“Order-by-disorder” in the Hilbert space and anomalous thermalization in Rokhshar-Kivelson models

Arnab Sen

Indian Association for the Cultivation of Science, Kolkata

Frustrated Metals and Insulators, ICTS, Bangalore

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Saptarshi Biswas (IISER K) **Debasish Banerjee (SINP)**

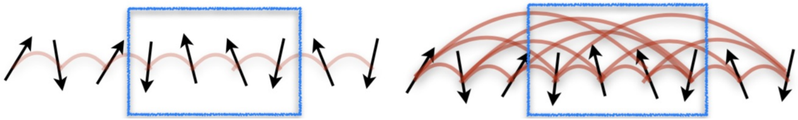
- Banerjee and AS, Phys. Rev. Lett. 126, 220601 (2021)
- Biswas, Banerjee and AS, SciPost Phys. 12, 148 (2022)

Mukherjee, Banerjee, Sengupta and AS, Phys. Rev. B 104, 155117 (2021) [PRB Editors' Suggestion]

Thermalization of macroscopic systems

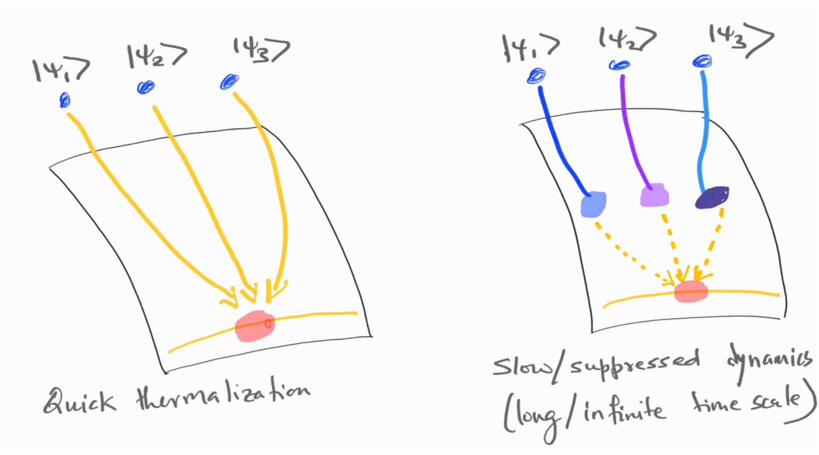
- Non-equilibrium states go to unique equilibrium state when left on their own (i.e. become macroscopically indistinguishable)

How do macroscopic quantum systems self-thermalize?

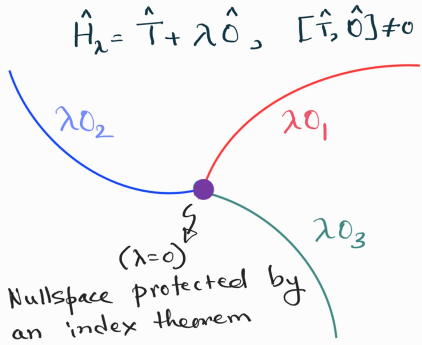


- $|\psi(t)\rangle = U(t)|\psi(0)\rangle = e^{-iHt}|\psi(0)\rangle$
  Paradox: unitary evolution cannot erase quantum information
- Spreading of quantum entanglement moves information about the initial state to highly non-local operators [Nandkishore, Huse, Annu. Rev. Condens. Matter Phys. (2015)]

Eigenstate **T**hermalization **H**ypothesis: Many-body eigenstates of local Hamiltonians with a finite energy density appear "thermal" for local operators [Deutsch (1991), Srednicki (1994), Rigol, Dunjko, Olshanii(2008)]

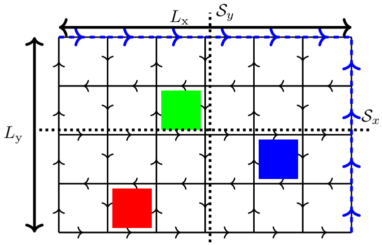


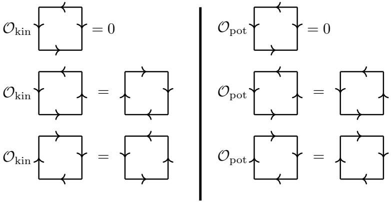
- Can thermalization be evaded is disorder-free systems? Answer: YES. Focus on some paradigmatic models of quantum magnetism

