Supersymmetric Chern-Simons Theories with Vector Matter

27. Sep. 2012

Subrahmanian Chandrasekhar discussion meeting

"Scattering without space-time"

@ International Center for Theoretical Science (ICTS)

Shuichi Yokoyama

Tata Institute of Fundamental Research (TIFR)

S.Jain_S.P.Trivedi_S.R.Wadia_SY (arXiv:1207.4750)

Ref. S.Giombi_S.Minwalla_S.Prakash_S.Trivedi_S.Wadia_X.Yin (arXiv:1110.4386)

Why Chern-Simons theory?

Why Chern-Simons theory?

Chern-Simons (CS) theories play many important roles.

① (Mathematics)

Knot theory, Jones polynomial **[Witten '89]**

② (string theory)

Cubic string field theory, Open topological string theory **[Witten '85]**

③ (M_theory)

Effective field theories of membranes. **[BLG '07, ABJM '08]**

④ (3d CFT)

Infinitely many interacting CFT (conformal zoo). **[Moore_Seiberg '89]**

⑤ (AdS/CFT correspondence)

Dual CFT3 of (HS) gravity on AdS4

Pure [HS] gravity on AdS3 **[Gaberdiel_Gopakumar '11]**

(Pure) Chern-Simons theory

$$iS_{cs} = \frac{ik}{4\pi}\int \mathrm{tr}(\widetilde{A}d\widetilde{A} + \frac{2}{3}\widetilde{A}^3)$$

NOTE

① CS coupling constant (k) is protected as an integer.

② Independent of metric. (Topological).

③ Exact CFT parametrized by (k,N) or λ=N/k. N: rank of gauge group

④ Exactly soluble. (Wilson loop ⇔ Knot) . **[Witten '89]**

Plan

✓ 1. Introduction

2. AdS4/CFT3 duality

Klebanov_Polyakov conjecture

3. Chern-Simons-Vector model

3.1 Vector fermion matter

3.2 SUSY vector matter

4. Summary & Discussion

AdS4/CFT3 correspondence

3d O(N)(U(N)) vector model ⇔ HS gravity on AdS4

[Klebanov_Polyakov '02]

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(Bosonic case)
Singlet sector of 3d free/critical vector boson model is dual to a HS theory in the large N limit with suitable bc.

Parity-invariant theory with parity even scalar (A_type Vasiliev theory)

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Parity-invariant theory with parity even scalar (A_type Vasiliev theory)

(Fermionic case)
Singlet sector of 3d free/GN vector fermion model is dual to a HS theory in the large N limit with suitable bc.

Parity-invariant theory with parity odd scalar (B_type Vasiliev theory)

AdS4/CFT3 correspondence

[Klebanov_Polyakov '02]

A scalar field conformally coupling to AdS4

$$S = \int d^4x \sqrt{g} \left((\partial_\mu h)^2 - 2h^2 \right)$$

(AdS radius set to unity)

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∃ 2 solutions: $h \sim z^\Delta$

(i) Δ=1

(ii) Δ=2

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Free N bosons

$$\Delta_{\mathcal{O}_\mathcal{B}} = 1$$

Free N fermions

$$\Delta_{\mathcal{O}_F} = 2$$

AdS4/CFT3 correspondence

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Free vector boson

$$\Delta_{\mathcal{O}_\mathcal{B}} = 1$$

GN vector fermion

$$\Delta_{\mathcal{O}_F} = 1 + \ldots$$

(ii) Δ=2

Critical vector boson

$$\Delta_{\mathcal{O}_\mathcal{B}} = 2 + \ldots$$

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Free vector boson

$$\Delta_{\mathcal{O}_B} = 1$$

$$\longrightarrow \quad \mathcal{O}_B^2 = (\bar{\phi}\phi)^2$$

Critical vector boson

$$\Delta_{\mathcal{O}_B} = 2 + \ldots$$

GN vector fermion

$$\Delta_{\mathcal{O}_F} = 1 + \ldots$$

$$\longrightarrow \quad \cancel{\mathcal{O}_F^2 = (\bar{\psi}\psi)^2}$$

Free vector fermion

$$\Delta_{\mathcal{O}_F} = 2$$

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(AdS radius set to unity)

∃ 2 solutions: $h \sim z^\Delta$

(i) Δ=1 $\longleftrightarrow$ (ii) Δ=2 cf. [Witten '01]

changing bc

Free vector boson $\longrightarrow$ Critical vector boson (A_type)

$\Delta_{\mathcal{O}_B} = 1$ $\mathcal{O}_B^2 = (\bar{\phi}\phi)^2$ $\Delta_{\mathcal{O}_B} = 2 + \ldots$

GN vector fermion $\longrightarrow$ Free vector fermion (B_type)

$\Delta_{\mathcal{O}_F} = 1 + \ldots$ $\cancel{\mathcal{O}_F^2 = (\bar{\psi}\psi)^2}$ $\Delta_{\mathcal{O}_F} = 2$

HS gravity on AdS4 ⇔ O(N)(U(N)) vector model

[Klebanov_Polyakov '02]

AdS4	???	CFT3
A_type Vasiliev	⟷	Free/Critical boson
B_type Vasiliev	⟷	Free/GN fermion
[Vasiliev]	[Giombi_Yin '11] [Sezgin_Sundell '02]	[Giombi_Prakarsh_Yin '11]

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SUSY Vasiliev	⟷	SUSY vector model
[Vasiliev et al]	[Sezgin_Sundell '03]	
~~Parity~~ Vasiliev	⟷	CS-vector model

[Giombi_Minwalla_Prakash_Trivedi_Wadia_Yin '11]

[Aharony_Gur-Ari_Yacoby '11]

HS gravity on AdS4 ⇔ O(N)(U(N)) vector model

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[Chang_Minwalla_Sharma_Yin '12] [Sezgin_Sundell '12]		[Jain_Trivedi_Wadia_SY '12]

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3.1 Chern-Simons-Fermion system

Action on R3 (euclidean)

$$S = \int d^3x \left[i\varepsilon^{\mu\nu\rho} \frac{k}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + \bar{\psi}\gamma^\mu D_\mu \psi - \mu\bar{\psi}\gamma^3\psi \right]$$

$$D_\mu \psi = \partial_\mu \psi - iA^a T^a \psi.$$

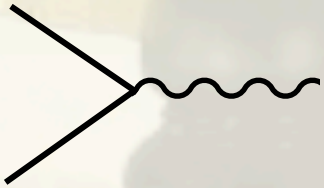
Feynman rule

① gauge propagator

② fermion propagator

③ gauge 3pt vertex

④ fermion_gauge vertex



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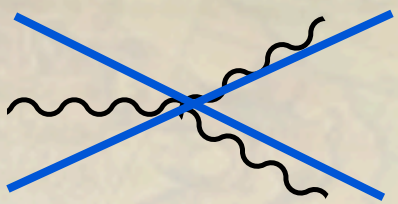
Feynman rule

gauge: $A_- = 0$

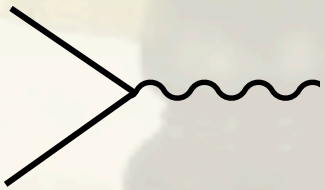
① gauge propagator

② fermion propagator

③ gauge 3pt vertex



④ fermion_gauge vertex



3.1 Chern-Simons-Fermion system

2 point function

① gauge_gauge

 = ???

