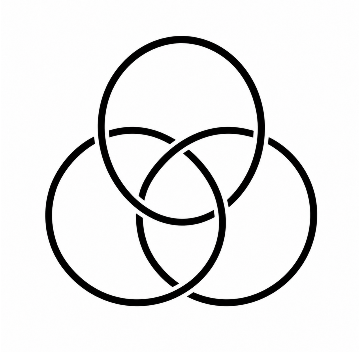
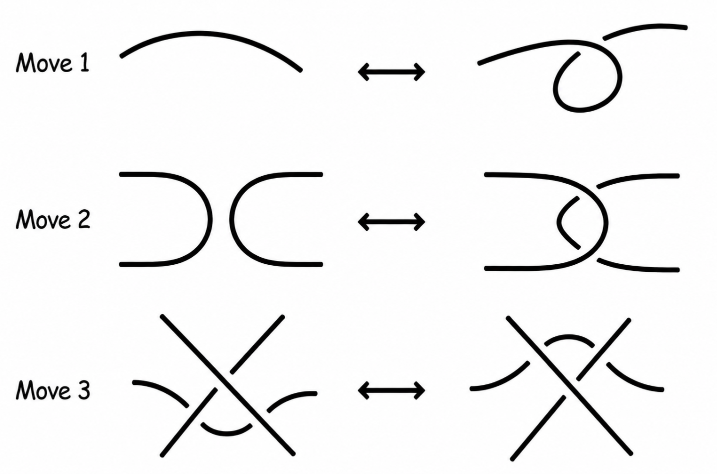
ICTS-RRI Maths Circle
July 25, 2026

Last time, we learned how to study a tangle of ropes by looking only at its shadow.

To remember which strand passes *over* and which passes *under*, we invented diagrams like the one below.



We also discovered that if we continuously deform the ropes without cutting them, or simply change the direction from which we project the shadow of the tangle, then its shadow changes through the following three local moves:



Mathematicians have proved something remarkable.

Any two shadows of the same tangle can be related by a sequence of these three moves.

Several of you invented ways of attaching numbers to a tangle in order to decide whether it can be undone.

If such a method is to work, the quantity you assign should remain unchanged whenever one of the three moves above is performed.

Do the ideas your group proposed pass this test?

If not, which move causes them to fail?

Can you think of a better one?

Now imagine a world with only three colours:

The inhabitants count

$$0, 1, 2, 0, 1, 2, \ldots$$

instead of

$$0, 1, 2, 3, 4, \ldots$$

For example,

$$2 + 1 = 0, \qquad 2 + 2 = 1, \qquad 2 \times 2 = 1.$$

Work out a few more examples until this arithmetic begins to feel natural.

What happens if we had 4 colors instead? Or some other whole number m?
Does the case $m = 12$ seem familiar from everyday life?
Can you think of any other cases that you have encountered?

A whole number p is *prime* if it is impossible to write it as a product of two whole numbers, neither of which is 1.

Let us call the arithmetic world associated to a prime number a "prime world".

Can you tell whether your arithmetic world is a prime world simply by looking at its multiplication table?

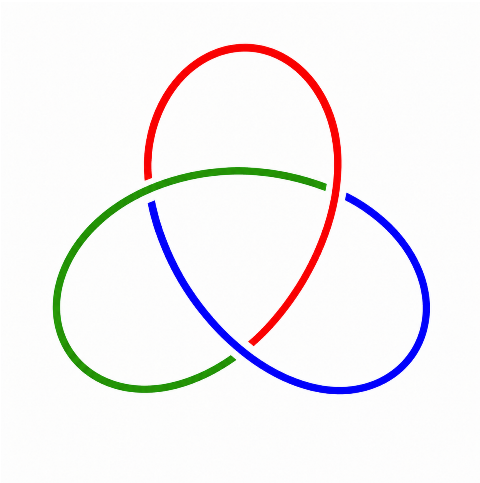
A geometer carries a tiny equilateral triangle whose three sides are coloured red, green and blue.

Whenever she walks once around a strand of the knot and returns to her starting point, the triangle mysteriously comes back rotated or reflected, as though the space around the rope remembered the rope's presence.

If she slowly changes her journey without ever crossing a strand, while always returning to the same starting point, the final position of the triangle doesn't change.

If she first makes journey a, and then journey b (that is, the combined journey $a * b$ from our previous session), the effect on the triangle is obtained by first applying the effect of a, and then the effect of b.

She colours the strand by the colour that appears along the base of her triangle after completing the journey.



Three strands meet at every crossing.

What relationships between their colours are possible?

Can you describe the rule?

Now test your colouring rule against the three local moves you discovered.

Does every colouring before a move correspond to exactly one colouring after the move?

If so, what does this tell you? Can you use this information to help tell apart different knots?

One final question.

Last time we discovered that one rope can be linked with another.

Suppose the ropes are replaced by two insulated loops of wire that can carry electric current.

Could one loop somehow detect the presence of the other, even though they never touch?

And if wires can do this...

Can one prime number feel the presence of another?