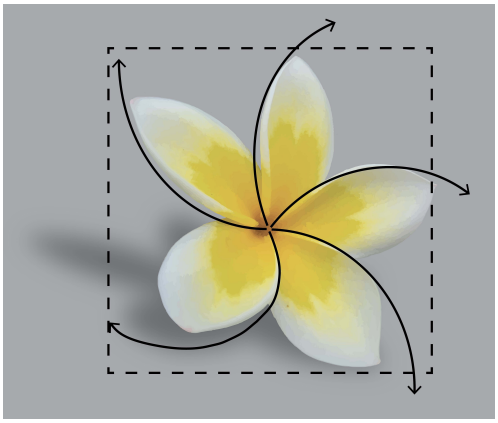
VORTEX MODULI

Bengaluru, February 6–17, 2023



The Virtual Booklet

ICTS
INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Minicourses

Óscar García-Prada (ICMAT, Spain)

Geometry of vortices on Riemann surfaces

Mon 6 14:30, Wed 8 11:30, Thu 9 11:30, Fri 10 14:30

This series of lectures is devoted to the geometry of moduli spaces of vortices on compact Riemann surfaces. Vortices can be regarded as solutions to some gauge theoretic equations involving a unitary connection on a fibre bundle and a Higgs field, or alternatively as pairs consisting of a holomorphic bundle and a holomorphic section of some associated bundle. To establish the link, a stability condition is required on the Higgs pair. Themes to be treated include abelian and non-abelian vortices, dimensional reduction of instantons and vortices, vortices and Hodge bundles, the Kähler–Yang–Mills equations, and gravitating vortices.

Niklas Garner (University of Washington, USA)

Categorical aspects of vortices

Tue 7 10:00, Wed 8 10:00, Thu 9 10:00, Fri 10 16:30

Vortex equations arise from supersymmetric quantum field theory (QFT) in describing certain BPS field configurations in three spacetime dimensions (or two spatial dimensions). In this minicourse, we will describe aspects of the corresponding QFTs and will extract from these vortex equations a category describing certain BPS line defects therein. Time permitting, we will use this physical setup to connect to notions in generalized affine Springer theory.

Nuno Romão (University of Augsburg, Germany)

Quantization of vortices

Mon 6 11:30, Tue 8 14:30, Thu 9 14:30, Fri 10 11:30

Vortex moduli spaces support intrinsic geometry (their L^2 Kähler metric), which has been used to approximate the classical dynamics of vortices in low-energy field theory, e.g. via the associated geodesic flow. An extension of this idea is to use the same L^2 geometry to address the quantum mechanics of vortices in 2+1 dimensions via quantisation of their moduli spaces. There are various ways of doing this, depending on how the vortex dynamics is set up, and this series of lectures is meant to illustrate some of the possibilities. The holomorphic quantization of the moduli space (relevant for first-order dynamics) will be discussed in the simplest example of vortices in line bundles. Some rudiments of the canonical quantisation of vortex moduli will also be presented, specifically the counting of states via L^2-Betti numbers, which is the viewpoint appropriate in a supersymmetric extension of second-order dynamics.

Martin Speight (University of Leeds, UK)

L^2 geometry of moduli spaces of vortices and lumps

Mon 6 10:00, Tue 7 11:30, Wed 8 14:30, Fri 10 10:00

The low energy classical dynamics of topological solitons can often be modelled as geodesic motion in the space of static solitons with respect to a natural Riemannian metric called the L^2 metric. In this minicourse we will develop techniques to calculate this metric, and extract information about the resultant dynamics, for sigma model lumps and abelian vortices. A recurrent theme will be interesting phenomena arising due to noncompactness of the moduli space of static solitons.