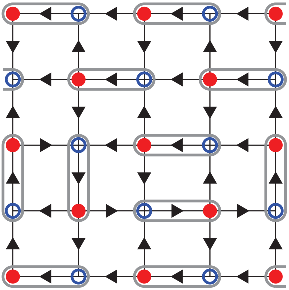


- An index theorem protects a macroscopic number of exact zero modes at $\lambda = 0$ (the mother point)
- Adding suitable non-commuting interactions may provide a route to anomalous thermalization in such theories

Quantum link and dimer models

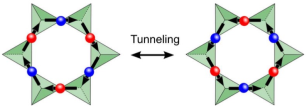


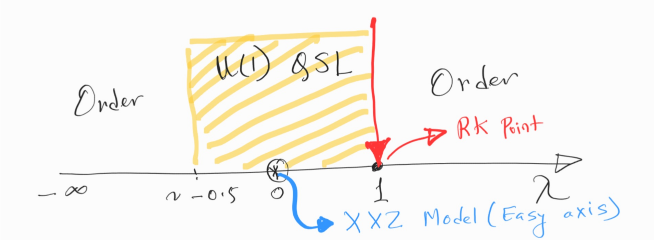




Rokshar-Kivelson models

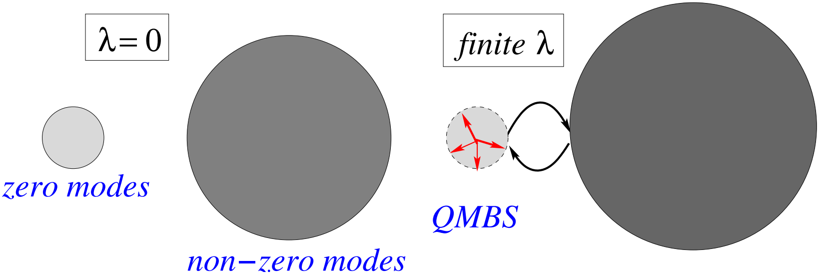
$$\mathcal{H}_{\mathrm{RK}} = -\sum_{\text{plaq.}} [|\circlearrowright\rangle\langle\circlearrowleft| + |\circlearrowleft\rangle\langle\circlearrowright|] + \lambda \sum_{\text{plaq.}} [|\circlearrowright\rangle\langle\circlearrowright| + |\circlearrowleft\rangle\langle\circlearrowleft|]$$





[Banerjee, Isakov, Damle, Kim, PRL (2008); Shannon, Sikora, Pollmann, Penc, Fulde, PRL (2012)]

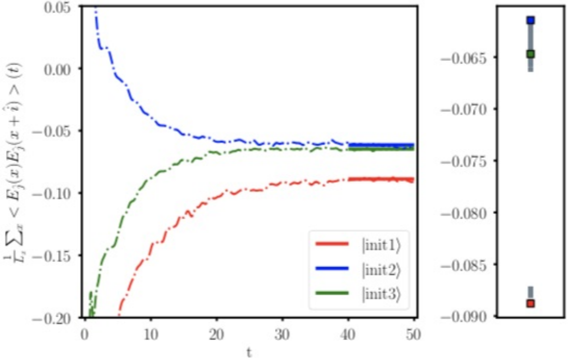
“Order-by-disorder” in the Hilbert space



- Some [$O(1)$/$O(V^a)$/$O(e^V)$] very special linear combinations of the zero modes also diagonalize $\mathcal{O}_{\mathrm{pot}}$ and hence $\mathcal{H}_{\mathrm{RK}}$ at any λ
- Such anomalous zero modes violate the **ETH** (also called **Q**uantum **M**any-**B**ody **S**cars) terminology of QMBS introduced in Turner et al., Nat Phys (2018)

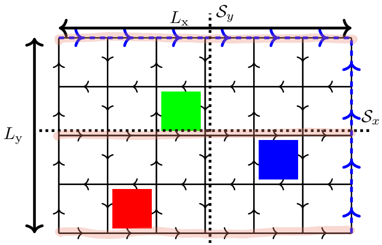
[Banerjee, AS, PRL (2021); Biswas, Banerjee, AS, SciPost Phys. (2022)]