② fermion_fermion

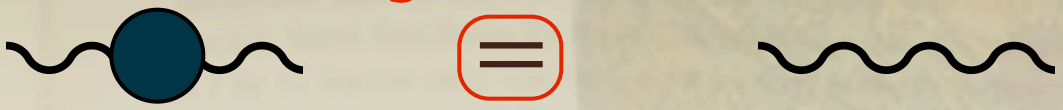
 = ???

3.1 Chern-Simons-Fermion system

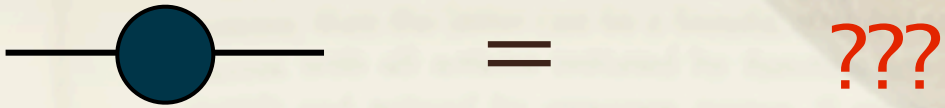
2 point function

① gauge_gauge

large k limit!!



② fermion_fermion



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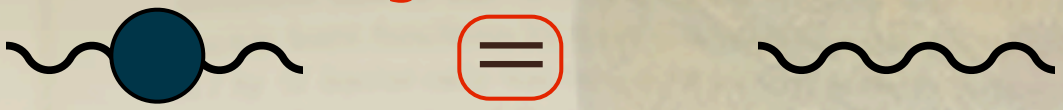

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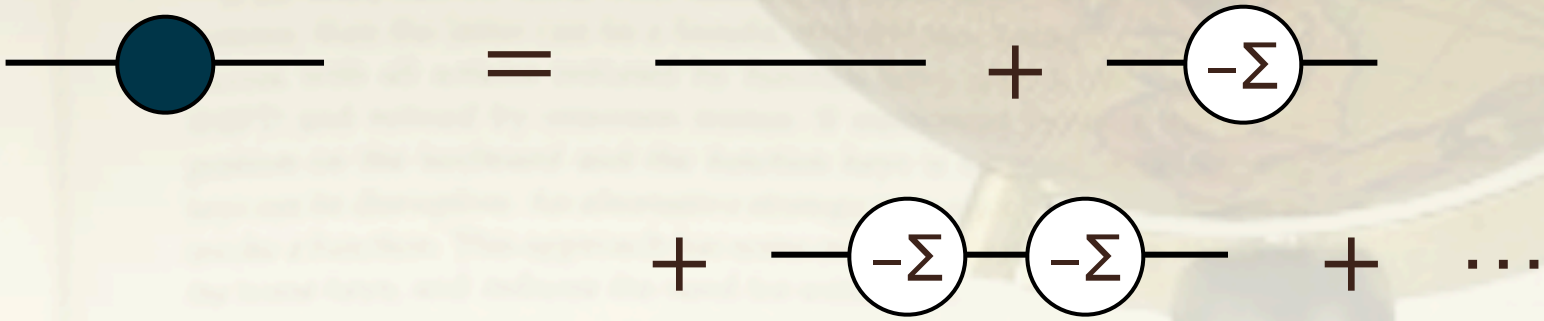

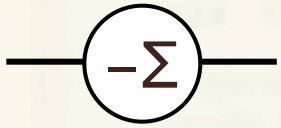

= ???

Fermion self-energy

[S.Giombi_S.Minwalla_S.Prakash_S.Trivedi_S.Wadia_X.Yin]

= ???

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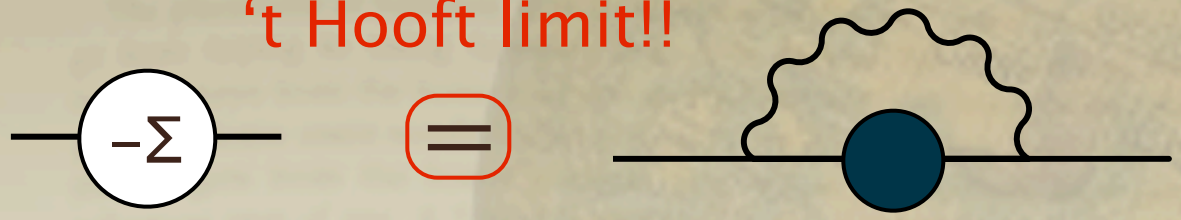
't Hooft limit!!



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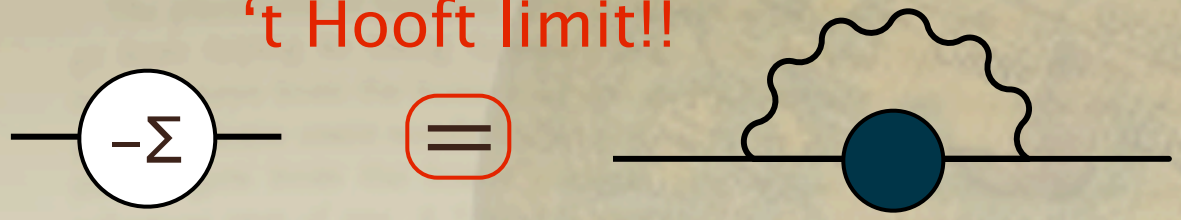

$$-\Sigma_F(p) = N \int \frac{d^3q}{(2\pi)^3} i\gamma^\mu G_{\mu\nu}(p-q) \frac{1}{i\gamma^\mu q_\mu + \Sigma_F(q)} i\gamma^\nu$$

$$G_{\mu\nu}(p) = \frac{2\pi}{kip_-}(\delta_{\mu,+}\delta_{\nu,3} - \delta_{\mu,3}\delta_{\nu,+}).$$

Schwinger_Dyson equation

Fermion self-energy

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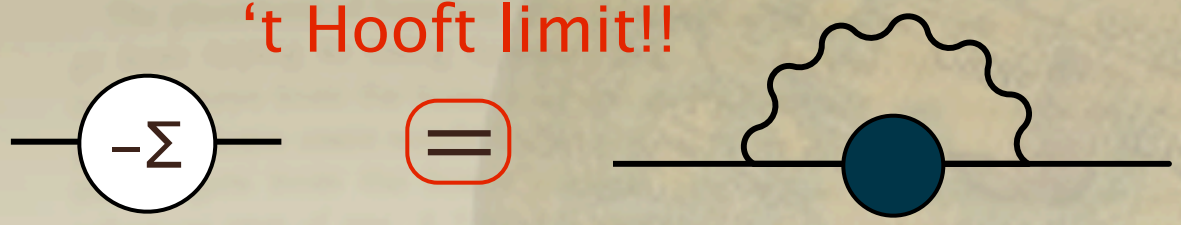

