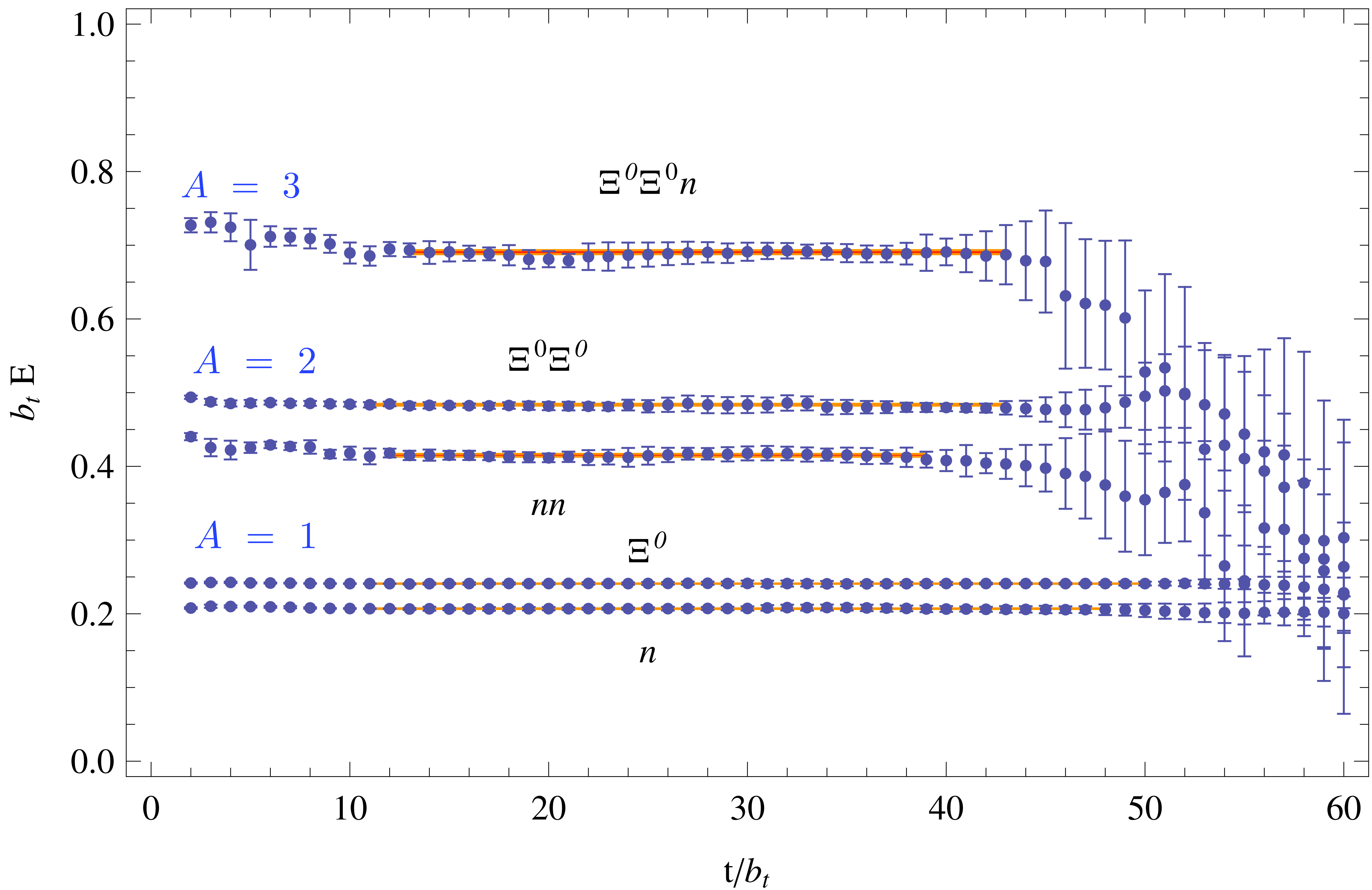


$$bM_{\Xi} = 0.24112 \pm 0.00021 \pm 0.00006$$

Is there a signal/noise problem?

Not anymore!

related to sign problem?