Chris Woodward (Rutgers University, USA)

Symplectic vortices and the quantum Kirwan map

Mon 6 18:30, Tue 7 18:30, Wed 8 18:30, Thu 9 18:30 (online)

This minicourse is devoted to the relationship between gauged pseudoholomorphic maps to a symplectic manifold with Hamiltonian group actions and the pseudoholomorphic maps to its symplectic quotient. We will discuss the large-area Gaio–Salamon adiabatic limit of gauged maps as well as the small-area limit in which the moduli space essentially reduces to a classical symplectic quotient. In the Gaio–Salamon limit the Ziltener compactification of the moduli space of affine vortices naturally arises and the quantum Kirwan map is defined by integration over the Ziltener compactification. We will explain the theory using the example of toric varieties as symplectic quotients of vector spaces.

Keynote Lectures

Tudor Dimofte (University of Edinburgh, UK)

3d gauge theories: vortices and vertex algebras

Mon 13 10:00, Tue 14 10:00

3d $\mathcal{N}$=4 gauge theories admit two topological twists, often called A and B, that are expected to lead mathematically to fully extended 3d topological quantum field theories (TQFTs). I will review some aspects of these putative TQFTs, some of their known and expected connections to representation theory, and (especially) connections to moduli spaces of vortices and their cohomology. I will then present some recent work on accessing these 3d TQFTs via boundary vertex algebras — much as was done for Chern–Simons TQFT using the WZW model in the '80s and '90s. In particular, I will discuss using boundary vertex algebras to define braided tensor categories of line operators and to prove their equivalence under 3d mirror symmetry. (These developments joint with Andrew Ballin, Thomas Creutzig and Wenjun Niu.)

David Favero (University of Alberta, Canada + University of Minnesota, USA)

Cohomological field theories from GLSMs

Tue 14 18:30, Wed 15 18:30 (online)

Gauged linear sigma models (GLSMs) serve as a means of interpolating between Kähler geometry and singularity theory. In enumerative geometry, they should specialize to both Gromov–Witten and Fan–Jarvis–Ruan–Witten theory. In joint work with Bumsig Kim (see `arXiv:2006.12182`), we constructed such enumerative invariants for GLSMs. Furthermore, we proved that these invariants form a cohomological field theory (CohFT). In this lecture sequence, I will describe GLSMs and CohFTs, review the history of their development in enumerative geometry, and discuss the construction of these general invariants. Briefly, the invariants are obtained by forming the analogue of a virtual fundamental class which lives in the twisted Hodge complex over a certain "moduli space of maps to the GLSM". This virtual fundamental class roughly comes as the Atiyah class of a "virtual matrix factorization" associated to the GLSM data.

Jeongseok Oh (Imperial College, UK)

Quasimaps, their wall-crossings and mirror symmetry[†]

Mon 13 11:30, Tue 14 17:30

Inspired by the role of loop spaces in the proof of the mirror theorem by his advisor (Alexander Givental), Bumsig Kim and his collaborators developed the theory of spaces of quasimaps. Interestingly, the change of their generating functions of invariants (defined by integration over these spaces) according to their stability conditions, known as a wall-crossing formula, has exactly the same form as mirror symmetry for certain potential functions predicted by physics. So it gives a geometric interpretation of mirror symmetry. A huge computational advantage is that the moduli spaces of quasimaps have less boundary components than those of stable maps, so

[†]Tribute to Bumsig Kim

that they become easier to work with. Together with the wall-crossing formula, we can compute genus g Gromov–Witten invariants for quintics, up to a few low degree invariants which remain unknown.
In this tribute talk to Bumsig Kim, I shall review his achievements in mirror symmetry through quasimaps.

Constantin Teleman (UC Berkeley, USA)

Gromov–Witten theory and gauge theory

Thu 16 10:00, Fri 17 10:00 (online)

We will review the close relation between 3-dimensional gauge theory ($\mathcal{N} = 4$ SUSY 3d gauge theory, in physics language) and the 2-dimensional A-model, focusing on the role of the Toda integrable system. The latter plays the role, in 3d, that the classifying space BG of a compact Lie group G plays in lower dimension. Compact symplectic manifolds with Hamiltonian G-action give boundary conditions for the 3d theory. We will review the Floer-theoretic construction of the latter, as well as a key geometric application to the quantum cohomology of GIT quotients of Fano manifolds. Much of this material is joint work with Dan Pomerleano. In the last portion of the lectures, we will quickly review the GLSM construction of Coulomb branches for 3d gauge theory and their relation to Gromov–Witten theory, along with more speculative comments on the case of quaternionic matter.