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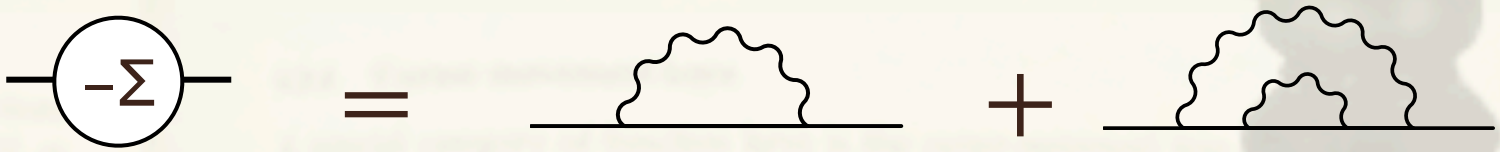
Schwinger_Dyson equation

Solution

$$\Sigma(p) = fp_s + ig\gamma^+ p_+ \qquad f = \lambda,\ g = -\lambda^2$$



Large N Effective action

① Integrate out gauge field with gauge: $A_- = 0$.

② Introduce auxiliary singlet fields Σ to kill 4 fermi interaction.

(Hubbard–Stratonovich transformation)

③ Integrate out ψ.

④ Evaluate it by saddle point approx. under translationally inv. config.

$$S = -NV \int \frac{d^3q}{(2\pi)^3} \mathrm{Tr}\left[\ln\left[i\gamma^\mu q_\mu + \Sigma(q)\right] - \frac{1}{2}\Sigma(q)\left(\frac{1}{i\gamma^\mu q_\mu + \Sigma(q)}\right)\right]$$

where saddle point equation is

$$\Sigma(p) = -2\pi i\lambda \int \frac{d^3q}{(2\pi)^3}\left(\gamma^3 \frac{1}{i\gamma^\mu q_\mu + \Sigma}\gamma^+ - \gamma^+ \frac{1}{i\gamma^\mu q_\mu + \Sigma}\gamma^3\right)\frac{1}{(p-q)^+}$$

[S.Giombi_S.Minwalla_S.Prakash_S.Trivedi_S.Wadia_X.Yin]

Finite temperature

→ Consider theory on $R^2 \times S^1$ (length $\beta = T^{-1}$).

$$\int \frac{dp_3}{2\pi} F(p_3) \to \frac{1}{\beta} \sum_{p_3 : F} F(p_3) := \frac{1}{\beta} \sum_{n \in \mathbf{Z}} F\left(\frac{2\pi(n + \frac{1}{2})}{\beta}\right), \quad \text{(anti-periodic bc)}$$

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(1) Schwinger_Dyson equation.

$$\Sigma(p) = \frac{1}{\beta} \sum_{q_3:F} \int \frac{d^2q}{(2\pi)^2} \left(\gamma^+ \frac{1}{i\gamma^\mu q_\mu + \Sigma(q)} \gamma^3 - \gamma^3 \frac{1}{i\gamma^\mu q_\mu + \Sigma(q)} \gamma^+ \right) \frac{2\pi\lambda}{i(p-q)_-}$$

(2) Free energy

$$S_{eff} = NV_2 \sum_{q_3:F} \int \frac{d^2q}{(2\pi)^2} \mathrm{tr} \left[-\log\left[i\gamma^\mu q_\mu + \Sigma(q)\right] + \frac{1}{2}\Sigma(q) \left(\frac{1}{i\gamma^\mu q_\mu + \Sigma(q)} \right) \right]$$

Finite temperature

(1) Solution of Schwinger_Dyson equation.

$$\Sigma = f \cdot p_s I + g \cdot i p_+ \gamma^+$$

$$f(\hat{p}) = \frac{\lambda}{\hat{p}} \log\left(2(\cosh\sqrt{\hat{p}^2 + c} + \cosh\hat{\mu})\right),$$

$$g(\hat{p}) = \frac{c}{\hat{p}^2} - f(\hat{p})^2, \qquad \hat{p} = \frac{p_s}{T}$$

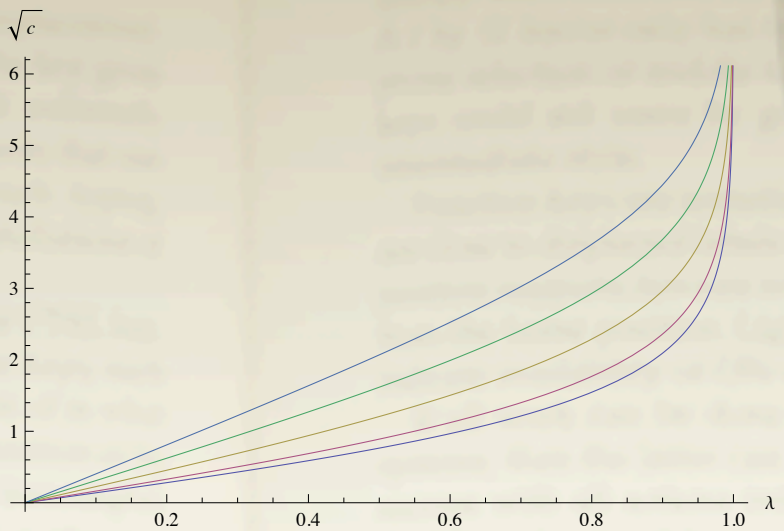
$$e^{\frac{\sqrt{c}}{|\lambda|}} = e^{\sqrt{c}} + e^{-\sqrt{c}} + e^{\hat{\mu}} + e^{-\hat{\mu}},$$

(2) Free energy density

$$F(T) = -\frac{NT^3}{6\pi}\left(\frac{\sqrt{c}^3}{|\lambda|} - \sqrt{c}^3 + 6\int_{\sqrt{c}}^{\infty} dy\; y \ln\left(1 + e^{-y}\right)\right).$$

