Non-equilibrium and periodically driven quantum systems

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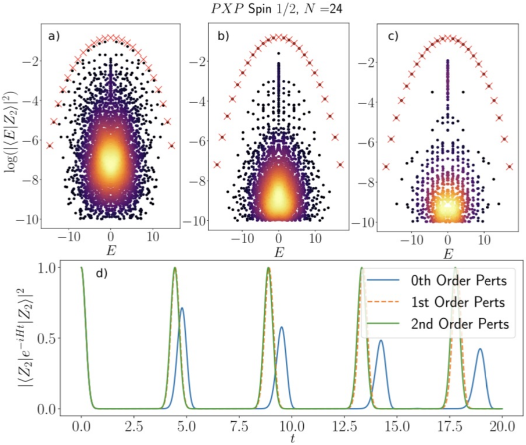
BSSP XII

(Lecture 4)

Fig from Bull, Desaules and Papić, arXiv:2001.08232

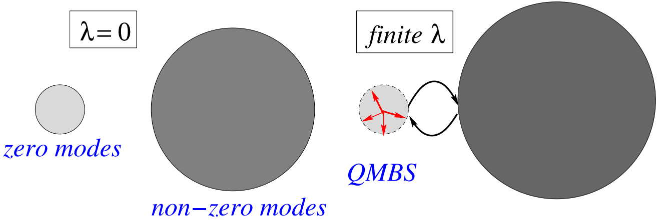


Similar approach of introducing small perturbations greatly enhance revivals from $|\mathbb{Z}_3\rangle$ and $|\mathbb{Z}_4\rangle$ as well

- Why so many zero modes? What protects this massive degeneracy?
- The model has translational symmetry and discrete spatial inversion symmetry (e.g., $j \to L - j + 1$). The Hamiltonian commutes with both these symmetries
- The Hamiltonian also anticommutes with $Q = \prod_{j=1}^{L} \sigma_j^z$. Each eigenstate with energy E, say $|E\rangle$, has a partner at energy $-E$ which is $Q|E\rangle$.

- An index theorem (Schecter, Iadecola, 2018) ensures that if $\{H, QI\} = 0$, then degeneracy of these zero modes scale as $\exp(\alpha N)$ with N being number of dof in system
- These exact zero modes seem to satisfy ETH based on some studies on finite sizes

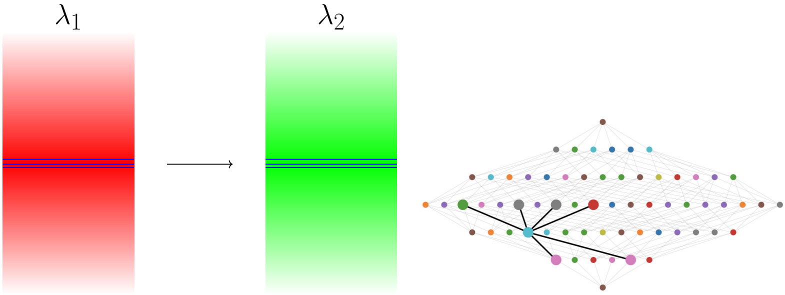
Alternate Mechanism for scarring



- Exponentially many exact zero-modes at mid-spectrum for $\lambda = 0$. Protected by an index theorem Schecter and Iadecola (2018)
- At $\lambda \neq 0$, this massive degeneracy lifted and zero modes hybridize with the non-zero modes
- However, some special linear combinations survive that also diagonalize $\mathcal{O}_{\text{pot}}$ and hence $H = \mathcal{O}_{\text{kin}} + \lambda \mathcal{O}_{\text{pot}}$.

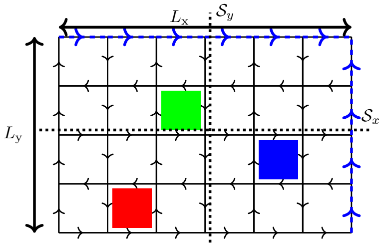
  Banerjee, Sen, arXiv:2012.08540

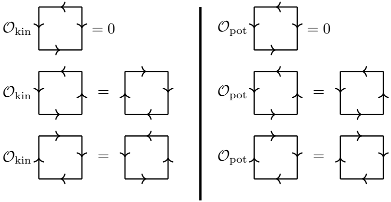
Anomalous high-energy eigenstates



- $|\psi_{\mathrm{QMBS}}\rangle = \sum_\alpha c_\alpha |\mathrm{ZM}_\alpha\rangle$, $\mathcal{O}_{\mathrm{kin}}|\psi_{\mathrm{QMBS}}\rangle = 0$, $\mathcal{O}_{\mathrm{pot}}|\psi_{\mathrm{QMBS}}\rangle = \mathcal{N}|\psi_{\mathrm{QMBS}}\rangle$, $H(\lambda)|\psi_{\mathrm{QMBS}}\rangle = \lambda\mathcal{N}|\psi_{\mathrm{QMBS}}\rangle$
- QMBS unchanged in spite of exp. small level spacing
- QMBS localized in the Hilbert space while individual zero modes are not

