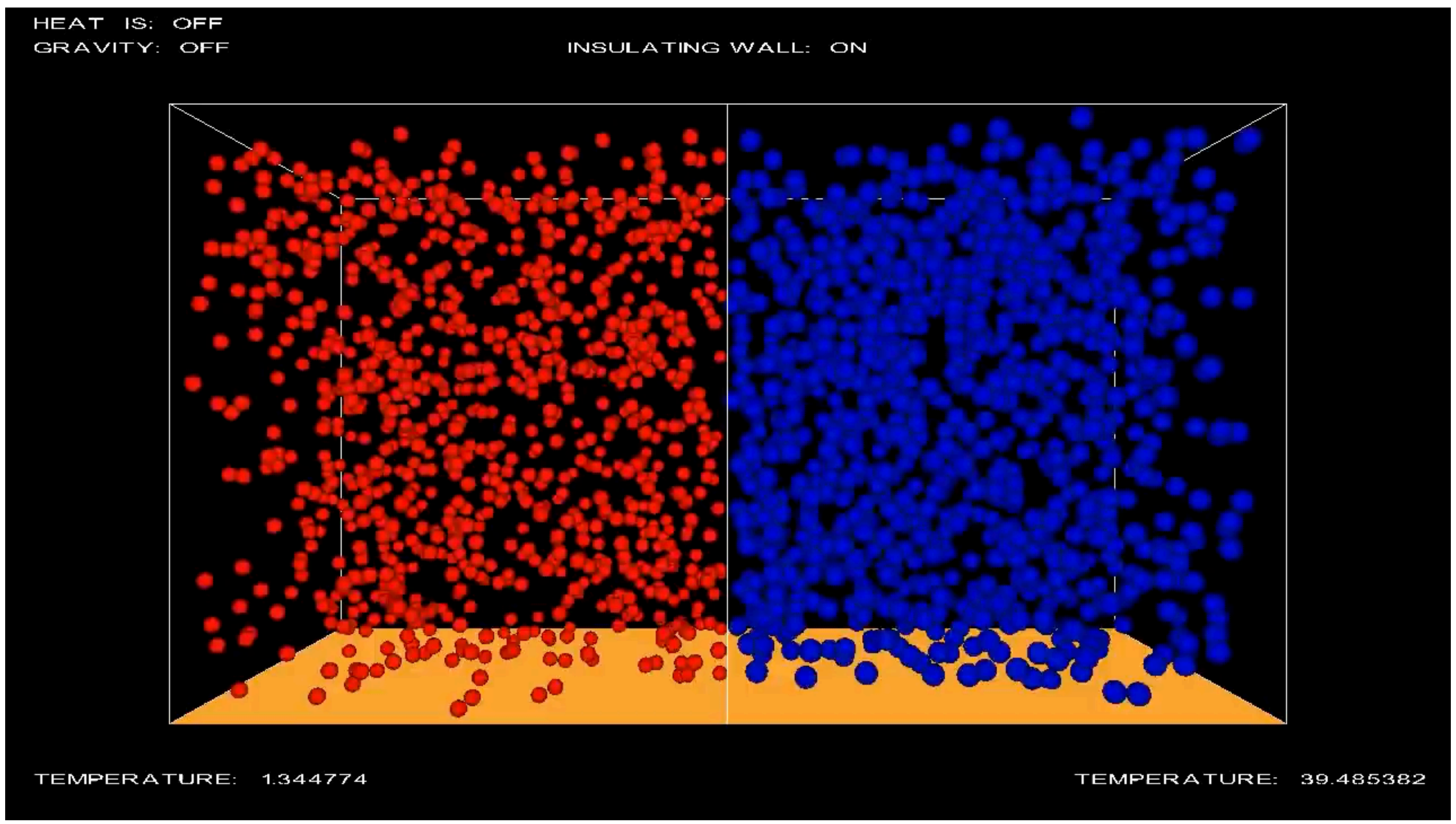
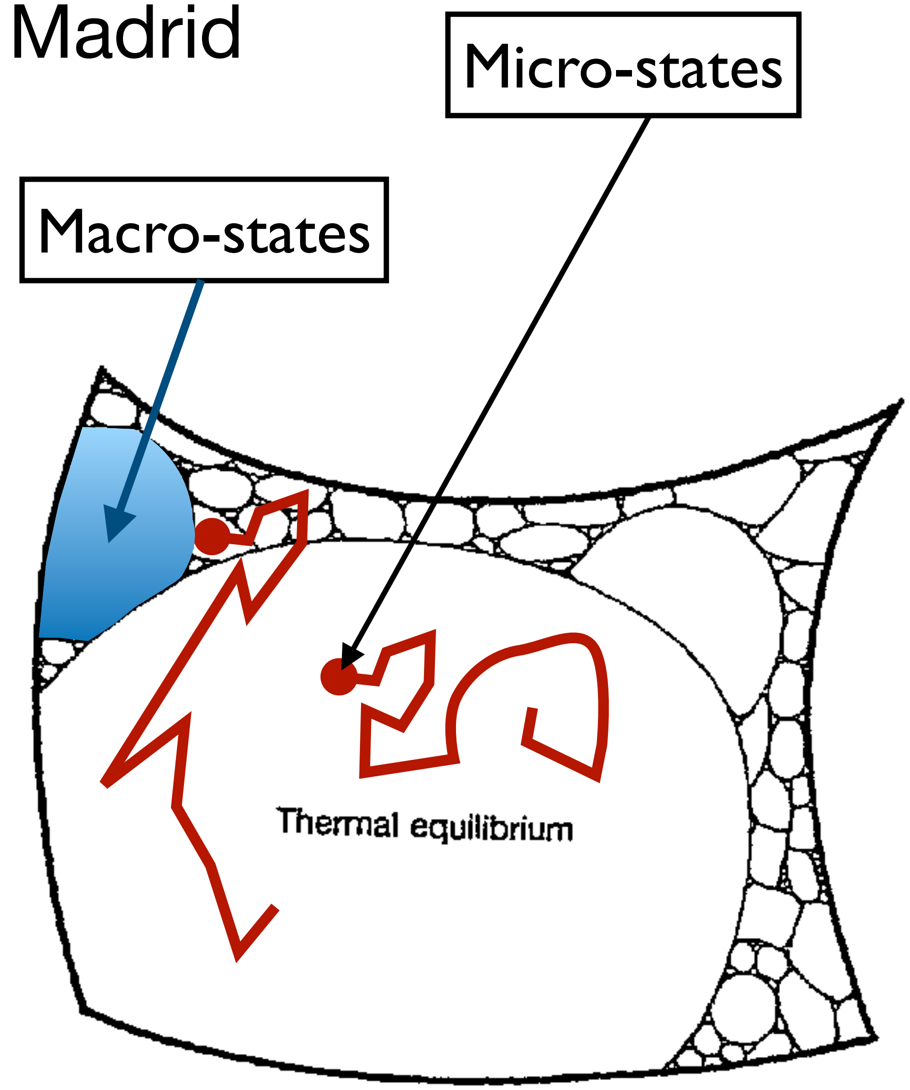
The irreversibility problem revisited: objectivity and giant fluctuations