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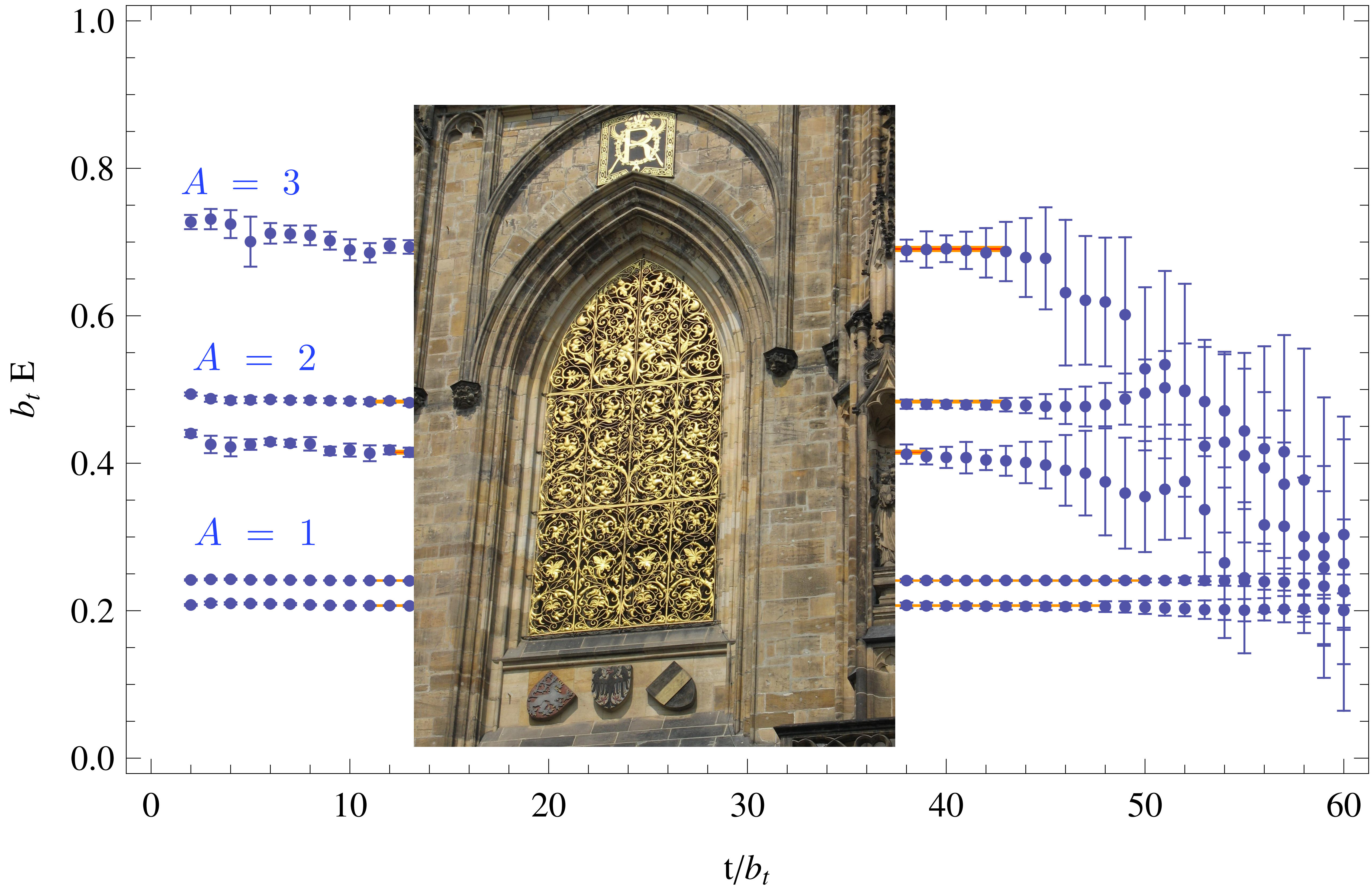
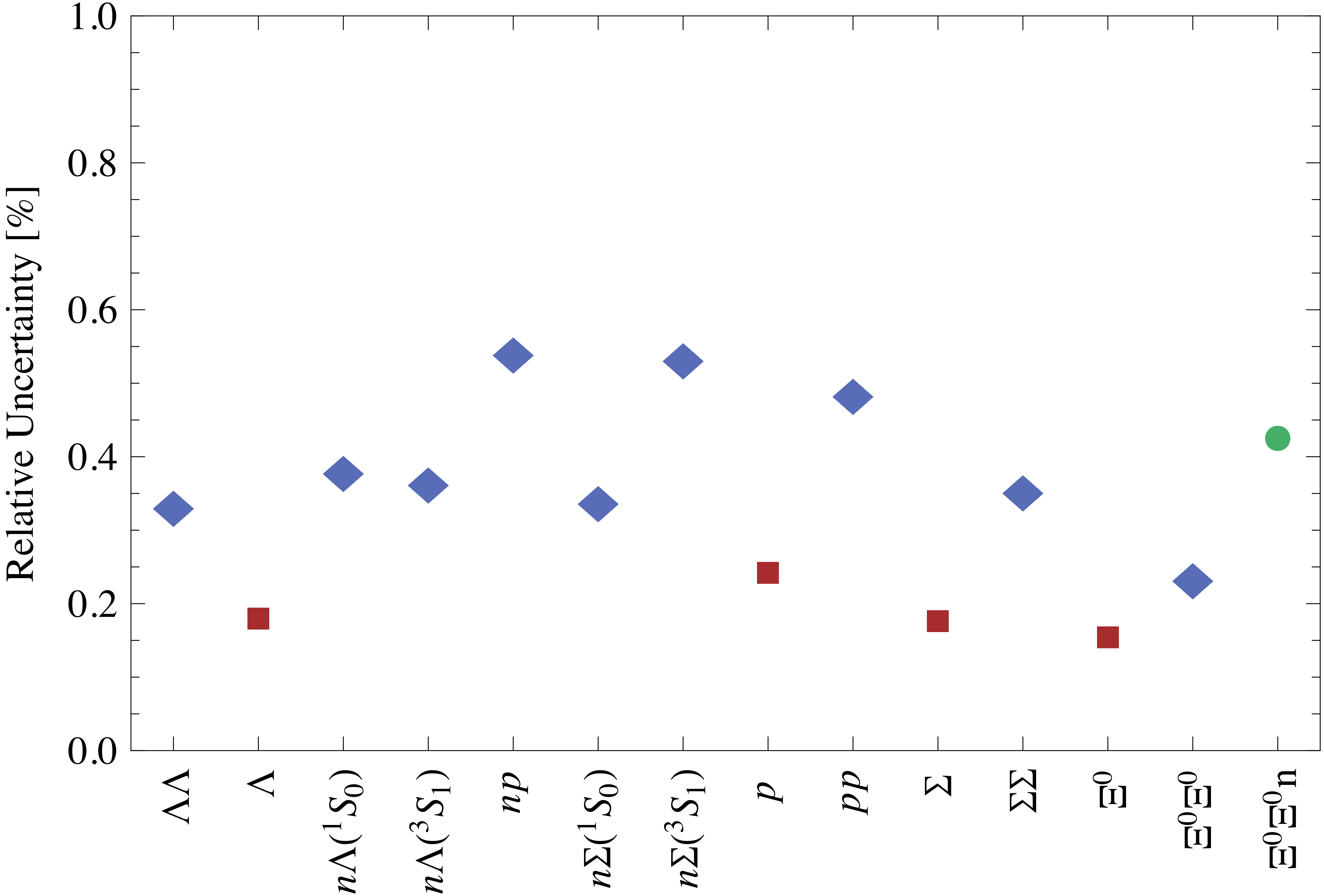




