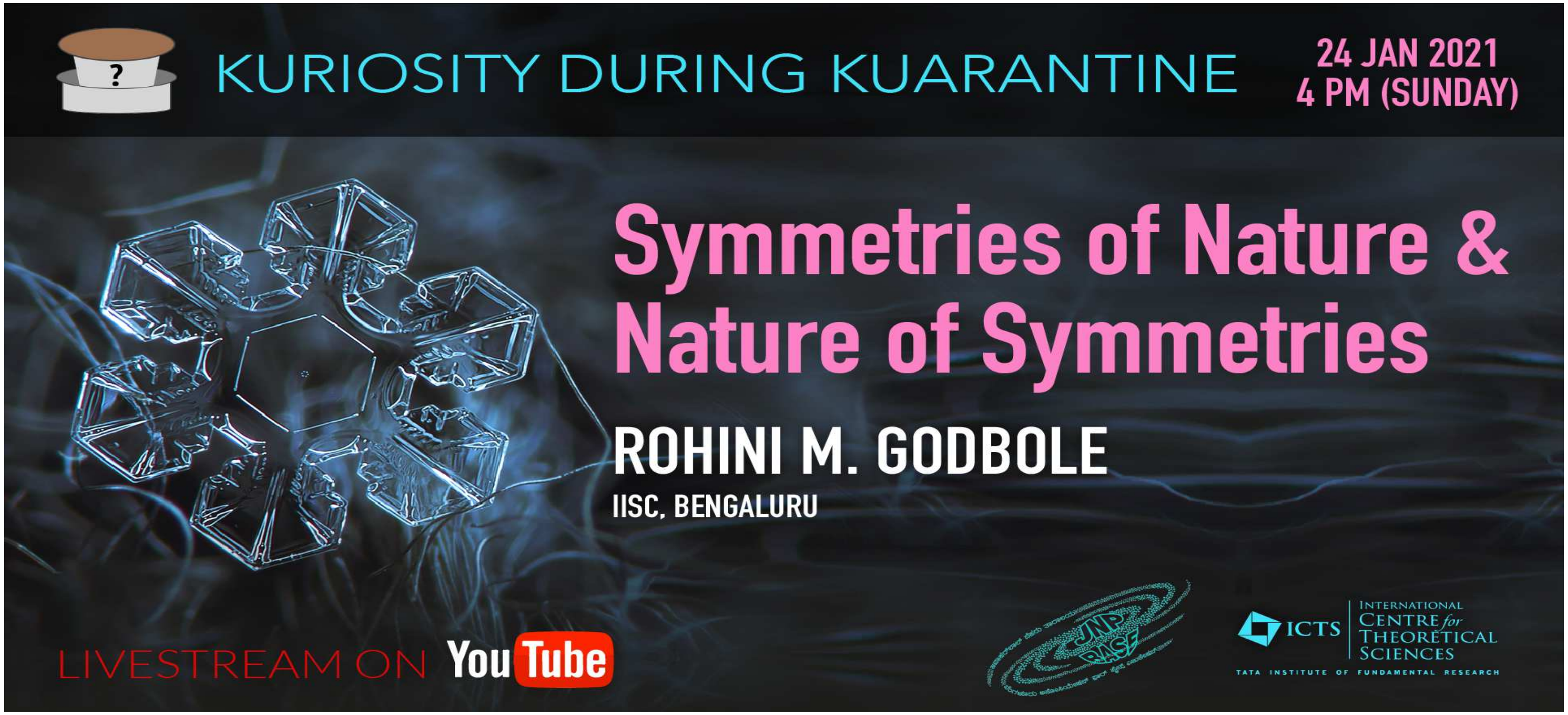
Symmetries of Nature and Nature of Symmetries.

Rohini M. Godbole
Centre for High Energy Physics, IISc, Bangalore, India



Kaapi With Kuriosity $\Longrightarrow$ Kuriosity During Kuarantine

KwK $\Longrightarrow$ KdK

Pandemic.

- What do we mean by symmetry? Symmetry, invariance and all that.

- Symmetry, invariance and conservation laws : Contribution of Emmy Noether.

- Symmetry Breaking: Higgs and all that.

Symmetries of Nature: Many naturally occurring patterns are symmetric!

$\Downarrow$

Symmetric things are Beautiful!

$\Downarrow$

Truths of nature are always beautiful! (we have it on good authority: Aristotle, A. Einstein, S. Chandrasekhar)

$\Downarrow$

So hunting for symmetries for laws of nature has led us to uncover truth about laws of nature.

Symmetries of Nature:

a) Symmetries we observe in nature all around us!

b)Symmetries that the equations encapsulating laws of nature governing the physical world exhibit!

Nature of symmetries:

A mathematical framework to describe the symmetry in terms of things remaining the same under specific operations. (Mathematician Galois!).

$$\Downarrow$$

In what sense 'things' remain the same? (Emmy Noether: Giant of Mathematics)

$$\Downarrow$$

Helped theoretical physicists to uncover many a secrets of Nature!

Symmetry is very simple to understand and at the same time very profound

Symmetry can be put to practical use: design planes or devices that are left right symmetric! (Animal Kingodom!)

Symmetry produces things of beauty which please the eye, mind in art: painting, music...

Symmetry ideas have provided a clarity to our scientific thinking and lit up paths for future explorations of the truth of nature.

Things that are symmetric are quite often quite beautiful!

Is truth always beautiful?

"Beauty is truth, truth beauty,"-that is all
Ye know on earth, and all ye need to know.

-JOHN KEATS, Ode on a Grecian Urn

Aristotle:

Nature always chooses the best option

The chief forms of beauty are order and symmetry and definiteness, which the mathematical sciences demonstrate in a special degree.

(The Complete Works of Aristotle Barnes ed., volume 2, 1705, 1078a36)

H. Bondi ,talking about EInstein (1879-1955):

What I remember most clearly was that when I put down a suggestion that seemed to me cogent and reasonable, Einstein did not in the least contest this, but he only said, "Oh, how ugly." As soon as an equation seemed to him to be ugly, he really rather lost interest in it and could not understand why somebody else was willing to spend much time on it. He was quite convinced that beauty was a guiding principle in the search for important results in theoretical physics.

H. Weyl (1885-1955)

I always tried to unite truth with the beautiful but when I had to choose one or the other I always chose beautiful

S. Chandrasekhar(1910-1995)

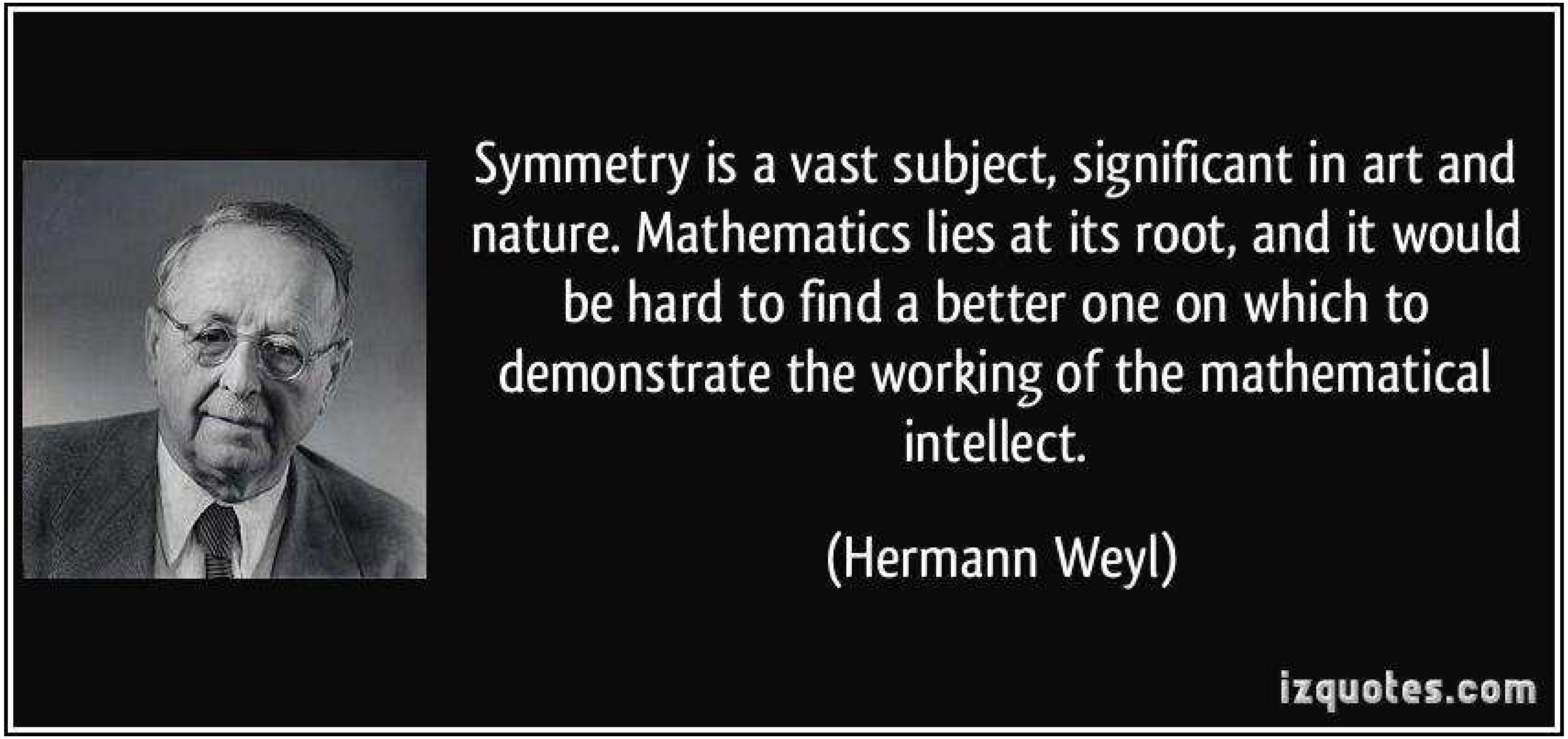
Book :Truth and Beauty: Aesthetics and Motivations in Science

A discussion of symmetries and realising how understanding symmetries have driven our search at the heart of matter, will actually cover almost all of the 20th century theoretical physics and some very important mathematical developments.

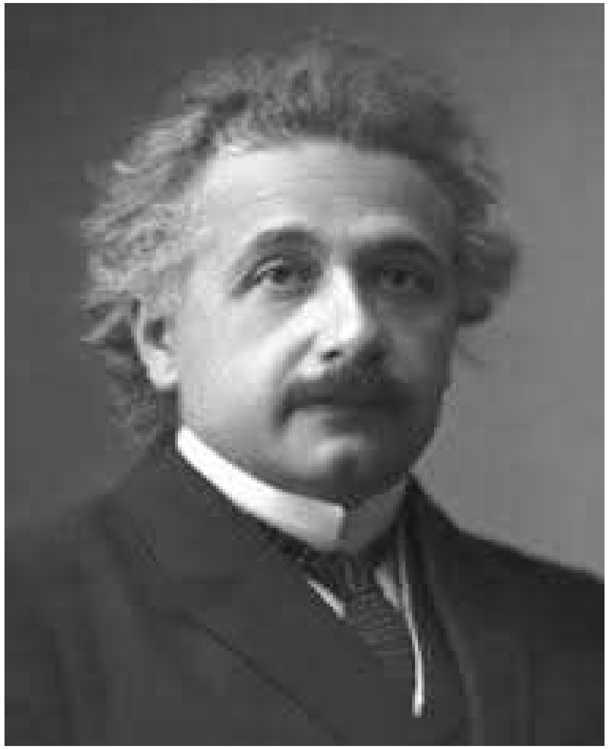
In fact it is this subject where the intertwining threads of all branches of sciences theoretical physics, biology, chemistry and fundamental mathematics are very obvious.