Juan MR Parrondo
Universidad Complutense de Madrid





Phase space

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BOLTZMANN'S ENTROPY AND TIME'S ARROW

Given that microscopic physical laws are reversible, why do all macroscopic events have a preferred time direction? Boltzmann's thoughts on this question have withstood the test of time.

Joel L. Lebowitz (Physics Today, 1993)

of particles of each type in each cube. To completely specify M so that its further evolution can be predicted, we would also need to state the total energy of the system and any other macroscopically relevant constants of the motion, also within some tolerance. While this specification of the macroscopic state clearly contains some arbitrariness, this need not concern us too much, since all the statements we are going to make about the evolution of M are independent of its precise definition as long as there is a large separation between the macro- and microscales.

Given the success of Ludwig Boltzmann's statistical approach in explaining the observed irreversible behavior of macroscopic systems in a manner consistent with their reversible microscopic dynamics, it is quite surprising that there is still so much confusion about the problem of irreversibility. (See figure 1.) I attribute this confusion to the originality of Boltzmann's ideas: It made them difficult for some of his contemporaries to grasp. The controversies generated by the misunderstandings of Ernst Zermelo and others have been perpetuated by various authors. There is really no excuse for this, considering the clarity of Boltzmann's later writings.[1] Since next year, 1994, is the 150th anniversary of Boltzmann's birth, this is a fitting moment to review his ideas on the arrow of time. In Erwin Schrödinger's words, "Boltzmann's ideas really give an understanding" of the origin of macroscopic behavior. All claims of inconsistencies that I know of are, in my opinion, wrong; I see no need for alternate explanations. For further reading I highly recommend Boltzmann's works as well as references 2–7. (See also PHYSICS TODAY, January 1992, page 44, for a marvelous description by Boltzmann of his visit to California in 1906.)

Arbitrariness did concern me!! → Can we define macro-states in an objective way?