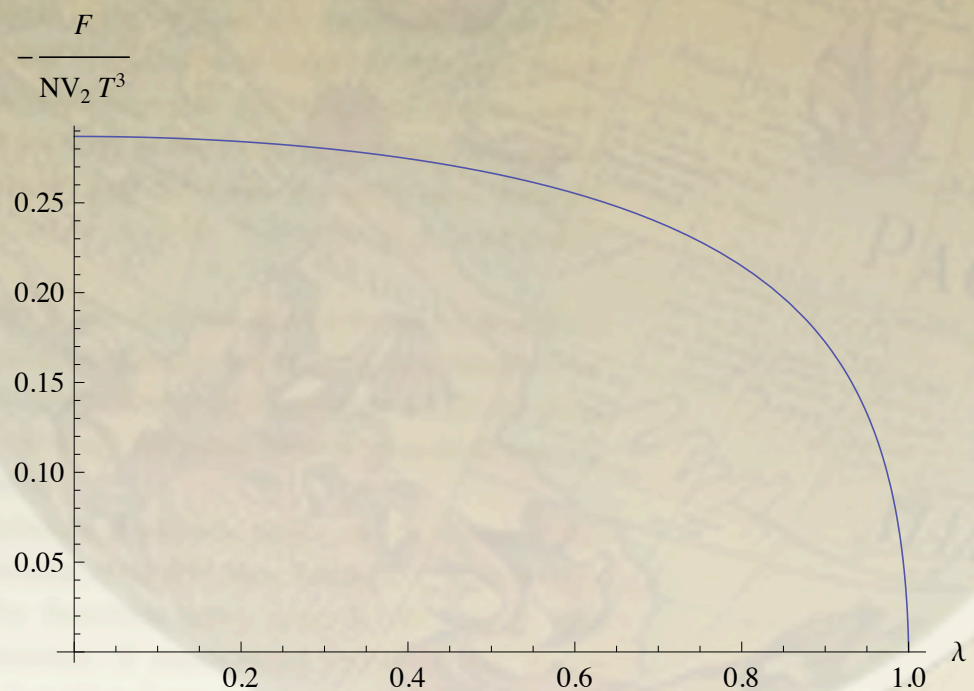
Finite temperature

(1) $\sqrt{c}$



(a)

(2) Free energy



$|\lambda|<1.$

Why $|\lambda|<1$?

Why |λ|<1?

→ UV completion by Yang_Mills term!?

IR		UV
SU(N) CSF	←	SU(N) YMCSF
level k_{IR}		level k_{UV}

$$|k_{IR}| = |k_{UV}| + N$$

Why |λ|<1?

→ UV completion by Yang_Mills term!?

IR		UV
SU(N) CSF	←	SU(N) YMCSF
level k_{IR}		level k_{UV}

$$|k_{IR}| = |k_{UV}| + N$$

$$\lambda = \frac{N}{k_{IR}} \quad \lambda_{UV} = \frac{N}{k_{UV}} \quad \Downarrow \quad |\lambda_{UV}| \in [0, \infty)$$

$$\boxed{|\lambda| = \frac{|\lambda_{UV}|}{1 + |\lambda_{UV}|} \leq 1 \ \textbf{!!}}$$

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3.2 SUSY CS-vector model

CS + 1 fund. chiral mult.

$$S = \int d^3x \Big[i\varepsilon^{\mu\nu\rho} \frac{k}{4\pi} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) + D_\mu \bar{\phi} D^\mu \phi + \bar{\psi} \gamma^\mu D_\mu \psi$$

$$+ \lambda_4 (\bar{\psi}\psi)(\bar{\phi}\phi) + \lambda_4' (\bar{\psi}\phi)(\bar{\phi}\psi) + \lambda_4'' \big((\bar{\psi}\phi)(\bar{\psi}\phi) + (\bar{\phi}\psi)(\bar{\phi}\psi) \big) + \lambda_6 (\bar{\phi}\phi)^3 \Big].$$

$$\lambda_4 = \frac{x_4}{\kappa}, \quad \lambda_4' = \frac{x_4'}{2\kappa}, \quad \lambda_4'' = \frac{x_4''}{4\kappa}, \quad \lambda_6 = \frac{x_6}{(2\kappa)^2}, \qquad \kappa = \frac{k}{4\pi}$$

In the ‘t Hooft limit, x_4, x'_4, x''_4, x_6 are of O(1).

3.2 SUSY CS-vector model

CS + 1 fund. chiral mult.

$$S = \int d^3x \Big[i\varepsilon^{\mu\nu\rho}\frac{k}{4\pi}\mathrm{Tr}(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho) + D_\mu\bar{\phi}D^\mu\phi + \bar{\psi}\gamma^\mu D_\mu\psi$$

$$+ \lambda_4(\bar{\psi}\psi)(\bar{\phi}\phi) + \lambda_4'(\bar{\psi}\phi)(\bar{\phi}\psi) + \lambda_4''\left((\bar{\psi}\phi)(\bar{\psi}\phi) + (\bar{\phi}\psi)(\bar{\phi}\psi)\right) + \lambda_6(\bar{\phi}\phi)^3\Big].$$

$$\lambda_4 = \frac{x_4}{\kappa}, \quad \lambda_4' = \frac{x_4'}{2\kappa}, \quad \lambda_4'' = \frac{x_4''}{4\kappa}, \quad \lambda_6 = \frac{x_6}{(2\kappa)^2}, \qquad \kappa = \frac{k}{4\pi}$$

In the ‘t Hooft limit, x_4, x'_4, x''_4, x_6 are of O(1).

