Non-equilibrium and periodically driven quantum systems

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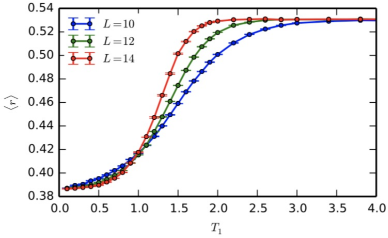
(Lecture 5)

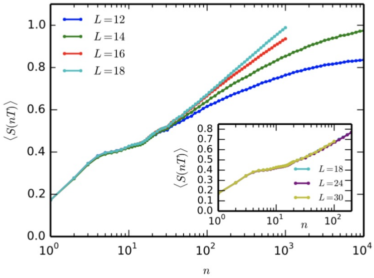
Floquet MBL

Fig from Ponte, Papić, Huveneers, Abanin, arXiv:1410.8518.
See also Lazarides, Das, Moessner, arXiv:1410.3455



- $H_0 = \sum_i h_i \sigma_i^z + J_z \sigma_i^z \sigma_{i+1}^z$ with $h_i \in [-W, W]$
- $H_1 = J_x \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y)$
  $U(T) = \exp(-iH_0 T_0) \exp(-iH_1 T_1)$ where $J_x = J_z = \frac{1}{4}$,
  $T_0 = 1$, $W = 2.5$ and T_1 is tuned

Fig from Ponte, Papić, Huveneers, Abanin, arXiv:1410.8518.



- Entanglement entropy shows a $\ln(n)$ behaviour in the Floquet-MBL phase
- Does FM expansion have a finite radius of convergence in this case?
- $[U(T), \tau_i^z] = 0$ and $[\tau_i^z, \tau_j^z] = 0$

Time Crystals

* In equilibrium, $\hat{\rho} = \frac{1}{Z} e^{-\beta \hat{H}}$ is time-independent by construction. Cannot break time-translation symmetry.

How are symmetries (other than time translation) broken in equilibrium?

* Note that $\hat{\rho}$ preserves all symmetries of $\hat{H}$ which would suggest that no symmetry of $\hat{H}$ can be broken.

* Resolution of this paradox well-known

E.g. consider a Z_2 Ising ferromagnet

The states $|\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\cdots\uparrow\uparrow\rangle \pm |\downarrow\downarrow\cdots\downarrow\downarrow\rangle\right)$

respect all symmetries.

But these are long-ranged correlated "cat" states

$$\langle m_z(x)\, m_z(x')\rangle - \langle m_z(x)\rangle\langle m_z(x')\rangle \not\to 0$$

as $|x-x'| \to \infty$

"Physical" states like $|\uparrow\uparrow\cdots\uparrow\uparrow\rangle$ (which are short-ranged correlated by the above definition) will take exponentially long in time to tunnel to $|\downarrow\downarrow\cdots\downarrow\downarrow\rangle$

These "physical" states break Z_2 symm. (Ergodicity breaking)

Does this kind of intuition work for time crystals?

↳ Answer: Yes

* In Floquet time crystals, the discrete time translation symmetry is spontaneously broken.

$(t \longrightarrow t+T)$

What are Floquet time crystals?

(see Else, Bauer, Nayak, arXiv: 1603.08001 &
Khemani, Lazarides, Moessner, Sondhi, arXiv: 1508.03344)

Definition 1: System fails to synchronize with the drive period in a precise manner for all "short-ranged correlated" initial states

(at least some local operator)

$$\langle \psi(t_1+T) | \hat{O} | \psi(t_1+T) \rangle \neq \langle \psi(t_1) | \hat{O} | \psi(t_1) \rangle$$

even at large t_1

But

$$\langle \psi(t_1+n_0T) | \hat{O} | \psi(t_1+n_0T) \rangle = \langle \psi(t_1) | \hat{O} | \psi(t_1) \rangle$$

for some integer $n_0 > 1$ [Simplest case $n_0 = 2$]

Definition 2: "All" eigenstates of the Floquet unitary $U(T)$ are necessarily 'cat states', i.e., entangled superpositions of macroscopically distinct states.

