

# ICTS-RRI Math Circle, Saturday 25th March 2023

## The Harmonic Series

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The Harmonic progression  $\{1/n, n = 1, 2, 3, \dots\}$  derives its name from music. Musical notes consist of a fundamental frequency  $f$  with its multiples  $f_n = nf$ , called overtones. The wavelength of these frequencies given by  $\lambda = \frac{c_s}{nf}$  ( $c_s$  is the speed of sound). These wavelengths form a Harmonic progression: The Harmonic series  $\sum_n 1/n$ , also appears in the

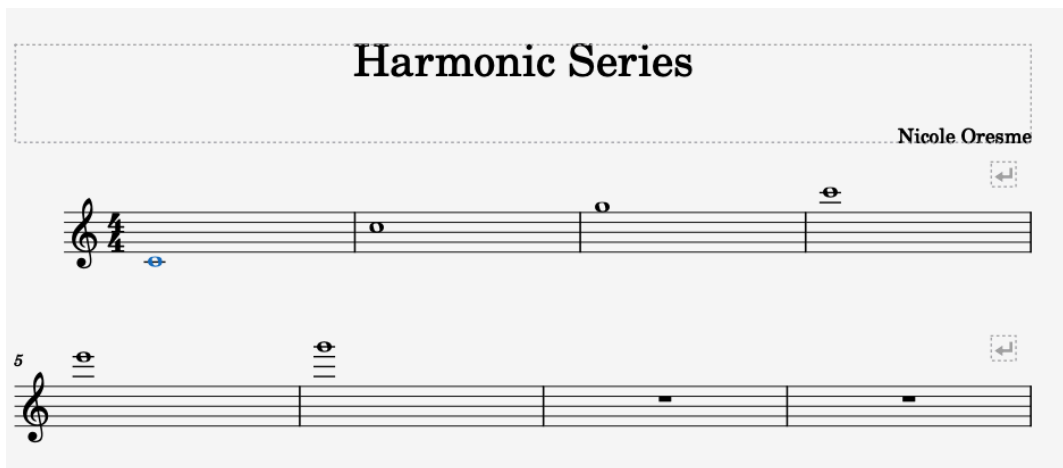


Figure 1: The Harmonic progression in music: the notes are multiples of a fundamental frequency

solution of some physical and mathematical problems. Here are two:

Problem 1: Crossing a desert

There are  $N$  litres of fuel stored at a fixed base. The jeep can carry at most 10 litres of fuel at any time, and can travel 200 km of distance on 10 litres of fuel. (The jeep's fuel consumption is assumed to be constant.) At any point in a trip the jeep may leave any amount of fuel that it is carrying at a fuel dump, or may collect any amount of fuel that was left at a fuel dump on a previous trip, as long as its fuel load never exceeds 100 litres. The problem is: Crossing the desert. How big a desert can the jeep cross? The jeep must return to the base at the end of every trip except for the final trip, when the jeep travels as far as it can before running out of fuel.

Problem 2: Stacking books

You are given a set of  $n$  identical wooden blocks or books. How far can a stack of  $n$  books protrude over the edge of a table without the stack falling over?

Try solving these problems before you come to the Math Circle on Saturday. And please come on time. You will miss most of the preparation if you come late.

Exploration: With these motivating problems, we will explore the problem of summing the Harmonic series. This problem has a rich history and teaches us much about infinite series and also brings out the perils of naive reasoning. The discussion will be guided by the questions you ask and the answers you find.

**Tea break:** 11:15