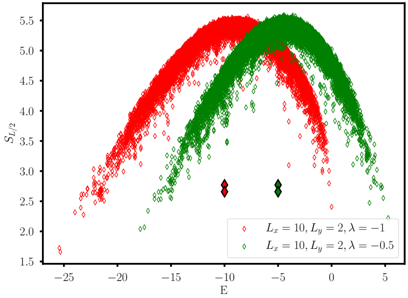
$U(1)$ quantum link model

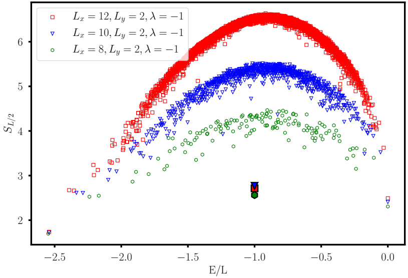




- Ice manifold (2 in, 2 out rule) with quantum fluctuations, also a $U(1)$ lattice gauge theory
  Chandrasekharan, Wiese (1997), Hermele, Fisher, Balents (2004)
- A $U(1)$ quantum link, $U_{\mathbf{r},\hat{\mu}} = S^{+}_{\mathbf{r},\hat{\mu}}$, is a raising operator of the electric flux $E_{\mathbf{r},\hat{\mu}} = S^{z}_{\mathbf{r},\hat{\mu}}$ and $H = \mathcal{O}_{\rm kin} + \lambda O_{\rm pot}$

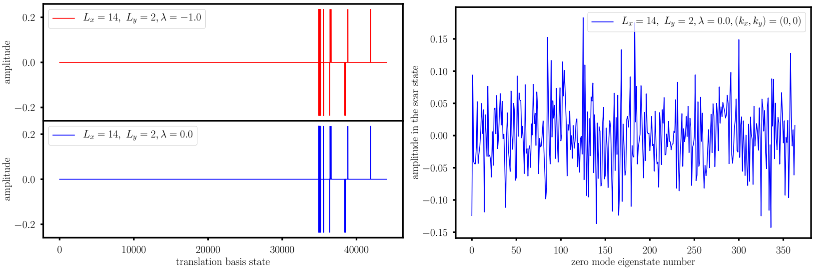
QMBS in ladders of width $L_y = 2$





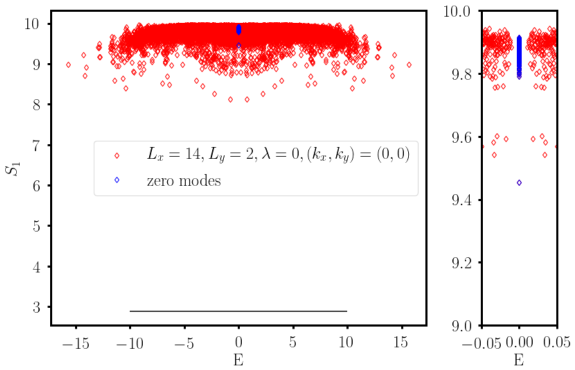
- Clear evidence of scars as outliers of (half-ladder) entanglement entropy $S_{L/2}$
- 4 QMBS each with $(N_p/2, 0)$ as eigenvalues of $(\mathcal{O}_{\text{pot}}, \mathcal{O}_{\text{kin}})$ where N_p equals no. of elementary plaquettes

QMBS from zero modes



- Construct matrix for $\mathcal{O}_{\text{pot}}$ in the zero mode subspace and diagonalize it. Only 4 states with integer eigenvalues $(= N_p/2)$ while rest are non-integers.
- QMBS resembles pseudo-random superposition of zero modes!