The concept has indeed been a very important part of intellectual development of humankind.

The developments in particle theory have been driven by our understanding of the symmetries



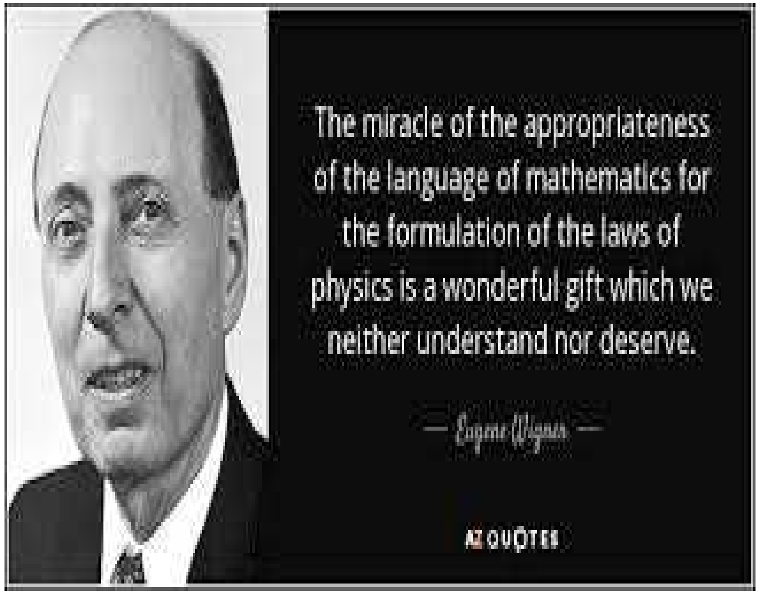
A mathematician. (Group theory: Mathematical Language to discuss symmetry)



Poincare Einstein

Sp. Theory of relativity Special and General Theory of relativity

First time: Symmetries and invariances were used to understand observed laws of nature and arrive at a theory.



Wigner Nobel Prize 1963:
"for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles".
He developed the mathematical machinery and then discovered a lot of things about nuclear dynamics through application of this symmetry. A major player which changed attitude of physics towards symmetries and the major mathematical tool Group Theory.

We all use the word symmetry in many different ways.

[a] Symmetry and Beauty: "The chief forms of beauty in nature are order, symmetry and definiteness. ": Aristotle 350 BC.

[b] Symmetry and Philosophy: "Symmetry,..., is one idea through the ages that human being has tried to comprehend and create order, beauty and perfection." H. Weyl (1952)

[c] Static Geometrical Symmetries: Shapes which look the same under translation, rotation and reflections. Human beings have engaged themselves in studying these since the time of the Babylonians and even before.

[d] Symmetry of laws of nature : "Scientists encapsulated the observed facts about nature in few equations which were seen to posses certain symmetries which gave an understanding why the laws of nature what they are! Abstracting these symmetries from the particular situation where they were observed paved the way for deciphering the laws of nature."

This last statement is the most complex connection of them all.

[e] Symmetries and constancy/invariant/conservation : The symmetries imply constancy of measurable quantities associated with a system.

Two types of symmetries of laws of nature, according to Wigner:

1)Geometrical Symmetries : Space Time symmetries.

2) Dynamical symmetries : These are the bread butter of Nuclear Physicists, Particle Physicists and in general theoretical physicists.

These were inferred and postulated from observed patterns in properties. Using the mathematical framework developed to understand (1) actually paved the way to imagine these symmetries of the unobservable!.

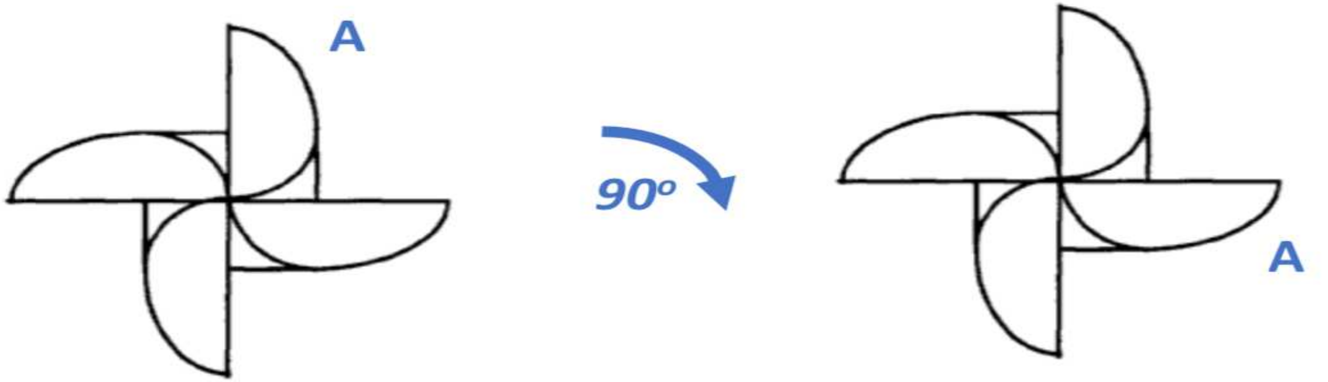
When formulating laws at scales much smaller or much bigger than ourselves we need to trust the messages given to us by these abstract ideas.

Symmetry : An operation/transformation that you perform that leaves the object unchanged / invariant /the same or similar

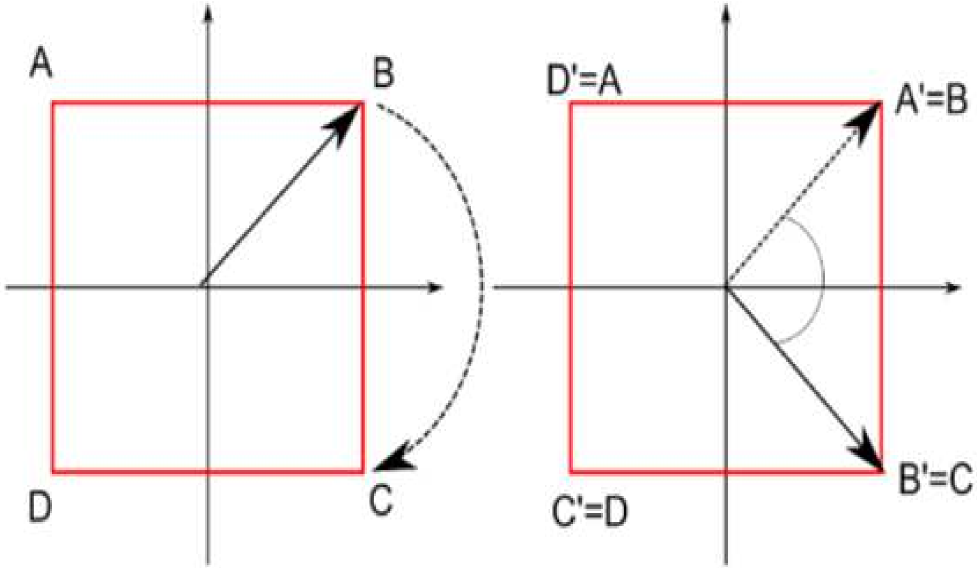
Let us see examples.

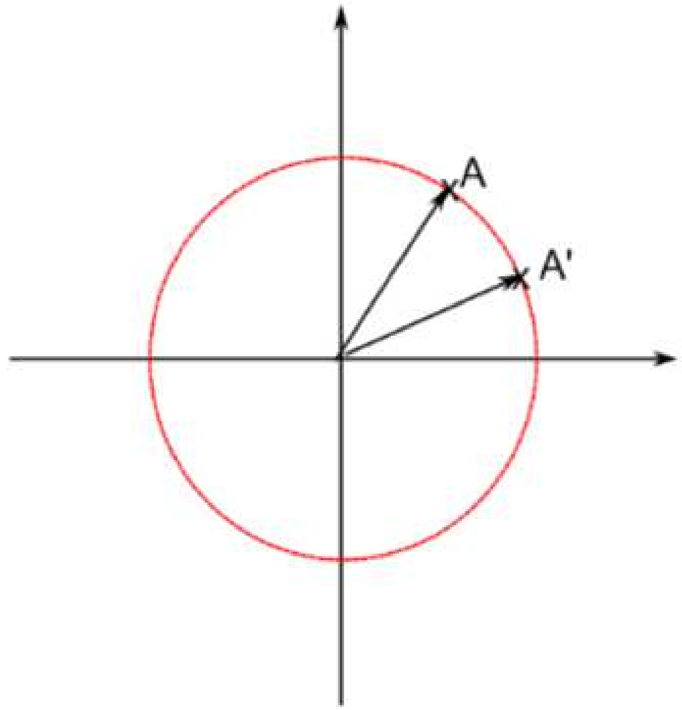
 

The group of two pictures are symmetric under an exchange of the two people.



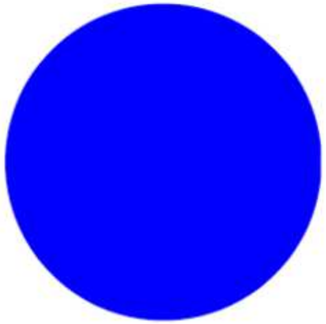
Rotation through 90 degrees is a symmetry of this figure.





