Chiral anomaly and its implications for hadron physics

Sayantan Sharma

The Institute of Mathematical Sciences

February 6, 2024

Probing Hadron Structure at the Electron-Ion Collider

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \rightarrow SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.

  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous

$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} tr\left[F^{\mu\nu} F_{\mu\nu}\right] \ .$$

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \to SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.
  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous

$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} tr \left[F^{\mu\nu} F_{\mu\nu}\right] \ .$$

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \to SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.

  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous

$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} tr\left[F^{\mu\nu} F_{\mu\nu}\right] .$$

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \rightarrow SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.

  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous

$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} tr\left[F^{\mu\nu} F_{\mu\nu}\right] \ .$$

What are the interesting questions in finite T QCD?

- The magnitude of the $U_A(1)$ breaking term can affect the order of the chiral phase transition in 2-flavor QCD as $m_{u,d} \to 0$.
  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

- Since $U_A(1)$ part is related to the index of the gauge fields, hence its origin is topological.

- An answer to the question holds the key towards understanding the topological origin of chiral symmetry breaking.

What are the interesting questions in finite T QCD?

- The magnitude of the $U_A(1)$ breaking term can affect the order of the chiral phase transition in 2-flavor QCD as $m_{u,d} \to 0$.
  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

- Since $U_A(1)$ part is related to the index of the gauge fields, hence its origin is topological.

- An answer to the question holds the key towards understanding the topological origin of chiral symmetry breaking.

What are the interesting questions in finite T QCD?

- The magnitude of the $U_A(1)$ breaking term can affect the order of the chiral phase transition in 2-flavor QCD as $m_{u,d} \to 0$.

  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

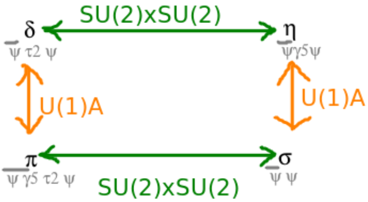
- Since $U_A(1)$ part is related to the index of the gauge fields, hence its origin is topological.

- An answer to the question holds the key towards understanding the topological origin of chiral symmetry breaking.

What are the interesting questions in finite T QCD?

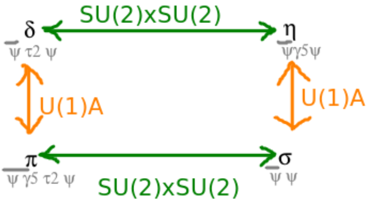
- The magnitude of the $U_A(1)$ breaking term can affect the order of the chiral phase transition in 2-flavor QCD as $m_{u,d} \to 0$.
  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]
- Do singlet and non-singlet chiral symmetries gets restored simultaneously?
- Since $U_A(1)$ part is related to the index of the gauge fields, hence its origin is topological.
- An answer to the question holds the key towards understanding the topological origin of chiral symmetry breaking.

How to quantify $U_A(1)$ breaking?



- Since $U_A(1)$ is not a symmetry of the partition function $\rightarrow$ no order parameter.
- Can be measured only from properties of hadrons. The observable are integrated correlation functions of hadrons of different quantum numbers [Shuryak, 94].

How to quantify $U_A(1)$ breaking?

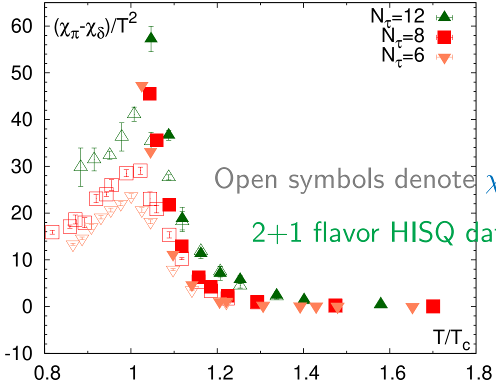


- Since $U_A(1)$ is not a symmetry of the partition function $\rightarrow$ no order parameter.
- Can be measured only from properties of hadrons. The observable are integrated correlation functions of hadrons of different quantum numbers [Shuryak, 94].

- It's a hard problem! The most used lattice Dirac action preserves only a remnant of the continuum chiral symmetries.