Relative Uncertainty [%]
1.0
0.8
0.6
0.4
0.2
0.0
ΛΛ
Λ
nΛ(1S0)
nΛ(3S1)
np
nΣ(1S0)
nΣ(3S1)
p
pp
Σ
ΣΣ
Ξ0
Ξ0Ξ0
Ξ0Ξ0n

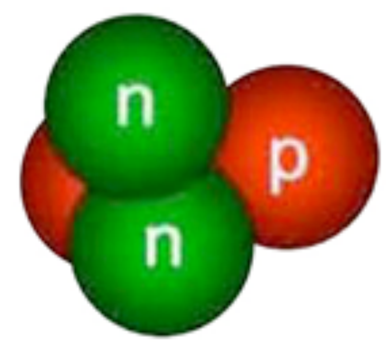
Contraction bottleneck for $A \gg 2$?

Naive factorial growth!

np: 36

nnp: 2880

npnp: 518400



$\vdots$

(A,Z): (A+Z)! (2A-Z)!

$\boxed{\text{Recursion relations for mesons } \rightarrow A \text{ growth!}}$

Baryon recursion relations in development!

Contraction bottleneck for $A \gg 2$?

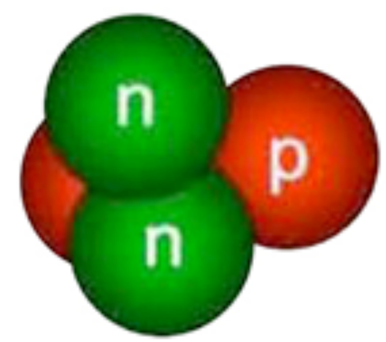
Naive factorial growth!