Localization in the Hilbert space



- The individual zero modes have a much larger Shannon entropy (S_1) than the QMBS
- Thus, the very special linear combination of the zero modes (that generates the scar) is also much more localized in the Hilbert space

Floquet phases in interacting matter

- Consider driving protocol of form $H(t) = H(t+T)$ where $T = 2\pi/\omega_D$
- Stroboscopic dynamics at times $t = nT$ controlled by **Floquet Hamiltonian** H_F which is defined as the generator of the single-period time evolution operator, or the **Floquet unitary** $U(T) = \mathcal{T}\left[e^{-i\int_0^T dt H(t)}\right] = e^{-iH_F T}$
- Time-ordering makes it notoriously difficult to calculate H_F for interacting systems and one generally resorts to various approximations (for a review of some of the analytic approaches, see Sen, Sen, Sengupta, arXiv:2102.00793)

- Floquet version of ETH?
- For periodically driven many-body quantum systems, energy conservation is lost. $\rho_S \propto \mathrm{Trace}_{\overline{S}}(\mathbb{I})$
- $U(T)$ resembles a random matrix with all its eigenstates mimicking random states as far as local quantities are concerned
- Eigenvectors of $U(T)$ have the form $|\psi_\alpha(t)\rangle = e^{-i\epsilon_\alpha t}|\phi_\alpha(t)\rangle$ where $|\phi_\alpha(t)\rangle = |\phi_\alpha(t+T)\rangle$ and $\epsilon_\alpha = \epsilon_\alpha + \frac{2\pi}{T}$ (quasienergy)

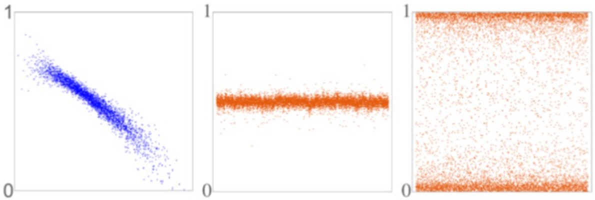
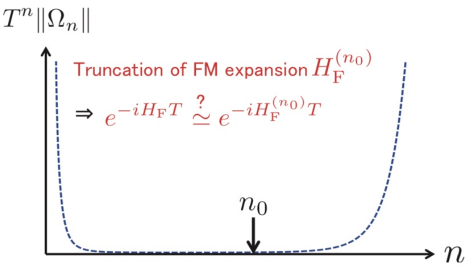


Fig. from Moessner and Sondhi's review: arXiv:1701.08056

Fig from Kuwahara, Mori, Saito, arXiv:1508.05797



- H_F cannot be written as a short-ranged Hamiltonian even though $H(t)$ is short-ranged
- E.g., do a Floquet-Magnus expansion $H_F = \sum_{m=0}^{\infty} T^m \Omega_m$
- $\exp(-iH_F T) = \exp(-iH_1 T_1) \exp(-iH_2 T_2)$. Now use BCH formula
- $Z = \ln(\exp(X) \exp(Y)) = (X+Y) + \frac{1}{2}[X, Y] + \frac{1}{12}[X - Y, [X, Y]] + \cdots$ where $X = -iH_1 T_1$, $Y = -iH_2 T_2$, $Z = -iH_F T$ with $T = T_1 + T_2$

Floquet systems seem to always approach ITE for local properties (heat death scenario)

How to see any non-trivial examples of Floquet matter?

Undriven

Non-ETH → Integrable; Many-body Quantum scars; Many-body localization; ⋮

Floquet

→ may escape Floquet ETH

periodic GGEs, Floquet versions of quantum scars, time crystals, ...

ETH → various prethermalization mechanisms (e.g. high drive frequencies) → prethermal phases → Floquet ETH

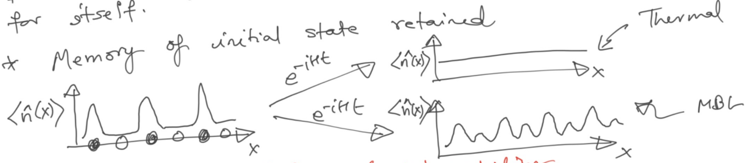
prethermal phases → non-trivial possibilities e.g. time crystals without disorder

Many-body localization (quick review)

* See excellent reviews by Nandkishore, Huse (arXiv: 1404.0686) and Abanin, Altman, Bloch, Serbyn (arXiv: 1804.11065)

* Excitations are localized due to disorder in MBL. Due to this, system cannot act as an effective heat bath for itself.