Rotations through discrete angles

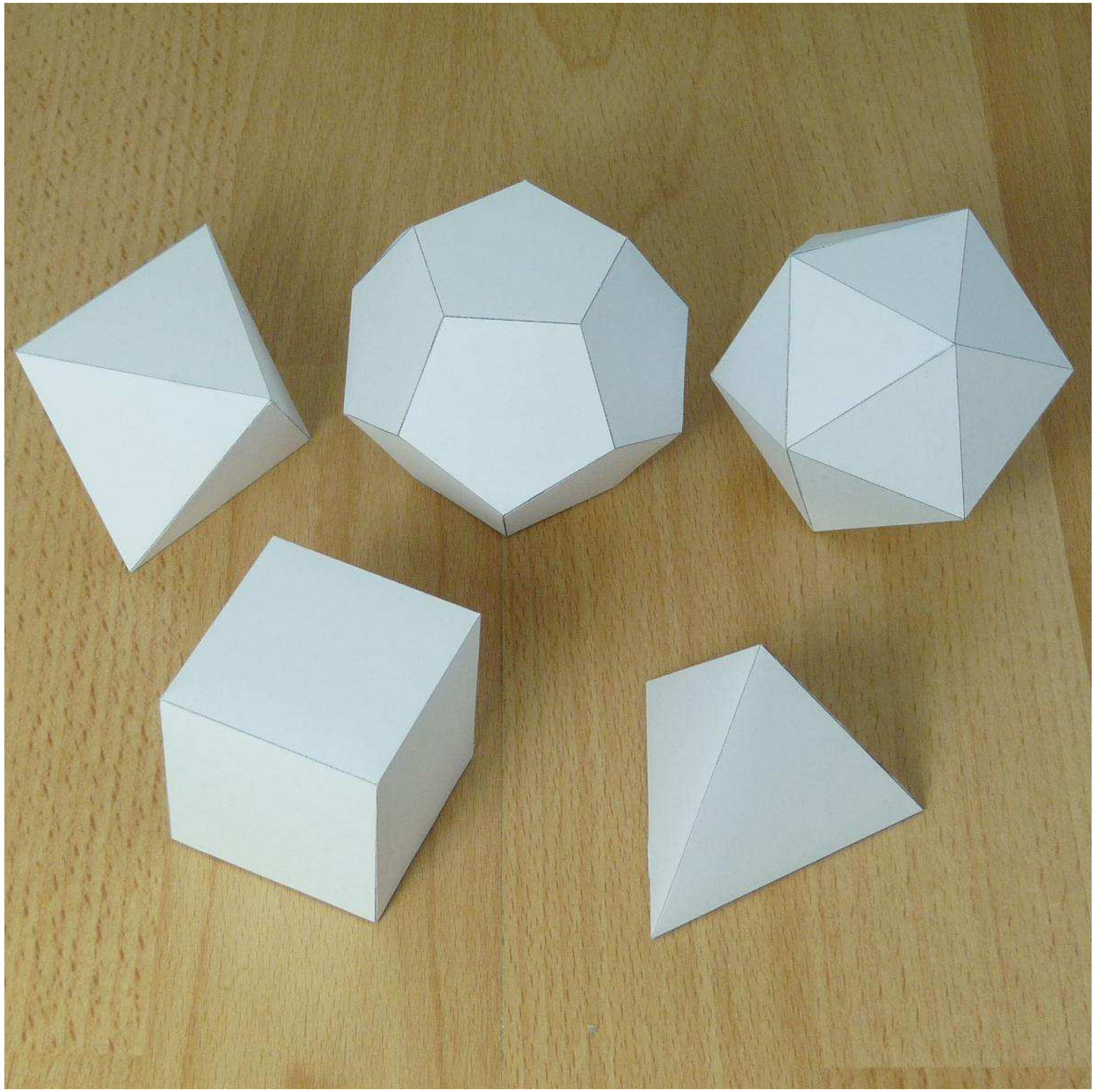
Continuous Rotations

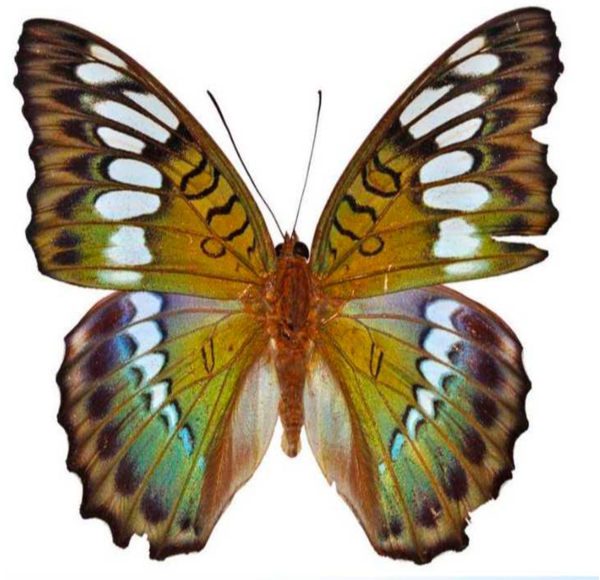


Circle

Platonic Solids:
Known since B.C.

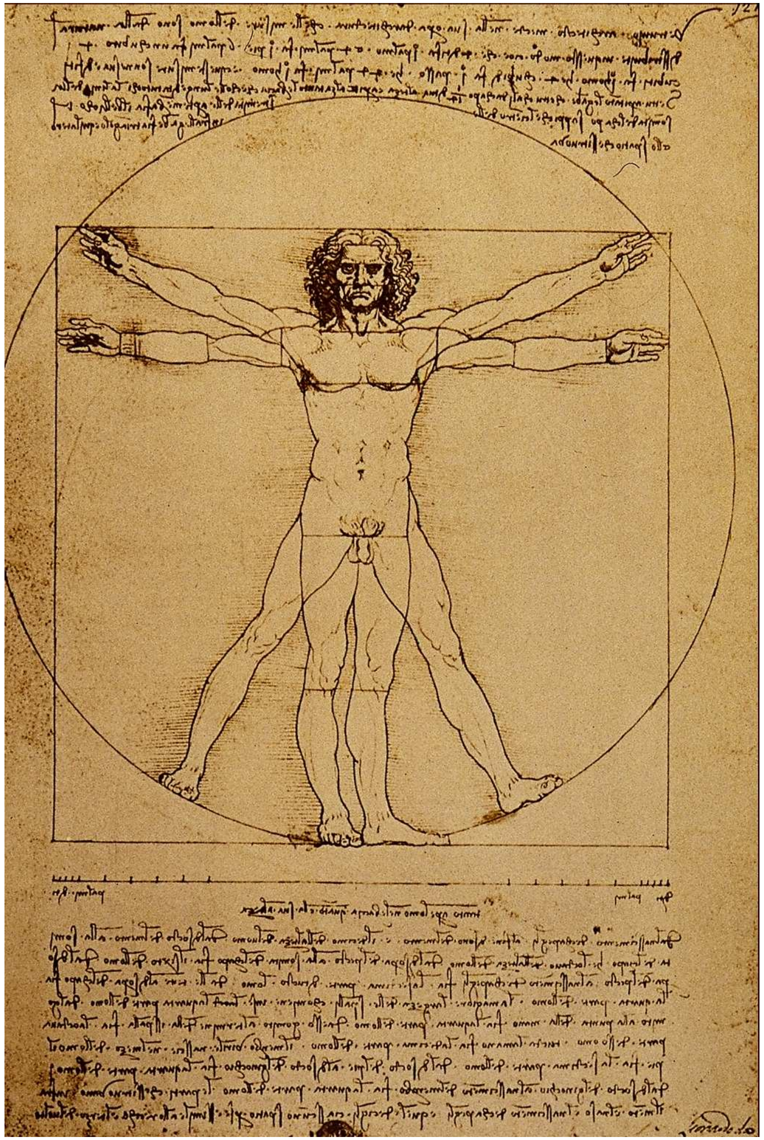
Sphere



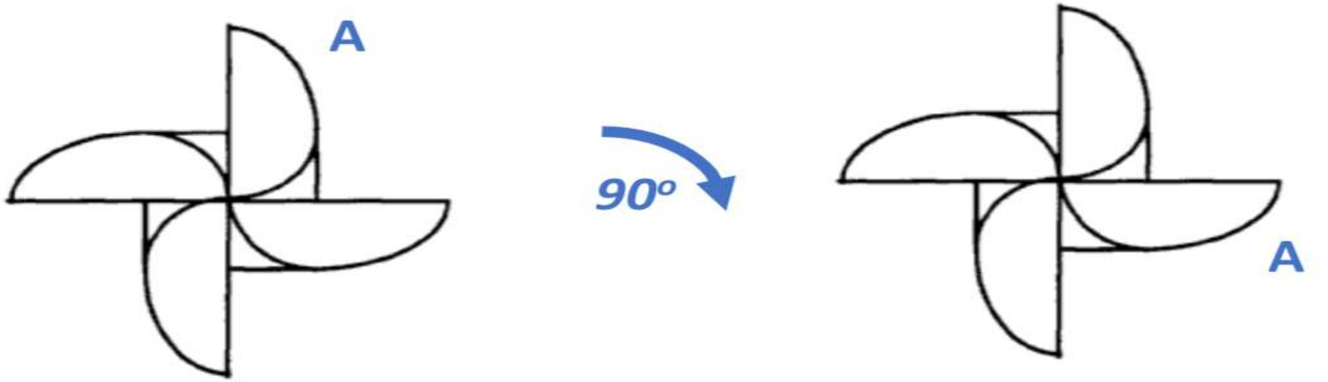


Butterfly: LR symmetric

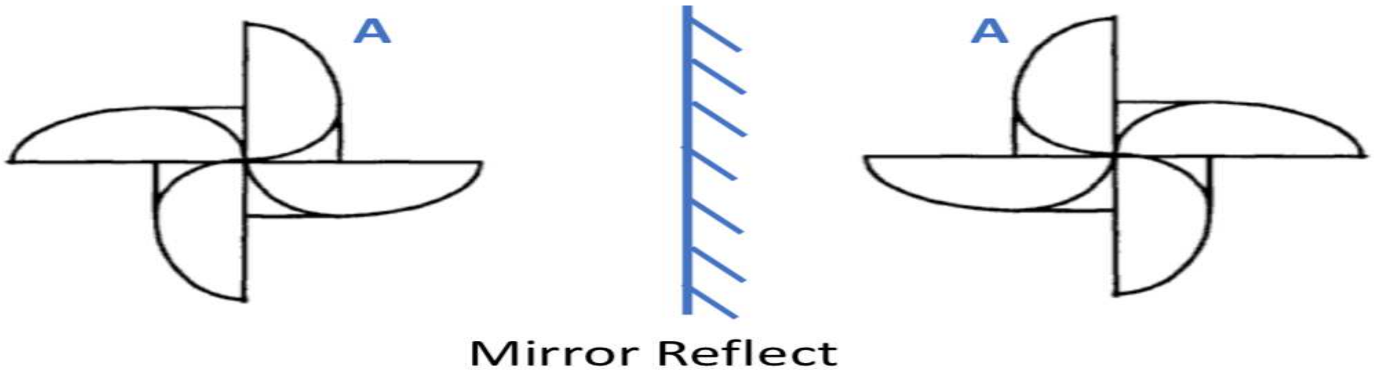
Snowflake: 6-fold symmetry



Leonardo da vinci painting
Ideal Human body proportions.