np: 36

Symmetries

nnp: 2880 $\longrightarrow$ 93

npnp: 518400 $\longrightarrow$ 1107



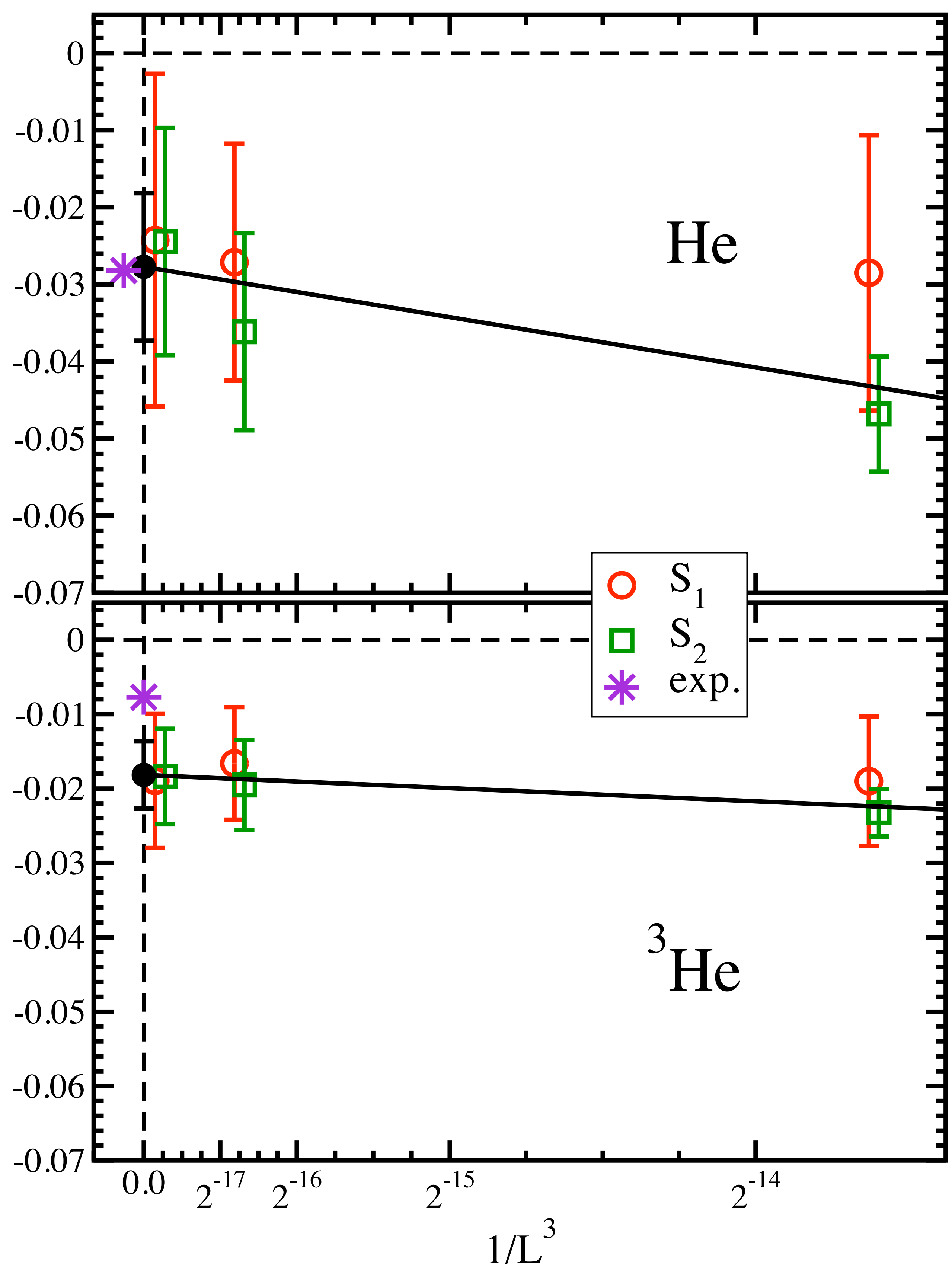
$\vdots$

(A,Z): (A+Z)! (2A-Z)!

Recursion relations for mesons $\rightarrow A$ growth!