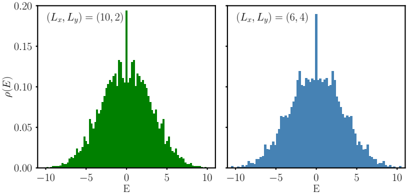
Memory effect in unitary dynamics



- Out of **6433** initial states with average energy $\lambda N_p/2$, 18 have overlap with the anomalous zero modes for QLM with $(L_x, L_y) = (14, 2) = 56$ spins
- Gauss Law $\rightarrow$ 4815738, $(W_x, W_y) = (0, 0) \rightarrow$ 1232454, $(k_x, k_y) = (0, 0) \rightarrow$ 44046

$\lambda = 0$ (The mother point)





- H anticommutes with $\mathcal{C} = \prod_{\mathbf{r},\hat{\mu}} E_{\mathbf{r},\hat{\mu}} \Rightarrow \mathcal{C}|E\rangle = |-E\rangle$
- H also commutes with space reflections about certain axes
- This point-group symmetry $\mathcal{S}_{x,y}$ commutes with $\mathcal{C}$
- $\rho(E) = \alpha\delta(E = 0) + \rho_{\text{reg}}(E)$, $E = 0$ states protected by an index theorem [Schecter and Iadecola (PRB, 2018)]

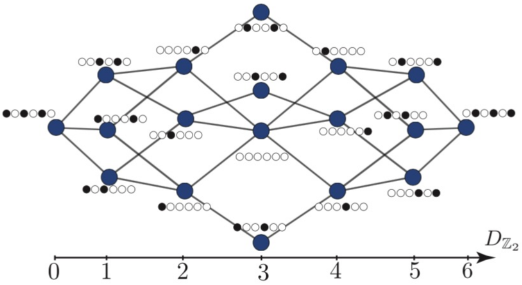
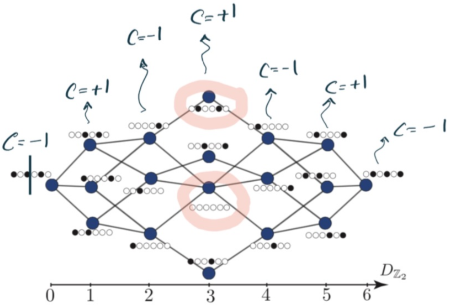


Fig. from Turner, Michailidis, Abanin, Serbyn, Papić, arXiv: 1806.10933 for PXP model

- Emergent bipartite hopping structure in Fock space with two "sublattices" ($\mathcal{C} = \pm 1$) due to $\{H, \mathcal{C}\} = 0$
- For any bipartite lattice s.t. $N_A \neq N_B$ where particle can only hop between A to B (assume zero onsite energy), $\#$ of zero modes $\geq |N_A - N_B|$ [Sutherland, PRB 34, 5208 (1986)]



- Let $N_{P\mathcal{C}}$ be the $\#$ of states with $P = \pm$ and $\mathcal{C} = \pm$. Given $|\psi\rangle$, superpositions $|\psi\rangle \pm P|\psi\rangle$ have $P = \pm$
- Let $\mathcal{N}$ be $\#$ of Fock states with $P = +$ (but without partner at $P = -$). Such states necessarily have $\mathcal{C} = +$
- $N_{++} = N_{-+} + \mathcal{N}$ and $N_{+-} = N_{--}$ from which $\#$ of zero modes $|N_{++} - N_{+-}| + |N_{-+} - N_{--}| \geq \mathcal{N}$ ($\mathcal{N} \sim \alpha^{V/2}$ if total HSD $\sim \alpha^{V}$)

- From ETH, a "typical" zero mode should resemble a featureless $T \to \infty$ state

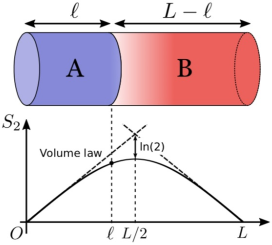
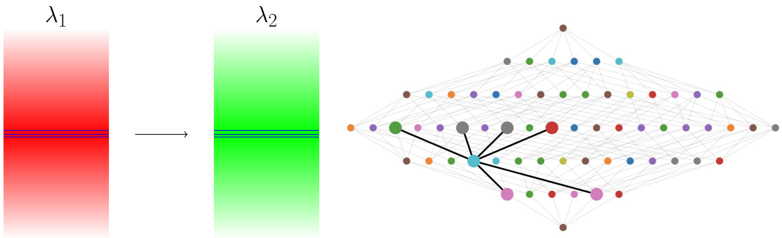


