Maths Circle September 12, 2026, session 68

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In the last session, we talked about the basic problem of knot theory. How to distinguish between knots? This is a hard problem at the forefront of research in topology. But we can begin to understand the problem in simple situations. We represent knots by their projections on a plane (flattening!). We saw the basic three Reidemeister moves (See Fig. 1) that take you from a knot diagram to an equivalent one.

One way to tell knots apart is to find quantities that do not change under these moves. These are called invariants. To be useful, an invariant has to be easy to calculate. We will try to find simple invariants that bring out the basic idea. The wide open problem of the field is to find invariants that can distinguish between *any* two knots. But we can try to find invariants that tell apart *some* knots.
Tangles: We saw how braiding N tubes and connecting the ends randomly can give rise to a tangle. Try to find a tangle invariant.
Coloring knots: A knot is tricolorable if you can color the arcs (strands between undercrossings) of its diagram using up to three colors, following two main rules.

1. Crossing Rule: At each crossing, the three incident strands (the overstrand and the two understrands) must be either all the same color or all different colors. No crossing can have exactly two colors meeting.

2. Diversity Rule: At least two different colors must be used across the entire knot diagram. (Using only one color is a ”trivial” coloring and does not count as tricolorable).

links: Given two closed loops which are each unknotted, find a rule for counting the crossings to construct an invariant measuring how the two loops are linked.

Next we look at applications to biology: knots and links come into play in Biology. In particular, we will discuss how it appears in DNA Elasticity. This is a continuation of our theme of applications of the mathematical ideas on knots and links developed by Pranav in July and August. We will use the materials listed below to illustrate our ideas.

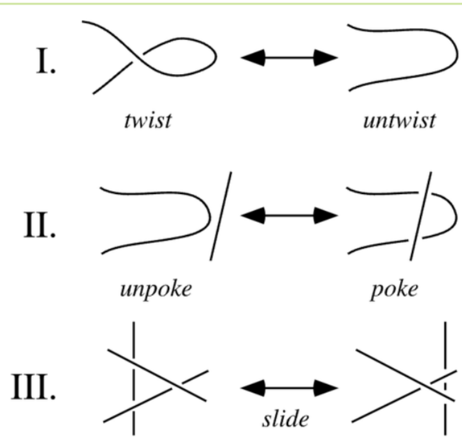

Figure 1: Reidemeister moves.

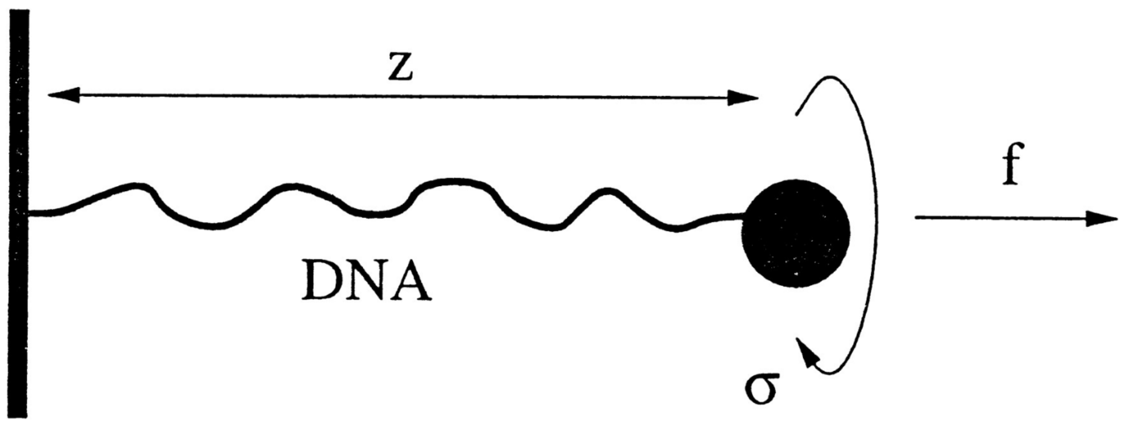

Figure 2: Experimental setup for measuring twist elasticity of a DNA molecule

Materials Needed: Paper, Pencils, cello tape, string, scissors, balls of wool.

In recent years researchers have developed techniques to manipulate a single molecule of DNA to measure its elastic properties by stretching and twisting it (See Fig. 2). During the session we will see how the twist elasticity of a DNA molecule will help us understand concepts of Knots and Links that you encountered in the past few sessions.

We will try to understand some properties of links by using paper strips.

We will introduce three notions central to understanding DNA elasticity: Link, Twist and Writhe.

In the next session (i.e. in the session of September 26), we will discuss how these three ideas are related to one another.