Effective field theories to model binaries

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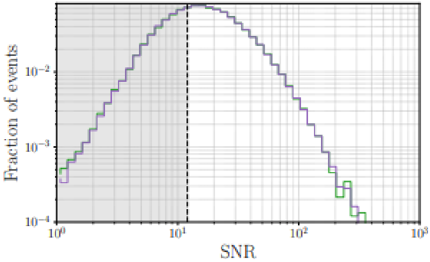
Overview

1. Introduction
2. How well do we need to know the waveform?
3. Waveform $\leftrightarrow$ two body dynamics
4. The EFT view on the 2 body problem in GR (NRGR)

Outline

SNR distribution 3G



Iacovelli+, APJ (2022), arXiv:2207.02771

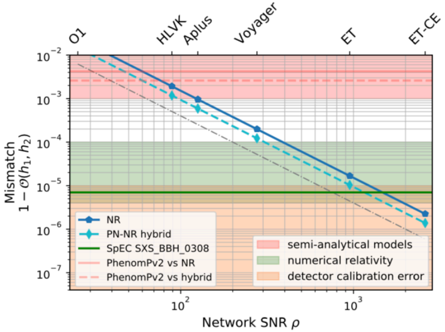
Given h_{true}, h_1, general requirement for $\delta h \equiv h_{true} - h_1$: $\langle \delta h | \delta h \rangle < 1$
wf distance at most equal to *noise length* $\Longrightarrow$ necessary condition on $\Delta \equiv \delta h_1 - \delta h_2 = h_1 - h_2$: $|\Delta| < 2$

Hu, Veitch, arXiv:2205.08448

How "precise" waveforms need to be?

$$\underbrace{\mathcal{M}(h_1,h_2)}_{mismatch} \equiv 1 - \frac{\langle h_1|h_2\rangle}{\underbrace{|h_1||h_2|}_{overlap}}\,, \qquad \langle h_1|h_2\rangle \equiv 4Re\int_0^\infty \frac{\tilde h_1*(f)\tilde h_2(f)}{S_n(f)}df$$

Given $|\Delta| \simeq 2|h_1|\,|h_2|\mathcal{M} < 2$ rule of thumb is $\mathcal{M} \lesssim \frac{1}{SNR^2}$ to be below statistical error



Pürrer, Haster PRD (2020), arXiv:1912.10055

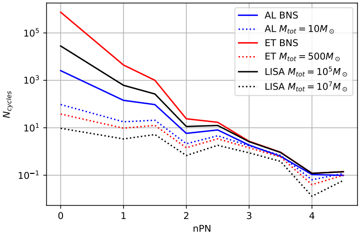
PN/PM order contributions

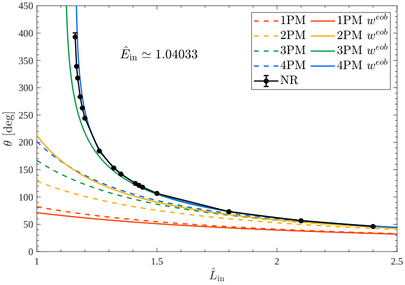
For $h_1 = Ae^{i\psi}$, $h_2 = Ae^{i\psi+\delta\psi}$

$$\mathcal{M} = 1 - \frac{\int_0^\infty \left(h_1 h_2^* + h_2 h_1^*\right) S_n^{-1} df}{2\left[\left(\int A^2 S_n^{-1} df\right)\left(\int A^2 S_n^{-1} df\right)\right]^{1/2}} \simeq \frac{\int (\delta\psi)^2 A^2 S_n^{-1}}{2\int A^2 S_n^{-1} df} = \frac{1}{2}\langle \delta\psi^2 \rangle$$

How much each PN order contribute to (time-domain) phasing?

From Blanchet+ PRL (2023) arXiv:2304.11185





Scattering angle comparison for unbound encounters

Rettegno+ : PRD (2023) arXiv:2307.06999

Outline

Fundamental GR: inspiral analytic model

Inspiral $h = A\cos(\phi(t))$ $\quad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M\pi f_{GW})^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 \left(1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^4 + \ldots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} \left(1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^3 + \ldots\right) \end{aligned}$$

E(v)(P(v)) known up to 4(4.5)PN Damour+ 1601.01283, Blanchet+ 1706.08480, Foffa+ 1903.05113/18 (Blanchet+ 2304.11186, 2407.18295)

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)}dv \\ &\sim \int \left(1 + \#(\eta, S_i/m_i^2)v^2 + \ldots + \#(\eta, S_i/m_i^2)v^6 + \ldots\right)\frac{dv}{v^6} \end{aligned}$$

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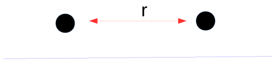
PN Coefficients (absorption $\sim v^8$, tidal $\sim v^{10}$)