Consider the $|+\rangle = \dfrac{|\uparrow\uparrow\cdots\uparrow\uparrow\rangle + |\downarrow\downarrow\cdots\downarrow\downarrow\rangle}{\sqrt{2}}$ and

$|-\rangle = \dfrac{|\uparrow\uparrow\cdots\uparrow\uparrow\rangle - |\downarrow\downarrow\cdots\downarrow\downarrow\rangle}{\sqrt{2}}$ again

Say, $U(T)|+\rangle = e^{i\omega_+}|+\rangle$ and $U(T)|-\rangle = e^{i\omega_-}|-\rangle$

Start with a "physical" initial state $|\uparrow\uparrow\cdots\uparrow\uparrow\rangle$

Then $(U^n)|\uparrow\uparrow\cdots\uparrow\uparrow\rangle \propto \cos(\omega n)|\uparrow\uparrow\cdots\uparrow\uparrow\rangle + \sin(\omega n)|\downarrow\downarrow\cdots\downarrow\downarrow\rangle$

where $\omega = \left(\dfrac{\omega_+ - \omega_-}{2}\right)$

Macroscopic "cat" states such as $|+\rangle$ and $|-\rangle$ also show signature in entanglement measures.

E.g., mutual information

$$I(A,B) = S(A) + S(B) - S(A \cup B) \geq 0$$

generally

vanishes iff A and B unent.

$I(A,B) \to \ln 2$ even when A and B distant

* Generic interacting systems (Floquet) locally reach infinite temperature ensemble $\Rightarrow$ $U(T)$ cannot have eigenvectors that look like macroscopic cat states

* What about some Floquet version of MBL?

Let us start with a soluble limit.

The Floquet unitary $U = \exp(-i t_0 H_{MBL}) \underbrace{\exp\left(-i\frac{\pi}{2}\sum_i \sigma_i^x\right)}$

$U(\theta) = \exp\left(-i\frac{\vec{\theta}\cdot\vec{\sigma}}{2}\right) \leftarrow$ π- pulse

Thus, $\exp\left(-i\frac{\pi}{2}\sum_i \sigma_i^x\right)|\{s_i^z\}\rangle = |\{-s_i^z\}\rangle$

$H_{MBL} = \sum_i J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z + h_i^x \sigma_i^x$ (say all couplings disordered)

$h_i^x \in [-h, h]$

When h small compared to J_i, h_i^z, then MBL stabilized without drive.

Consider the solvable limit of $h=0$ first.

For $h=0$, eigenstates of $H_{MBL} \to |\{s_i^z\}\rangle$

Take any such state as initial state and now look at system stroboscopically with

$$U = \exp(-i t_0 H_{MBL}) \exp\left(-i\frac{\pi}{2}\sum_i \sigma_i^x\right)$$

$\underbrace{\uparrow\uparrow\cdots\uparrow\uparrow}_{n=0}$ } $\underbrace{\downarrow\downarrow\cdots\downarrow\downarrow}_{n=1}$ } $\underbrace{\uparrow\uparrow\cdots\uparrow\uparrow}_{n=2}$ } $\underbrace{\downarrow\downarrow\cdots\downarrow\downarrow}_{n=3}$

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Let $E^+(\{s_i^z\}) = \sum_i J_i s_i^z s_{i+1}^z$ (same for $\{-s_i^z\}$)

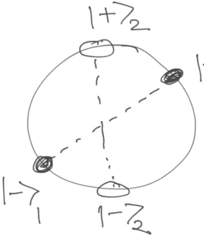
$E^-(\{s_i^z\}) = \sum_i h_i^z s_i^z$ (changes sign for $\{-s_i^z\}$)

$$U|\{s_i^z\}\rangle = e^{-it_0 E^+(\{s_i^z\})}\, e^{-it_0 E^-(\{-s_i^z\})}\, |\{-s_i^z\}\rangle$$

$$U|\{-s_i^z\}\rangle = e^{-it_0 E^+(\{s_i^z\})}\, e^{-it_0 E^-(\{s_i^z\})}\, |\{s_i^z\}\rangle$$