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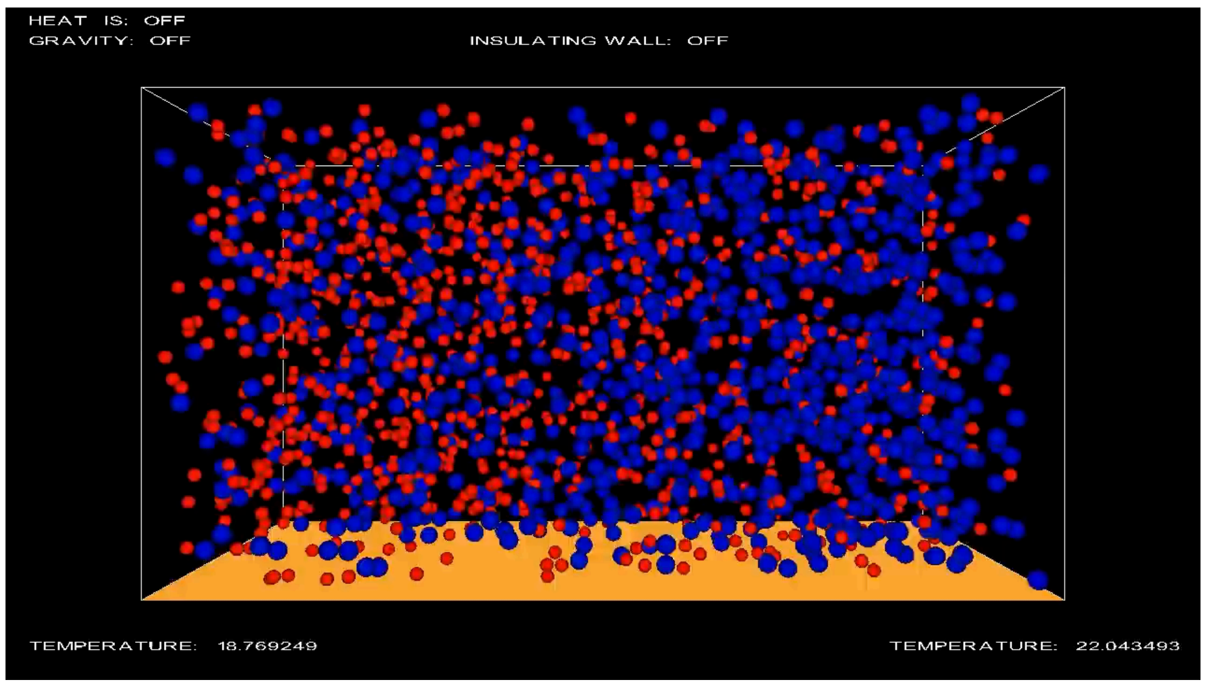
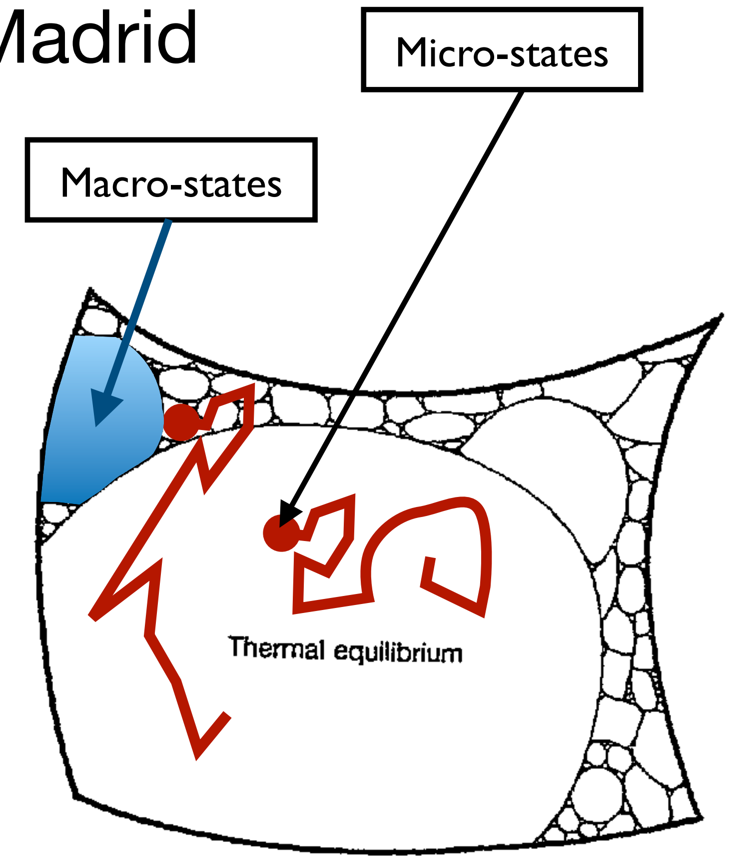
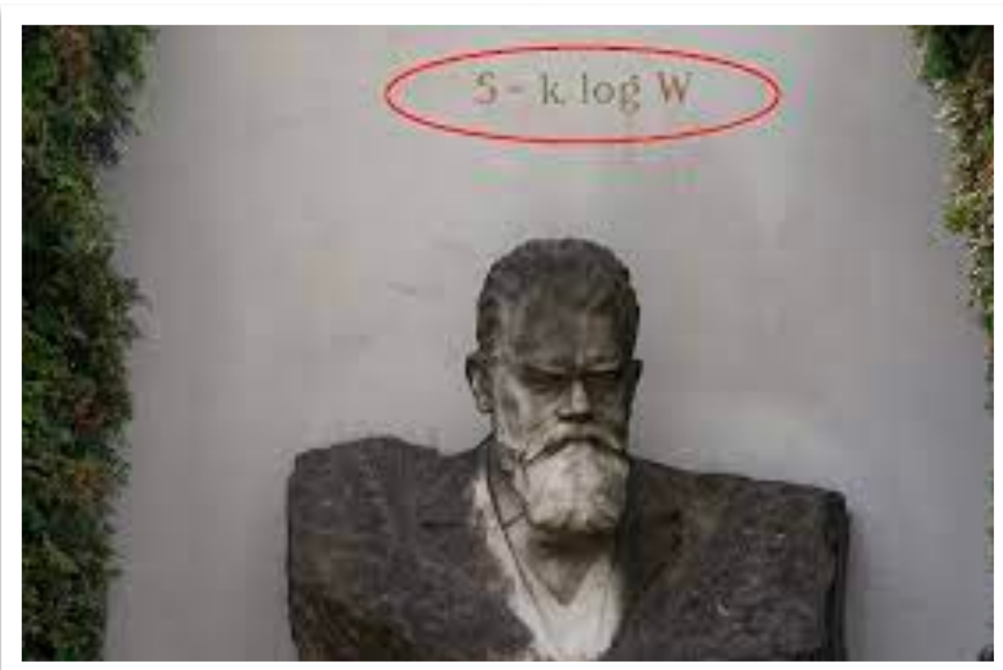


