Sublattice scars and triangle relation in an Abelian lattice gauge theory

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Stability of quantum matter in and out of equilibrium at various scales, ICTS

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Collaborators

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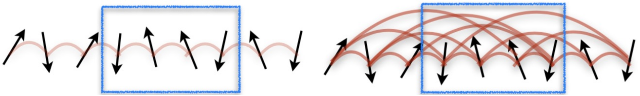
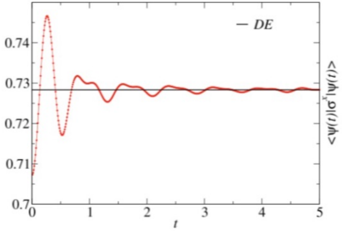
- Sau, Banerjee, AS, *to appear*
- Sau, Stornati, Banerjee, AS, arXiv:2311.06773
- Biswas, Banerjee, AS, SciPost Phys. 12, 148 (2022)
- Banerjee, AS, Phys. Rev. Lett. 126, 220601 (2021)

How do macroscopic quantum systems self-thermalize?

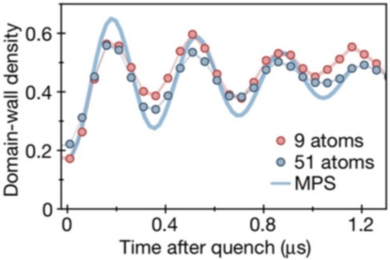


- $|\psi(t)\rangle = U(t)|\psi(0)\rangle = e^{-iHt}|\psi(0)\rangle$
  Paradox: unitary evolution cannot erase QI



Eigenstate **T**hermalization **H**ypothesis: Many-body eigenstates of local Hamiltonians with a finite energy density appear "thermal" for local operators [Deutsch (1991), Srednicki (1994), Rigol, Dunjko, Olshanii(2008)]

Experimental realization of weak ergodicity breaking



- Bernien et al., Nature 551, 579 (2017) realized programmable quantum spin model with tunable interactions $\sim$ 51 qubits.
- Certain initial conditions took much longer to relax

Persistant oscillations in $|1010\cdots\rangle \rightarrow |0101\cdots\rangle \rightarrow |1010\cdots\rangle$ while $|0000\cdots\rangle$ rapidly thermalized

PXP model (approximate scars)

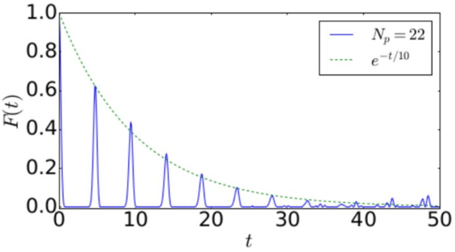
Mott insulators in strong electric fields, Sachdev, Sengupta, Girvin (2002)

- L spin-$1/2$ on a 1D lattice with a constrained Hilbert space, not all 2^L configurations allowed

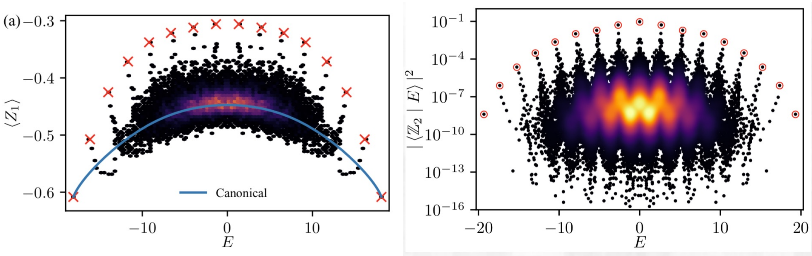
  $\cdots\downarrow\downarrow\cdots,\ \cdots\uparrow\downarrow\cdots,\ \cdots\downarrow\uparrow\cdots,\ \cdots\uparrow\uparrow\cdots$

- HSD for L sites equals $F_{L-1}+F_{L+1}$ where $F_1=F_2=1$ and $F_n+F_{n+1}=F_{n+2}$; *HSD* $=\tau^L$ for $L\gg 1$ where $\tau=\frac{\sqrt{5}+1}{2}$

- $H_{\mathrm{PXP}}=-w\sum_i P^{\downarrow}_{i-1}\sigma^x_i P^{\downarrow}_{i+1}$ $\cdots\downarrow\uparrow\downarrow\cdots\Leftrightarrow\cdots\downarrow\downarrow\downarrow\cdots$



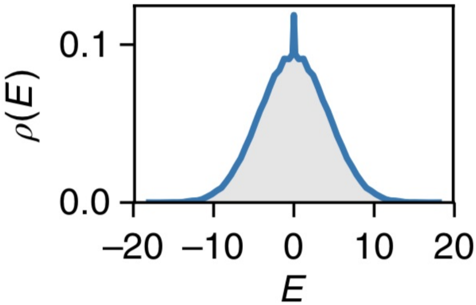
Quantum many-body scars



Turner *et al.*, Nat. Phys. 14, 745 (2018), PRB 98, 155134 (2018)