Eigenvectors of $U \rightarrow \dfrac{e^{it_0 E^-(\{-s_i^z\})}|\{s_i^z\}\rangle \pm e^{it_0 E^-(\{s_i^z\})}|\{-s_i^z\}\rangle}{\sqrt{2}}$

with eigenvalues $\pm e^{-it_0 E^+(\{s_i^z\})}$



Now turn on a small $h_i^x \in [-h, h]$ and also change

$$\underbrace{e^{-i\frac{\pi}{2}\sum_i \sigma_i^x}}_{\text{Call } \tilde{X} \text{ (}\pi\text{-pulse)}} \text{ to } e^{-i\left(\frac{\pi}{2}-\epsilon\right)\sum_i \sigma_i^x}$$

The resulting problem has many-body localization (small perturbations given in 1D with strong disorder)

We know that local in space perturbations perturb locally in MBL

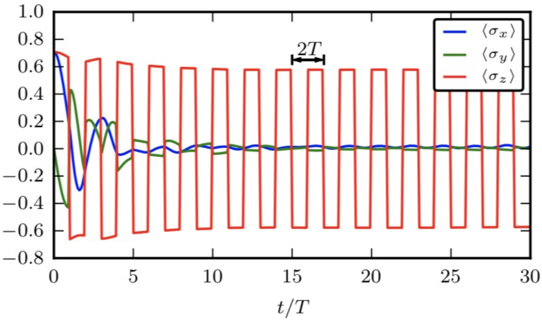
$\therefore$ $\exists$ a local unitary, $\mathcal{U} = e^{-iS}$

S is local

s.t. $U_F^{\text{perturbed}} = \mathcal{U}^\dagger \left(e^{-it_0 H_{MBL}} X \right) \mathcal{U}$

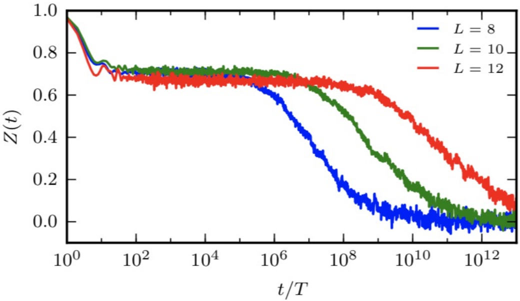
Such a local unitary $\mathcal{U}$ cannot connect short-ranged correlated states with the long-ranged correlated eigenstates of $(h=0, \epsilon=0)$. For small (h, ϵ), U_F^{per} necessarily has all eigenstates to be macroscopic cat states.

Fig from Else, Bauer, Nayak, arXiv: 1603.08001



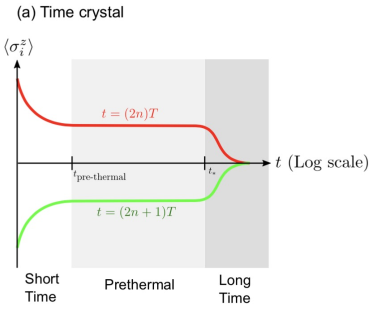
- $[\cos(\pi/8)|\uparrow\rangle + \sin(\pi/8)|\downarrow\rangle]^{\otimes L}$ for system size $L = 200$ and $h = 0.3$ and $t_0 = 1$ using time-evolving block decimation.
- Disorder averaged over 146 disorder configurations

Fig from Else, Bauer, Nayak, arXiv: 1603.08001



- ED data for small system sizes clearly show a timescale that diverges exponentially in the system size
- $Z(t) = \overline{(-1)^t \langle \sigma_i^z(t) \rangle \mathrm{sign}(\langle \sigma_i^z(0) \rangle)}$ starting from random initial product states $\{s_i^z\}$ and computing the average over 500 disorder realizations for a fixed position i

Fig from Else, Bauer, Nayak, arXiv: 1607.05277



- Prethermal Floquet time crystals can be stabilized without disorder in dimensions higher than one or with long-ranged interactions in 1D

Floquet prethermalization at large ω_D

* Suppose $\omega_D \gg J_{loc}$ where J_{loc} denotes the energy scale associated with local rearrangements of dof in an interacting problem

* Then, there appears a large transien time below which the system is in a prethermal Floquet state.