SUSY

N=1 $$x_4 = \frac{1+w}{2},\ x_4' = w,\ x_4'' = w-1,\ x_6 = w^2,$$

N=2 $$w = 1 - \frac{1}{N}$$

3.2 SUSY CS-vector model

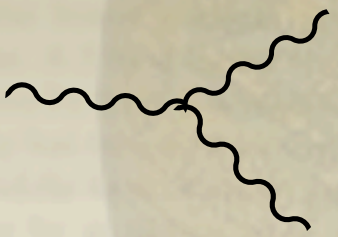
Feynman rule

① gauge propagator

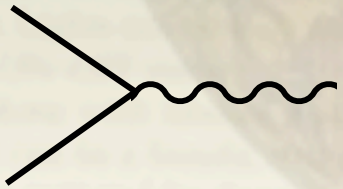
② fermion propagator

③ scalar propagator

④ gauge 3pt vertex

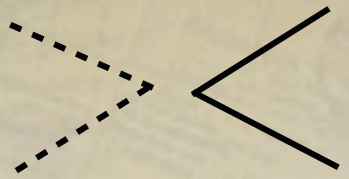


⑤ fermion_gauge vertex

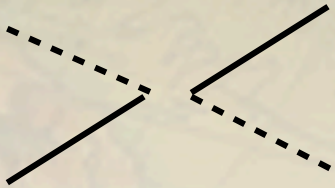


⑥ scalar_gauge vertex

⑦ scalar_fermion vertex



⑧ scalar_fermion vertex



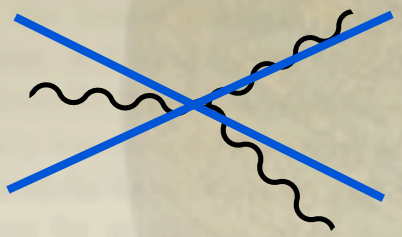
⑨ 6 scalar vertex

3.2 SUSY CS-vector model

Feynman rule

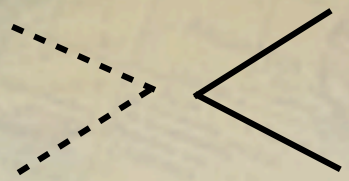
① gauge propagator

④ gauge 3pt vertex



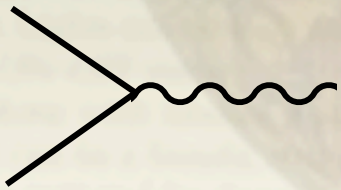
gauge: $A_- = 0$

⑦ scalar_fermion vertex

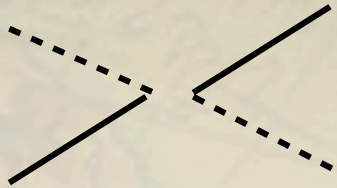


② fermion propagator

⑤ fermion_gauge vertex

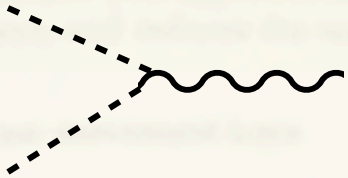


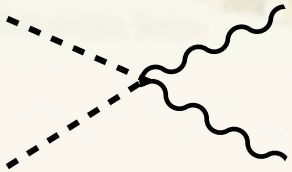
⑧ scalar_fermion vertex



③ scalar propagator

⑥ scalar_gauge vertex





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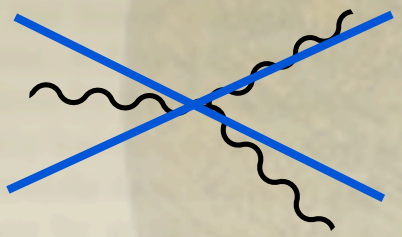

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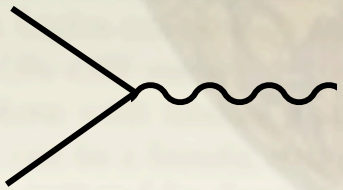
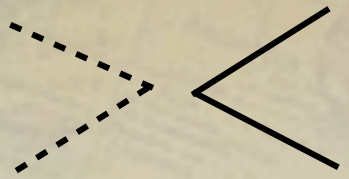
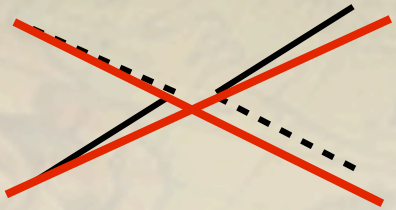


⑥ scalar_gauge vertex

⑦ scalar_fermion vertex



⑧ scalar_fermion vertex



t' Hooft limit

⑨ 6 scalar vertex

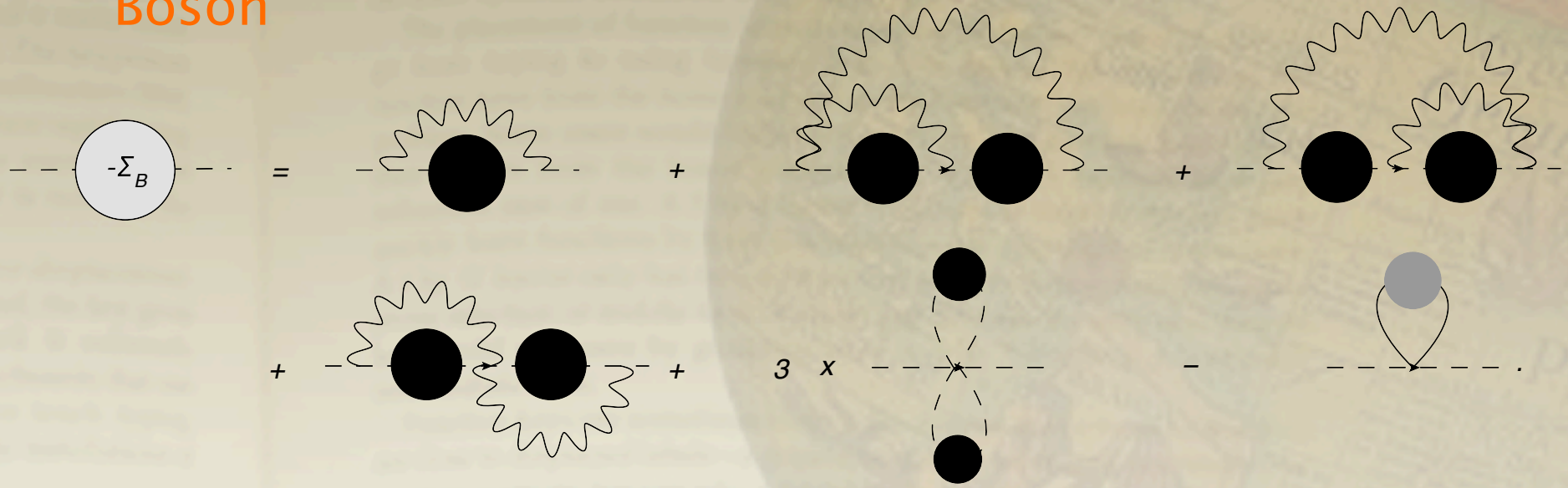
1PI self energy diagram

Boson

= ???

1PI self energy diagram

Boson



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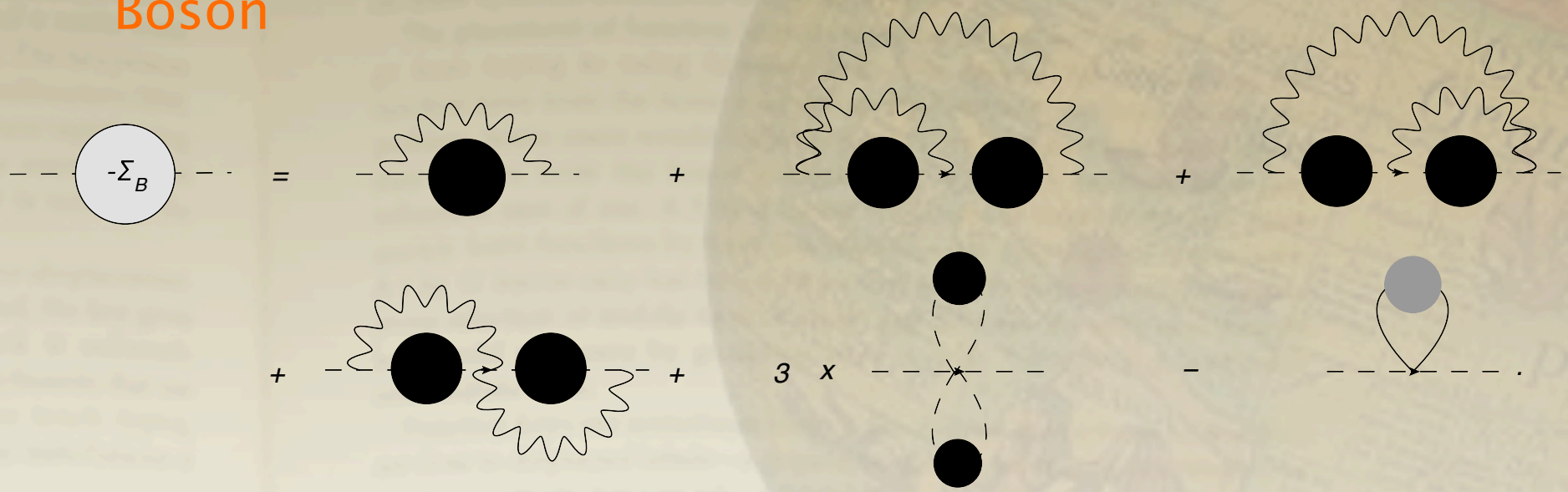
Boson