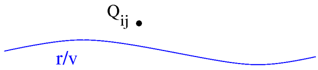
Fundamental aspects of the EFT formalism

NRGR & PN approximation to GR: small expansion parameter v, related to metric perturbation $v^2 \sim \frac{GM}{r}$

Near zone, $D \sim r$

Far zone, $D \gtrsim \lambda = r/v$





Describe conservative dynamics

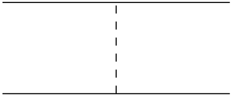
conservative + dissipative

EFT framework pioneered by W. Goldberger & I. Rothstein, hep-th/0409156
for spin R. Porto, gr-qc/0511061 (no spin in this talk)

Inspired by NRQCD, see e.g. Manohar & Stewart hep-ph/0605001

Scale separation → Near + Far

The full (I), Near, Far zone integrals:



$$
\begin{aligned}
I &\equiv e^{-i\omega t}e^{i\mathbf{k}\cdot\mathbf{x}}\frac{1}{\mathbf{k}^2-\omega^2}\,, \\
N &\equiv e^{-i\omega t}e^{i\mathbf{k}\cdot\mathbf{x}}\frac{1}{\mathbf{k}^2}\sum_{m\geq 0}\left(\frac{\omega^2}{\mathbf{k}^2}\right)^m = e^{-i\omega t}e^{i\mathbf{k}\cdot\mathbf{x}}\frac{1}{\mathbf{k}^2}\sum_{m\geq 0}\left[-\left(\frac{d^2}{dt^2}\right)\mathbf{k}^2\right]^m ,(v^2) \\
F &\equiv e^{-i\omega t}\sum_{n\geq 0}\frac{(i\mathbf{k}\cdot\mathbf{x})^n}{n!}\frac{1}{\mathbf{k}^2-\omega^2}\,, \qquad (v)
\end{aligned}
$$

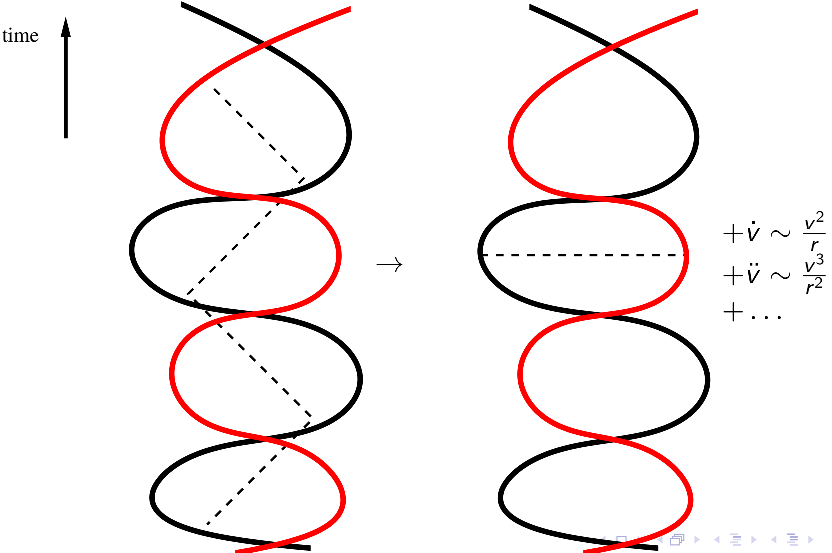
Decomposing

$$
\begin{aligned}
\int_{\mathbf{k}} I &= \int_{\mathbf{k}} F + \int_{\mathbf{k}} N + \int_{\mathbf{k}<\bar{\mathbf{k}}}\left(\underbrace{I-F}_{=0}-N\right) + \int_{\mathbf{k}>\bar{\mathbf{k}}}\left(\underbrace{I-N}_{=0}-F\right) \\
&= \int_{\mathbf{k}} F + \int_{\mathbf{k}} N - \int_{\mathbf{k}<\bar{\mathbf{k}}} N - \int_{\mathbf{k}>\bar{\mathbf{k}}} F \\
&= \int_{\mathbf{k}} F + \int_{\mathbf{k}} N - 2\underbrace{\int_{\mathbf{k}}\sum_{n,m}\frac{(i\mathbf{k}\cdot\mathbf{x})^n}{n!}\left(\frac{\omega^2}{\mathbf{k}^2}\right)^m}_{=0 \ in\ dim.\ reg.}
\end{aligned}
$$

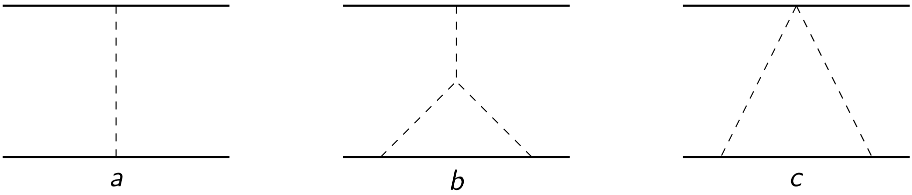
PN approximation for compact binary systems