- Subspace spanned by $O(L)$ special eigenstates which account for most of the Néel state
- Approximately equally spaced eigenvalues
- These eigenstates are highly atypical
  (half-chain entanglement entropy $S \sim \ln L$ instead of $S \sim L$)

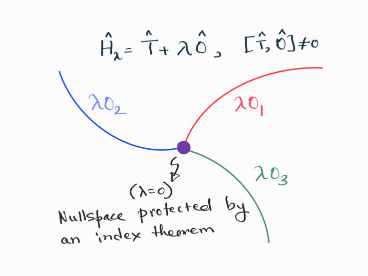
Zero modes in the PXP model

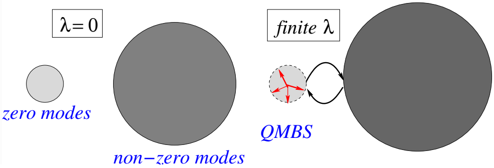


- $\mathcal{C} = \prod_{i=1}^{L} \sigma_i^z$ anticommutes with H
- $\mathcal{C}|E\rangle$ has negative energy wrt $|E\rangle$
- Both H and $\mathcal{C}$ commute with a spatial inversion operator $I : i \to L - i + 1$
- No. of zero modes $\geq \sqrt{\mathrm{HSD}}$

  Protected by an index theorem Schecter and Iadecola (2018)

“Order-by-disorder” in the Hilbert space

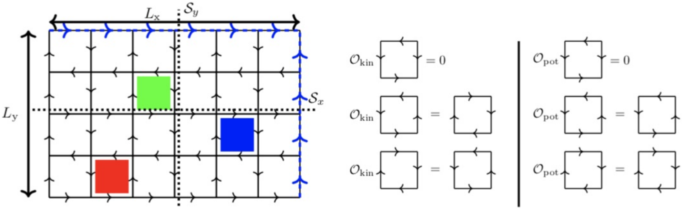


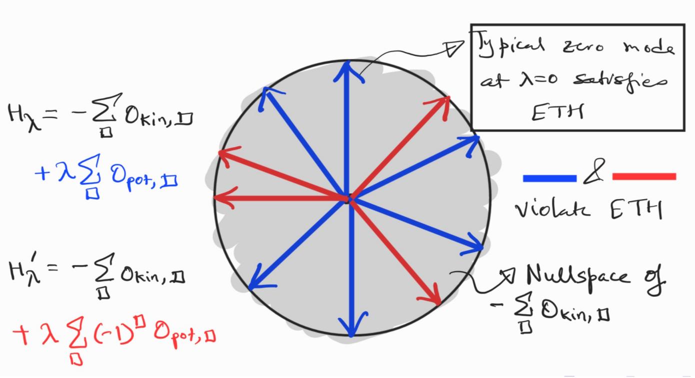


- $\mathcal{T}$ ($\mathcal{O}$) typically has continuous (discrete) spectrum
- Some [$O(1)$/$O(V^a)$/$O(e^V)$] special zero modes of $\mathcal{T}$ also diagonalize $\mathcal{O}$ and hence $\mathcal{H}_\lambda$ at any λ
- Such anomalous zero modes violate the **ETH**

[Banerjee, AS, PRL (2021); Biswas, Banerjee, AS, SciPost Phys. (2022), Sau, Stornati, Banerjee, AS, arXiv: 2311. 06773 (2023)]

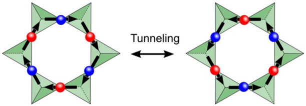
Quantum link model

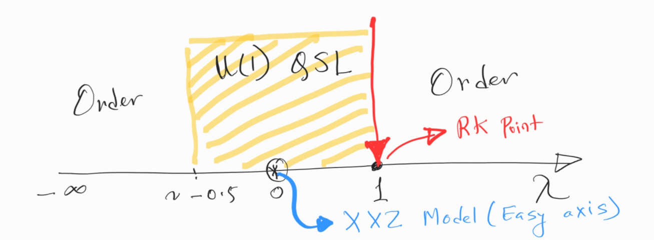




Rokshar-Kivelson models

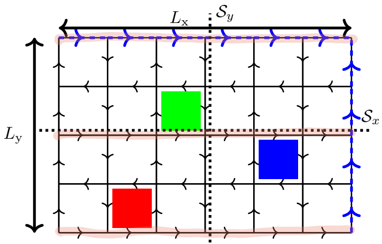
$$\mathcal{H}_{\rm RK} = -\sum_{\text{plaq.}} \left[|\circlearrowright\rangle\langle\circlearrowleft| + |\circlearrowleft\rangle\langle\circlearrowright| \right] + \lambda \sum_{\text{plaq.}} \left[|\circlearrowright\rangle\langle\circlearrowright| + |\circlearrowleft\rangle\langle\circlearrowleft| \right]$$