- One important check is to find how well the chiral Ward identities are reproduced [L. Giusti, G. C. Rossi, M. Testa, 04, HotQCD 1205.3535]

$$\chi_\pi - \chi_\delta = 2\ \chi_{5,disc} = 2\ \chi_{disc}\ .$$



Open symbols denote χ_{disc}

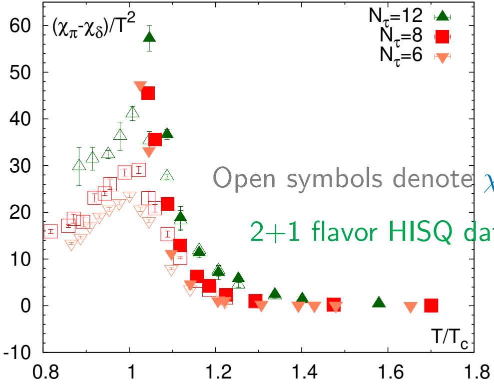
2+1 flavor HISQ data $m_\pi = 160$ MeV

[Fig. from Petreczky, Schadler, S.S. 16].

- It's a hard problem! The most used lattice Dirac action preserves only a remnant of the continuum chiral symmetries.

- One important check is to find how well the chiral Ward identities are reproduced [L. Giusti, G. C. Rossi, M. Testa, 04, HotQCD 1205.3535]

$$\chi_\pi - \chi_\delta = 2\ \chi_{5,disc} = 2\ \chi_{disc}\ .$$



Open symbols denote χ_{disc}

2+1 flavor HISQ data $m_\pi = 160$ MeV

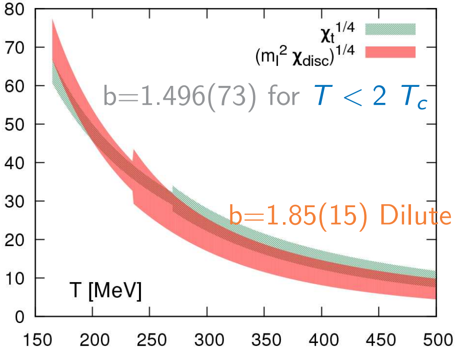
[Fig. from Petreczky, Schadler, S.S. 16].

Topological origin of the $U_A(1)$ breaking

- The topological susceptibility is related to $U_A(1)$ breaking through $\chi_t = \ m^2 \ \chi_{disc} = m^2(\chi_\pi - \chi_\delta)/4$.

- Characterizing, $\chi_t^{1/4}(T) \sim (T_c/T)^b$ [Petreczky, Schadler, S.S. 16].

[See also C. Bonati et. al., 15, 18, Sz. Borsanyi et. al., 16, F. Burger et. al, 18]



b=1.496(73) for $T < 2\ T_c$

b=1.85(15) Dilute instanton gas $T > 2\ Tc$

- Integrated correlators can be written in terms of the eigenvalue density of QCD Dirac operator

$$\chi_\pi - \chi_\delta \stackrel{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Very little known about $\rho(\lambda)$. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, Ward identities of n-point function of scalar & pseudo-scalar currents in chiral symmetry restored phase can be related in terms of $\rho(\lambda)$.
- $\rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Is the eigenvalue spectrum always a polynomial in λ?

- Integrated correlators can be written in terms of the eigenvalue density of QCD Dirac operator

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Very little known about $\rho(\lambda)$. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, Ward identities of n-point function of scalar & pseudo-scalar currents in chiral symmetry restored phase can be related in terms of $\rho(\lambda)$.
- $\rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Is the eigenvalue spectrum always a polynomial in λ?

- Integrated correlators can be written in terms of the eigenvalue density of QCD Dirac operator

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Very little known about $\rho(\lambda)$. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, Ward identities of n-point function of scalar & pseudo-scalar currents in chiral symmetry restored phase can be related in terms of $\rho(\lambda)$.
- $\rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Is the eigenvalue spectrum always a polynomial in λ?

- Integrated correlators can be written in terms of the eigenvalue density of QCD Dirac operator

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Very little known about $\rho(\lambda)$. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, Ward identities of n-point function of scalar & pseudo-scalar currents in chiral symmetry restored phase can be related in terms of $\rho(\lambda)$.
- $\rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Is the eigenvalue spectrum always a polynomial in λ?