	Near	Far
World-line	$-m_a \int dt + \int dx^\mu S^{ij}\omega_{\mu ij}$ $\int dt\left(Q_{ij}(S)R_{0i0j} + R^2_{\mu\nu\rho\sigma} + \dots\right)$	$\int d^4x \Big(E h_{00} + \frac{1}{2}S^{ij}h_{0i,j}$ $+Q_{ij}\underbrace{E^{ij} + O_{ijk}}_{v}E^{ij,k} + J_{ij}B_{ij}\dots\Big)$
Bulk	$\frac{1}{16\pi G}\int d^4x\left[R - \frac{1}{2}\left(g^{\alpha\beta}\Gamma^\mu_{\alpha\beta}\right)^2\right]$	
5PN	Blumlein, Maier, Marquard, Schäfer 2010.13672	Foffa, RS 2103.03190 Almeida, Foffa, Muller, RS 2307.05327, 2410.10565 Porto, Riva, Yang 2409.05860

Near zone approximation trades knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time (PN approximation)



Near Zones examples



Examples of NZ processes at LO and NLO. From *Riemann*^{2} (diag c) in the world-line one has $(\partial^2)^2 1/r^2 \sim 1/r^6 \sim 5PN$ finite size effect

Method of regions: Internal graviton momentum can be expanded following the scaling:

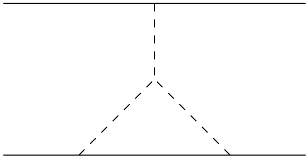
hard	(m, m)	quantum	×
soft	$(\lvert\vec{q}\rvert, \lvert\vec{q}\rvert)$	quantum	×
potential	$(v/r, 1/r)$	classical	✓
radiation	$(v/r, v/r)$	classical	✓

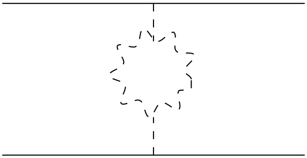
and then integrated over the full phase space

Only potential and radiation gravitons exchanged in classical processes: theory in terms of world lines selects diagrams that do not send source off-shell

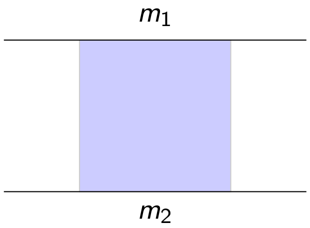
Potential graviton $\rightarrow$ small change in energy wrt momentum, dominate classically

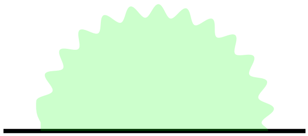
Ex. of classical/quantum connected diagrams



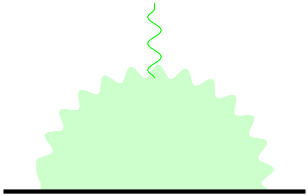


Near vs. Far zone graphs

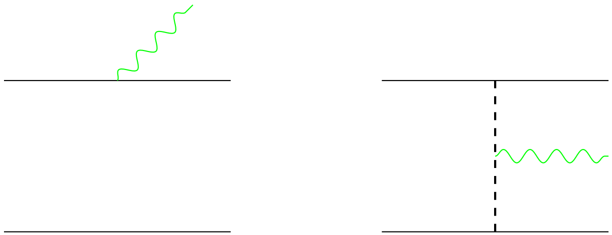




And 1 pt diagrams → radiation



Matching: Multipoles in terms of two-body variables



Expanding in powers of the external momentum one obtains the coupling between long wavelength and multipole moments

Outline

Summary: 2 body dynamics expansions (spin-less)

Post-Minkowskian expansion parameter is $G_N M/r$, vs PN expansion

$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots]$$

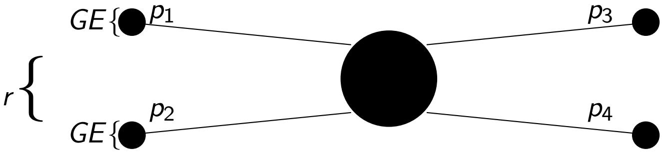
		N	1PN	2PN	3PN	4PN	5PN*	6PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	v^{14}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	v^{12}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	v^{10}/r	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	v^8/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	v^6/r^4	...
5PM*						$1/r^5$	v^2/r^5	v^4/r^5	...
6PM							$1/r^6$	v^2/r^6	...
...								$1/r^7$	...