Fig. from Nakagawa, Watanabe, Fujita, Sugiura, arXiv: 1703.02993

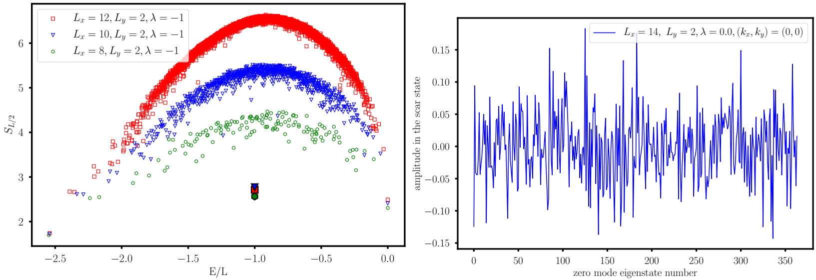
- $\rho_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{B}} |\Psi\rangle\langle\Psi| \Rightarrow S_{L/2} = -\mathrm{Tr}[\rho_{\mathrm{A}} \ln \rho_{\mathrm{A}}]$
- Typical high-energy eigenstates follow volume law scaling for entanglement entropy unlike ground states (area law)

Anomalous Zero modes (Quantum many-body scars)



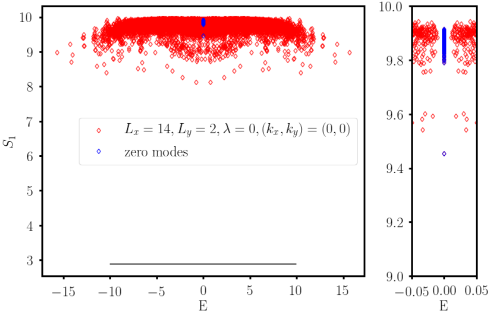
- $|\psi_{\rm QMBS}\rangle = \sum_\alpha c_\alpha |{\rm ZM}_\alpha\rangle$,
  $\mathcal{O}_{\rm kin}|\psi_{\rm QMBS}\rangle = 0$, $\mathcal{O}_{\rm pot}|\psi_{\rm QMBS}\rangle = N|\psi_{\rm QMBS}\rangle$ $(N \sim V)$
  $\mathcal{H}_{\rm RK}(\lambda)|\psi_{\rm QMBS}\rangle = \lambda N|\psi_{\rm QMBS}\rangle$
- QMBS localized in the Hilbert space while *typical* zero modes are not

QMBS in ladders of width $L_y = 2$



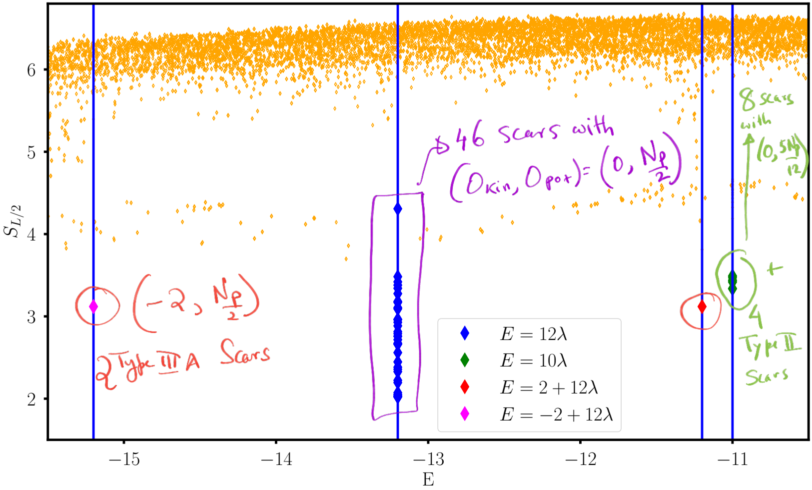
- 4 QMBS each with $(N_p/2, 0)$ as eigenvalues of $(\mathcal{O}_{\text{pot}}, \mathcal{O}_{\text{kin}})$ where N_p equals total # of elementary plaquettes
- QMBS resembles pseudo-random superposition of zero modes!