- Integrated correlators can be written in terms of the eigenvalue density of QCD Dirac operator

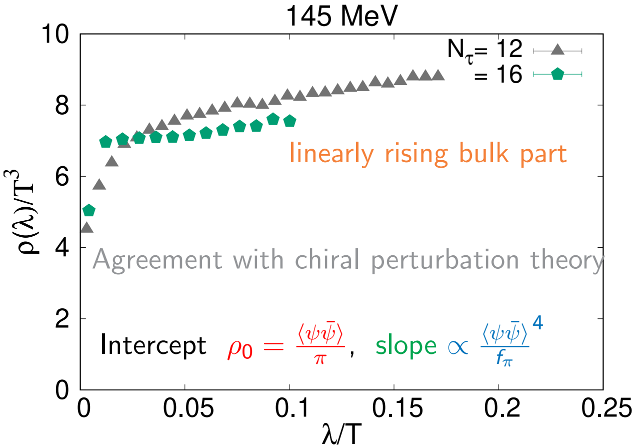
$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2+m_f^2)^2}$$

- Very little known about $\rho(\lambda)$. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, Ward identities of n-point function of scalar & pseudo-scalar currents in chiral symmetry restored phase can be related in terms of $\rho(\lambda)$.
- $\rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Is the eigenvalue spectrum always a polynomial in λ?

- The spectral density can be characterized as:

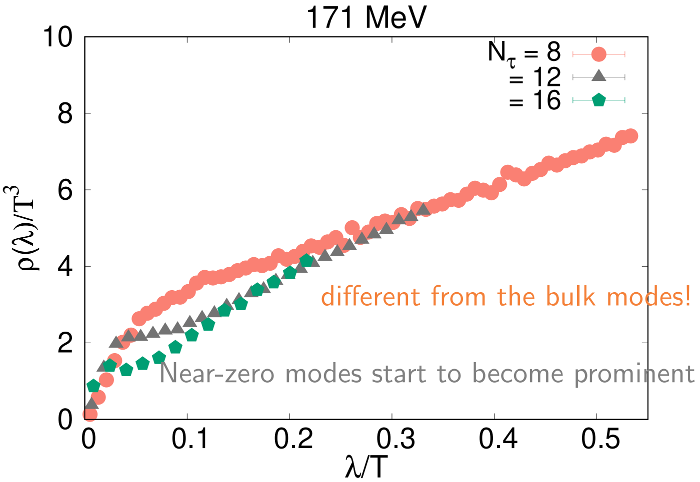
$$\frac{\rho(\lambda)}{T^3} = \frac{\rho_0}{T^3} + \frac{\lambda}{T}.\frac{c_1(T,m)}{T^2} + \frac{\lambda^2}{T^2}.\frac{c_2(T,m)}{T} + \frac{\lambda^3}{T^3}c_3(T,m)\ .$$

[Ref: O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023].



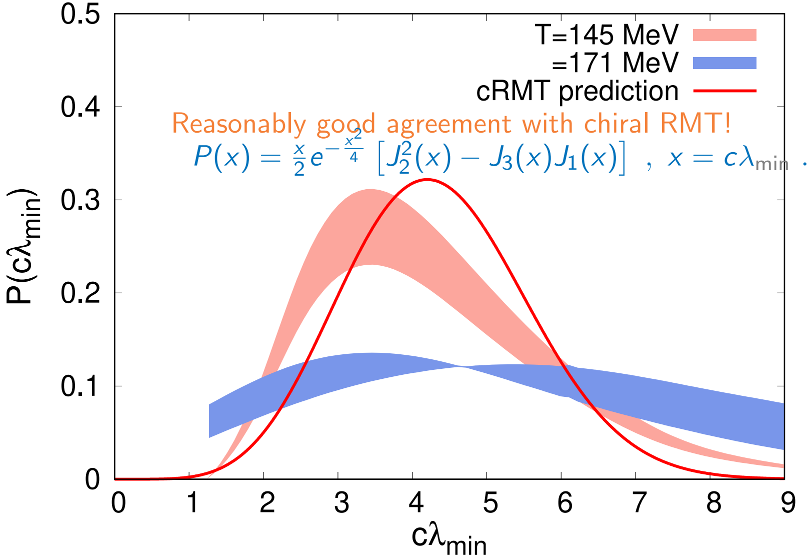
Spectral Density when chiral symmetry is restored

- The bulk modes show a linear rise in λ characterized by $c_1(T, m) = 16.8(4)\,T^2 + \mathcal{O}(m^2/T^2)$. Not λ^3!