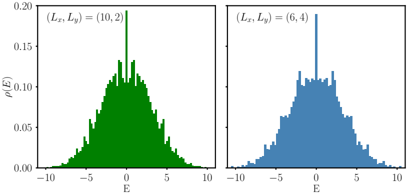




[Banerjee, Isakov, Damle, Kim, PRL (2008); Shannon, Sikora, Pollmann, Penc, Fulde, PRL (2012)]

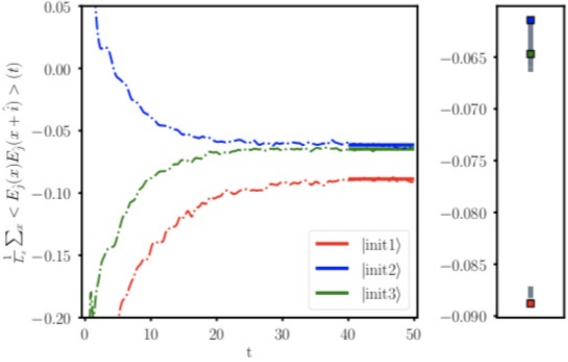
$\lambda = 0$ (The mother point)





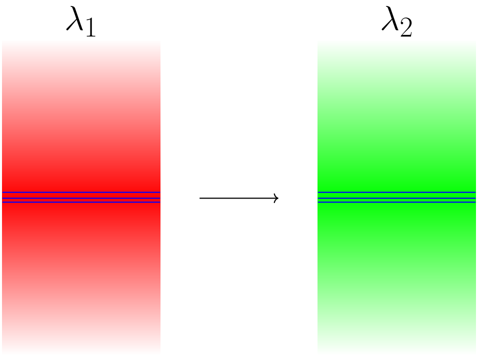
- H anticommutes with $\mathcal{C} = \prod_{\mathbf{r},\hat{\mu}} E_{\mathbf{r},\hat{\mu}} \Rightarrow \mathcal{C}|E\rangle = |-E\rangle$
- H also commutes with space reflections about certain axes
- This point-group symmetry $\mathcal{S}_{x,y}$ commutes with $\mathcal{C}$
- $\rho(E) = \alpha\delta(E = 0) + \rho_{\text{reg}}(E)$, $E = 0$ states protected by an index theorem [Schecter and Iadecola (PRB, 2018)]

Memory effect in unitary dynamics



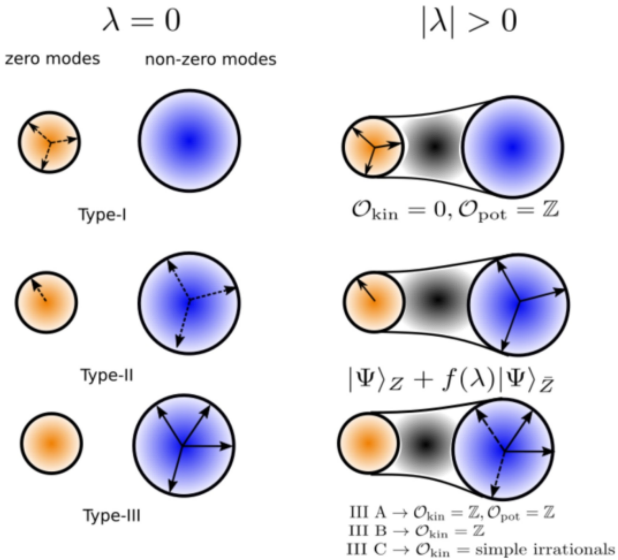
- Out of **6433** initial states with average energy $\lambda N_p/2$, 18 have overlap with the anomalous zero modes for QLM with $(L_x, L_y) = (14, 2) = 56$ spins
- Gauss Law $\rightarrow$ 4815738, $(W_x, W_y) = (0, 0) \rightarrow$ 1232454, $(k_x, k_y) = (0, 0) \rightarrow$ 44046

Anomalous Zero modes (Quantum many-body scars)