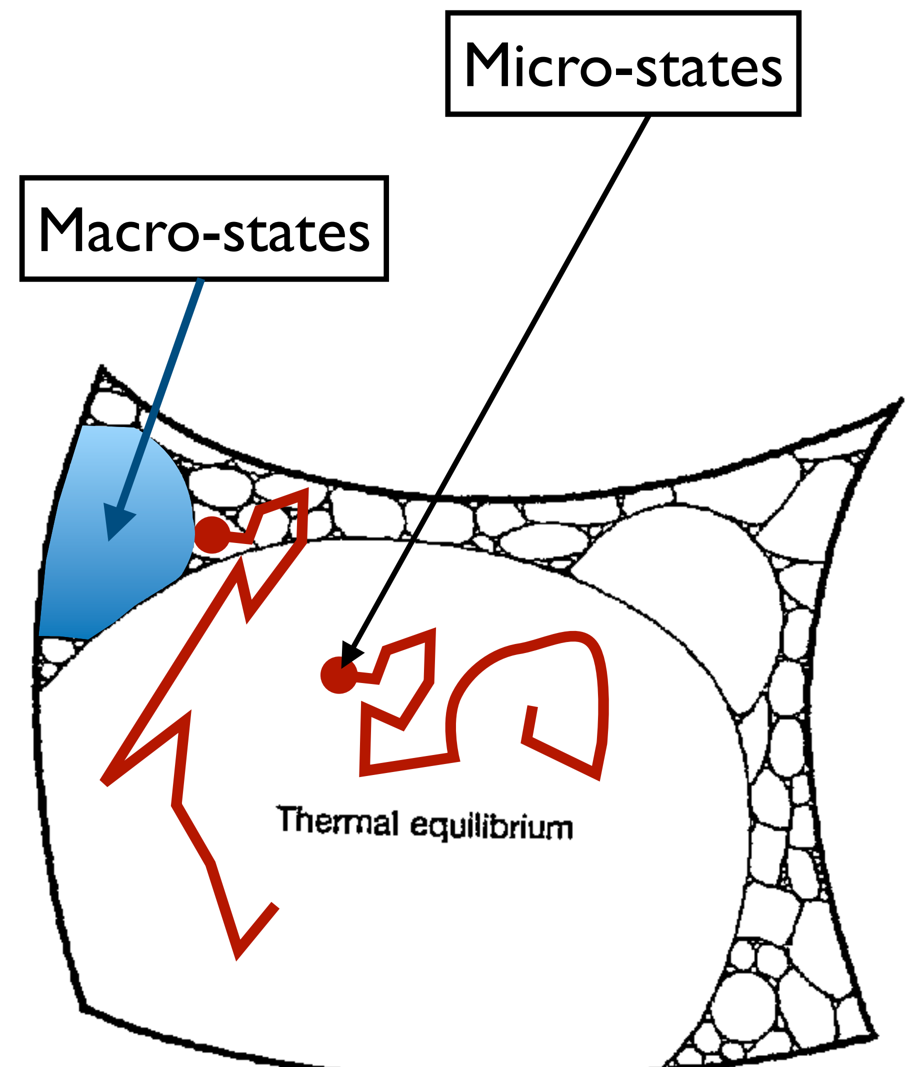


- An **objective reformulation** of Boltzmann's explanation of irreversibility based on observables.
- Symmetry breaking as a possible origin of **giant fluctuations**.

Boltzmann's explanation of irreversibility



Can we define macro-states in an objective way?



$$S = k \log W$$

Entropy of a given macrostate

Boltzmann constant

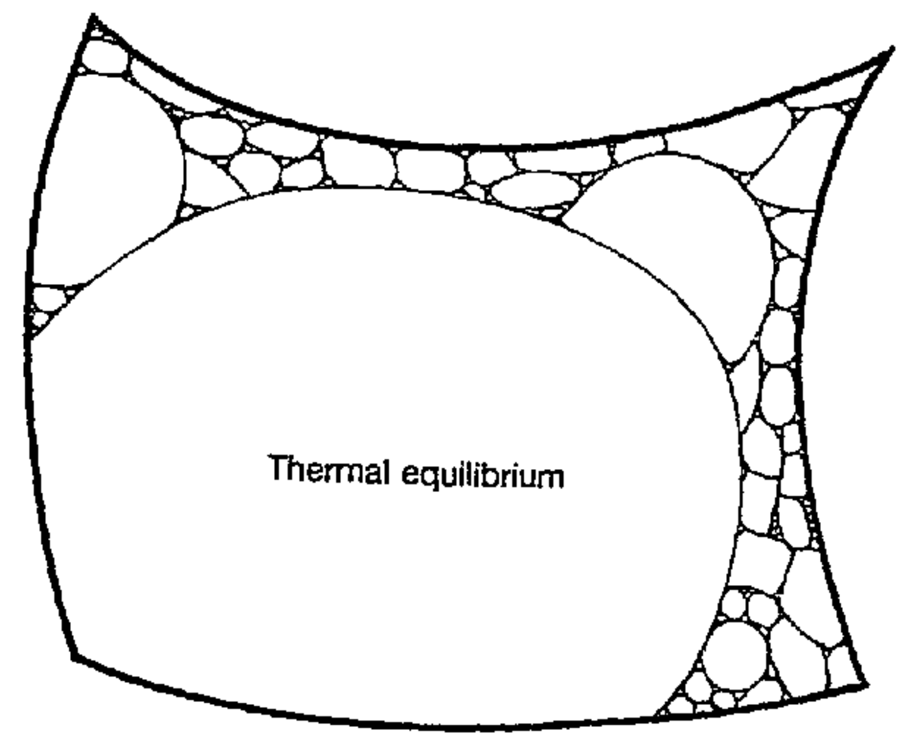
Number of compatible micro-states

Observable-dependent entropy

We define an entropy function
for an observable:

