

Math-Circle: Session 2

TIFR-CAM and ICTS

November 7, 2022

Problem 1 (Revisiting π). *Theme: Don't take anything for granted. Revisit even those things which may seem obvious!!*

It is now a common knowledge that length of circumference of a circle with diameter d is πd . But, if you think for a moment, what does π mean? It is just a symbol assigned to a number whose exact value can never be known. We do know it is approximately $22/7$ or 3.14 . So what is π ? In fact, one of the earliest definition of π is the ratio of the length of circumference to the diameter of any circle. But, if you are a mathematician, to make sure that this definition is a proper definition, you must verify that ratio of the length of circumference to the diameter is same for any two circle. Then, you would give a name to that ratio which is common for any two circle, and that name is the π .

Verify that ratio of the length of circumference to the diameter is the same for any two circle.

Problem 2 (Carom/Billiard Problem). *Theme: Observe the world around yourself and ask questions!!*

All of us have played carom at some point in our life. Some people are extremely gifted in playing rebound shots. Let's play a mathematically idealised carom game. Suppose you are given a carom board which is a square $ABCD$. Assume that carom pockets are point sized and placed at corners A , B , C , and D . Also assume that your striker is point sized. You are sitting on the side AB , and you pick a point P on the line segment AB where you place your striker. You hit the striker in the direction making an angle θ with the line segment AB . Suppose that the striker moves indefinitely, i.e. there is no friction, and it reflects from sides like light. Can you try to understand this system by asking any question that comes to your mind. Some question that come to my mind are:

- (a) *How can I play so that I eventually hit one of the corners ?*
- (b) *How can I play so that my striker eventually repeats its original move, i.e. it comes back to the original position P and leaves with the same angle θ ?*
- (c) *How can I play so that my striker never repeats its original move?*
- (d) *Now assume that the striker is not point sized and it has some positive radius. Then, can we hit the striker so that it touches every point within the carom board?*

Problem 3. *Can you generalize the above problem? For example, imagine that your carom board is of triangular shape similar to a given triangle ABC . Ask the above questions in this setting as well. Investigate by considering different types of triangles ABC .*

Note: The part-(b) of the above question for a general triangle ABC is still an open problem till date. This line of investigation is currently a very active area of research. For example, you can consider more general shapes such pentagon, hexagon etc., and ask the same questions. A lot of interesting results have recently been proven about such systems.