DTP-Math-Circle: Session 2—Probability

Sept 16 2022

1 Tossing coins and dice

If you toss a coin four times in succession, what is the probability that you get an even number of heads?

If you do this hundred times in a row, can you generalize and see what is the probability that you get an even number of heads?

In five dice throws what is the probability of rolling the same number (i) at least three times (ii) exactly three times?

A magician holds one die in his left hand and two dice in his right. What is the probability that the number on the die in his left hand is greater than the sum of the dice in his right?

2 Paul the Octopus

"Paul the Octopus" is said to have predicted the results of 12 football games out of 14 correctly. What is the probability of this happening just by chance ?

3 Making excuses safely

Four employees reach office very late after a holiday trip. To excuse their lateness, they make up an excuse on the spur of the moment: They tell the boss that one tire of their car burst on their way back.

Their boss takes each to a different room, and simply asks: "Which tire?" What is the chance of them getting away with this excuse?

If instead of a car, they had said they were in an autorickshaw and one tire burst, how would the probability change?

4 Counting to get odds

The probability of a boy or a girl being born is identical. If Raj has two children and at least one of them is a boy, what is the probability that he also has a daughter ? If Shashi has two children and the elder is a boy, what is the probability that the younger is a daughter?

For a best of 3 set tennis match between you and your friend, you have probability p of winning any set regardless of the outcomes of previous sets. Would you bet on the match finishing in two sets or three sets?

An airplane is built to be able to fly on one engine. If the plane's two engines operate independently, and each has a 1% chance of failing in any given four-hour flight, what is the chance the plane will fail to complete a four-hour flight due to engine failure?

5 Library of Babel

Imagine that in some universe, certain intelligent creatures have a writing system that has 28 characters: 26 letters of the Roman alphabet, a full stop, and a space. They have an unusual library that houses a collection of books each containing a single "sentence". A sentence is any combination of the letters and spaces, always beginning with a letter and ending with a full stop. In every room in the library, all the books contain sentences with the same number of characters. One of the rooms has all possible books that contain sentences of 13 characters. What's the probability of finding a book in this room that says "HELLO READER."?

6 Birthdays

In a class of 35 pupils, what is the probability that at least two have the same birthday? Assume there are no leap years and a year has 365 days.

How will you handle leap years?

7 Playing cards cleverly

There are two decks of cards, one deck is a normal deck of 52 cards and the other has 104 cards, with two copies of each card. You pick two cards from the same deck and if both of the cards are red, you win. Which pack will you choose to pick from?

Another card problem: In the game of poker, each player is given 5 cards from a regular pack of 52 cards. The less probable hand wins. Two possible special hands are (A) "four of a kind", i.e. four 5's or four 7's or 4 Queens, etc., the fifth card can be anything. (B) "Full House", which is 3 of a kind and 2 of another kind. Which hand would win ? What about (C) "Flush", i.e. five cards of the same suit ?

What is the probability of getting "Queen of hearts + King of hearts + Ace of spades + 8 of diamonds + 3 of Clubs" ? Why is this not taken to be the winning hand in the game?

8 Throwing coins on a grid

A 1-inch-diameter coin is thrown on a table covered with a grid of lines two inches apart. What is the probability the coin lands in a square without touching any of the lines of the grid?

9 Average score

You are playing a game (like Ludo) with a friend in which each of you takes turns tossing a die, with the following rule: If you get a six, you get to toss again, but in any case no one gets more than three consecutive tosses. If you don't get a six, it is your friend's turn to toss the die.

If you play the game for a very long time, what is the average score you expect to get (averaged over all your turns).

10 Three-way paint-ball duel

Amar, Akbar and Antony agree to enter into a three-way paint-ball duel, or a paint-ball "truel". They stand at the vertices of an equilateral triangle, so each person is the same distance from the other two. In paint-ball, you shoot balls of paint out of a gun, and if you are hit by a paint-ball shot at you by someone else, you are out. If you are the last to not be hit, you win. So each person tries to maximize the chance that they win by not getting hit by a paint ball before the others.

In the case of the paint-ball truel, the way they play is that they take turns taking one shot at a time with their paint=ball gun. Amar is the worst shot, and when he aims at a target he only hits it 1/3 of the time, so he gets to shoot first. Akbar is second-worst, hitting 2/3 of the time, so he gets to shoot second. Antony is a sharpshooter who always hits his target, so he will have to wait and shoot third. The three men will take turns shooting their paint-ball guns in this order until only one man is left without paint on him, and he will of course be the winner.

Amar is up first. What should he do to secure his best odds of winning?

11 School bus seating

There are four children who go to school by a minivan that holds four of them, apart from the driver. Each of them has an assigned seat in the minivan, so the first boy has seat 1, second has seat 2 and so on.

The first boy is in a bad temper on Monday, and instead of sitting on his assigned seat, goes and randomly sits at some seat without checking (so it could be his or not, he doesn't care). The other three children are careful and when they enter, they try to sit in their own seat if available, or if not, they sit on one of the available seats randomly.

What is the probability that the last child gets his own seat?

Can you repeat this for a larger school van with 5 children, and for an even larger van with 6 children?

Can you generalize your solution to a big school bus with assigned seating?

Now, go back to the first situation with the four children. What is the probability that the second-last child gets his own seat? And the third-last?

Can these answers also be generalized?