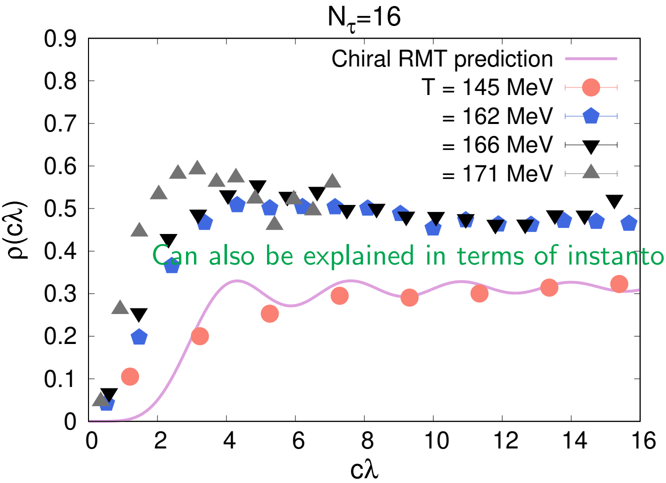
[Fig. from O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023].

Lowest eigenvalue and chiral random matrix theory



What more does the eigenspectrum tell us?

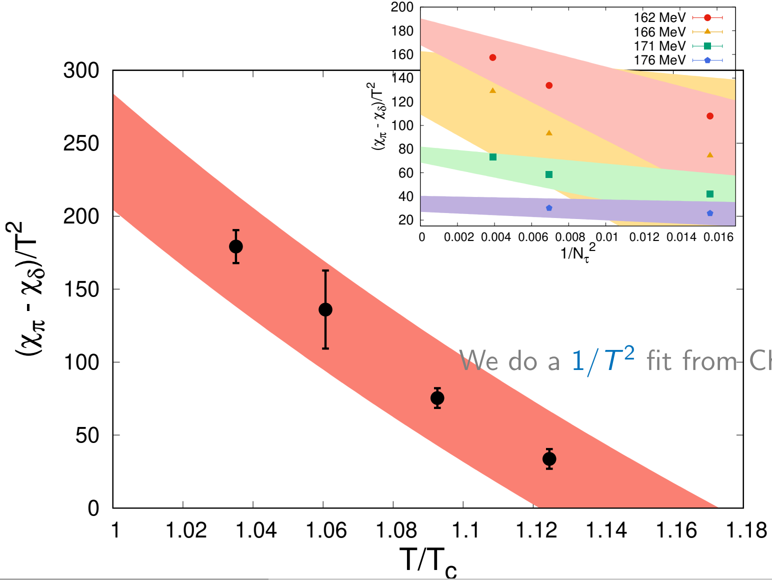
- $T < T_c$: random matrix theory predicts eigenvalues of QCD $\rightarrow$ disordered phase [O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023].
- $T > T_c$: disorder decreases: interactions become short ranged.



Can also be explained in terms of instanton-liquid model.

$U_A(1)$ effectively restored at $1.15\,T_c$

[O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023]



We do a $1/T^2$ fit from ChiPT

Summary of our results

- From hadronic correlators axial part of chiral symmetry is effectively restored at 1.15 T_c $\rightarrow$ near-zero and the bulk modes of the QCD Dirac operator disentangle.

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields i.e. instantons

- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential responsible for λ-dep. in the quark Dirac spectrum. No temperature dep.

- The eigenvalue spectrum can be explained in terms of $T = 0$ chiral RMT of GUE universality class. Such a dependence can be interpreted in terms of an interacting ensemble of instantons

  [J. Verbaarschot, 94]

Summary of our results

- From hadronic correlators axial part of chiral symmetry is effectively restored at 1.15 T_c $\rightarrow$ near-zero and the bulk modes of the QCD Dirac operator disentangle.

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields i.e. instantons

- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential responsible for λ-dep. in the quark Dirac spectrum. No temperature dep.

- The eigenvalue spectrum can be explained in terms of $T = 0$ chiral RMT of GUE universality class. Such a dependence can be interpreted in terms of an interacting ensemble of instantons [J. Verbaarschot, 94]

Summary of our results

- From hadronic correlators axial part of chiral symmetry is effectively restored at 1.15 T_c $\rightarrow$ near-zero and the bulk modes of the QCD Dirac operator disentangle.
- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields i.e. instantons
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential responsible for λ-dep. in the quark Dirac spectrum. No temperature dep.
- The eigenvalue spectrum can be explained in terms of $T = 0$ chiral RMT of GUE universality class. Such a dependence can be interpreted in terms of an interacting ensemble of instantons

  [J. Verbaarschot, 94]

Summary of our results

- From hadronic correlators axial part of chiral symmetry is effectively restored at 1.15 T_c $\rightarrow$ near-zero and the bulk modes of the QCD Dirac operator disentangle.