Research Talks

Luis Álvarez-Cónsul (ICMAT, Spain)

Obstructions to the existence of gravitating vortices

Tue 14 11:30

In this talk, I will start by explaining some background about the self-dual Einstein–Maxwell–Higgs equations on a compact surface, and then analyze obstructions to the existence of solutions, expressed in terms of the multiplicities of the zeroes of the Higgs field and the vortex number. Joint work with Mario García-Fernandez, Oscar García-Prada, Vamsi Pritham Pingali and Chengjian Yao.

Mathew Bullimore (Durham University, UK)

Vortices and generalised symmetry

Mon 13 16:30 (online)

There is much recent interest in generalised or categorical symmetries that go beyond the paradigm of groups and into the realm of higher groups and fusion categories. I will discuss aspects of such symmetries in the context of three-dimensional abelian GLSMs and moduli spaces of vortices. I will then speculate on generalised notions of equivariance in quasimap K-theory of toric stacks.

Mykola Dedushenko (SCGP Stony Brook, USA)

(i) **Elliptic stable envelopes as interfaces in a 3d QFT**

Wed 15 11:30

I will review the physical realization of elliptic stable envelopes within 3d $\mathcal{N} = 4$ gauge theories. This consists of two steps: (1) explaining why the (Higgs phase) SUSY vacua on a two-torus are captured by the elliptic cohomology of the Higgs branch; (2) observing that a natural SUSY interface exists that interpolates between the massive and massless regimes of the theory.

(ii) **Symplectic duality and the vortex partition function**

Thu 16 17:30

Elliptic stable envelopes are known to appear in the 3d mirror symmetry of the "vertex function" (which encapsulates the K-theoretic count of vortices). Looking instead at the Higgs–Coulomb phase transition in a given 3d $\mathcal{N} = 4$ theory (also known as symplectic duality), we explain this via the properties of the cigar partition function (or half-index) and the Janus interfaces from talk (i).

Aleksander Doan (University College London, UK)

Holomorphic curves and the ADHM vortex equations

Thu 16 14:30 (online)

I will discuss the problem of counting embedded holomorphic curves in Calabi–Yau manifolds of complex dimension three or, more generally, in symplectic six-manifolds. While the naïve count does not typically lead to an interesting geometric invariant, I will outline an ongoing project with Thomas Walpuski whose goal is to define an invariant of symplectic six-manifolds by counting embedded holomorphic curves with weight given by the number of solutions to certain gauge-theoretic equations called the ADHM vortex equations.

Andrea Ferrari (Durham University, UK)

Vortices, other saddles, and wall-crossing

Mon 13 14:30

The enumerative geometry of vortex moduli spaces plays a key role in the study of supersymmetric gauge theories in three dimensions. For instance, in the presence of eight supercharges, field configurations contributing to the path integral can often be localised to moduli spaces of solutions of generalised vortex equations. In the presence of less supersymmetry, however, the path integral can receive additional contributions, which interplay in a remarkable way with those originating from vortex moduli spaces. In this talk, I will introduce some of these phenomena in simple, abelian examples, and comment on the expected relation to potential implications for topics in mathematics such as wall-crossing. (Joint work with M. Bullimore, H. Kim and G. Xu.)

Mario García-Fernandez (ICMAT + UA Madrid, Spain)

Gravitating vortices with positive curvature

Tue 14 14:30 (online)

In this talk I will give an overview of joint work with V. Pingali and C. Yao in `arXiv:1911.09616`, where we give a complete solution to the existence problem for gravitating vortices on acompact Riemann surface with non-negative topological constant $c > 0$.

