Non-equilibrium and periodically driven quantum systems

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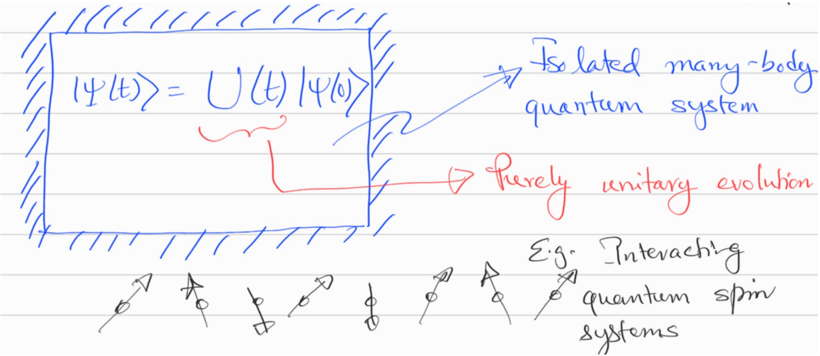
BSSP XII

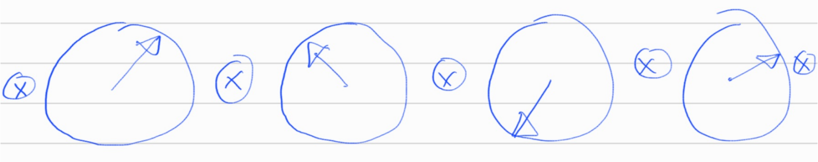
(Lecture 1)

Outline of the course

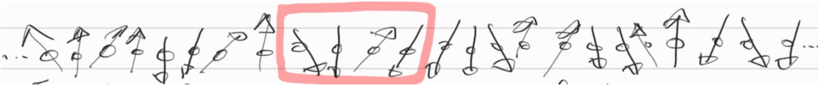
- Basic notions of thermalization in isolated many-body quantum systems
- Dynamics in $S = 1/2$ transverse field Ising model in 1D
- Many-body quantum scars
- Floquet time crystals in 1D with disorder (many-body localization)
- Prethermal time crystals without disorder



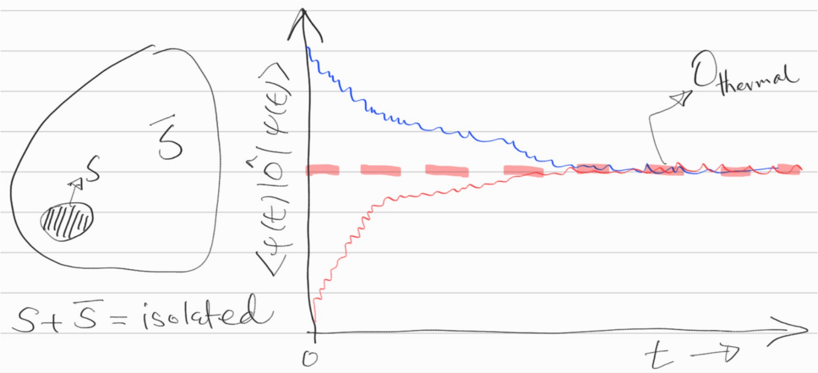
- Experimental platforms like ultracold atoms, trapped ions, NV centres, superconducting qubits etc [tremendous progress in last decade or so]
- For our purpose, unitary dynamics generated from either $H(t) = H$ (**quench**) or $H(t) = H(t+T)$ (**Floquet**)



- Simple starting wavefunction
  For example $|\psi(0)\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_L\rangle$
- Many-particle system with a local in space Hamiltonian H
- $|\psi(t)\rangle = \exp(-iHt)|\psi(0)\rangle$, $\langle\psi(t)|O_A|\psi(t)\rangle$ where O_A local
- $O = \sum_{A_i} O_{A_i}$ also local



- Extensive value of conserved quantities like $\langle\psi(0)|H|\psi(0)\rangle$ (sum of local energy densities) and subextensive uncertainities in such quantities