$$
\begin{aligned}
-\Sigma_B(p) = {} & N \int \frac{d^3q}{(2\pi)^3} (p+q)^\mu G_{\mu\nu}(p-q) \frac{1}{q^2+\Sigma_B(q)} (q+p)^\nu \\
& - 2 \times N^2 \int \frac{d^3q}{(2\pi)^3} \frac{d^3q'}{(2\pi)^3} \frac{1}{q'^2+\Sigma_B(q')} G_{\mu\nu}(q-q')(q'+q)^\nu \frac{1}{q^2+\Sigma_B(q)} G^\mu{}_\rho(p-q)(p+q)^\rho \\
& - N^2 \int \frac{d^3q}{(2\pi)^3} \frac{d^3q'}{(2\pi)^3} (p+q')^\nu \frac{1}{q'^2+\Sigma_B(q')} G_{\nu\mu}(p-q') \frac{1}{q^2+\Sigma_B(q)} G^\mu{}_\rho(p-q)(p+q)^\rho \\
& - 3\lambda_6 N^2 \int \frac{d^3q}{(2\pi)^3} \frac{d^3q'}{(2\pi)^3} \frac{1}{q^2+\Sigma_B(q)} \frac{1}{q'^2+\Sigma_B(q')} \\
& - \lambda_4 N \int \frac{d^3q}{(2\pi)^3} \left(-\mathrm{tr} \frac{1}{i\gamma^\mu q_\mu + \Sigma_F(q)} \right),
\end{aligned}
\tag{E.1}
$$

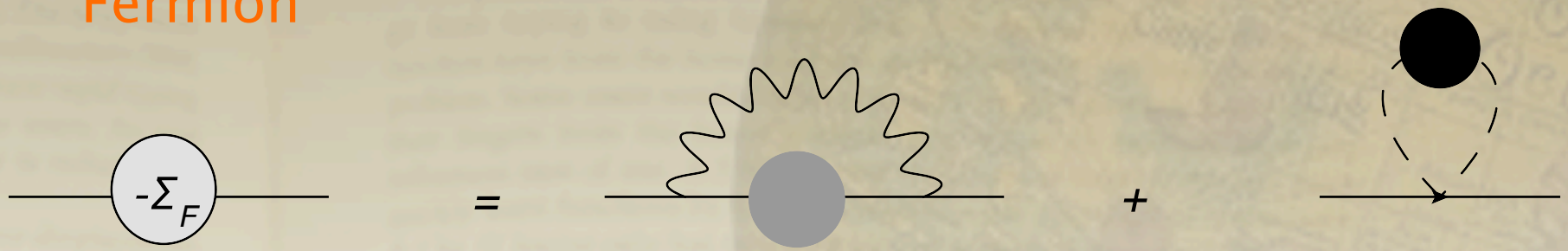
1PI self energy diagram

Fermion

$=$???

1PI self energy diagram

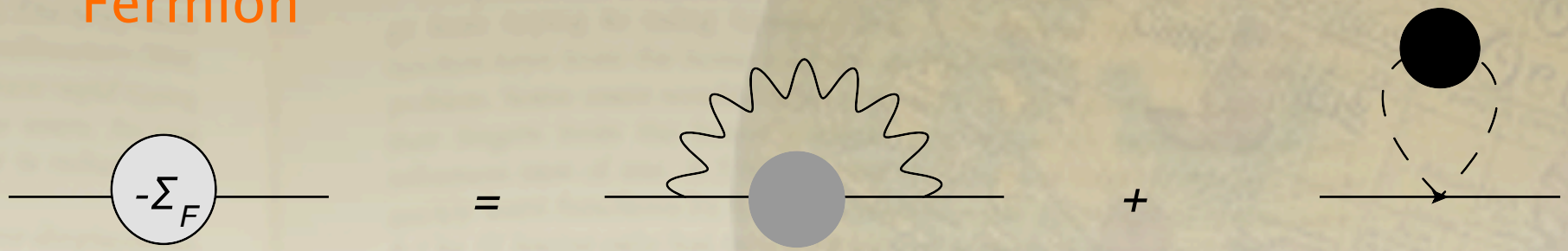
Fermion



$$-\Sigma_F(p) = N\int \frac{d^3q}{(2\pi)^3} i\gamma^\mu G_{\mu\nu}(p-q)\frac{1}{i\gamma^\mu q_\mu + \Sigma_F(q)} i\gamma^\nu - \lambda_4 N \int \frac{d^3q}{(2\pi)^3}\frac{1}{q^2+\Sigma_B(q)}.$$

1PI self energy diagram

Fermion



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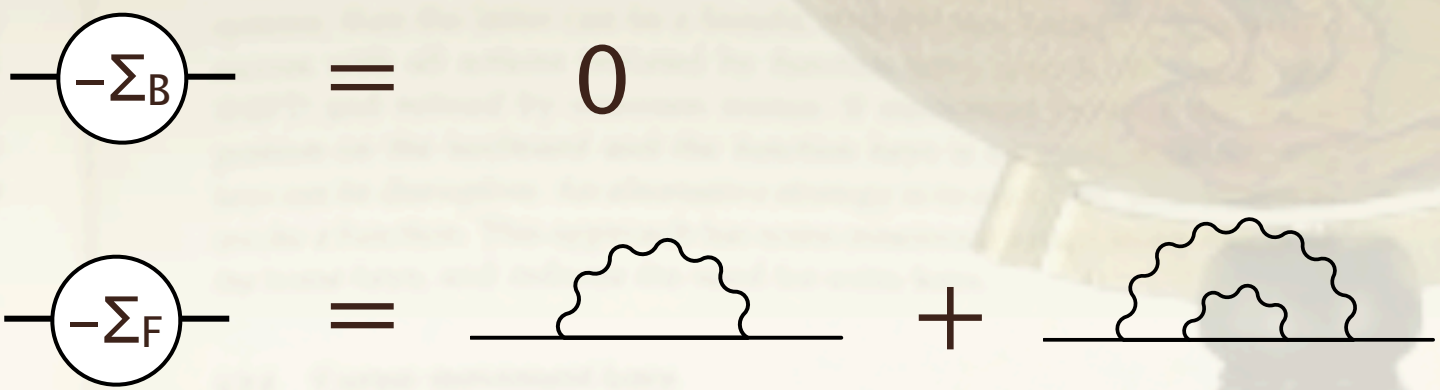
Solution