* Memory of initial state retained



MBL has an "emergent" form of integrability

Consider a 1D $S=\frac{1}{2}$ model with disorder:

$$H_{XXZ} = \frac{J_\perp}{2} \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \frac{J_z}{2} \sum_{i=1}^{L} \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^{L} h_i^z \sigma_i^z$$

$h_i^z \in [-W, W]$

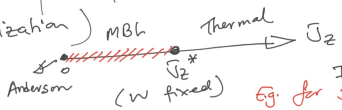
using JW transformation

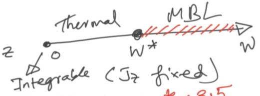
$$= t \sum_i \left(c_i^\dagger c_{i+1} + h.c.\right) + V \sum_i \hat{n}_i \hat{n}_{i+1} + \sum_i \epsilon_i n_i$$

where $\epsilon_i \in [-W, W]$

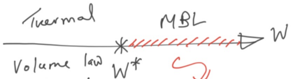
When $J_z = 0$, free fermions with disorder in 1D (Anderson localization)

Say $J_\perp = 1$





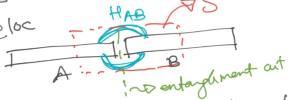
E.g. for $J_z = J_\perp = 1$, $W^* \approx 3.5$



Volume law scaling of entanglement entropy of eigenstates

Area law scaling for entanglement entropy of eigenstate (almost all eigenstates)

Intuition: In MBL phase, a local perturbation (in space) only significantly affects dof within a localization length ξ_{loc}



$H = H_A + H_B + \cancel{H_{AB}}$

Clearly, $|\psi_{AB}\rangle = |\phi_A\rangle \otimes |\phi_B\rangle$

Then $S_{ent} \propto \xi_{loc} \to$ Area law

$$H_{XXZ} = \frac{J_\perp}{2} \sum_{i=1}^{L} \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \frac{J_z}{2} \sum_{i=1}^{L} \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^{L} h_i \sigma_i^z$$

Consider $J_\perp \to 0$. Then, $\hat{H}$ commutes with $\hat{\sigma}_i^z$

Eigenstates are

$|\{\sigma\}\rangle = |\sigma_1^z \sigma_2^z \ldots \sigma_N^z\rangle$ → Each eigenstate labelled by a string of +1 and −1

Turn on a small $J_\perp$ → system in MBL

Remarkably, $\tau_i^z = Z\sigma_i^z + \sum_{jk}\sum_{a,b=x,y,z} f_{i,jk}^{ab}\,\sigma_j^a\sigma_k^b + \ldots$

$f_{i,jk}^{ab}$ ↓ decays with ξ_{loc}

$Z \neq 0$

τ_i^z → Dressed operators

→ (Somewhat reminiscent of Fermi-liquid theory)

Furthermore, $UHU^\dagger = H_{diag}$, $\tau_i^z = U\sigma_i^z U^\dagger$

s.t. $[\tau_i^z, H] = 0$, $[\tau_i^z, \tau_j^z] = 0$ ⇒ (Emergent integrability)

U is quasi-local → $U = \prod_i \ldots \hat{U}^{(3)}_{i,i+1,i+2}\,\hat{U}^{(2)}_{i,i+1}$ s.t. $\| 1 - \hat{U}^{(n)}_{i,i+1,\ldots,i+n} \|_F^2 < e^{-n/\xi_{loc}}$

* $\hat{U}$ is quasi-local because it rotates product states (with zero entanglement) to area law eigenstates of MBL
* Generically $\hat{U}$ for non-MBL thermalizing phases should be highly non-local since it rotates product states into eigenstates with volume-law entanglement.

$$\hat{H}_{MBL} = \sum_i \tilde{h}_i \tau_i^z + \sum_{i>j} J_{ij} \tau_i^z \tau_j^z + \sum_{i>j>k} J_{ijk} \tau_i^z \tau_j^z \tau_k^z + \cdots$$

$$J_{ij} \sim J_0 e^{-|i-j|/\xi}$$

- In principle, τ_i^z can be obtained even for a thermal system (satisfying ETH) by finding U numerically but then τ_i^z will be highly non-local in real space with a vanishing overlap ($\not{Z}$) with the original operators σ_i^z

$\rightsquigarrow$ Possibility of stabilizing long-ranged order in 1D even for high-energy eigenstates since excitations are no longer mobile (usual stat. mech. arguments fail here)

$\rightsquigarrow$ E.g. H_{XXZ} with $J_\perp$ small compared to J_z, and h_i^z should have "spin-glass order" in eigenstates

$$\langle \sigma_i^z \sigma_j^z \rangle_{\text{Eigenstate}} \neq 0 \quad \text{for } |i-j| \to \infty$$