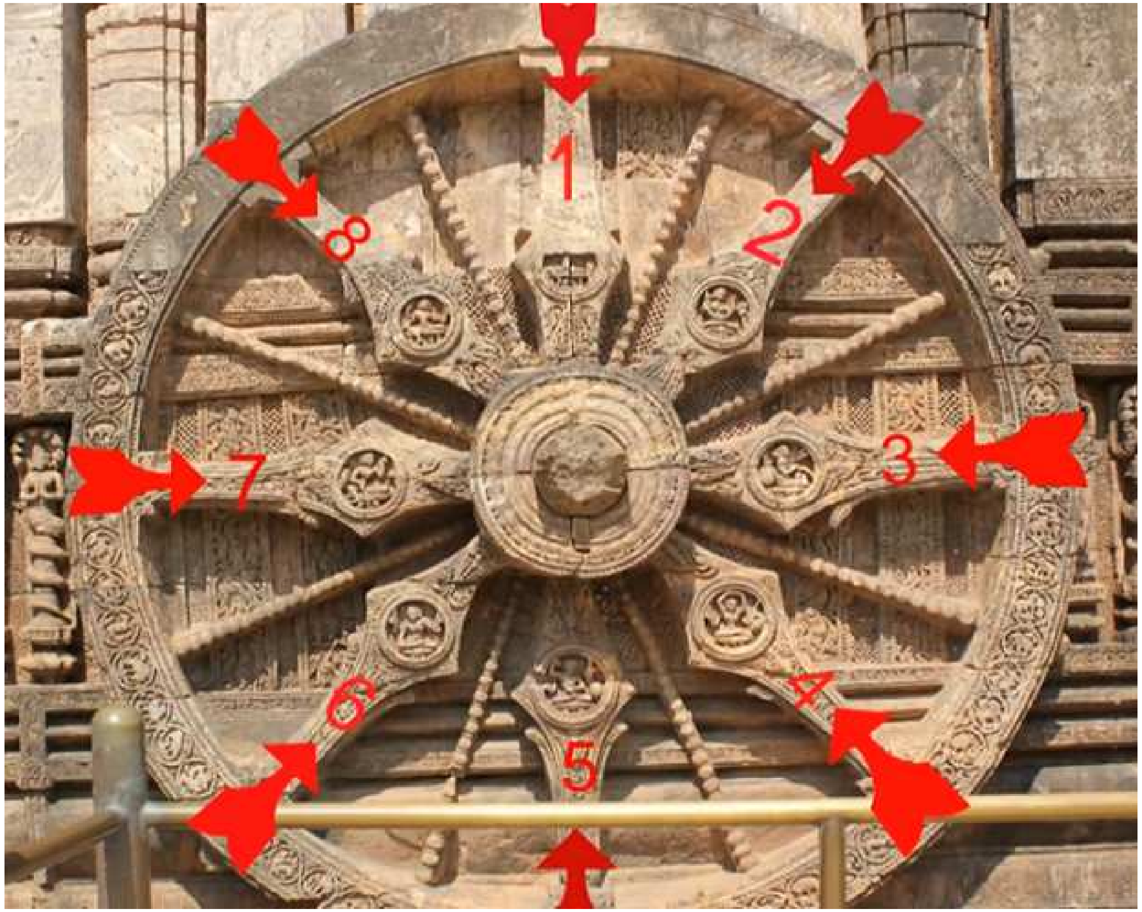


Rotation through 90 degrees is a symmetry of this figure.



Mirror reflection is not a symmetry of this figure.

If we zoom we will see symmetries in difft. panels, friezes etc.



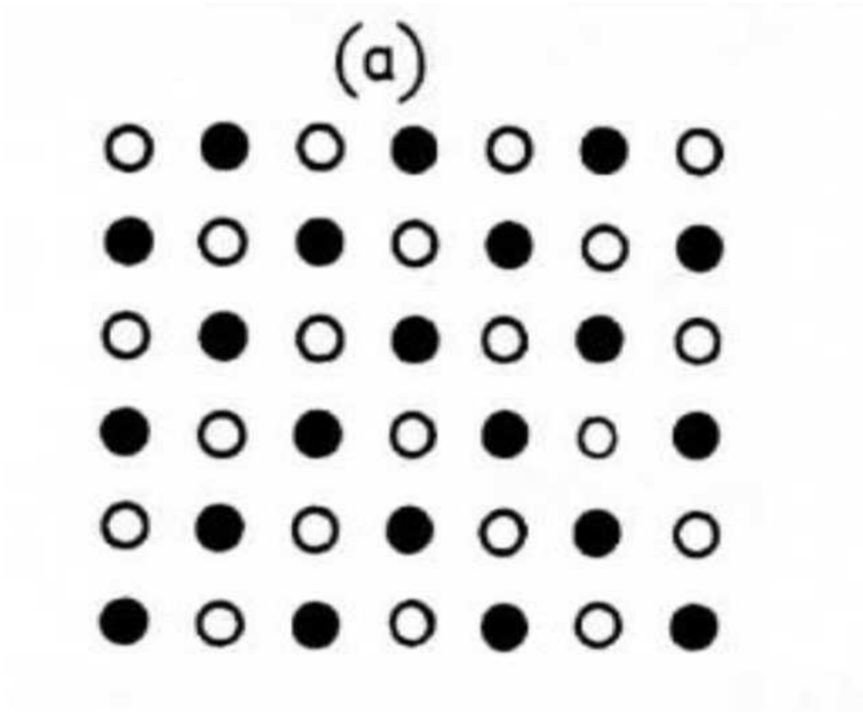
Left-right symmetry

Rotation through 45 deg (neglect figs)

(Konark Temple)

In the decorations of churches/temples/mosques (for example) the designs will repeat. That is translational symmetry : symmetry under translation of a pattern.

For example a NaCl crystal:



Symmetry here is translation, rotation and scaling!

Roots of any algebraic equation satisfy certain relations among themselves. Like for the roots of a a quadratic equation $ax^2 + bx + c = 0$, the product or two roots is c/a. We can tell this without knowing the roots.

Look at the permutation of the roots which will leave these relationships unchanged.

Considering the properties of such operations he was able to answer the question why some quintic equations can be solved by method of radicals and some can not!

In principle absolute space does not allow us to define right and left. Choice of right and left is pretty arbitrary.

Only when space is endowed with a direction can one define right as the sense in which the rotation of a screw will move it upwards and left is the opposite sense of rotation!

But then reversing that direction would interchange left and right!

Left-Right symmetry observed in animal kingdom mostly. .

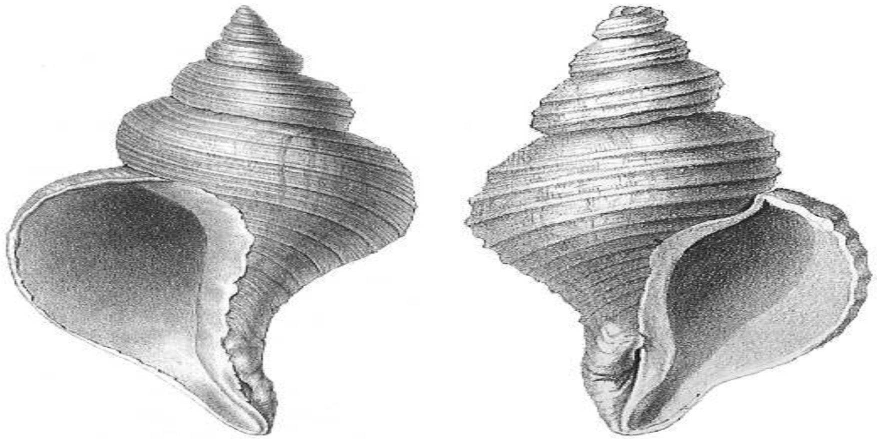
Development of biological shapes into LR symmetric bodies is due to the fact that effect of of gravitation as well as direction of motion for animals capable of motion, up and down, forward and backward is defined but only L-R remains arbitrary. Lower organisms are in fact spherical! {Symmetry: Herman Weyl, 1952}

Kant, Leibtnitz: philosophical discussion about 'right' and 'left'.

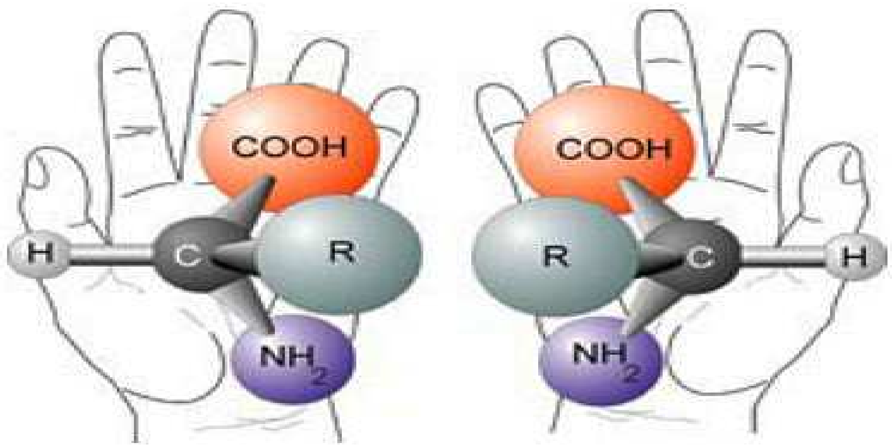
In fact philosophically 'right' was considered 'right' and the word for left in Latin in sinister.

This is an important concept in Physics, Chemistry and Biology.

Sometimes we see only shells (say) with only one handedness (gastropdes) Neptunea Angulata.



Amino acids can have two inequivalent isomers: with opposite handedness. Chemically both can be synthesised and both are stable. But Amino acids in Protein are only L Amino acids.



Casimir: Once published a paper in Nature showing that the cows either chew by moving their jaws either left circular motion OR right circular motion

T.A. Davis (a student of J.B. Haldane) found that leaves of coconut trees arrange in either clockwise or anticlockwise and one of them has higher yield than the other!

10 Percent people in the world are left handed : ie their left hand is dominant!

In Cricket people love to have a Left-Right combination!

The last Australia vs India Match:

The net result : India won!

A $L \leftrightarrow R$ transformation would take R. Pant to W. Sundar: the result is thus invariant under this transformation.

In case of R. Pant to C. Pujara, the $L \leftrightarrow R$ transformation will have to be accompanied by a aggressive $\leftrightarrow$ defensive batting too!

Invariances are thus complicated story!

Anyway back to business at hand!

Does the observation that proteins have ONLY L-Amino acids is indicative of some symmetry breaking?

As long as both the amino acids can be chemically synthesised the observed prevalence of a particular handedness is only due to self reinforcing processes. This does not say that laws of physics forbid it.

Example of ants following directions where more ants have gone before.

Evolutionary biologists worry about such things though!

Sometimes differences between left and right handed compounds can be lethal on organic systems. L-glucose can not be used by body.

Thalidomide: chemically synthesised sample contained both and L and R molecules.

Laboratory tests were done with only one isomer that was produced by other organisms.

The what is symmetry breaking?

One can define a right handed electron and left handed electron as the states which have spin parallel and antiparallel to the direction of motion.

Only left handed type of electrons are ejected in beta decays whereas both the left and the right handed electrons exist in nature in general!.

This is indeed breaking of L-R symmetry.

We will come to this later. Both symmetry and its breaking are important.

In solids defects breaking the symmetry of the arrangements actually change the material properties completely.

Thus this symmetry breaking (defects introduced by hand) can be used to manipulate the material properties at will almost!