Large N effective action

$$\tilde{S} = \tilde{S}_B + \tilde{S}_F + \tilde{S}_{BF}$$

$$\widetilde{S}_B = NV \int \frac{d^3q}{(2\pi)^3} \left(\log\left(q^2 + \Sigma_B(q)\right) - \frac{2}{3} \frac{\tilde{\Sigma}_B(q)}{q^2 + \Sigma_B(q)} \right) \tag{25}$$

$$\widetilde{S}_F = NV \int \frac{d^3q}{(2\pi)^3} \mathrm{Tr}\left(-\ln\left(i\gamma^\mu q_\mu + \Sigma_F(p)\right) + \frac{1}{2}\Sigma_F(q) \frac{1}{i\gamma^\mu q_\mu + \Sigma_F(q)} \right) \tag{26}$$

$$\widetilde{S}_{BF} = -\frac{\lambda_4 N^2 V}{6} \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 + \Sigma_B(q)} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left(\frac{1}{i\gamma^\mu p_\mu + \Sigma_F(p)}\right) \tag{27}$$

where saddle point equation (or e.o.m.) is

$$\Sigma_B(q) = \int \frac{d^3q_2}{(2\pi)^3} \frac{d^3q_3}{(2\pi)^3} \left[C_2(q, q_2, q_3) + C_2(q_2, q, q_3) + C_2(q_3, q_2, q) \right] \frac{1}{q_2^2 + \Sigma_B(q_2)} \frac{1}{q_3^2 + \Sigma_B(q_3)}$$
$$- \lambda_4 N \int \frac{d^3q'}{(2\pi)^3} \mathrm{tr}\left(\frac{1}{i\gamma^\mu q'_\mu + \Sigma_F(q')}\right). \tag{B.7}$$

$$\Sigma_F(p) = -2\pi i\lambda \int \frac{d^3q}{(2\pi)^3} \left(\gamma^3 \frac{1}{i\gamma^\mu q_\mu + \Sigma_F} \gamma^+ - \gamma^+ \frac{1}{i\gamma^\mu q_\mu + \Sigma_F} \gamma^3 \right) \frac{1}{(p-q)_-}$$
$$+ \lambda_4 N \int \frac{d^p}{(2\pi)^3} \frac{1}{p^2 + \Sigma_B(p)}, \tag{B.12}$$

Finite temperature

Finite temperature

(1) Solution of Schwinger_Dyson equation.

$$\Sigma_B(p) = \sigma T^2, \quad \Sigma_{F,I}(p) = f(\widetilde{p})p_s, \quad \Sigma_{F,+}(p) = ig(\widetilde{p})p_+,$$

$$f(\widetilde{p}) = \frac{2\lambda}{\widetilde{p}}\left(\log(2\cosh(\frac{\sqrt{\widetilde{p}^2+c}}{2})) - x_4\log(2\sinh(\frac{\sqrt{\sigma}}{2}))\right).$$

$$g(\widetilde{p}) = \frac{c}{\widetilde{p}^2} - f(\widetilde{p})^2. \qquad \widetilde{p} = \beta p_s.$$

$$c = \left\{2\lambda\left(\log(2\cosh(\frac{\sqrt{c}}{2})) - x_4\log(2\sinh(\frac{\sqrt{\sigma}}{2}))\right)\right\}^2.$$

$$\sigma = \lambda^2\Bigg[(1+3x_6)\left(\log(2\sinh(\frac{\sqrt{\sigma}}{2}))\right)^2 - 8x_4^2\log(2\sinh(\frac{\sqrt{\sigma}}{2}))\log(2\cosh(\frac{\sqrt{c}}{2}))$$
$$+ 4x_4\left(\log(2\cosh(\frac{\sqrt{c}}{2}))\right)^2\Bigg].$$

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$$+ 4x_4\left(\log(2\cosh(\frac{\sqrt{c}}{2}))\right)^2\Bigg].$$

(2) Free energy density

$$F = \frac{NT^3}{6\pi}\Bigg[-\sqrt{\sigma}^3 + \sqrt{c}^3 + 2\sigma\log(2\sinh(\frac{\sqrt{\sigma}}{2})) - \frac{3}{2}c\log(2\cosh(\frac{\sqrt{c}}{2}))$$
$$- 2\lambda^2\log(2\cosh(\frac{\sqrt{c}}{2}))\left\{\log(2\cosh(\frac{\sqrt{c}}{2})) - x_4\log(2\sinh(\frac{\sqrt{\sigma}}{2}))\right\}^2$$
$$+ 6\int_{\sqrt{\sigma}}^{\infty} dy y\log(1-e^{-y}) - 6\int_{\sqrt{c}}^{\infty} dy y\log(1+e^{-y})\Bigg].$$

Finite temperature

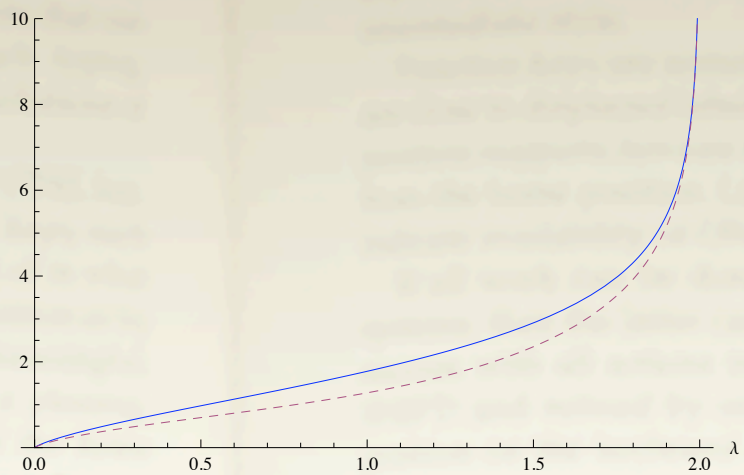
N=1 case $x_4 = \frac{1+w}{2}, \; x_6 = w^2$