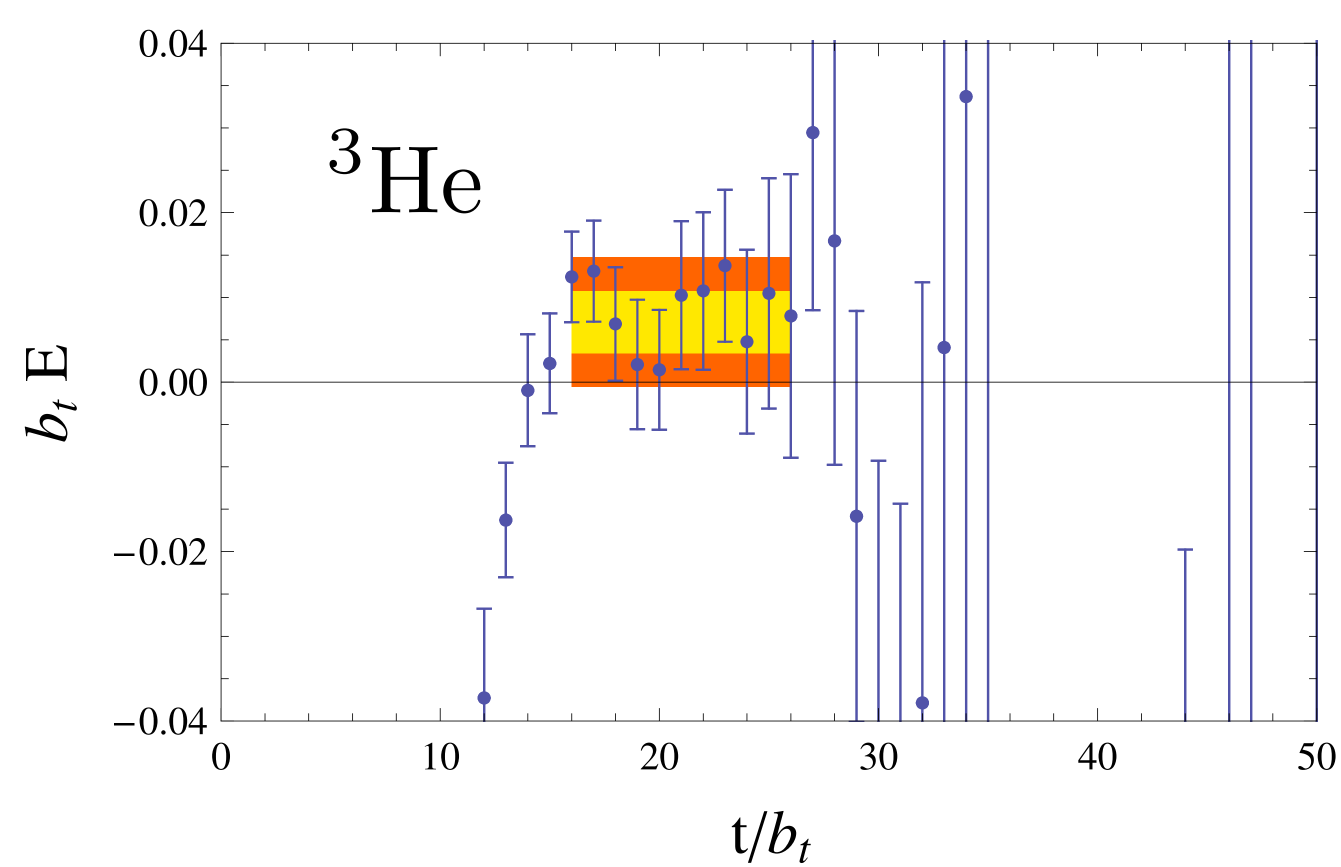
Baryon recursion relations in development!

(Yamazaki *et. al.* (2009))