Dan Halpern-Leistner (Cornell University, USA)

Infinite-dimensional geometric invariant theory and gauged Gromov–Witten theory

Wed 15 17:30 (online)

Harder–Narasimhan (HN) theory gives a structure theorem for vector bundles on a smooth projective curve. A bundle is either semistable, or it admits a canonical filtration whose associated graded bundle is semistable in a graded sense. After reviewing recent advances in extending HN theory to arbitrary algebraic stacks, I will discuss work in progress with Andres Fernandez Herrero to apply this general machinery to the stack of "gauged" maps from a curve C to a G-scheme X, where G is a reductive group and X is projective over an affine scheme. Our main application is to use HN theory for gauged maps to compute generating functions for K-theoretic enumerative invariants known as gauged Gromov–Witten invariants. This problem is interesting more broadly because it can be formulated as an example of an infinite dimensional analog of the usual setup of geometric invariant theory, which has applications to other moduli problems.

Justin Hilburn (Perimeter Institute, Canada)

2-categorical aspects of 3d mirror symmetry

Wed 15 16:30

By now it is known that many interesting phenomena in geometry and representation theory can be understood as aspects of mirrorsymmetry of 3d $\mathcal{N} = 4$ SUSY QFTs. Such a QFT is associated to a hyperkähler manifold X equipped with a hyperhamiltonian action of a compact Lie group G and admits two topological twists. The first twist, which is known as the 3d B-model or Rozansky–Witten theory, is a TQFT of algebro-geometric flavor and has been studied extensively by Kapustin, Rozansky and Saulina. The second twist, which is known as the 3d A-model or 3d Seiberg–Witten theory, is a more mysterious TQFT of symplecto-topological flavor. In this talk I will discuss what is known about the 2-categories of boundary conditions for these two TQFTs. They are expected to provide two distinct categorifications of category $\mathcal{O}$ for the hyperkähler quotient $X///G$ and 3d mirror symmetry is expected to induce a categorification of the Koszul duality between categories $\mathcal{O}$ for mirror symplectic resolutions. For abelian gauge theories this picture is work in progress with Ben Gammage and Aaron Mazel-Gee. This generalizes works of Kapustin–Vyas–Setter and Teleman on pure gauge theory.

Chiu-Chu Melissa Liu (Columbia University, USA)

Higgs–Coulomb correspondence and wall-crossing in abelian GLSMs

Thu 16 18:30 (online)

We define and compute I-functions and central charges for abelian GLSMs using virtual factorizations of Favero and Kim. In the Calabi–Yau case we provide analytic continuation for central charges by explicit integral formulas. The integrals in question are called hemisphere partition functions and we call the integral representation Higgs–Coulomb correspondence. We then use it to prove GIT stability wall-crossing for central charges.

Nick Manton (University of Cambridge, UK)

Statistical mechanics of vortices

Mon 13 17:30 (online)

Classical or quantized statistical mechanics of critically-coupled Abelian Higgs vortices can be modelled by free dynamics on the N-vortex moduli space, with N large. Vortex interactions are captured by the non-trivial moduli space geometry. To avoid boundary effects and satisfy Bradlow's constraint, the vortices are defined on a compact surface of large area A, with $A/N > 4\pi$. The classical partition function depends only on the moduli space volume, and the first quantum correction at high temperature T depends on the integrated scalar curvature. Using these known geometrical quantities, we deduce the high-T equation of state of the vortex gas. When A/N is only slightly larger than 4π, the moduli space simplifies to complex projective space with its Fubini–Study geometry. Here the quantum partition function and equation of state can be calculated for any temperature. (NSM thanks S. Nasir, J. Baptista, J.M. Speight and S. Wang for their collaboration and contributions.)

Michael McBreen (Chinese University of Hong Kong, China)

Uniformizing the elliptic stable envelopes of a hypertoric variety

Wed 15 10:00 (online)

Elliptic stable envelopes were introduced by Aganagic and Okounkov as a key tool in the study of K-theoretic quasimap invariants for Nakajima quiver varieties and other symplectic resolutions. They bear an interesting relation to symplectic duality: the stable envelopes of a dual pair of resolutions M and N should arise from a "duality interface" living on the product $M \times N$. I will give a gentle introduction to some of these ideas. I will then describe joint work with Artan Sheshmani and Shing-Tung Yau which uniformizes the duality interface of a hypertoric dual pair, yielding a distinguished K-theory class on an affine analogue of $M \times N$.