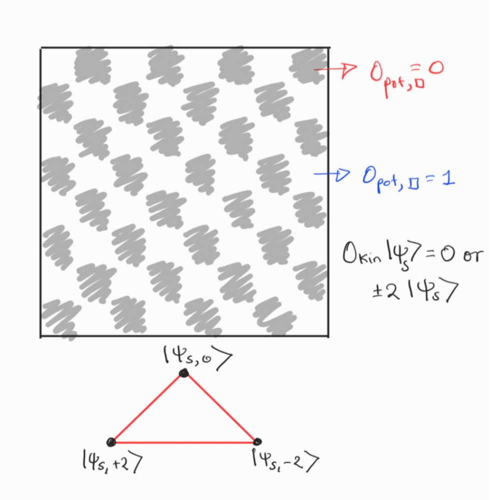
- $|\psi_{\mathrm{QMBS}}\rangle = \sum_\alpha c_\alpha |\mathrm{ZM}_\alpha\rangle$,
  $\mathcal{O}_{\mathrm{kin}}|\psi_{\mathrm{QMBS}}\rangle = 0$, $\mathcal{O}_{\mathrm{pot}}|\psi_{\mathrm{QMBS}}\rangle = N|\psi_{\mathrm{QMBS}}\rangle$ $(N \sim V)$
  $\mathcal{H}_{\mathrm{RK}}(\lambda)|\psi_{\mathrm{QMBS}}\rangle = \lambda N|\psi_{\mathrm{QMBS}}\rangle$
- QMBS localized in the Hilbert space while *typical* zero modes are not

The full variety



Biswas, Banerjee, AS, SciPost Phys. 12, 148 (2022)

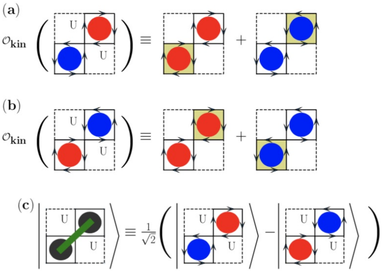
Sublattice scars in QLM

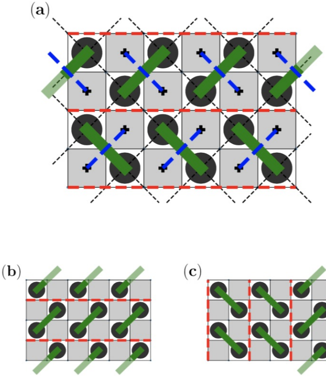


- $\mathcal{O}_{\text{pot},\square}|\psi_s\rangle = 1$, for one sublattice and $\mathcal{O}_{\text{pot},\square}|\psi_s\rangle = 0$ for other sublattice

Sau, Stornati, Banerjee, Sen, arXiv: 2311.06773

Short singlet sublattice scars





- Subset of sublattice scars have short singlet representation
- These arise as sols. of a tiling problem
- For $(L_x, 2)$, **4** such scars. For $(L_x, 4)$, $\mathbf{O(2^{L_x/2})}$ such scars

Sau, Stornati, Banerjee, Sen, arXiv: 2311.06773

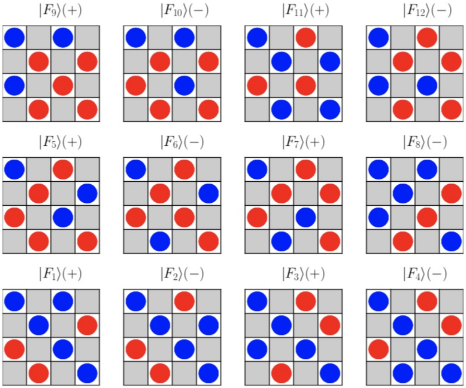
Nonsinglet sublattice scars in QLM

Lattice	Scars in equal A-C sector		Scars in unequal A-C sector
	Singlet scars	Non-singlet scars	
$L_x \times 2$	2	0	0
4×4	10	0	3
6×4	19	3	1
8×4	35	17	1
10×4	67	62	1
6×6	28	1	1

Lattice	Scars with $\mathcal{O}_{\rm kin} = +2$	Scars with $\mathcal{O}_{\rm kin} = -2$
4×4	3	3
6×4	1	1
8×4	1	1
6×6	1	1

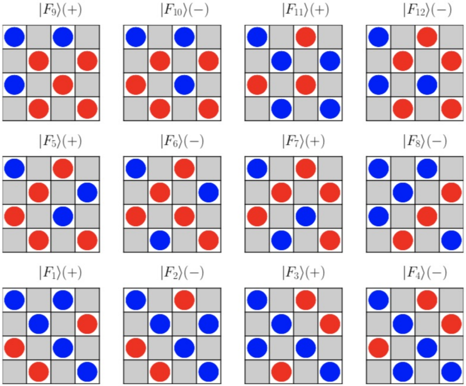
- Sublattice scars are composed of Fock states with one sublattice of unflippable plaquettes and another of flippable plaquettes
- Active sublattice may have $\left(\frac{N_p}{4}, \frac{N_p}{4}\right)$, $\left(\frac{N_p}{4}+1, \frac{N_p}{4}-1\right)$, or $\left(\frac{N_p}{4}-1, \frac{N_p}{4}+1\right)$ clockwise and anticlockwise plaquettes
- Applying $\mathcal{C}$ on Fock states with $\left(\frac{N_p}{4}, \frac{N_p}{4}\right)$ $\left[\left(\frac{N_p}{4}\pm 1, \frac{N_p}{4}\mp 1\right)\right]$ active plaquettes given eigval +1 [-1]