Localization in the Hilbert space



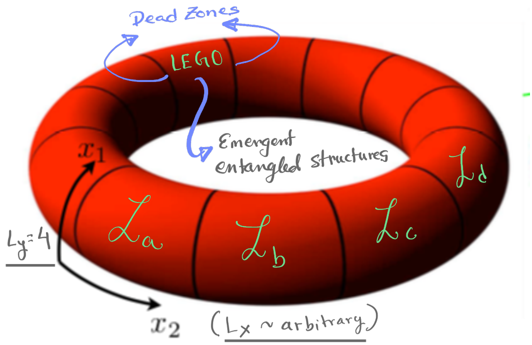
- Shannon entropy $S_1 = -\sum_\alpha |\psi_\alpha|^2 \ln |\psi_\alpha|^2$ where $|\Psi\rangle = \sum_{\alpha=1}^{\mathcal{N}_T} \psi_\alpha |\alpha\rangle$
- $S_1 \sim \ln(\mathcal{N}_T)$ for delocalized state and $S_1 \sim \text{const}$ for localized state
- The individual zero modes at $\lambda = 0$ have a much larger Shannon entropy (S_1) than the QMBS

Wider QLM ladders to approach 2D, more variety!

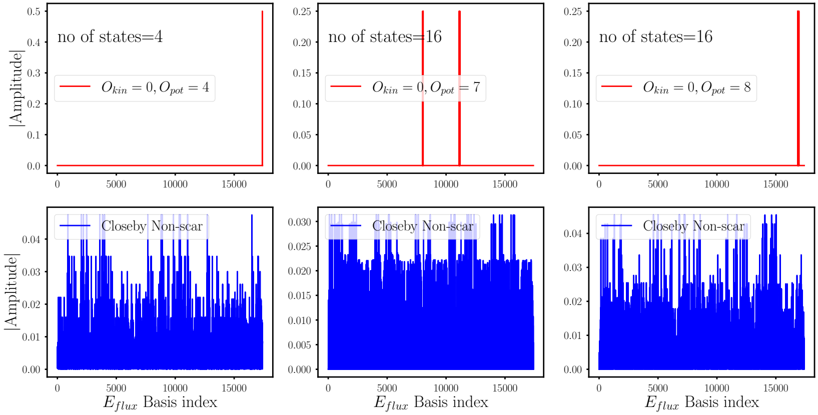


- 46 (106) scars with $\mathcal{O}_{\text{pot}} = \frac{N_p}{2}, \mathcal{O}_{\text{kin}} = 0$ for $L_y = 4$ and $L_x = 6(8)$
- Data for $(L_x, L_y) = (6, 4)$ shown above

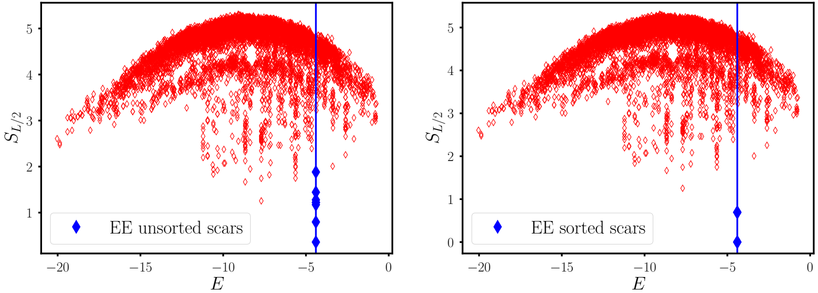
Exact "Lego Scars" in QDM in thin torus limit