Vamsi Pingali (IISc Bangalore, India)

The vortex ansatz as a fertile testing ground for certain systems of PDEs

Fri 17 11:30

I shall review the vortex bundle construction of García-Prada. Then I shall proceed to discuss the vortex ansatz as a way to dimensionally reduce several interesting systems of PDEs like the Kähler–Yang–Mills equations (and its offshoots) and the vector bundle Monge–Ampère equation. I shall discuss the latter in detail and end with an advertisement for studying fully nonlinear systems of PDEs arising from vector bundles (and the fact that the vortex ansatz can prove to be a powerfultesting ground for them).

Sushmita Venugopalan (IMSc Chennai, India)

Vortices on non-compact Riemann surfaces

Thu 16 11:30

Symplectic vortices on punctured Riemann surfaces are related by a Hitchin–Kobayashi correspondence to gauged maps satisfying a semistability condition. These objects show up in various settings in gauged Gromov–Witten theory, such as in the definition of the quantum Kirwan map by Ziltener and Woodward using affine vortices; and in quasimaps to GIT quotients defined by Kim, Ciocan-Fontanine and Maulik.

Richard Wentworth (University of Maryland, USA)

Conformal limits of parabolic Higgs bundles

Fri 17 14:30 (online)

Gaiotto introduced the notion of a conformal limit of a Higgs bundle and conjectured that these should identify the Hitchin component with the oper stratum in the de Rham moduli space. In the case of closed Riemann surfaces this result was proven by Dumitrescu et al., and the limits were shown to exist much more generally by Collier and the speaker. In this talk I will report on progress in the case of parabolic Higgs bundles, which were the context of Gaiotto's original conjecture. (This is joint work with B. Collier and L. Fredrickson.)

Schedule

The program will adopt the following daily schedule throughout the two weeks:

08:00–10:00	breakfast (served at the cafeteria)
10:00–10:50	lecture slot 1
11:00–11:30	morning coffee/tea break (served by the lecture hall)
11:30–12:20	lecture slot 2
12:45–14:00	lunch (served by the lecture hall)
14:30–15:20	lecture slot 3
15:30–16:30	evening tea/snack (served at the cafeteria)
16:30–17:20	lecture slot 4
17:30–18:20	lecture slot 5
18:30–19:20	lecture slot 6
19:00–21:00	dinner (served at the cafeteria)

The assignment of slots will be as follows.

Week 1: *Minicourses*

Slots	Monday 6	Tuesday 7	Wednesday 8	Thursday 9	Friday 10
1	Speight I	Garner I	Garner II	Garner III	Speight IV
2	Romão I	Speight II	García-Prada II	García-Prada III	Romão IV
3	García-Prada I	Romão II	Speight III	Romão III	García-Prada IV
4					Garner IV
5					
6	Woodward I	Woodward II	Woodward III	Woodward IV	

Additional events on Monday 6:

09:50	Informal welcoming address by the Organizers
15:30	Official welcoming session by ICTS Director Prof. Rajesh Gopakumar

Week 2: *Keynote lectures and research talks*

Slots	Monday 13	Tuesday 14	Wednesday 15	Thursday 16	Friday 17
1	Dimofte I	Dimofte II	McBreen	Teleman I	Teleman II
2	Oh I	Álvarez-Cónsul	Dedushenko (i)	Venugopalan	Pingali
3	Ferrari	García-Fernandez		Doan	Wentworth
4	Bullimore		Hilburn		
5	Manton	Oh II	Halpern-Leistner	Dedushenko (ii)	
6		Favero I	Favero II	Liu	

Weekend: Suggestions

Excursion to Belur, Halebid and Shravanabelagola

We will organize a one-day trip on Saturday 11th to visit some highlights of Hindu architecture in Karnataka: notably, the highly ornate temples that still remain at Belur and Halebid, historical capitals of the flourishing Hoysala kingdom, after their destruction in the 14th century by the armies of Delhi. These sites are within relatively close distance of Hassan, a small city by Indian standards, where we will have our main meal (i.e. dinner) at a local restaurant before heading back.