R.P. Feynman has a wonderful statement about it:

So our problem is to explain where symmetry comes from. Why is nature so nearly symmetrical? No one has any idea why. The only thing we might suggest is something like this: There is a gate in Japan, a gate in (Neiko) Nilkko, which is sometimes called by the Japanese the most beautiful gate in all Japan; ..... But when one looks closely he sees that in the elaborate and complex design along one of the pillars, one of the small design elements is carved upside down; otherwise the thing is completely symmetrical. ..... they purposely put an error in there, so that the gods would not be jealous and get angry with human beings.

We might like to turn the idea around and think that the true explanation of the near symmetry of nature is this: that God made the laws only **nearly** *symmetrical so that we should not be jealous of His perfection!"*


copyright 2004 eve@eveandersson.com


a
b
b


b


a

Summary:

If a system is unchanged by an operation we call that operation symmetry of the system

We discussed mainly static symmetries of bodies, shapes, designs..except in the last case of L-R symmetry.

What symmetries did we discuss?

1)Translation of origin of space and time through a fixed distance,

2)Rotation of coordinates through a fixed number of angles or continuous rotations.

3)Reflection in a mirror.

These are in fact symmetries of equations describing motion of particles or system of particles!

The physical laws are extracted from a large amount of data on time evolution of physical systems from a set of given initial conditions. These are the laws of nature (physics).

The fact that can be thus extracted already says that 1 and 2 are certainly true properties .

The physical space may not be able to distinguish between Left and Right, but it is possible that physics may provide a way if some physical phenomena does not.

We look at to how laws of physics look when one changes the choice of origin of space and time OR orientation of the coordinate axes. Space by itself does not distinguish ANY one point as special.

Consider Newton's second law of motion in a vector form:

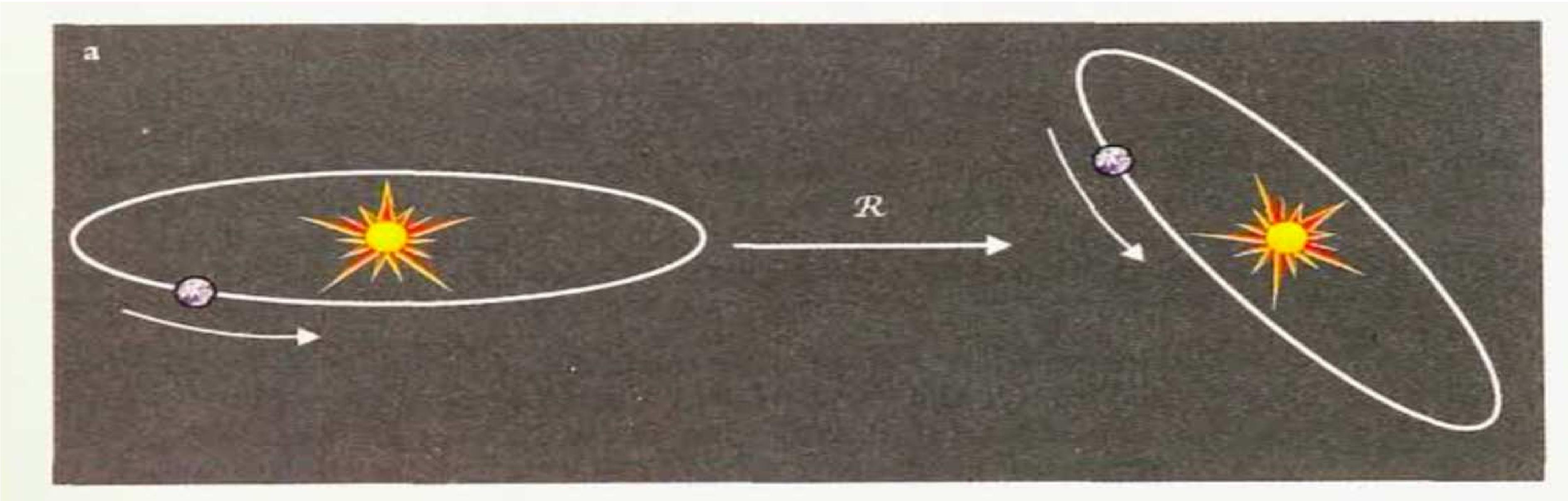
$$m\frac{d^2x}{dt^2} = F_x, m\frac{d^2y}{dt^2} = F_y, m\frac{d^2z}{dt^2} = F_z.$$

OR equivalently

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}$$

These equations are invariant under rotation and translation of coordinates!

What does it mean for physics?



Thus implication of rotational invariance is NOT that the orbits are necessarily circular But that the relative orientation of the sun and earth will be the same no matter what is the orientation of the observer's coordinate axes.

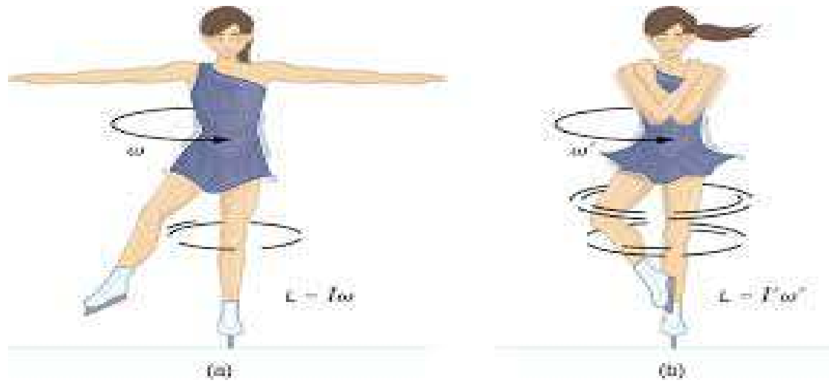
Along with equations of motion , people had observed conservation of various kinematical quantities:

1)Linear momentum is conserved in the absence of a force.

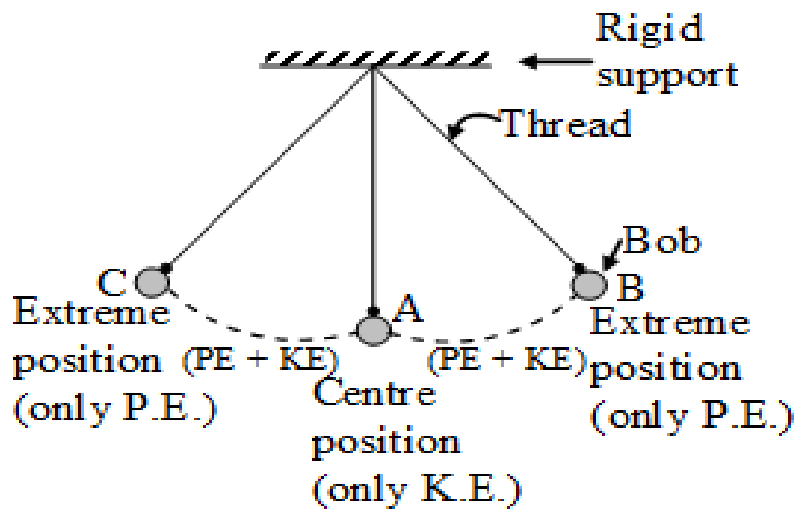
2)In the absence of a Torque Angular Momentum is conserved.

3)Total energy of a system in motion is also conserved : conversion of potential energy into kinetic energy and vice versa.

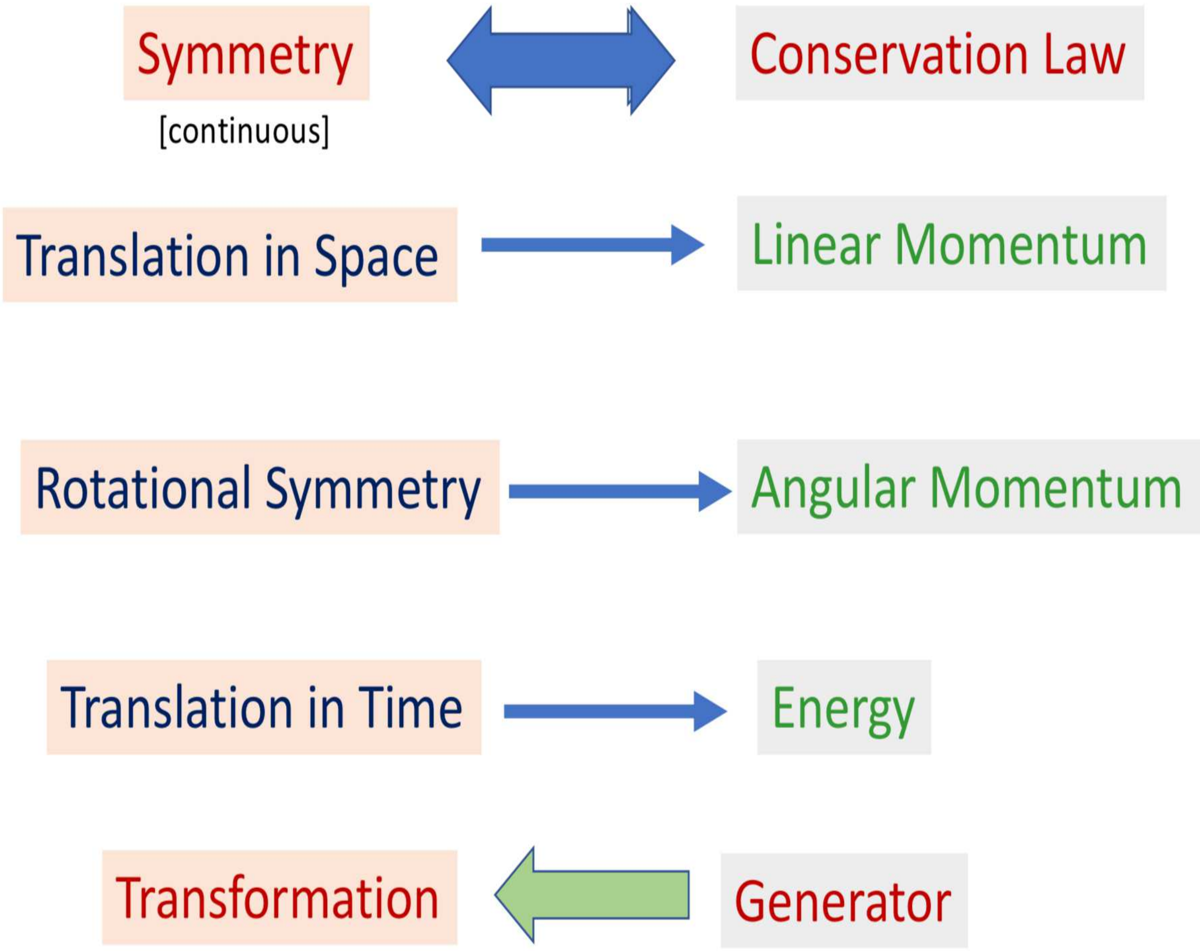
But these were just observations, the laws were 'empirical' !
Do they HAVE to be always true?



$$L = I\omega = I'\omega'$$



Emmy Noether proved in 1918 that for every symmetry operation generated by a continuous transformation, there was a conserved quantity.



Her theorem also removed an apparent contradiction in general relativity.