Finite temperature

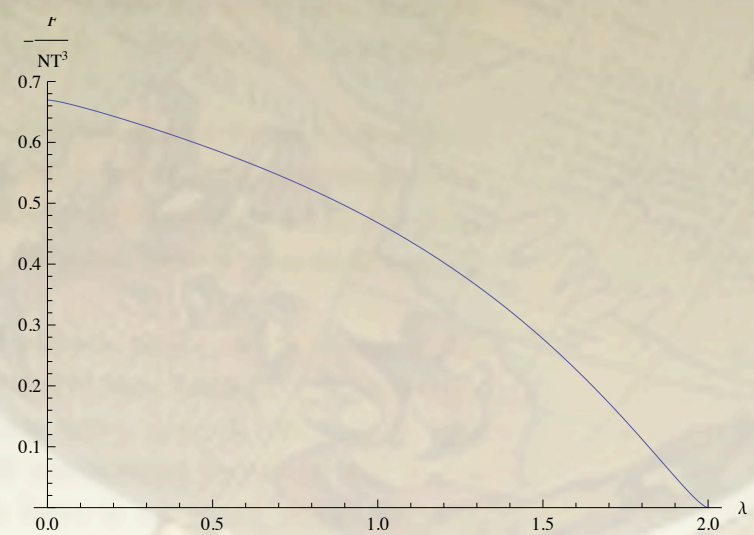
N=1 case $x_4 = \frac{1+w}{2}$, $x_6 = w^2$

w=0

(1) √c, √σ



(2) Free energy density



|λ|<2.

Finite temperature

N=2 case $x_4 = x_6 = 1$ (w=1)

Finite temperature

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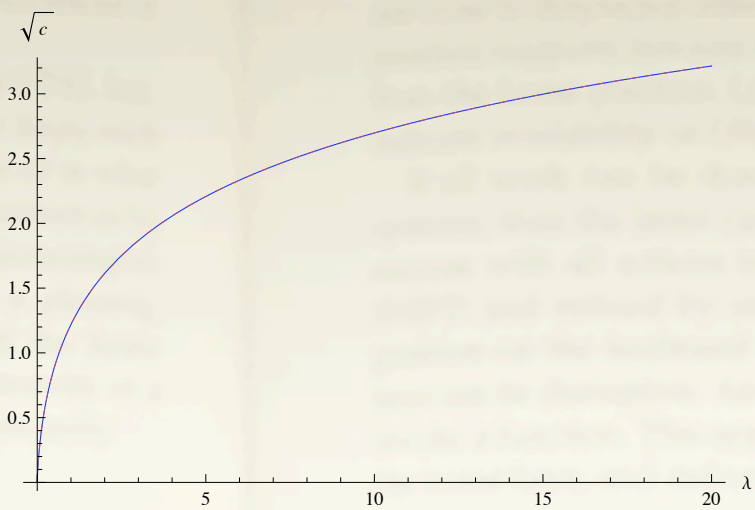
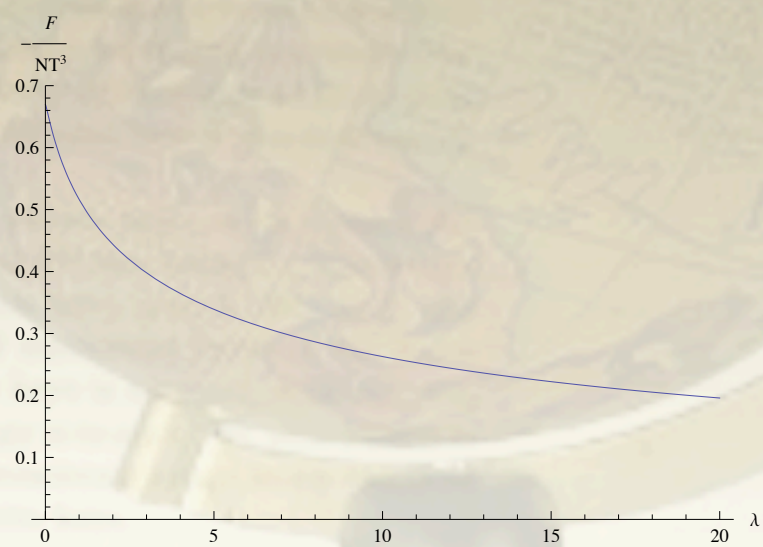
(1) $\sqrt{c}$, $\sqrt{\sigma}$

$$\sqrt{\sigma} = \sqrt{c} = 2|\lambda| \log(\coth \frac{\sqrt{c}}{2}).$$



(2) Free energy density

$$F = -\frac{NT^3}{6\pi}\left(\frac{\sqrt{c}^3}{|\lambda|} + 6\int_{\sqrt{c}}^{\infty} dy y \log(\coth \frac{y}{2})\right)$$



$|\lambda| < \infty$.

Why $|\lambda|<2$ when $N=1(w=0)$?

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→ UV completion by Yang_Mills term!?

IR		UV
N=1 SU(N) CSF	←	N=1 SU(N) YMCSF
level k_{IR}		level k_{UV}

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$$\lambda = \frac{N}{k_{IR}} \quad \lambda_{UV} = \frac{N}{k_{UV}} \quad \Downarrow \quad |\lambda_{UV}| \in [0, \infty)$$

$$|\lambda| = \frac{|\lambda_{UV}|}{1 + \frac{|\lambda_{UV}|}{2}} \leq 2 \text{ !!}$$

Why $|\lambda|<\infty$ when $N=2$?

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$$|\lambda_{IR}| = |\lambda_{UV}| < \infty \quad !!$$

Plan

- √ 1. Introduction
- √ 2. AdS4/CFT3 duality
  - √ Klebanov_Polyakov conjecture
- √ 3. Chern-Simons-Vector model
  - √ 3.1 Vector fermion matter
  - √ 3.2 SUSY vector matter
- 4. Summary & Discussion

Summary

- We study $SU(N)_k$ CS-vector model in the 't Hooft limit.

- We determined self-energy, free energy of matter fields exactly for all orders of λ.

- The thermal mass and free energy are well-behaved. as interaction stronger, correlation length shorter and DOF less.

- In N=1 case the solution stops existing at some λ.

- In N=2 case the solution exists for all values of λ, which is consistent with exact SC symmetry for $\forall$N, k.

  cf. N=4 SYM in 4d

- We discussed the relation of the system to UV theory incorporated with YM kinetic term.

Discussion

- Validity of our gauge choice, Lorentz invariance
- Finite volume analysis (ex. $S^2 x S^1$, S^3)

  cf. [Klebanov_Pufu_Sachdev_Safdi arXiv:1112.5342]
- Higher spin currents and their correlation functions

  [Aharony_Guri-Ari_Yacoby '12], [Maldacena_Ziboedov '12]
- 3d bosonization [Maldacena_Ziboedov '12]
- Seiberg-like duality cf. [Giveon_Kutasov '08], [ABJ '08]
- Dual SUSY ~~parity~~ Vasiliev theories

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- dS4/CFT3 (Sp(N) vector model)

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⋮

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Thank you.