One example in (4, 4) QLM



- Example: $|\psi_{s,0}\rangle = \frac{1}{2\sqrt{6}} \sum_{i=1}^{12} \mathrm{Sign(i)}(|F_i\rangle + \mathbb{C}_E|F_i\rangle)$ s.t. $\mathcal{O}_{\mathrm{kin}}|\psi_s\rangle = 0$ for (4,4) ladder using Fock states with $\left(\frac{N_p}{4}+1, \frac{N_p}{4}-1\right)$ and $\left(\frac{N_p}{4}-1, \frac{N_p}{4}+1\right)$ (A,C) plaquettes

Triangle relation



$\mathbb{O}(|\psi_{s,+2}\rangle - \mathcal{C}|\psi_{s,+2}\rangle) \propto |\psi_{s,0}\rangle \rightarrow |\psi_{s,+2}\rangle - |\psi_{s,-2}\rangle \propto \mathbb{O}|\psi_{s,0}\rangle$
where $\mathbb{O}$ changes (does not change) sign of Fock states with $\left(\frac{N_p}{4}+1, \frac{N_p}{4}-1\right)$ $\left(\left(\frac{N_p}{4}-1, \frac{N_p}{4}+1\right)\right)$ (A,C) plaquettes

(Long-ranged) Parent Hamiltonian for sublattice scars

$$\mathcal{H}_{\mathrm{LR}} = \frac{1}{N_p}\Big(\sum_{\text{plaq.}} \left[|\circlearrowright\rangle\langle\circlearrowleft| + |\circlearrowleft\rangle\langle\circlearrowright|\right]\Big)^2 \qquad (1)$$
$$+c\sum_{\text{plaq.}}(-1)^{x_i+y_i}\left[|\circlearrowright\rangle\langle\circlearrowright| + |\circlearrowleft\rangle\langle\circlearrowleft|\right]$$

where $c > 0$ ($c < 0$) gives one set of sublattice scars as groundstates

This long-ranged model has no simple mean field limit

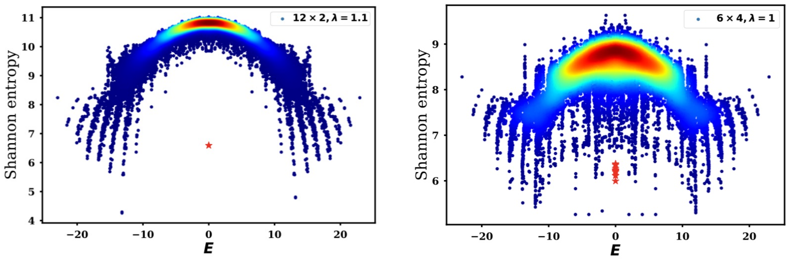
$$\mathbb{O}(|\psi_{s,+2}\rangle - \mathcal{C}|\psi_{s,+2}\rangle) \propto |\psi_{s,0}\rangle \rightarrow |\psi_{s,+2}\rangle - |\psi_{s,-2}\rangle \propto \mathbb{O}|\psi_{s,0}\rangle$$

can be interpreted as creation of an excited state ($|\psi_{s,+2}\rangle - |\psi_{s,-2}\rangle$) by acting $\mathbb{O}$ on a ground state ($|\psi_{s,0}\rangle$)

Zero energy eigenstates of $\mathcal{H}_{\mathrm{LR}}$ stay unchanged with c!

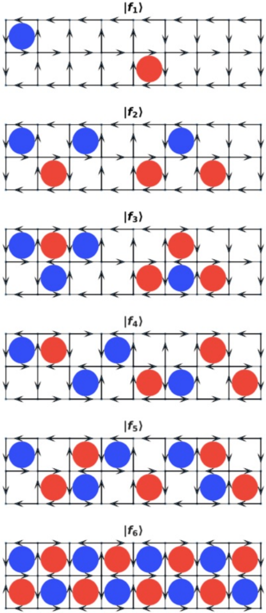
Staggered RK model

$$\mathcal{H}_{\mathrm{st}} = \mathcal{O}_{\mathrm{kin}} + \lambda \mathcal{O}_{\mathrm{pot}} = -\sum_{\square} \mathcal{O}_{\mathrm{kin},\square} + \lambda \sum_{\square} (-1)^{\square} \mathcal{O}_{\mathrm{pot},\square}$$