Hilbert and Einstein were puzzled by the non-conservation of energy in general theory of relativity.

Her theorem showed that there was nothing wrong with conservation of energy, one was missing the piece coming from space time curvature.

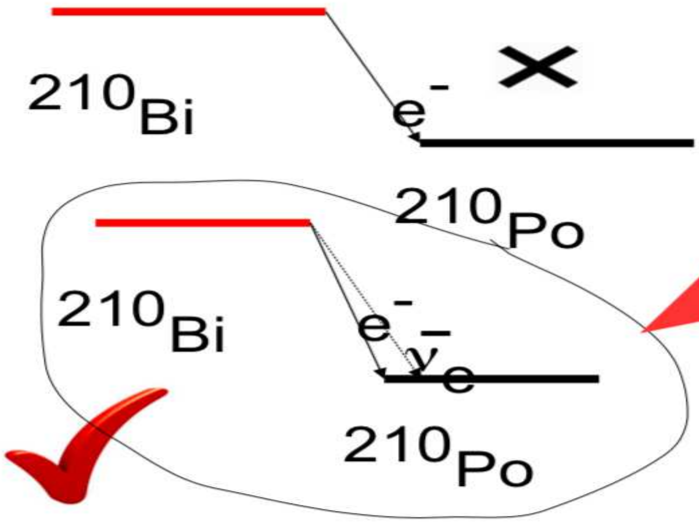
In fact Pauli used the energy and angular momentum conservation to postulate a new particle : the ν.

A symmetry principle predicted existence of a particle which was really very elusive!

Birth of idea of neutrino: desperation

- 1918: Chadwick noted an apparent violation of energy conservation in radioactive beta decay

$$^{210}Bi \rightarrow {}^{210}Po + \mathrm{e}^-$$



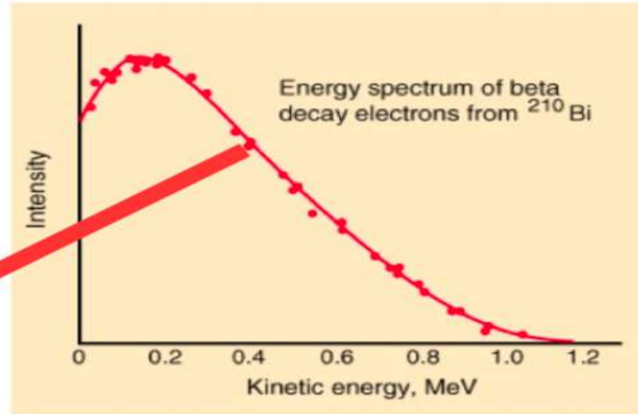


Figure 1.4 The continuous energy spectrum for β decay of ^{210}Bi, from G. J. Neary, Roy. Phys. Soc. (London), A175, 71 (1940).

Lorentz Transformation:

Lorentz derived that the electric and magnetic field should transform in a certain fashion in frames moving wrt each other in uniform motion if Maxwell's Equations were right.

Maxwell's equations in free space imply that velocity of light is determined by the dielectric constant and magnetic permeability of the medium.

How then can velocity of light depend on the state of motion of the observer? But this is what Galliean transformations of space time suggested.

That is Maxwell's equations were inconsistent with Galliean transformation.

But instead if the electric and magnetic fields as well as space-time in two frames were related to each other via Lorentz Transformations, Maxwell's Equations remained unchanged.

So Maxwell's equations are independent of the state of uniform motion of the observers, that is have relativistic invariance. The correct transformations are 'Lorentz Transformations'.

Maxwell's equations have the Relativistic Invariance built into them.

The 'right' equations were the ones which had the 'correct' symmetry.

Bargmann: (relativist)

"...these laws of physics which express a basic 'invariance' or 'symmetry' of physical phenomena seem to be our most fundamental ones."

They make the description of physical phenomena 'simpler' and more 'compact' and hence more beautiful to a physicist.

Poincare: (Mathematician, Mathematical Physicist)

"The scientist does not study nature because it is useful. He studies it because he takes pleasure in it, and he takes pleasure in it because it is beautiful"

Dirac Equation: Quantum Mechanical Equation to describe an electron moving at relativistic speeds at atomic distance scales!

Needs to give same results for all observers: needs to be relativistically invariant!

Unexpected fallout:

'Explains' why the gyromagnetic ratio (connecting magnetic moment μ and spin angular momentum $\vec{S}$) is 2.

Predicted existence of 'anti' particle of electron.

Second example of 'invariance'/'symmetry' predicting a particle.

Every Fermion has to have an anti fermion! (positron, antiproton, antineutron....) Fermion number is always conserved!

Energy an electron gains while passing through a region of potential difference depends ONLY on the DIFFERENCE in the potentials at the two ends!

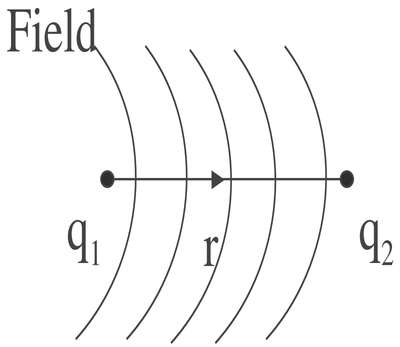
We can call the potentials at two ends V_1, V_2 or V_1', V_2' such that $V_1' = V_1 + V_0, V_2' = V_2 + V_0$ and nothing changes!

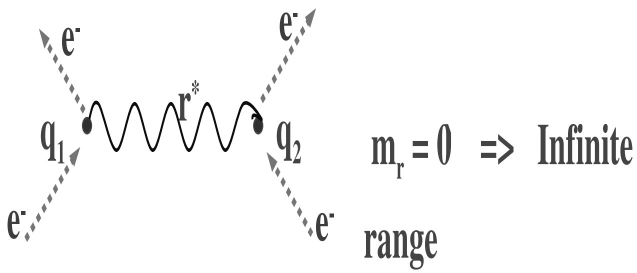
For the cognicenti, Maxwell's equations in Classical Electromagentism are unchanged under the change :

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha(x)$$

Believe it or not but it is this 'invariance' (which we call Gauge invariance) which is responsible for the $1/r$ law of Coulomb Potential!

In classical electromagnetism already action at a distance is 'explained' in terms of electromagnetic fields.





V(r) $= \frac{q_1 q_2}{r}$

Force between two particles can be understood either in terms of the field or in terms of exchange of a field quantum!

The properties of the quantum such as spin, mass govern the nature of the force such as range, dependence on $\vec{r}$ between the particles etc. If γ had mass m_γ, the potential $V(r) = exp(-m_\gamma r)\frac{q_1 q_2}{r}$

Recall also gauge invariance requires $m_\gamma = 0$. The observed zero mass and infinite range of coloumb potential understood as a result of gauge invariance.

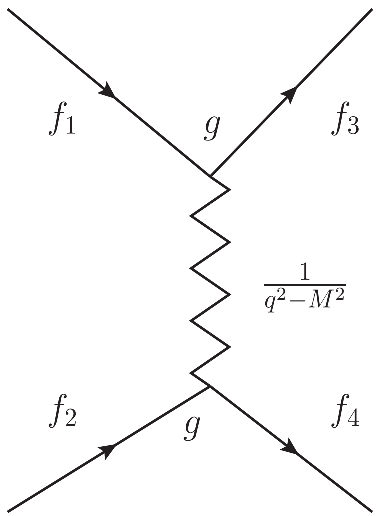
The Standard Model (SM) of particle physics:

Even though we call it a model it is actually the candidate for **the** 'theory' of the fundamental particles and interactions among them!

Built, brick by brick, over the last 50-60 years, combining information from a lot of different types of experiments and many many innovative theoretical ideas.

The basic mathematical framework is that of quantum field theories (QFT)

All the three interactions are described as local gauge theories based on gauge group $SU(3) \times SU(2)_L \times U(1)_Y$.



$$V(r) \propto \frac{e^{-Mr}}{r}$$

Analyse differences between the strong, weak and electromagnetic interactions, to appreciate differences between $g, W^{\pm}, Z^0$ and γ. All are gauge interactions but have different conservation laws. and different masses

Weak interactions violate parity maximally

The corresponding gauge bosons couple ONLY to the left handed fermions.

Interactions are short range and the corresponding gauge bosons are therefore massive

EW interaction description has a problem!

They are short ranged and hence the corresponding gauge bosons are massive.

But if they are massive the force laws can not be symmetric.

On the other hand all the properties of the weak interactions that have been measured to a high degree of accuracy tell us that the laws of motion ARE symmetric.

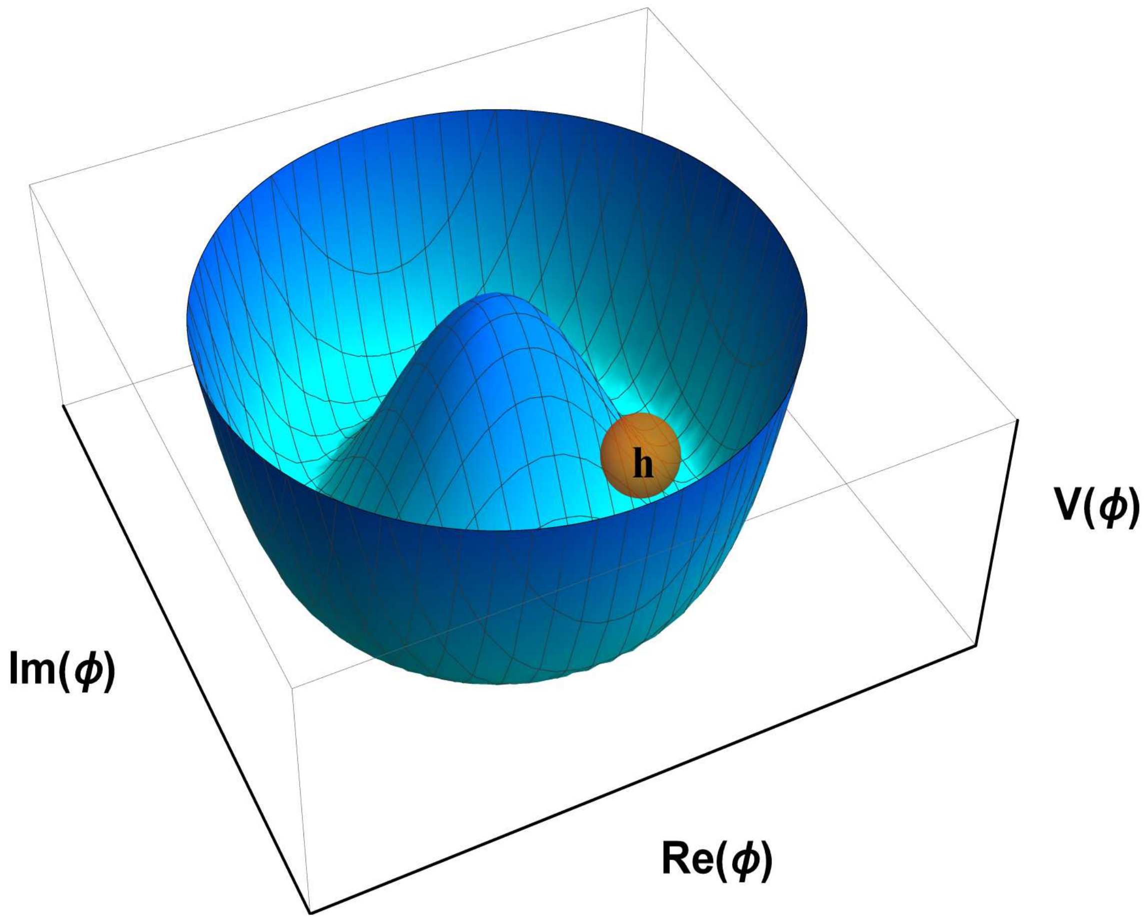
Non zero mass of the W/Z bosons indicate symmetry is broken,their properties indicate it is a good symmetry of weak interactions!