Please note that this is a full-day program, requiring departure from ICTS at 6:00 am, by air-conditioned minibus, to enjoy the daylight (but avoid the scorching heat) at the monuments. On our way out, we shall make a stop (around 7:30 am) at Shravanabelagola to hike in Indragiri and Chandragiri, two hills surrounding the village. Here, we will also be able to visit Jain temples featuring e.g. the famous 10th-century monolithic statue of Gommateshwara, one of the largest free-standing sculptures in the world; but only after a barefoot climb of 700 steps carved in a steep granite slope — no shoes allowed, so recommended for the early morning when the stone steps have not yet heated up. We should be back to ICTS around 11:00 pm.

Estimated cost per person: ₹2,800 (dinner not included).

Hampi adventure

Hampi is the name given today to the village next to the ruins of the once great city of Vijayanagara, the capital of a vast empire controlling most of Southern India and marking the golden age of medieval Hindu civilization. The magnificent old city, which might have reached one million inhabitants at its zenith, was plundered and then razed to the ground in 1565 after a clash of cultures with the sultanates of the Deccan. It is now a UNESCO site and arguably the most rewarding tourist destination in Karnataka. You need at the very least two full days to explore the richness of this site and to enjoy its pleasant rural surroundings. The trip is usually feasible by night train from Bangalore (with berths, leaving late on Friday and coming back in the early hours of Monday); however, we have just checked that the regular tickets for this so-called 'Hampi express' have already sold out for the weekend within our program. An alternative is to make the same route by coach, which usually takes less time than the train, and can be booked at short notice. You would need to organize the trip by yourself, including (very basic) accommodation for one night in the area.

Exploring Bangalore

The city of Bangalore has made its reputation for other reasons than being an obvious tourist destination, but it is nonetheless worth exploring for some colonial architecture, parklands and art galleries. Such an outing is straightforward to organize: you can just take advantage of the free shuttle connecting ICTS with IISc, another research institute in a more central location, and then perhaps move forward by cab/auto-rickshaw or using the underground, which is easy to navigate.

Most trending right now is the Museum of Art and Photography (MAP), which is scheduled to open to the public on February 18th, the Saturday *after* our program closes. If you cannot visit in person (perhaps because you are attending our program remotely!), we still encourage you to have a virtual tour of the collection on their well-developed website: `https://map-india.org/`

Practicalities

- Venue: Ramanujan Lecture Hall
- Address: ICTS, Survey No. 151, Shivakote, Hesaraghatta Hobli, Bengaluru 560 089, India
- Email contacts:
  - ICTS Programs Office & Organizers: `vortex@icts.res.in` (dedicated to our program)
  - ICTS Guest House: `guesthouse@icts.res.in` (a printer is available here)
- Phone contacts:
  - ICTS Front Desk: +91 91 6475 1100 (also on WhatsApp)
  - ICTS Programs Office: +91 80 4653 6052 (Anupama Murali)
  - ICTS Guest House Reception: +91 80 4653 6400 (available 24/7)
  - ICTS Covid Emergency Response Team: +91 91138 71394, +91 94482 87712
- Program webpage: `https://www.icts.res.in/program/Vort2023`

- Shuttle service ICTS ↔ IISc: `https://www.icts.res.in/facilities/transport`
- Train travel: `https://www.ixigo.com/trains`

  Train tickets have various price categories (depending on the train and level of comfort, among other things). Popular routes often sell out; in this case, one can consider getting a waitlist ticket. Alternatively, one could get a *tatkal* ticket; this category of tickets only becomes available on the day before the journey after 10 am, and it incurs an extra fee.
- Swimming pool: please note that wearing a swimming cap is compulsory. You can buy one from the reception for ₹160.
- Money: there is an ATM at the ICTS campus, by the entrance to the Sports Centre.
- Coffee: pehaps the best option within the ICTS campus is the bar/cafeteria of the academic block, which has a functioning espresso machine (₹10 for a cup).
- On-site Wi-Fi access: SSID: `VORT2023` Password: `Rtm@17m$`