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields i.e. instantons

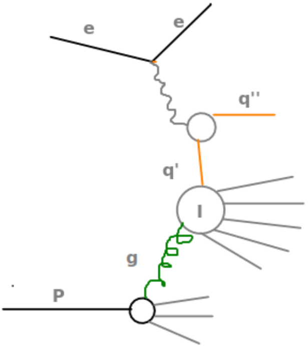
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential responsible for λ-dep. in the quark Dirac spectrum. No temperature dep.

- The eigenvalue spectrum can be explained in terms of $T = 0$ chiral RMT of GUE universality class. Such a dependence can be interpreted in terms of an interacting ensemble of instantons

  [J. Verbaarschot, 94]

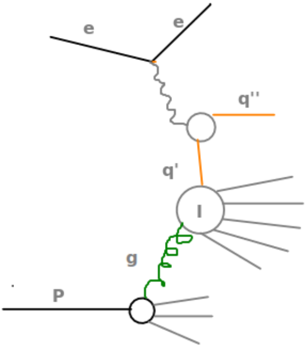
Can instanton effects be measured at the EIC?

- Instanton induced effects visible in the chirality violating process $g + \gamma^* \to \sum_{q \in u,d} (\bar{q}_L + q_R) + n_g g$ which leads to high multiplicity of charged and neutral particles [A. Ringwald & J. Schrempp, 98, 99].
- For HERA the typical kinematic region was $0.2 < y < 0.7$ and $150 < Q^2 < 15000$ GeV^2.
- Workflow: instanton perturbation theory, LSZ reduction and analytic continuation to Minkowski space



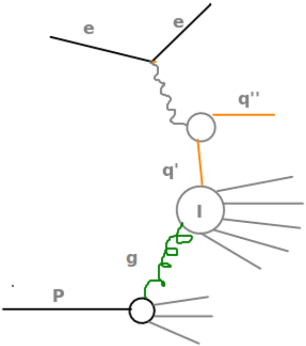
Can instanton effects be measured at the EIC?

- Instanton induced effects visible in the chirality violating process $g + \gamma^* \to \sum_{q \in u,d} (\bar{q}_L + q_R) + n_g g$ which leads to high multiplicity of charged and neutral particles [A. Ringwald & J. Schrempp, 98, 99].
- For HERA the typical kinematic region was $0.2 < y < 0.7$ and $150 < Q^2 < 15000$ GeV^2.
- Workflow: instanton perturbation theory, LSZ reduction and analytic continuation to Minkowski space



Can instanton effects be measured at the EIC?

- Instanton induced effects visible in the chirality violating process $g + \gamma^* \to \sum_{q\in u,d} (\bar{q}_L + q_R) + n_g g$ which leads to high multiplicity of charged and neutral particles [A. Ringwald & J. Schrempp, 98, 99].
- For HERA the typical kinematic region was $0.2 < y < 0.7$ and $150 < Q^2 < 15000$ GeV^2.
- Workflow: instanton perturbation theory, LSZ reduction and analytic continuation to Minkowski space



Can instanton effects be measured at the EIC?

- The amplitude of such a process is

$$\mathcal{M}^a_{\mu,\mu'} = \int_0^\infty \frac{d\rho}{\rho^5} D(\rho, \mu_R) \int \mathcal{D}U \; \mathcal{A}^a_{\mu,\mu'}(\rho, U) \; , \; a = 1-3.$$

- The typical input is the density distribution D which has been typically measured in dilute instanton gas regime [A. Ringwald & J. Schrempp, 98, 99].
- From our results on the onset of dilute regime we can constrain the size $\rho \sim 1/(900$ MeV$) \Rightarrow$ valid for $Q_\gamma > 10$ GeV. Size of instantons $\rho \lesssim Q_\gamma^{-1}$ hence IR-divergences can be controlled.
- Lattice QCD can do much more! It is possible to isolate the individual instanton-like objects and constrain the size distribution given the virtuality scale $\rightarrow$ one might need to go beyond the dilute regime and measure the interaction measure from $I-\bar{I}$ separations. [Work in progress with Nihar Sahoo & Swagatam Tah].