$$A : \Gamma \to \mathbb{R}$$
$$x \to A(x)$$

micro-state



$$\omega(a;E) = \int_\Gamma dx\, \delta(E - H(x))\delta(a - A(x))$$

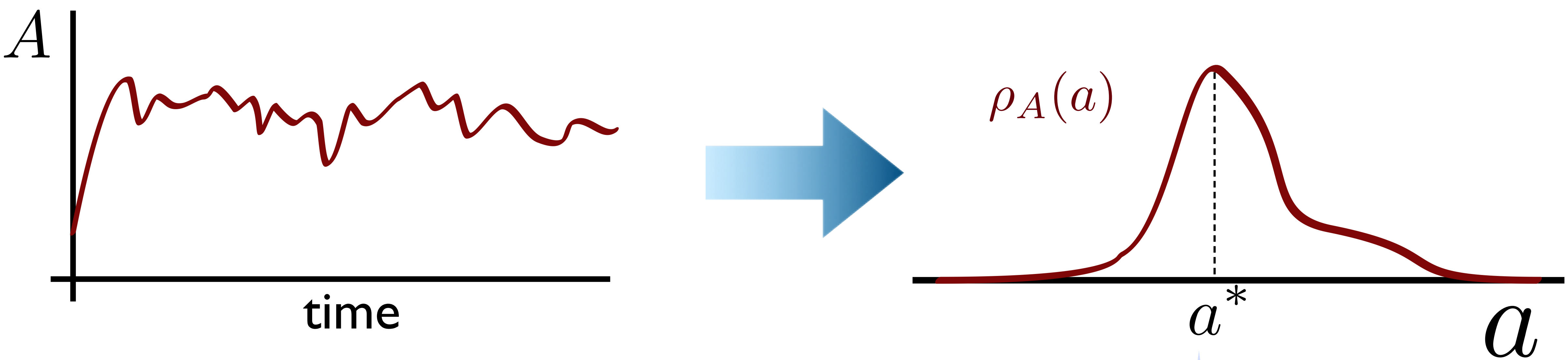
$$S(a;E) = k \log \omega(a;E)$$

Einstein's formula

Uniform sampling (ergodicity): $\rho_A(a) = \dfrac{\omega(a;E)}{\omega(E)} \propto e^{S(a;E)/k}$

Observable-dependent entropy

Uniform sampling (ergodicity): $\rho_A(a) = \dfrac{\omega(a;E)}{\omega(E)} \propto e^{S(a;E)/k}$



$$\max_a S(a;E) = S(a^*;E)$$

most likely value

An observable A exhibits irreversibility if: $S(a^*;E) - S(a_0;E) \gg k$

This condition is fully objective (although imprecise)

initial condition

Observable-dependent irreversibility

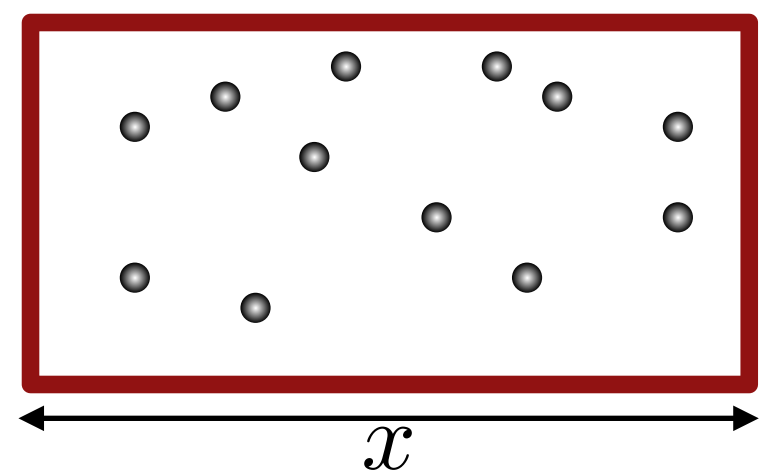
An observable *A* exhibits irreversibility if: $S(a^*;E) - S(a_0;E) \gg k$

initial condition

Two examples:

i) Position of a molecule in a gas:

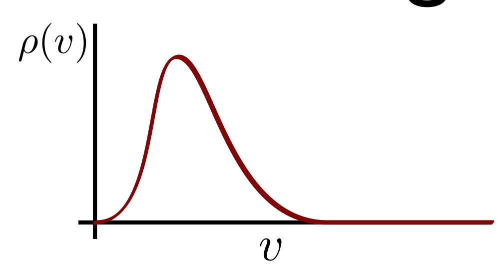
$S(x;E)$ is constant



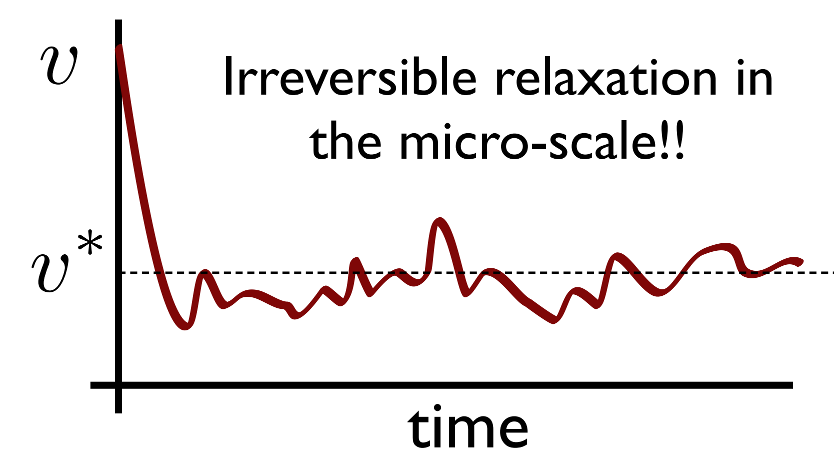
Position cannot exhibit irreversibility, because fluctuations are of the same order as the range of the observable (Reimann & Gemmer, Physica A 2020)

ii) Velocity (modulus) of a molecule in a gas:

$$S(v;E) = -\frac{mv^2}{2T} + \log(v^2) + \mathrm{const}$$



$v^* = \sqrt{\frac{4kT}{m}}$ Irreversibility if: $\frac{mv_0^2}{2} \gg kT$



Objective definition of equilibrium?