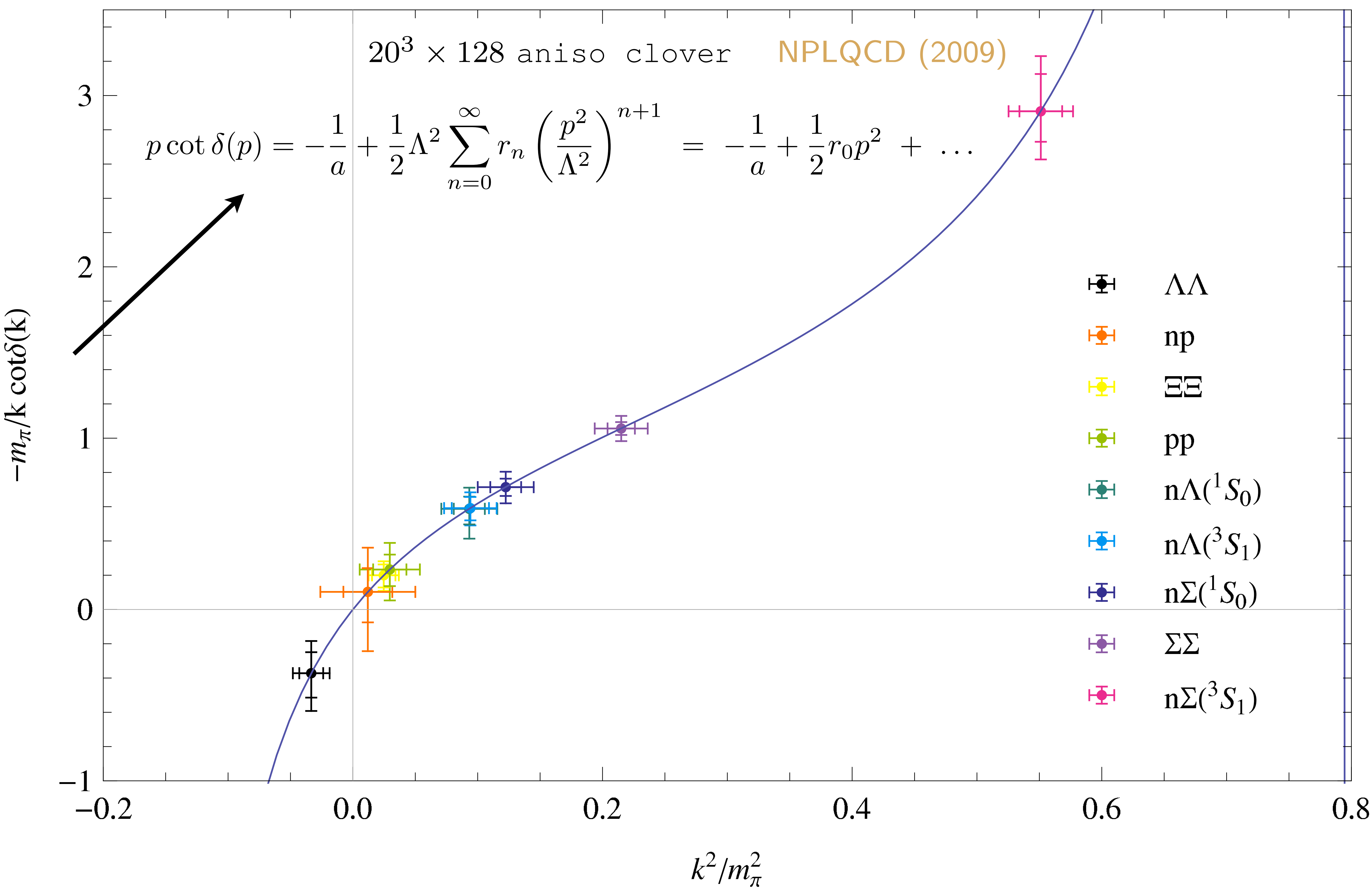
Quenched - Heavy pions

$20^3 \times 128$ aniso clover NPLQCD (2009)



Full QCD

Lattice QCD: Baryon-Baryon



Is there an H-dibaryon?

Need other volumes!