Can instanton effects be measured at the EIC?

- The amplitude of such a process is

$$\mathcal{M}^a_{\mu,\mu'} = \int_0^\infty \frac{d\rho}{\rho^5} D(\rho, \mu_R) \int \mathcal{D}U \; \mathcal{A}^a_{\mu,\mu'}(\rho, U) \; , \; a = 1 - 3.$$

- The typical input is the density distribution D which has been typically measured in dilute instanton gas regime

  [A. Ringwald & J. Schrempp, 98, 99].

- From our results on the onset of dilute regime we can constrain the size $\rho \sim 1/(900 \text{ MeV}) \Rightarrow$ valid for $Q_\gamma > 10$ GeV. Size of instantons $\rho \lesssim Q_\gamma^{-1}$ hence IR-divergences can be controlled.

- Lattice QCD can do much more! It is possible to isolate the individual instanton-like objects and constrain the size distribution given the virtuality scale $\rightarrow$ one might need to go beyond the dilute regime and measure the interaction measure from $I - \bar{I}$ separations.

  [Work in progress with Nihar Sahoo & Swagatam Tah].

Can instanton effects be measured at the EIC?

- The amplitude of such a process is

$$\mathcal{M}^{a}_{\mu,\mu'} = \int_0^\infty \frac{d\rho}{\rho^5} D(\rho, \mu_R) \int \mathcal{D}U \; \mathcal{A}^{a}_{\mu,\mu'}(\rho, U) \;,\; a = 1-3.$$

- The typical input is the density distribution D which has been typically measured in dilute instanton gas regime [A. Ringwald & J. Schrempp, 98, 99].
- From our results on the onset of dilute regime we can constrain the size $\rho \sim 1/(900$ MeV$) \Rightarrow$ valid for $Q_\gamma > 10$ GeV. Size of instantons $\rho \lesssim Q_\gamma^{-1}$ hence IR-divergences can be controlled.
- Lattice QCD can do much more! It is possible to isolate the individual instanton-like objects and constrain the size distribution given the virtuality scale $\rightarrow$ one might need to go beyond the dilute regime and measure the interaction measure from $I - \bar{I}$ separations. [Work in progress with Nihar Sahoo & Swagatam Tah].

Can instanton effects be measured at the EIC?

- The amplitude of such a process is

$$\mathcal{M}^a_{\mu,\mu'} = \int_0^\infty \frac{d\rho}{\rho^5} D(\rho, \mu_R) \int \mathcal{D}U \; \mathcal{A}^a_{\mu,\mu'}(\rho, U) \; , \; a = 1 - 3.$$

- The typical input is the density distribution D which has been typically measured in dilute instanton gas regime [A. Ringwald & J. Schrempp, 98, 99].
- From our results on the onset of dilute regime we can constrain the size $\rho \sim 1/(900 \text{ MeV}) \Rightarrow$ valid for $Q_\gamma > 10$ GeV. Size of instantons $\rho \lesssim Q_\gamma^{-1}$ hence IR-divergences can be controlled.
- Lattice QCD can do much more! It is possible to isolate the individual instanton-like objects and constrain the size distribution given the virtuality scale $\rightarrow$ one might need to go beyond the dilute regime and measure the interaction measure from $I - \bar{I}$ separations. [Work in progress with Nihar Sahoo & Swagatam Tah].

Outlook

- The effects of anomalous part of chiral symmetry can only be studied from hadron correlation functions. We now understand some of its consequences for the QCD phase transition.
- The precise microscopic origin of the effective restoration of $U_A(1)$ is not yet understood.
- Understanding and isolating the topological constituents is important and the EIC has a very promising prospect in this regard by looking at chirality violating processes.

Outlook

- The effects of anomalous part of chiral symmetry can only be studied from hadron correlation functions. We now understand some of its consequences for the QCD phase transition.
- The precise microscopic origin of the effective restoration of $U_A(1)$ is not yet understood.
- Understanding and isolating the topological constituents is important and the EIC has a very promising prospect in this regard by looking at chirality violating processes.

Outlook

- The effects of anomalous part of chiral symmetry can only be studied from hadron correlation functions. We now understand some of its consequences for the QCD phase transition.
- The precise microscopic origin of the effective restoration of $U_A(1)$ is not yet understood.
- Understanding and isolating the topological constituents is important and the EIC has a very promising prospect in this regard by looking at chirality violating processes.