An observable *A* exhibits irreversibility if: $S(a^*;E) - S(a_0;E) \gg k$

initial condition

An micro-state *x* is at equilibrium if: $S(a^*;E) - S(A(x);E) \lesssim k \quad \forall A$

It doesn't work: for any micro-state x_0, we can define the irreversible observable:

$$A(x) = \begin{cases} 1 & \text{if } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

Open question: can we obtain an objective definition of equilibrium for micro-states by restricting (in an objective way) the set of observables? (*quantum typicality?*)

Conclusions (1st part)

Objective

- The entropy function of an observable.
- The irreversible behavior of an observable.
- The second law ($\Delta S \geqslant 0$) for an observable.

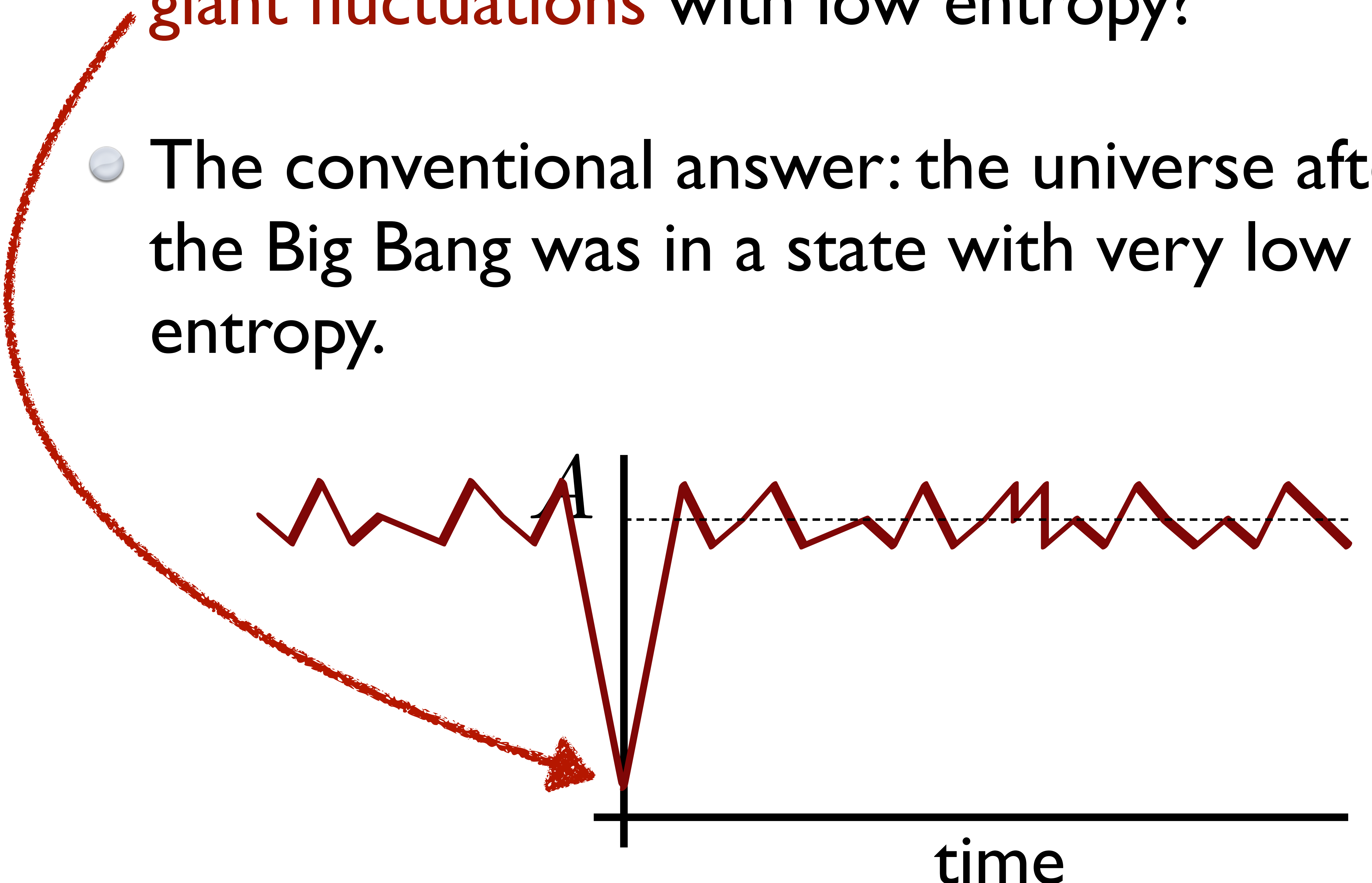
Subjective

- The choice of observables.
  Example: System in contact with a thermal bath.
  Observables= microstate of the system + average energy (temperature) of the bath.
- The entropy of a system.

Equilibrium?

An open question

- How can a system (isolated or not) generate giant fluctuations with low entropy?
- The conventional answer: the universe after the Big Bang was in a state with very low entropy.