$16^3 \times 128$

$20^3 \times 128$

$24^3 \times 128$

$32^3 \times 128$

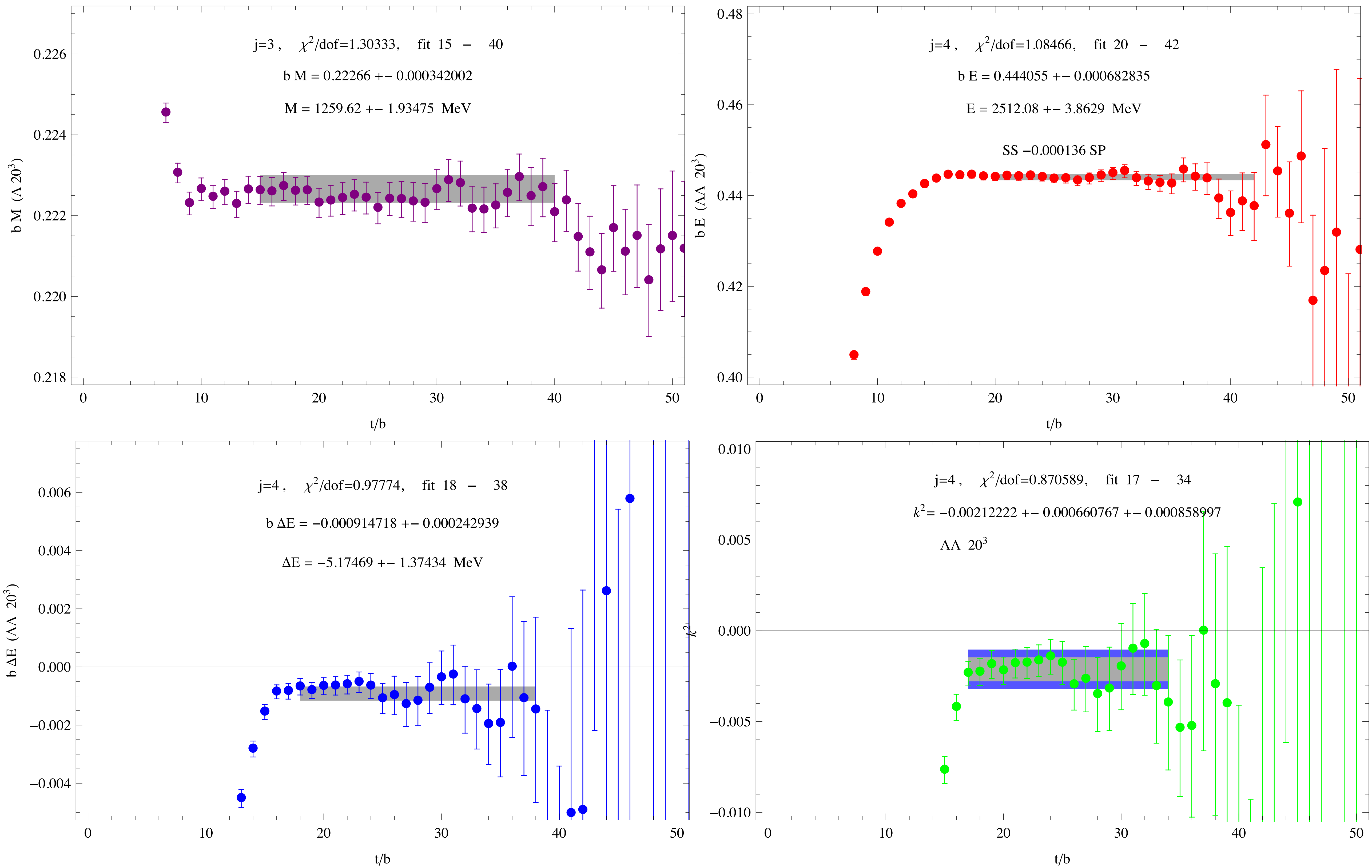
in progress

Recall:

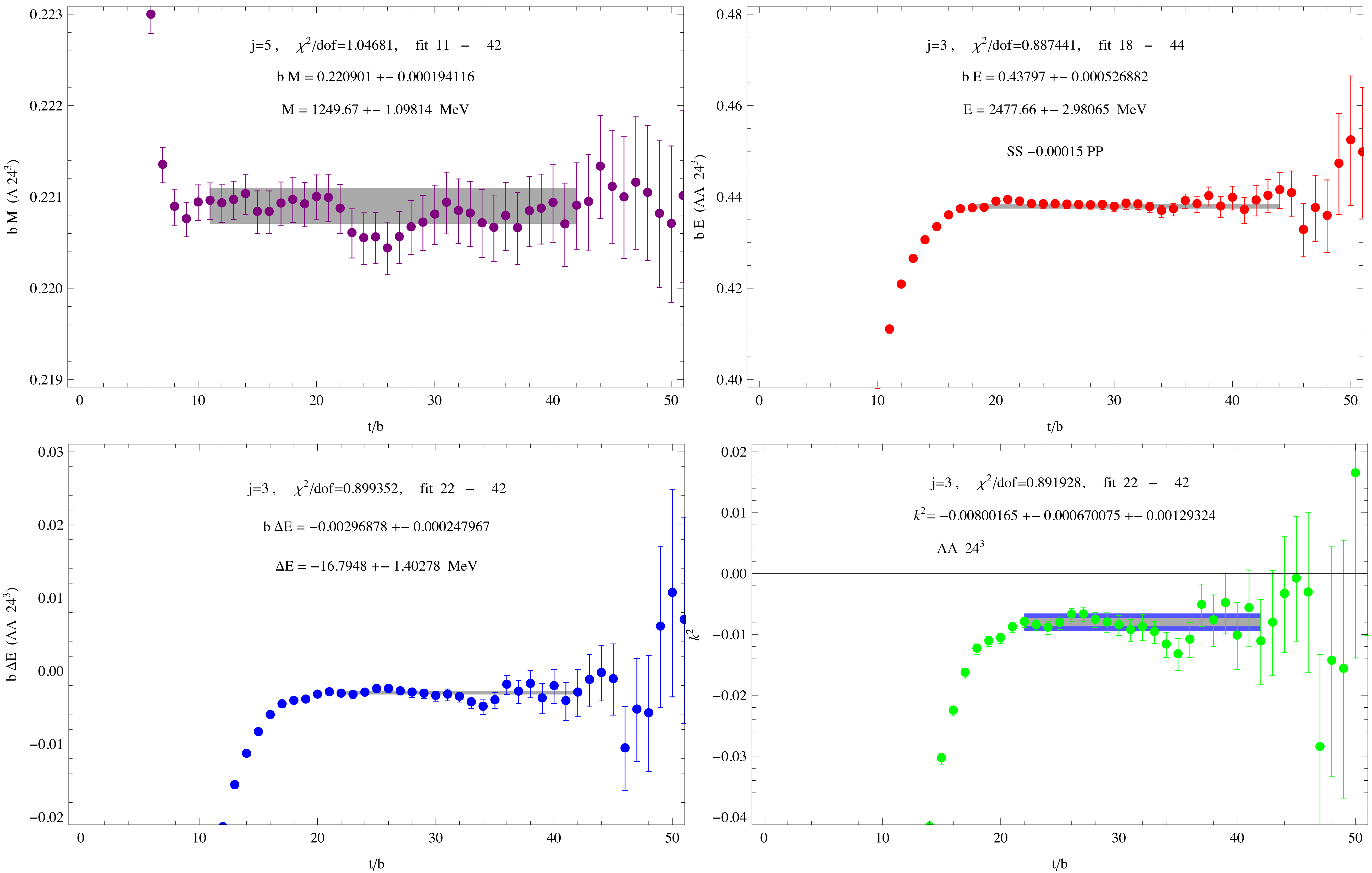
$$\kappa = \gamma + \frac{6}{L}\frac{e^{-\gamma L}}{1 - \gamma r_3} + \mathcal{O}(e^{-\sqrt{2}\gamma L})$$