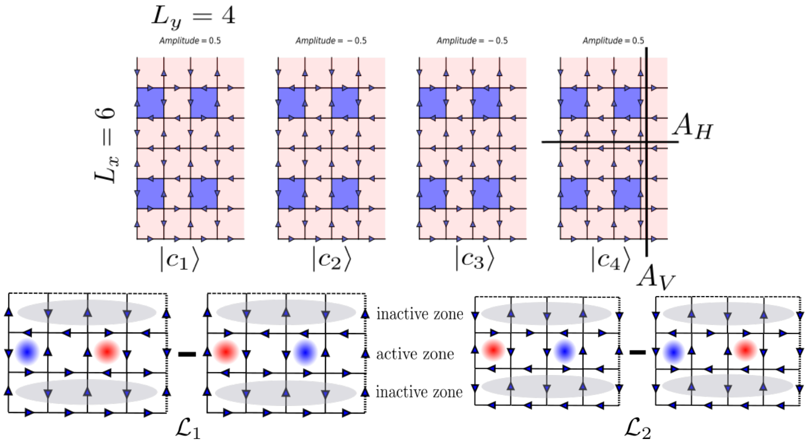
- $|\psi_L\rangle = |\mathcal{L}_i\rangle \otimes |\mathcal{L}_j\rangle \otimes \cdots \otimes |\mathcal{L}_k\rangle$ where we choose type of lego that can fit with each other at the boundaries
- Can be interpreted as states with excitations that exhibit sub-dimensional motion (reminiscent of fractonic models)
- Since legos come in multiple varieties, $\exp(L_x)$ lego scars when $L_x \gg 1$ with $L_y = 4$



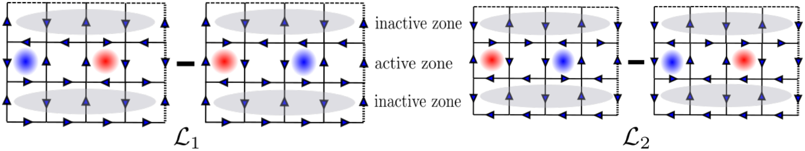
- QDM on ladder with dimension $(L_x, L_y) = (8, 4)$
- Scars with lowest EE (using a minimization algorithm introduced by Reuvers (2018)) shown in the basis of Fock states after using an EE minimization procedure



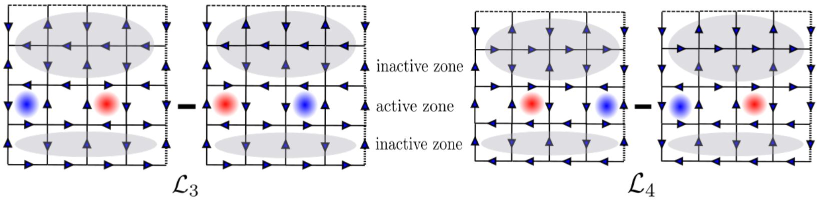
- Data for QDM in the $(W_x, W_y) = (1, 0)$ winding number sector with $\lambda = -1.1$ for $(L_x, L_y) = (8, 4)$
- The blue dots represent 8 quantum scars that have a well-defined $(\mathcal{O}_{\text{kin}}, \mathcal{O}_{\text{pot}}) = (0, 4)$



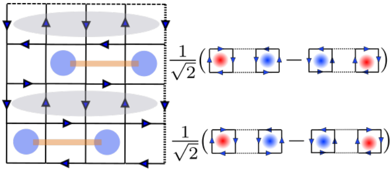
- $|\psi_L\rangle = |\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle$ gives this state for $L_x = 6$
- $(|\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle) \otimes (|\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle) \otimes \cdots (|\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle)$ for $L_x = 6n$ inactive zones



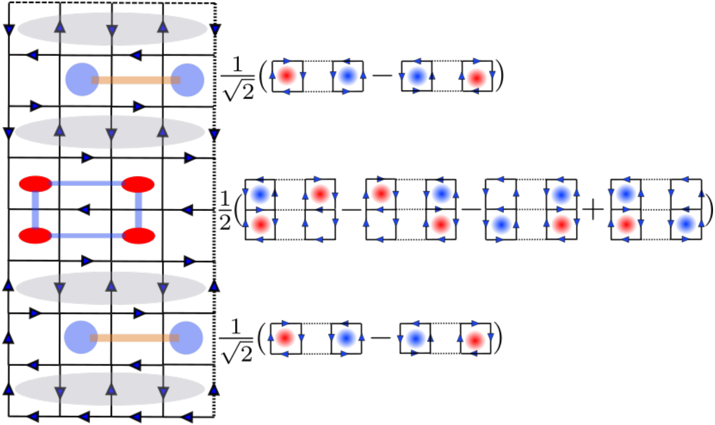
- The sign structure inside the legos crucial in giving rise to the inactive zones
- $\mathcal{O}_{\text{kin}}\left(\boxed{C}\boxed{U}\boxed{A}\boxed{U} - \boxed{A}\boxed{U}\boxed{C}\boxed{U}\right) = \left(\boxed{A}\boxed{C}\boxed{A}\boxed{C} + \boxed{C}\boxed{U}\boxed{C}\boxed{U} - \boxed{C}\boxed{U}\boxed{C}\boxed{U} - \boxed{A}\boxed{C}\boxed{A}\boxed{C}\right) = 0$
- $\mathcal{O}_{\text{kin}}\left(\boxed{A}\boxed{C}\boxed{A}\boxed{C}\right)$ would have taken the excitations through the inactive zones if these did not cancel due to the sign structure



