

## MCI QUESTIONS FOR AUGUST 5, 2022

### 1. THE ALGEBRA OF HEADLINES AND THE CASE OF A MISSING JAVELIN

Tired of the low standard of media coverage, the Logical News Network (LNN) decided to set an example and bring more transparency and accuracy in their news reports. To do that, they made the following rules.

- A headline is a declarative sentence that is either true or false (but not both).
- A headline is *publishable* if it is true and not publishable if it false.

In other words, a headline is simply another name for a (declarative) statement that is either true or false but not both.

**Example 1.1.** *The sentence “Neeraj Chopra won the Olympic gold medal in javelin throw in 2020” is a headline since it is true. It is also a publishable headline. Likewise, “2020 is an odd number” is a headline since it is false. It is, however, not publishable. On the other hand, sentences such as “are you crazy?” and “Please eat pizza everyday!” are not headlines since they are not declarative.*

**Problem 1.2.** *In each of the following, decide whether (according to LNN rules) the given statement is publishable. If not, is it at least a headline?*

- India’s population today is more than 100 crores.*
- The above statement is false.*
- This statement is false.*
- “Is this statement publishable?”*

Given a headline  $p$ , we denote by  $p'$ , the *negation* of  $p$ . This means that  $p'$  is true when  $p$  is false and  $p'$  is false when  $p$  is true.

**Example 1.3.** *Let  $p$  be the statement that “2020 is an odd number”. Then,  $p'$  is the statement that “2020 is an even number”. Clearly,  $p'$  is true.*

Let  $p$  and  $q$  be headlines. We denote by  $pq$  the headline that asserts that  $p$  and  $q$  hold simultaneously. In other words,  $pq$  is true if *both*  $p$  and  $q$  are true and false otherwise.

Similarly, we define  $p + q$  as a headline which is true if *either*  $p$  or  $q$  (or both) are true and false otherwise.

**Example 1.4.** *Let  $p$  be the headline that “Neeraj Chopra won the Olympic gold medal in javelin throw in 2020” and  $q$  be the headline that “2020 is an odd number.” Then  $p + q$  is true but  $pq$  is false.*

If a headline is true, we give it a *truth value* of 1 while if it is false, we give it a truth value of 0. If  $p$  is a headline then  $p = 0$  means that the statement  $p$  is false while  $p = 1$  means that the statement is true.

**Example 1.5.** *The following are easy to verify.*

$$p + p' = 1, \quad pp' = 0.$$

If two headlines  $p$  and  $q$  have the same truth values, then we write  $p = q$ .

**Problem 1.6.** *Let  $p, q$  and  $r$  be headlines. Show that the following hold.*

- $p + q = q + p, \quad pq = qp$*

- (ii)  $(p + q) + r = p + (q + r)$
- (iii)  $p(q + r) = pq + qr$
- (iv)  $p^2 = p$  and in general  $p^n = p$  for  $n \geq 1$
- (v)  $p + p = p$  and in general  $p + p + \dots + p = p$ .
- (vi)  $p + qr = (p + q)(p + r)$ .

**Problem 1.7.** *Neeraj is contesting in the javelin throw event at the world athletics championships. One day before the finals, he discovers that his javelin is missing. A CID team is called and they determine that there are six suspects, who, due to the confidential nature of the investigation, are given codes names Alpha, Beta, Gamma, Delta, Epsilon and Zeta. Based on available evidence, the investigating team comes to the conclusion that exactly two of these people were involved in the theft of the javelin. They find that Zeta has run away and is nowhere to be found. The remaining five suspects are brought to the CID office for questioning. Each of these five suspects is asked the question: "who stole the javelin?" Alpha says "Beta and Gamma", Delta says "Epsilon and Zeta", Epsilon says "Zeta and Beta", Gamma says "Alpha and Beta" and Beta says "Epsilon and Delta".*