$m_\pi \sim 389$ MeV $\quad b_s \sim 0.1227(8)$ fm $\quad b_s/b_t = 3.500(32)$

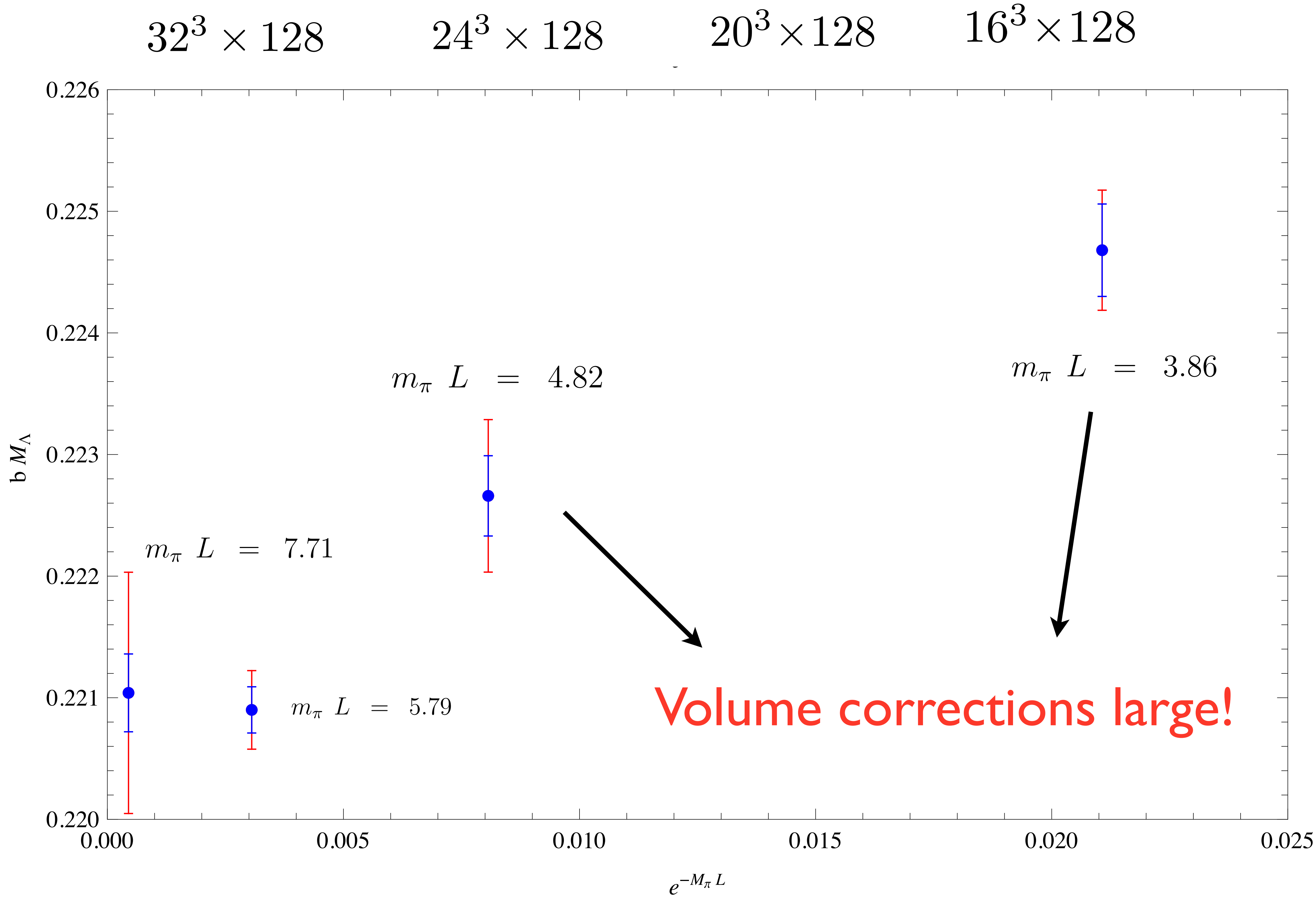
$20^3 \times 128$



$24^3 \times 128$



Volume dependence of the mass



Need more statistics on the large volume!!

Conclusion

- We are approaching a golden age where nuclear properties and reactions will be calculated using lattice QCD.
- Two-baryon systems are currently under intense investigation. Calculation of the deuteron is a major outstanding benchmark.
- Calculations of three-body systems are in progress.
- Lattice QCD requires:
  - ★ the resources to move beyond the benchmarking stage.
  - ★ a strong collaborative effort among physicists, computer scientists and applied mathematicians.