How do we we have our cake and eat it too?

The theory has the symmetry but the ground state does not.

Example of the ferromagnet: Lagrangian has rotational symmetry but the ground state does not.

Just like gauge symmetry of fermion interactions required existence of gauge bosons this mechanism requires existence of a spin 0, fundamental particle: Higgs boson.



This is where M/s Higgs, Englert, Brout, Kibble and Hagen make the apperance.

They invented a mechanism which can make the gauge bosons massive without breaking the symmetry!

The mechanism predicted existence of a spin zero particle and it is the interaction with this boson that made a mass for gauge boson possible keeping the symmetry intact!

This is called 'spontaneous symmetry breaking' and they used a mechanism invented by Nambu who got the Nobel prize in 2008.

Weinberg and Salam (independently) showed how the Higgs mechanism could be used to have a theory with gauge symmetry where photons are massless and the weak gauge bosons are massive!

So at this point the aim was only to explain how we can have masses for gauge bosons consistent with the gauge symmetry!

The route was via the interactions with this spin zero boson.

There was a windfall!

Just like nonzero masses for the weak gauge boson were inconsistent with the gauge symmetry, nonzero masses for all matter particles in the periodic table were also inconsistent with gauge symmetry.

Weinberg and Salam showed they could use the Higgs mechanism and the Higgs boson to solve this problem too.

Let me try to explain why and how!

Consider a spin half particle with spin pointing along z axis and moving along z axis.

This is called a right handed state of this spin half particle

If the particle is massive we can go from the right handed state to left handed state via a Lorentz Transformation.

Experimentlly it is known that the weak interactions treat left handed fermions and right handed fermions differently. This existence of weak interactions which differentiate between different handedness of the particle would produce conflicts with special theory of relativity for massive particles.!

Higgs mechanism provides a way to allow left handed fermions to change their handedness and hence can make the differential treatment of fermions with two handedness compatible with special theory of relativity.

So this is the origin of the staement that the Higgs boson gives masses to all the particles!

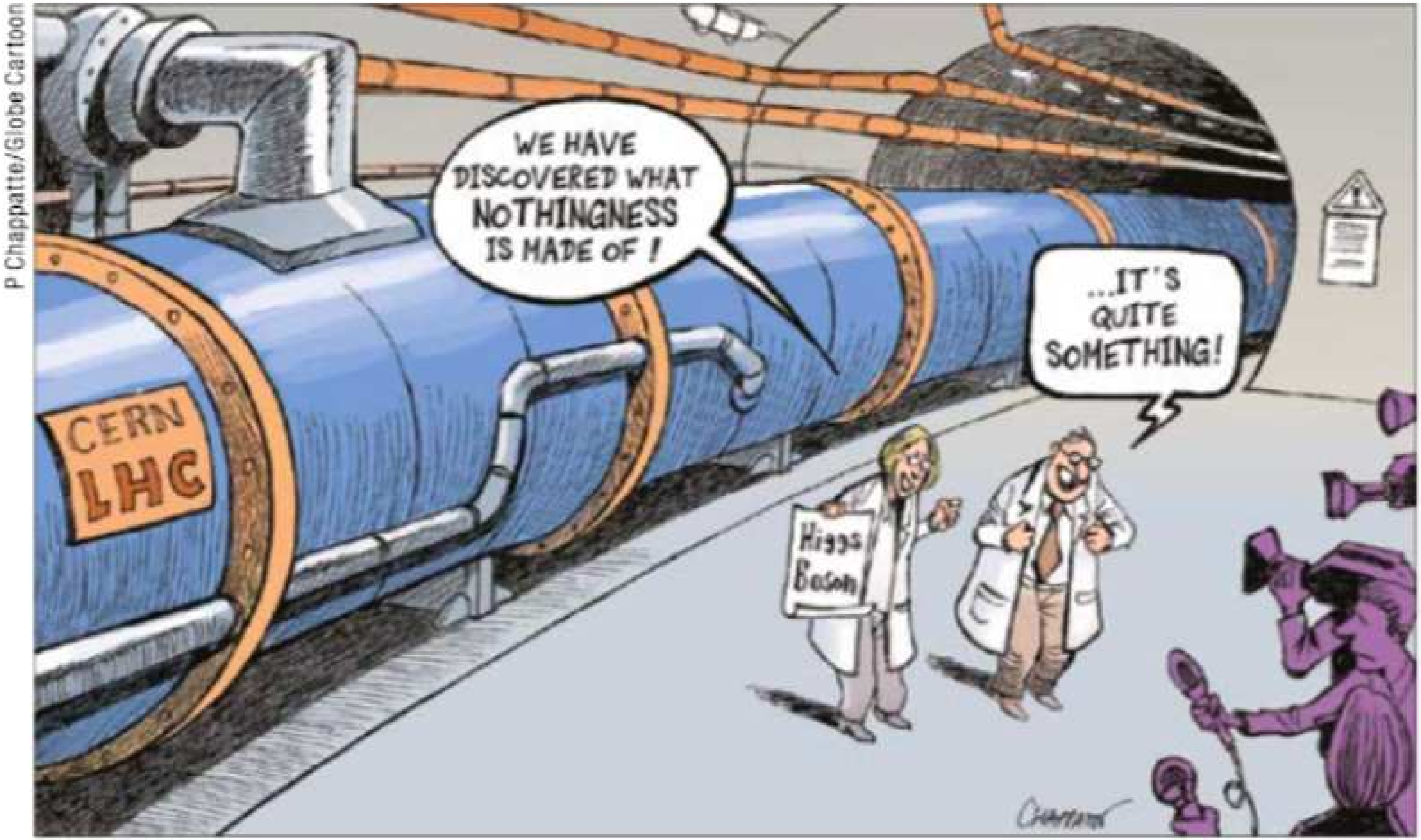
What it really means is that these interactions of the matter fermions with Higgs bosons make the existence of a nonzero mass consistent with different symmetries.

What will go wrong it the theory does not have these symmetries?

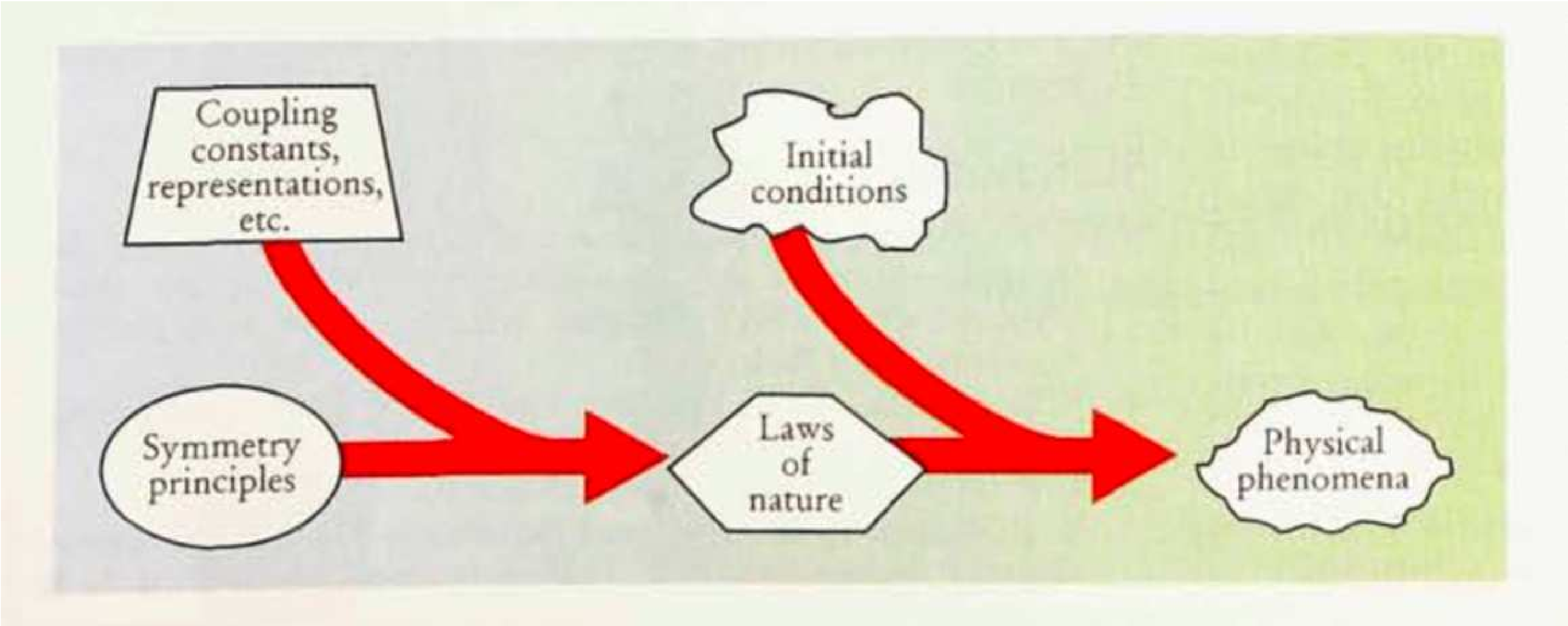
The theory HAS to have these symmetries if the quantum formulation of the theory has to make sense!

That it does makes sense was told to us in experiments performed at the Large Electron Positron collider (LEP) at CERN, Geneva, Switzerland, in the 90's and at Tevatron at Fermi Laboratory, Chicago, USA.