- For generic many-body quantum systems, thermalization expected for local subsystems (full system clearly remains a pure state)
- System acts as a “bath” for itself

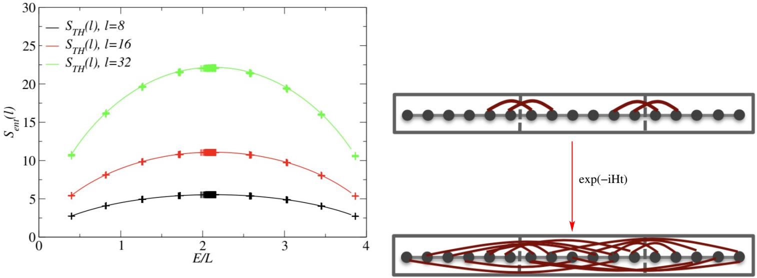
- Take the thermodynamic limit first and then consider a finite subsystem S
- Consider the reduced density matrix of subsystem S which equals $\rho_s(t) = \mathrm{Trace}_{\overline{S}}\rho(t)$ where the full density matrix of the system is $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$
- A local operator O confined strictly within S can be calculated as $\langle O(t)\rangle = \mathrm{Trace}[\rho_s(t)O]$
- After a "sufficiently long" time, $\rho_S(t) \to \mathrm{Trace}_{\overline{S}}(\rho^{\mathrm{eq}})$ if system acts as a bath for itself

- No time-averaging required here unlike in classical systems
- S is not weakly coupled to $\overline{S}$ at its boundaries
- ρ^{eq} determined by the local conservation laws.

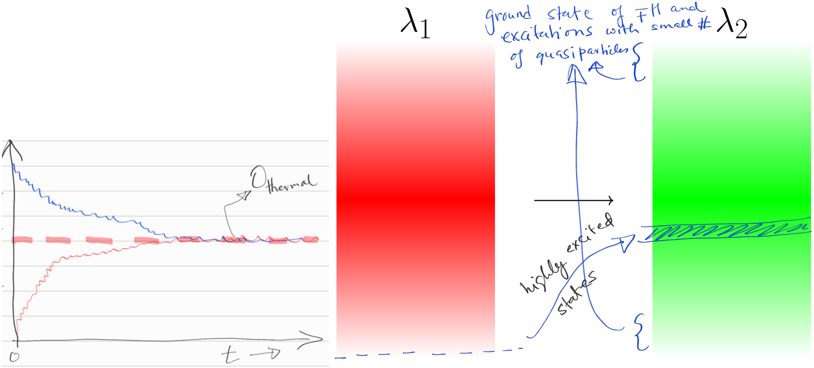
entropy from $\rho_s(t)$? $\rightarrow$ von Neumann entanglement entropy

- Consider a pure state $|\psi_{AB}\rangle \neq |\phi_A\rangle|\phi_B\rangle$
- $\rho_{A/B} = \mathrm{Trace}_{B/A}|\psi_{AB}\rangle\langle\psi_{AB}|$ is a mixed density matrix since A entangled to B
- $S(\rho_A) = -\mathrm{Trace}[\rho_A \ln \rho_A] = S(\rho_B) = -\mathrm{Trace}[\rho_B \ln \rho_B]$
- **Exercise**: consider a singlet of two spin-1/2 degrees of freedom $\frac{1}{\sqrt{2}}(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle)$. Show that $S(\rho_A) = S(\rho_B) = \ln(2)$.