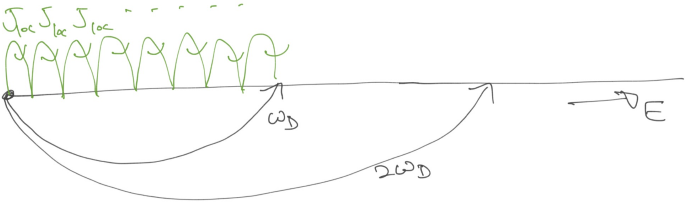
$$\boxed{t_* \sim \exp\left(\frac{\omega_D}{J_{loc}}\right)}$$

* This prethermal state well-described by $H_F^{(n)} = \sum_{m=0}^{n} T^m \Omega_m$ choosing an optimum n_0

* Heating prevented for $t \lesssim t_*$ because $H_F^{(n)}$ appears as a conserved quantity for stroboscopic times

* Importantly, dynamics of local observables described accurately by $H_F^{(n)}$ for $t \lesssim t_*$

Physically, a many-body system requires $O\left(\frac{\omega_D}{J_{loc}}\right) \gg 1$ correlated local rearrangements to absorb a single quantum of energy from the drive.



Now consider the case where ω_D is much greater than all but one of the local scales of H

$$H(t) = H_0(t) + V(t)$$ and $\lambda T \ll 1$ where λ is the scale of $V(t)$

* Imp, $H_0(t)$ [whose scale $\sim \omega_D$] has a special form

$$X = \mathcal{T} \exp\left(-i \int_0^T dt\, H_0(t)\right) \text{ and } \boxed{X^M = I}$$

Going to the interaction picture (where $V(t)$ is the "interaction term"), we get

$$U(MT) = \mathcal{T} \exp\left[-i \int_0^{MT} dt\, V_{int}(t)\right]$$

where $V_{int}(t) = U_0(t,0)^\dagger V(t) U_0(t,0)$

U_0 is propagator of $H_0(t)$

Since $U_0(MT) = X^M = \mathbb{I}$, the operator $U(MT)$ is identical in interaction and Schrödiger pictures

Rescale $t \to t/\lambda$, $U(MT)$ describes a Floquet system driven at large $\tilde{\omega}_D = \frac{2\pi}{\lambda M T}$ (remember $\lambda T \ll 1$) by a drive of local strength 1.

Standard Floquet prethermalization applies for $\underline{\underline{\tilde{\omega}_D \gg 1}}$ with prethermal time $\underline{\underline{t_F \sim \exp(\tilde{\omega}_D)}}$

Furthermore, $U(T) \approx \mathcal{D}^{\dagger}\left(X \exp(-iDT)\right)\mathcal{D}$

in the prethermal window where $\mathcal{D}$ is a time-independent local unitary rotation, D is a local H that has the further property of $[D, X] = 0$

(see Else, Bauer, Nayak, arXiv: 1607.05277)

emergent symm.

- Interesting prethermal phases can be stabilized when X can be interpreted as a symmetry that can be spontaneously broken due to choice of initial state and dim. of system.

To stabilize the prethermal time crystal, consider $d=2$ and

$$H_0(t) = -\sum_i h_x(t)\,\sigma_i^x$$

$$V = -J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - h_z\sum_i \sigma_i^z$$

→ note this term breaks Z_2

Choose $\int_0^T dt\; h_x(t) = \frac{\pi}{2} \longrightarrow U_0(T) = X = \prod_i \sigma_i^x$ (π pulse idea)

↳ $h_x(t) \sim \frac{1}{T} \sim \omega_D$ So $\underline{M=2}$

$$\boxed{h_z, J \ll \omega_D}$$

Then $[D, X] = 0 \Rightarrow D = -J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z + \cdots$

preserves Ising symm.

Cartoon picture