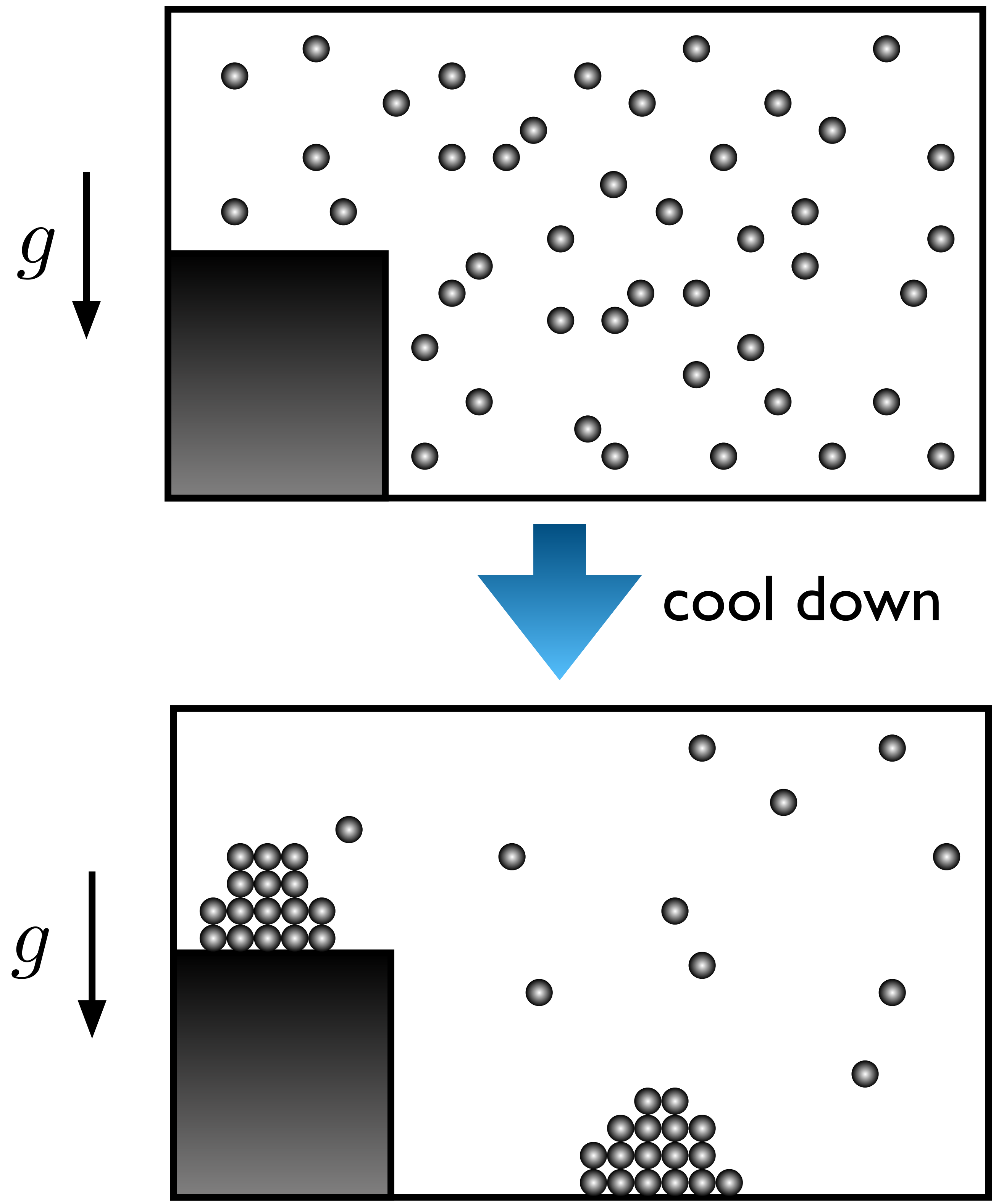


Symmetry breaking and giant fluctuations

The idea: condensation can occur anywhere. The maximum entropy state corresponds to the droplet forming at the bottom of the container. However, there is a non-negligible probability that the droplet forms at the top of the dark box. In this case, a giant fluctuation has been generated.