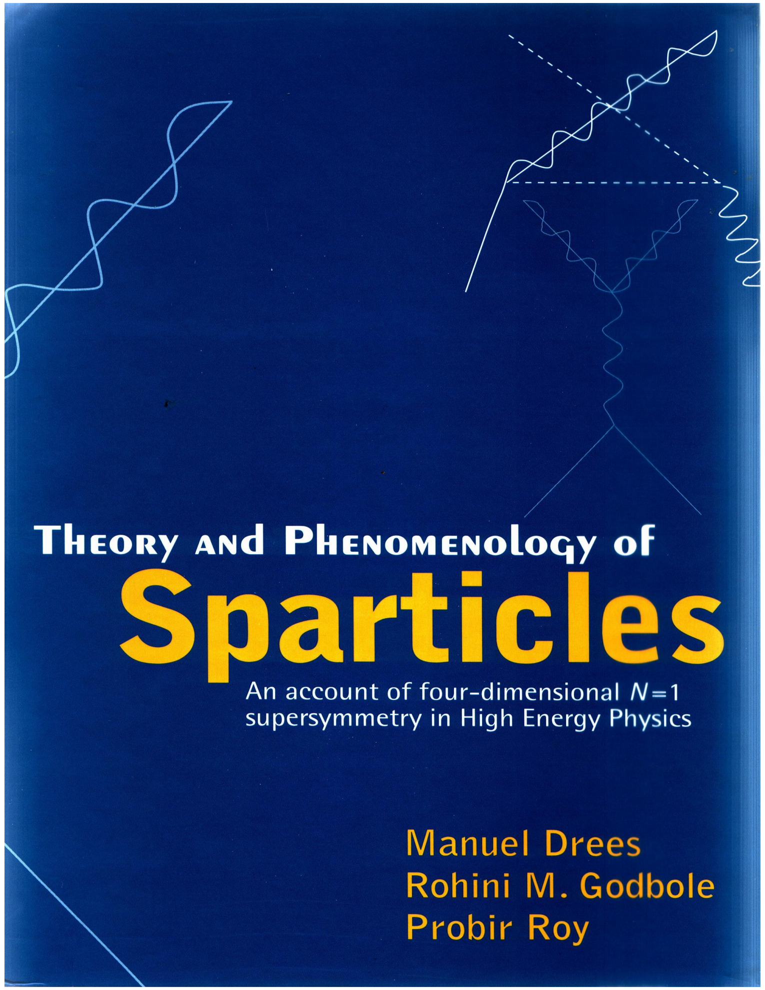
Hence the large hadron collider was constructed and it did find the Higgs boson.

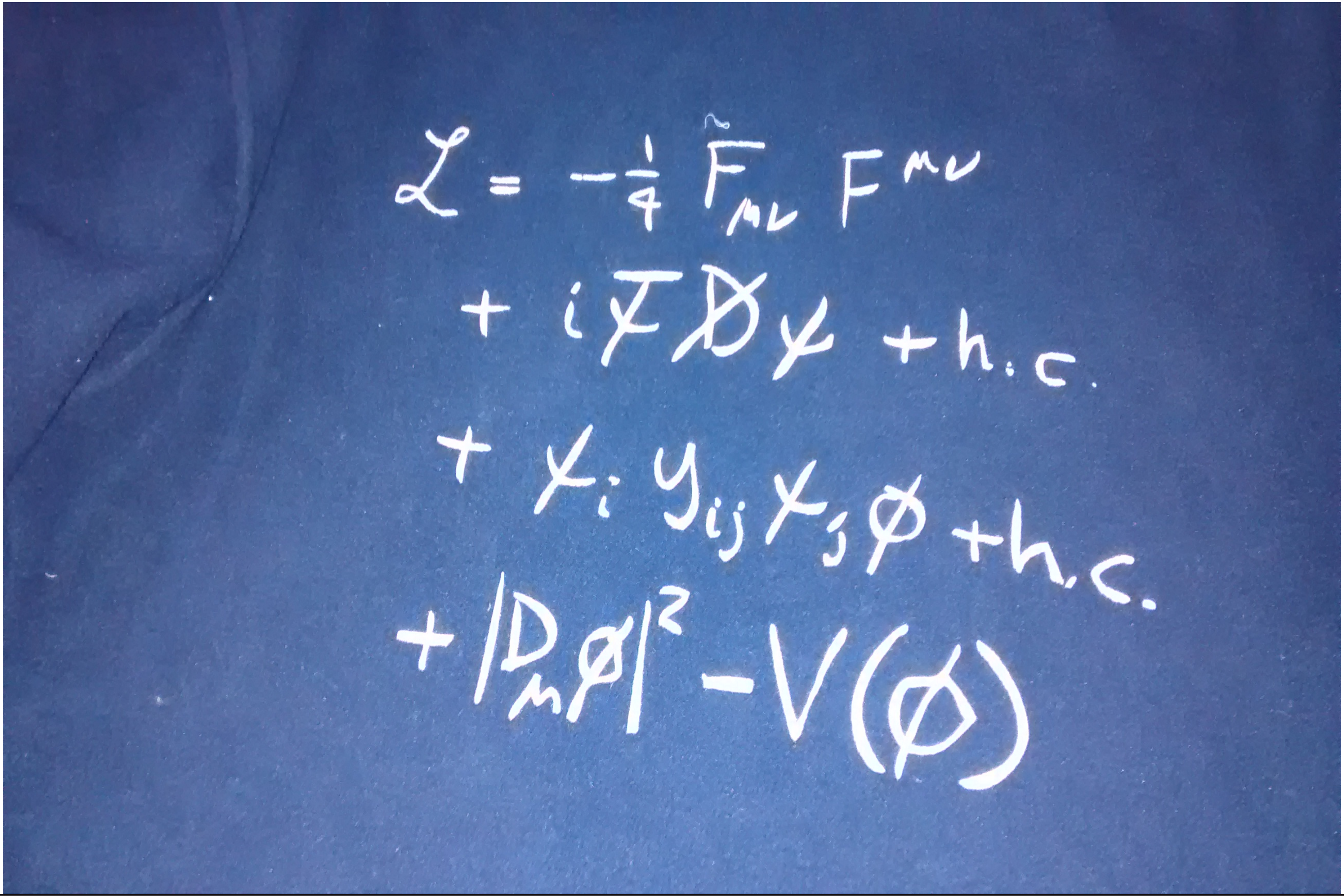

P Chappatte/Globe Cartoon
CERN
LHC
WE HAVE DISCOVERED WHAT NOTHINGNESS IS MADE OF !
...IT'S QUITE SOMETHING!
Higgs Boson

A summary is (a la Wigner, as presented by D. Gross in Physics Today

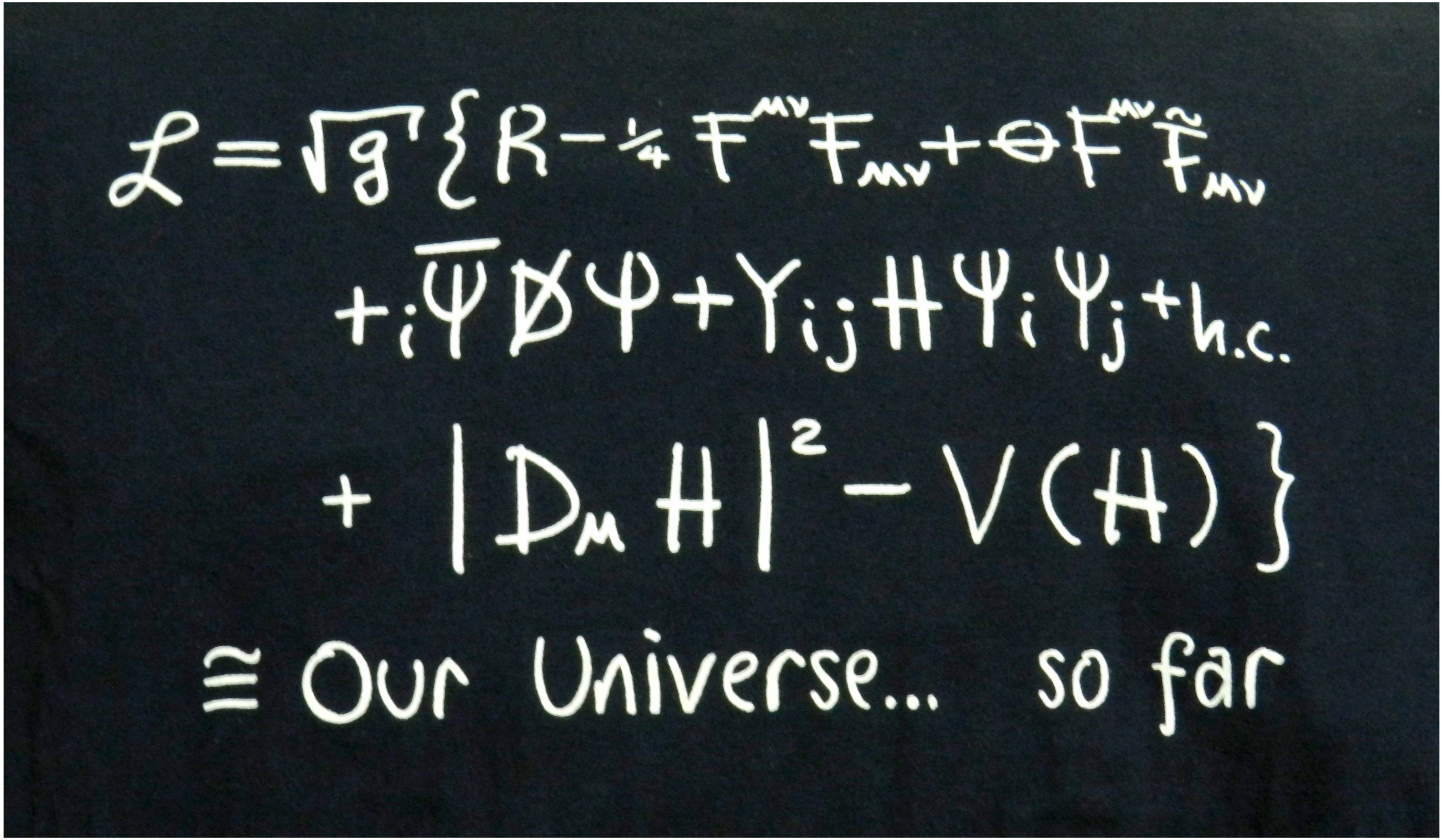


- Many ways of extending the SM. Some with new symmetries and some without.
- Some trying to include gravity in the description.
- Some of these had predictions which can be tested at the LHC.
- M. Drees, R.M. Godbole and P. Roy, Theory and phenomenology of sparticles, World Scientific, 2005




$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$


Top
&
Higgs
BSM
BSM


$\mathcal{L} = \sqrt{g}\{R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \theta F^{\mu\nu}\tilde{F}_{\mu\nu}$
$+ i\bar{\Psi}\not{D}\Psi + Y_{ij}H\Psi_i\Psi_j + h.c.$
$+ |D_\mu H|^2 - V(H)\}$
$\cong$ Our Universe... so far

- Symmetry , Herman Weyl, Princeton Univ. Press, (1952) (M, P)

- Essays by Eugene Wigner, 'Aspects of Symmetry', Indiana University Press (P)

- Fearful Symmetry, Princeton Univ. Press (P)

- Symmetry and Beauty : History of Symmetry, Ian Stewart (M)

- N. Mukunda, Images of Twentieth Century Physics, University Press (P)

- D.Gross, Physics Today 48, 12, 46 (1995) P

Articles in American Journal of Physics:

i) D. Park, AJP 36,577 (1968),

ii) J. Rosen, AJP, 49, 304 (1981),

iiii) Gauge Invariance, AJP 56, 586 (1988).