- $S(\rho_S(t))$ approaches the usual thermodynamic entropy for a finite subsystem S (in the thermodynamic limit)



- Left panel–entanglement entropy for (typical) eigenstates for the 1D $S = 1/2$ transverse field Ising model
- Right panel–spreading of quantum entanglement which moves the information about the initial state to highly non-local correlations rendering it inaccessible to local probes



- $|\psi(0)\rangle = \sum_i c_i |E_i\rangle$
- $\langle\psi(t)|O|\psi(t)\rangle =$
  $\sum_i |c_i|^2 \langle E_i|O|E_i\rangle + \sum_{i\neq j} c_i c_j^* \exp[-i(E_i - E_j)t]\langle E_j|O|E_i\rangle$
- Thus, $|E_i\rangle$ appear "thermal" for local operators (**Eigenstate thermalization hypothesis**) (Deutsch (1991), Srednicki (1994), Rigol, Dunjko, Olshanii (2008))

- Operators like $O = (|E_m\rangle\langle E_n| + \mathrm{h.c.})$ do not thermalize
- $\langle\psi(t)|O|\psi(t)\rangle \sim \cos(E_m - E_n)t$

$$\underbrace{\begin{pmatrix} \alpha_{1m} \\ \alpha_{2m} \\ \vdots \\ \alpha_{Nm} \end{pmatrix}}_{|E_m\rangle} \underbrace{\begin{pmatrix} \beta_{1n} & \beta_{2n} & \cdots & \beta_{Nn} \end{pmatrix}}_{\langle E_n|} = \begin{pmatrix} \alpha_{1m}\beta_{1n} & \alpha_{1m}\beta_{2n} & \cdots & \alpha_{1m}\beta_{Nn} \\ \alpha_{2m}\beta_{1n} & \alpha_{2m}\beta_{2n} & \cdots & \alpha_{2m}\beta_{Nn} \\ \vdots & \vdots & & \vdots \\ \alpha_{Nm}\beta_{1n} & \alpha_{Nm}\beta_{2n} & & \alpha_{Nm}\beta_{Nn} \end{pmatrix}$$

highly non-sparse matrix

- Unlike ground states of local H which follow area law, the highly-excited eigenstates follow volume law scaling for entanglement entropy
- Typically, such volume law states cannot be written as ground states of any local in space Hamiltonian
- ETH $\rightarrow$ Thermalization of subsystems

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Absence of Diffusion in Certain Random Lattices

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(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

I. INTRODUCTION

A NUMBER of physical phenomena seem to involve quantum-mechanical motion, without any particular thermal activation, among sites at which the mobile entities (spins or electrons, for example) may be localized. The clearest case is that of spin diffusion[1,2]; another might be the so-called impurity band conduction at low concentrations of impurities. In such situations we suspect that transport occurs not by motion of free carriers (or spin waves), scattered as they move through a medium, but in some sense by quantum-mechanical jumps of the mobile entities from site to site. A second common feature of these phenomena is randomness: random spacings of impurities, random interactions with the "atmosphere" of other impurities, random arrangements of electronic or nuclear spins, etc.

Our eventual purpose in this work will be to lay the foundation for a quantum-mechanical theory of transport problems of this type. Therefore, we must start with simple theoretical models rather than with the complicated experimental situations on spin diffusion or impurity conduction. In this paper, in fact, we attempt only to construct, for such a system, the reasonably well, and to prove a theorem about the model. The theorem is that at sufficiently low densities, transport does not take place; the exact wave functions are localized in a small region of space. We also obtain a fairly good estimate of the critical density at which the theorem fails. An additional criterion is that the forces be of sufficiently short range—actually, falling off as $r \to \infty$ faster than $1/r^3$—and we derive a rough estimate of the rate of transport in the $V \propto 1/r^3$ case.

Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as "the thermodynamic system of spin interactions" when there is no obvious contact with a real external heat bath.

- Violations of ETH in interacting systems?

- Integrable systems (*all* eigenstates have a quasiparticle description)
- Many-body localization (high-energy eigenstates satisfy area law scaling of entanglement entropy)
- Extensive number of emergent conservation laws, almost all initial conditions retain memory during dynamics
- Weak ergodicity breaking from quantum many-body scars embedded in an otherwise ETH-satisfying spectrum
- Leaves a dynamical signature–some initial states do not thermalize, while most do!