$$\frac{\Delta S}{k} \simeq \frac{mgh}{kT} \simeq 10^{17}$$

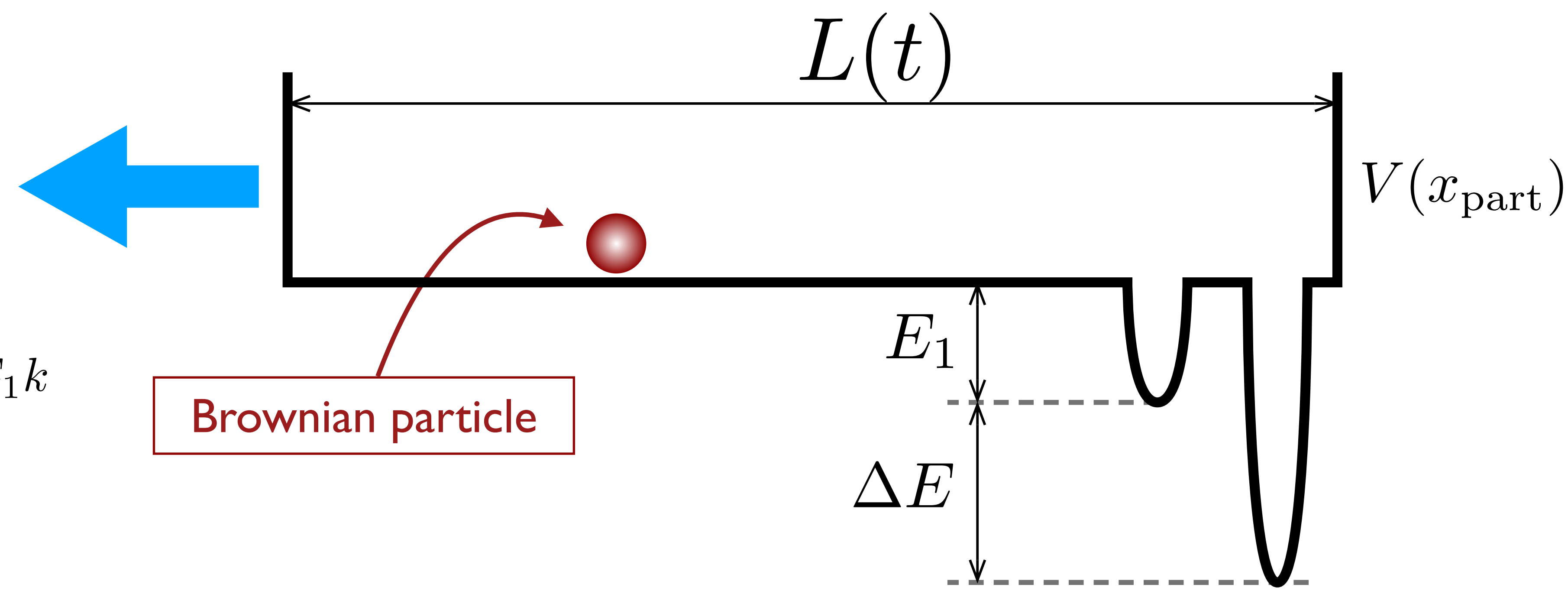
m=1 g
h=10 cm
T=300 K



Entropy and giant fluctuations

Cooling down can be achieved by adiabatic expansion (like the expansion of cosmic microwave background).

Adiabatic expansion (very slow for the fluid but very fast for the Brownian particle)



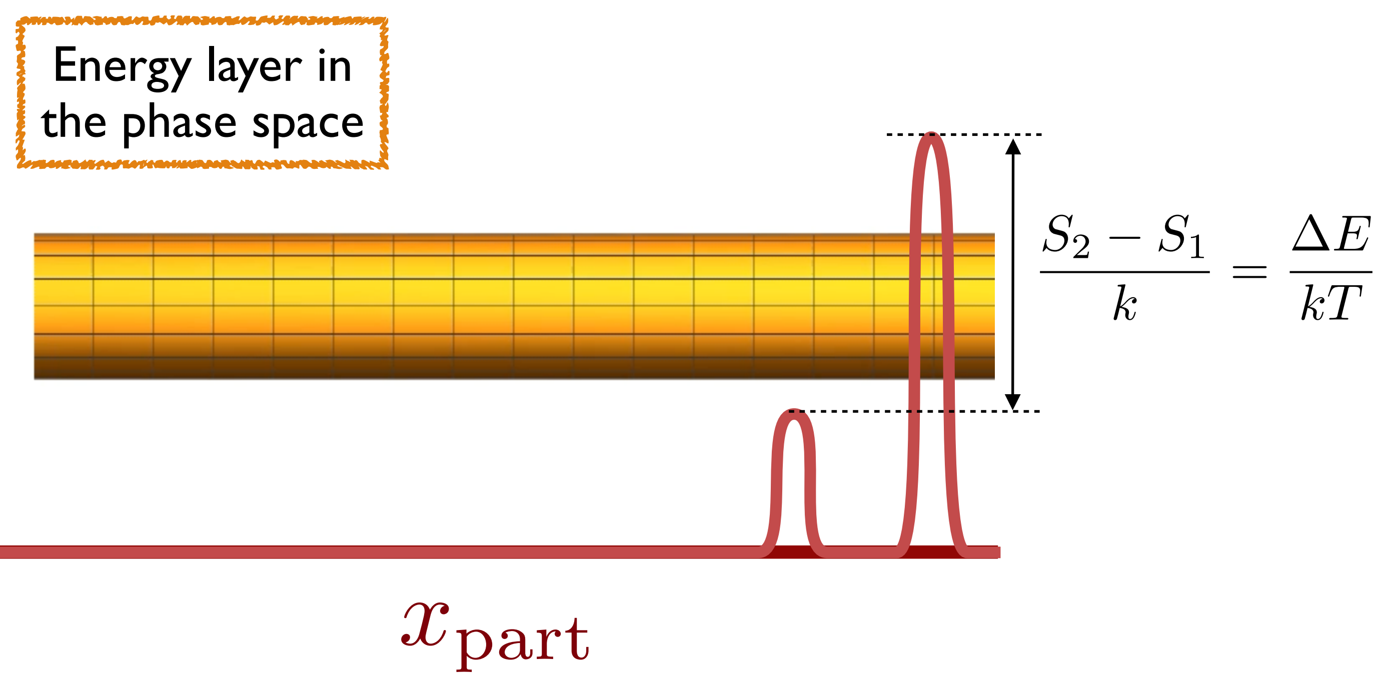
$$\frac{E_1 + \Delta E}{k} \ll T_{\mathrm{hot}} \to T \ll E_1 k$$

Entropy

$$\omega(E, x_{\mathrm{part}}) \propto L^N \left[E - V(x_{\mathrm{part}})\right]^{(N-1)/2}$$

$$S(E, x_{\mathrm{part}}) = \frac{N-1}{2} \log\left[E - V(x_{\mathrm{part}})\right] + C'$$

$$S(E, x_{\mathrm{part}}) \simeq -\frac{V(x_{\mathrm{part}})}{T} + C''$$



Conclusions (2nd part)

- Giant fluctuations (i.e. irreversibility) result from meta-stable states and gravitational aggregates induced by cooling down (adiabatic expansion of the microwave background radiation?)
- Continuous symmetry breaking creates **slow** degrees of freedom.
- Consequences on thermodynamics of information?