4PM by Bern, Parra-Martinez, Roiban, Ruf, Shen, PRL (2022) 2112.10750 $\rightarrow$ 4PM
Driess et al., Nature (2025), 2411.11846 $\rightarrow$ 5PM $O(\nu)$
Dlapa, Kälin, Liu, Porto, 2506.20665 $\rightarrow$ 5PM $O(\nu)$

Still missing non-linear radiative^2 terms contributing 5PM $O(\nu^2)$, which include missing 5PN non-linear memory terms.

Scattering amplitudes and PM approximation

In fundamental particle physics one would expect the perturbative parameter $GE^2 \ll 1$, $E^2 \sim (p_1 + p_2)^2$ (like in *Fermi* theory)



For BBH scattering we work under the condition

$$\underbrace{\lambda_{DB} \ll GE}_{\text{suppresses quantum effects}}$$

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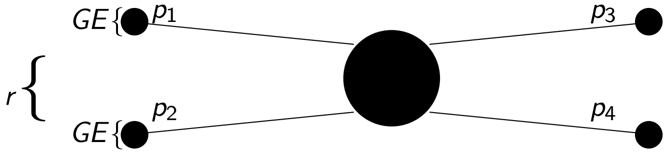


For BBH scattering we work under the condition

$$\lambda_{DB} \ll \underbrace{GE \ll r}_{\text{weakgravityexpansion}}$$

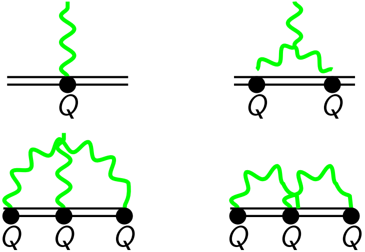
Processes $\frac{\hbar}{L} \ll 1$ and perturbative expansion in $\frac{GM}{b} \sim \frac{GE^2}{L} \ll 1$

Non-linear multipoles and memory

Most complicated sector so far: radiative multipoles self-interactions

Emission

Self-energy



$$h(t) \propto G T_{ij} \to \frac{G}{2} \ddot{Q}_{ij}$$

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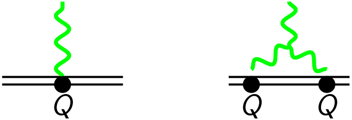
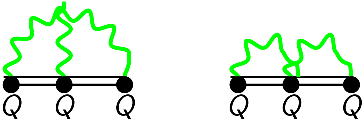


Self-energy



$$
\begin{aligned}
h(t) &\propto G\,\ddot{Q}_{ij} + G^2 \int_{-\infty}^{t} dt'\,\dddot{Q}_{ik}(t')\,\dddot{Q}_{jk}(t') \\
&\sim G\omega^2 \tilde{Q}_{ij} + \frac{G^2}{\omega} \int_{-\infty}^{\infty} d\omega' \omega'^3 (\omega-\omega')^3 \tilde{Q}_{ik}(\omega-\omega')\tilde{Q}_{kj}(\omega')
\end{aligned}
$$

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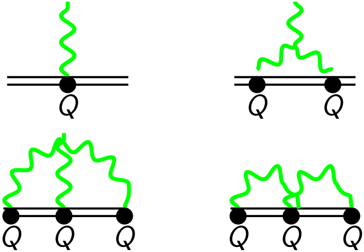

$$\tilde{h}(\omega) \propto G\,\ddot{Q}_{ij} + G^2 \int_{-\infty}^{t} dt'\,\dddot{Q}_{ik}(t')\,\dddot{Q}_{jk}(t')$$

$$\sim G\omega^2\,\tilde{Q}_{ij} + \frac{G^2}{\omega}\int_{-\infty}^{\infty} d\omega'\omega'^3(\omega-\omega')^3\,\tilde{Q}_{ik}(\omega-\omega')\,\tilde{Q}_{kj}(\omega')$$

Memory

Linear memory: After scattering, system conserves a memory of the interaction $h_{ij}|_{t\to-\infty}^{t\to\infty} \sim \Delta T_{ij} \neq 0$.

Christodoulou PRL '91

Relevant for bound system $\to$ remnant with recoil (though non-linear in radiative multpoles)

Non-linear memory: radiation generated by radiation scattering depends on the history of the source (causal e non-local):

$$h_{mem}(t) \sim \int_{\infty}^{t} \left[\frac{\partial^2 E}{\partial t' \partial \Omega'} \frac{\hat{n}'^i \hat{n}'^j}{(1 - \hat{n}' \cdot \vec{v})_r} d\Omega' \right]^{TT} dt'$$

Thorne Rev. Mod. Phys. '80

QCD meets gravity 2025

in São Paulo (ICTP-SAIFR) 15-19/12/2025

Pre-QCD meets gravity school, 8-12/12, lecturers:

- Alfredo Guevara
- Henrik Johansson
- Radu Roiban
- RS

THE END