*Four of the suspects who were interviewed mentioned exactly one thief correctly while the fifth one had told a complete lie (meaning that none of the two names mentioned were involved in the theft). Can you help the CID solve this mystery?*

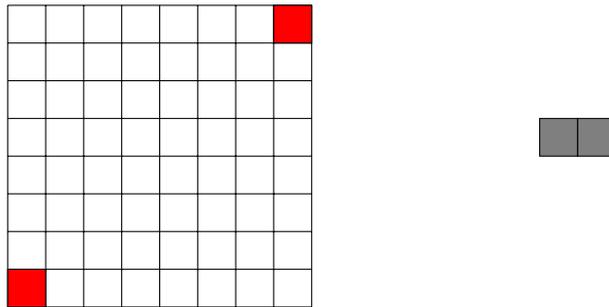
*(Hint: Write the given information in the form of statements and use the work done in Problem 1.6.)*

# Math Circles of India Questions

## 1 Tiling and Colouring

### 1.1 Chessboard

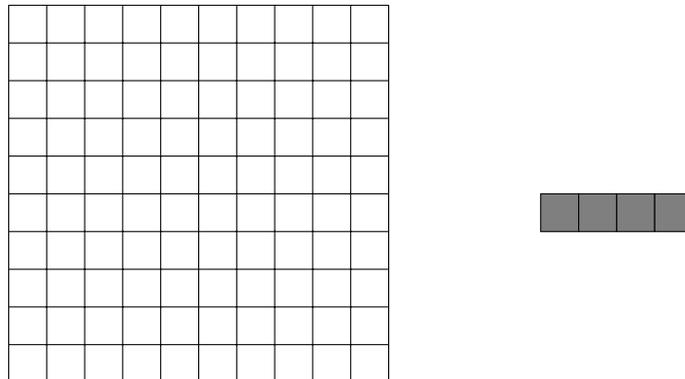
Consider an  $8 \times 8$  chessboard, and delete the tiles from two diagonally opposite corners. Can the remaining region be covered by dominoes? (a domino is a  $1 \times 2$  tile - it may be placed vertically or horizontally).



*Hint:* Think of an actual chessboard!

### 1.2 $10 \times 10$ board

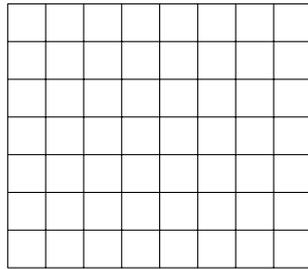
Consider a  $10 \times 10$  board. Can you cover it with  $1 \times 4$  rectangles?



### 1.3 An $m \times n$ board

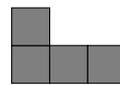
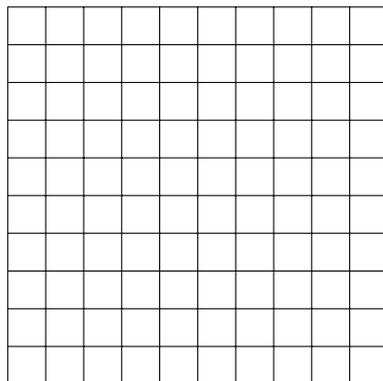
Consider an  $m \times n$  board, and an integer  $k$ . When is it possible to tile the board with  $1 \times k$  tiles?

For simplicity, try the case  $m = 7, n = 8$  and  $k = 3$ .



### 1.4 L-shaped tiles

Can one tile a  $10 \times 10$  grid with L-shaped tiles as shown below?



*Hint:* Use two colours, but not like a chessboard.

There are many variations and generalizations of these problems. Some examples:

- Can you 'tile' a given shape  $A$  with another given shape  $B$ ? Typically, both  $A$  and  $B$  are made from square tiles attached in some configuration.
- If such a tiling is possible, then how many different tilings (arrangements) are possible?
- Similar problems exist in higher dimensions - can one 'pack' a cube with 'bricks' of some prescribed height/width?