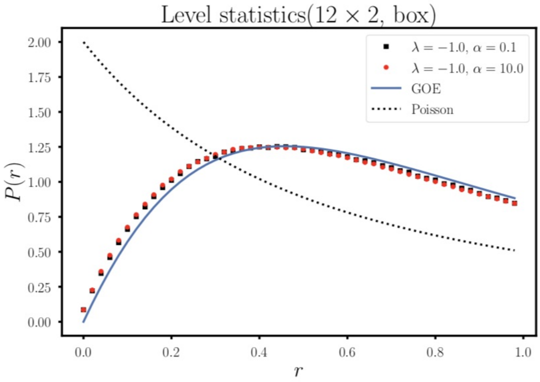


Lattice	Anomalous zero modes of $\mathcal{H}_{\mathrm{st}}$
$L_x \times 2$	2
4×4	8
6×4	14

Rep. states for anomalous zero modes of (8, 2) QLM

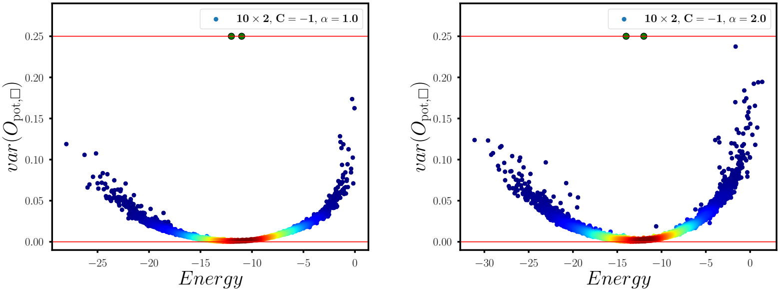


Disorder in QLM? Sau, Banerjee, Sen (to appear)



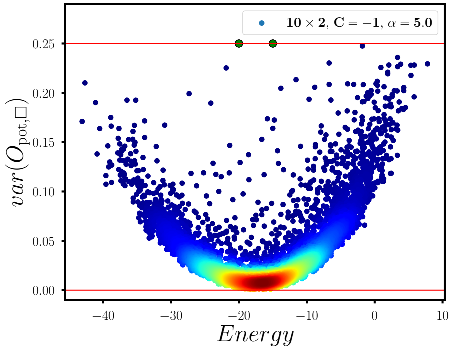
- $H = -\sum_{\square} \mathcal{O}_{\text{kin},\square} + \sum_{\square} \lambda_{\square} \mathcal{O}_{\text{pot},\square}$ where $\lambda_{\square} = \lambda(1 + \alpha r_{\square})$ with $r_{\square} \in (-1/2, 1/2)$. α: disorder strength
- $r = \frac{\min(s_n, s_{n-1})}{\max(s_n, s_{n-1})}$, $s_n = E_{n+1} - E_n$.
  $P(r) = \frac{27}{4} \frac{r+r^2}{(1+r+r^2)^{5/2}} \Theta(1-r)$
- No sign of MBL+ Sublattice scars stable with disorder

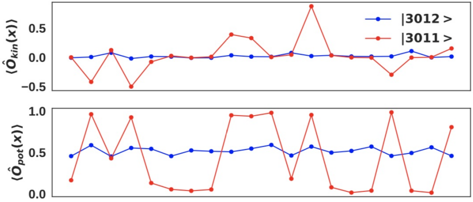
Disorder in QLM? Sau, Banerjee, Sen (to appear)



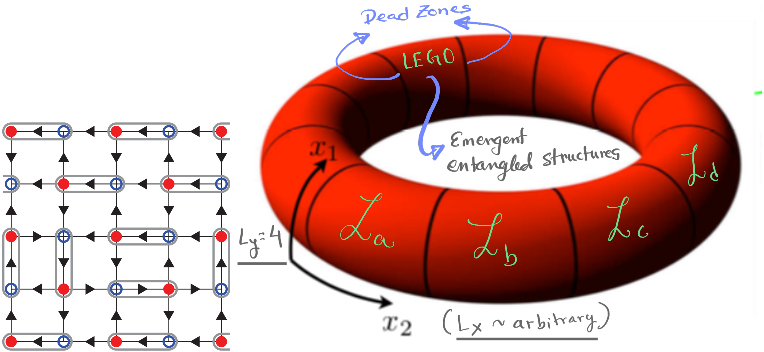
$\mathrm{var}(\mathcal{O}_{\mathrm{pot},\square}) = \frac{1}{\mathrm{N_p}} \sum_{\square}(\langle \mathcal{O}_{\mathrm{pot},\square} \rangle - 1/2)^2$ [Expected to approach 0 from ETH]

Disorder in QLM? Sau, Banerjee, Sen (to appear)