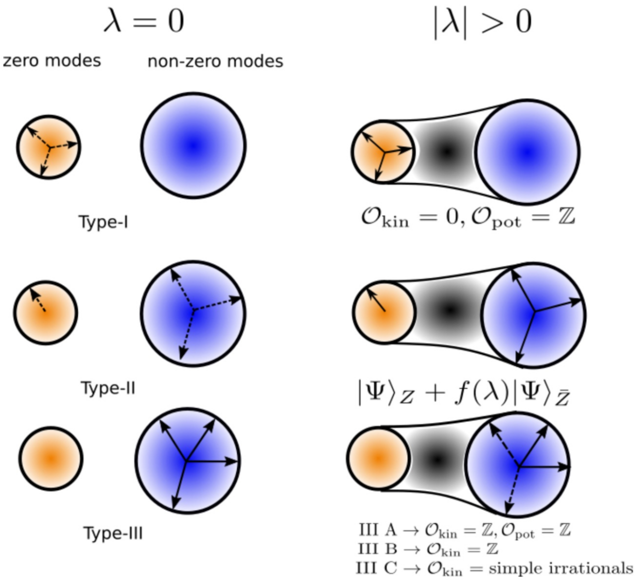
$$|\psi_{L1}\rangle = |\mathcal{L}_3\rangle \otimes |\mathcal{L}_4\rangle, \qquad |\psi_{L2}\rangle = |\mathcal{L}_5\rangle \otimes |\mathcal{L}_5\rangle$$



$$|\psi_{L3}\rangle = |\mathcal{L}_6\rangle$$

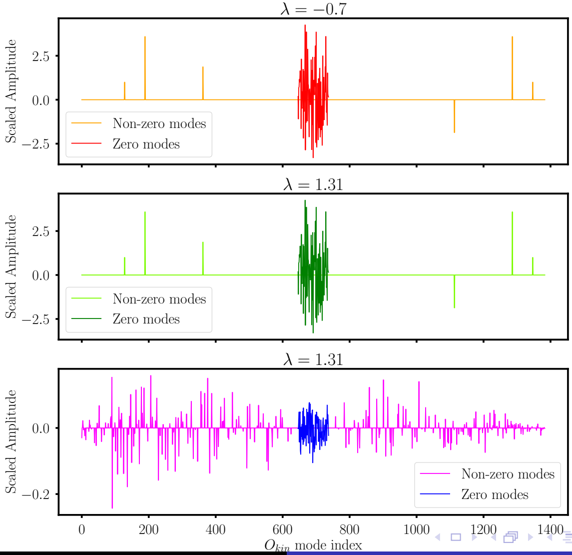


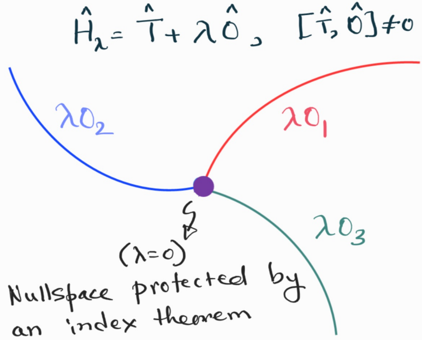
The full variety



Type II scars for QLM on $(L_x, L_y) = (6, 4)$

- $|\mathrm{Scar}\rangle = |\psi_{\mathrm{z}}\rangle + f(\lambda)|\psi_{\mathrm{nz}}\rangle$ with $H(\lambda)|\mathrm{Scar}\rangle = (5N_p/12)\lambda$
- $\langle \mathcal{O}_{\mathrm{pot}} \rangle = 5N_p/12$ and $\langle \mathcal{O}_{\mathrm{kin}} \rangle = 0$





- Several well-known LGTs and condensed matter systems harbor such points
- Non-Abelian LGTs and fermionic models with such index theorem points being explored