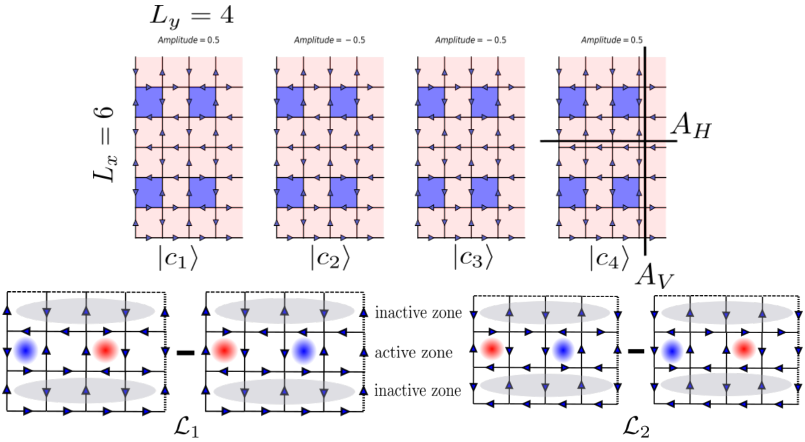


Exact "LEGO Scars" in Quantum Dimer Model in thin torus limit

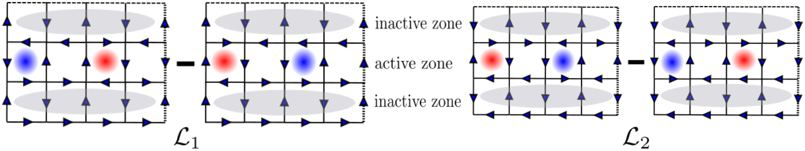


- $|\psi_L\rangle = |\mathcal{L}_i\rangle \otimes |\mathcal{L}_j\rangle \otimes \cdots \otimes |\mathcal{L}_k\rangle$ where we choose type of LEGO that can fit with each other at the boundaries

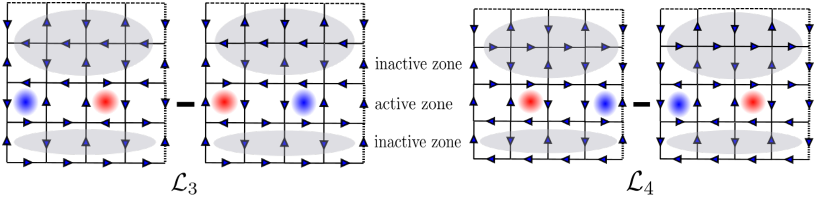
Biswas, Banerjee and AS, SciPost Phys. 12, 148 (2022)



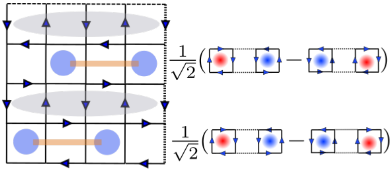
- $|\psi_L\rangle = |\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle$ gives this state for $L_x = 6$
- $(|\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle) \otimes (|\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle) \otimes \cdots (|\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle)$ for $L_x = 6n$



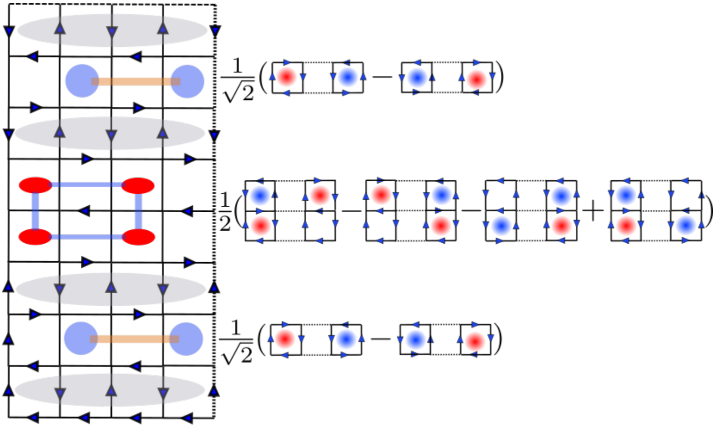
- The sign structure inside the LEGO crucial in giving rise to the inactive zones
- $\mathcal{O}_{\text{kin}}\left(\boxed{\text{C}}\boxed{\text{U}}\boxed{\text{A}}\boxed{\text{U}} - \boxed{\text{A}}\boxed{\text{U}}\boxed{\text{C}}\boxed{\text{U}}\right) =$ $\left(\boxed{\text{A}}\boxed{\text{C}}\boxed{\text{A}}\boxed{\text{C}} + \boxed{\text{C}}\boxed{\text{U}}\boxed{\text{C}}\boxed{\text{U}} - \boxed{\text{C}}\boxed{\text{U}}\boxed{\text{C}}\boxed{\text{U}} - \boxed{\text{A}}\boxed{\text{C}}\boxed{\text{A}}\boxed{\text{C}}\right)$ $= 0$
- $\mathcal{O}_{\text{kin}}\left(\boxed{\text{A}}\boxed{\text{C}}\boxed{\text{A}}\boxed{\text{C}}\right)$ would have taken the excitations through the inactive zones if these did not cancel due to the sign structure



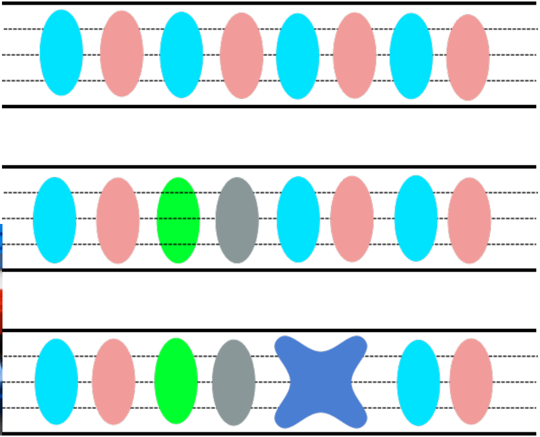
$$|\psi_{L1}\rangle = |\mathcal{L}_3\rangle \otimes |\mathcal{L}_4\rangle, \qquad |\psi_{L2}\rangle = |\mathcal{L}_5\rangle \otimes |\mathcal{L}_5\rangle$$



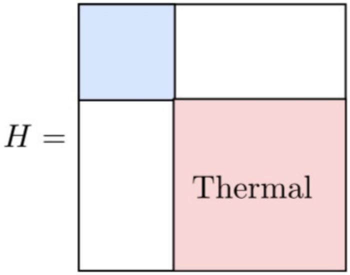
$$|\psi_{L3}\rangle = |\mathcal{L}_6\rangle$$

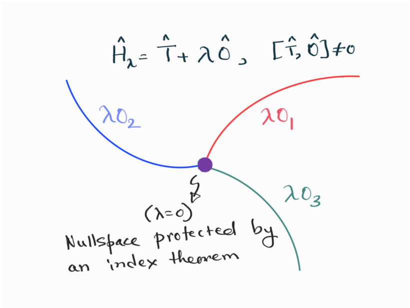


LEGO game for large L_x



Since LEGOs come in multiple varieties, $\exp(L_x)$ lego scars when $L_x \gg 1$ with $L_y = 4$





- *Order-by-disorder but in the Hilbert space*
- Sublattice scars and lego scars suggest emergent fractons
- Triangle relation connects scars with $\mathcal{O}_{\text{kin},\square} = \pm 2$ and $\mathcal{O}_{\text{kin},\square} = 0$ for QLM

Thank you!

Index Theorem (Supp)

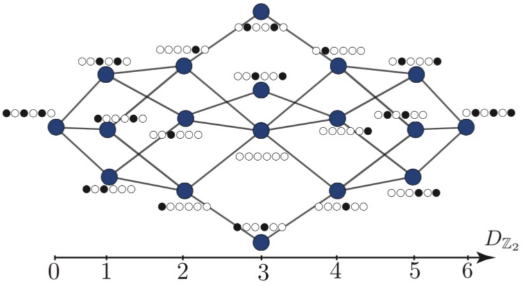
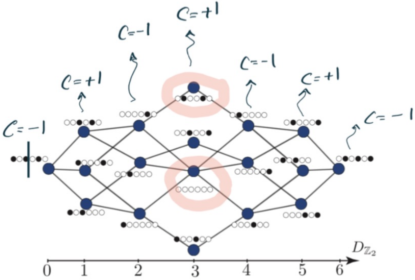


Fig. from Turner, Michailidis, Abanin, Serbyn, Papić, arXiv: 1806.10933 for PXP model

- Emergent bipartite hopping structure in Fock space with two "sublattices" ($\mathcal{C} = \pm 1$) due to $\{H, \mathcal{C}\} = 0$
- For any bipartite lattice s.t. $N_A \neq N_B$ where particle can only hop between A to B (assume zero onsite energy), # of zero modes $\geq |N_A - N_B|$ [Sutherland, PRB 34, 5208 (1986)]

Index Theorem (Supp)



- Let $N_{P\mathcal{C}}$ be the # of states with $P = \pm$ and $\mathcal{C} = \pm$. Given $|\psi\rangle$, superpositions $|\psi\rangle \pm P|\psi\rangle$ have $P = \pm$
- Let $\mathcal{N}$ be # of Fock states with $P = +$ (but without partner at $P = -$). Such states necessarily have $\mathcal{C} = +$
- $N_{++} = N_{-+} + \mathcal{N}$ and $N_{+-} = N_{--}$ from which # of zero modes $|N_{++} - N_{+-}| + |N_{-+} - N_{--}| \geq \mathcal{N}$ ($\mathcal{N} \sim \alpha^{V/2}$ if total HSD $\sim \alpha^V$)