

ICTS-RRI Math Circle, Saturday 24th June, 2023
A Math puzzle followed by Infinite Series

Sam

June 19, 2023

Puzzle

We will start with a puzzle featuring Sharmila and Prakash. There are two numbers n and m , less than 100, say. Sharmila is given their sum $S = n + m$ and Prakash their product $P = nm$. Their conversation goes as follows.

Sharmila: I know the sum, but I don't know the product.

Prakash: I know the product, but I don't know the sum.

Sharmila: I *knew* you wouldn't know. Now I know the numbers.

Prakash: Now, I too know the numbers.

What were the numbers n, m ? You can assume as usual that Sharmila and Prakash are both adept mathematicians.

Tea break: 11:15

Last time we talked about how apparently divergent series can be given a meaning. Today we will step back a bit and look more conservatively at infinite series. What does one mean when one says a series converges? How does one decide whether a series converges or not? What is the relation between a series and an integral?

Finally we will return to divergent series and see in what way they are useful to Physics.

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Addendum

Sam

June 28, 2023

There was a change of plan. We took advantage of the presence of a visitor Kshitij Gajjar to have a session on graph theory after the tea break. The infinite series part was accordingly postponed to the next session.

For the puzzle (Sharmada and Prakash) below, there was an objection from several of the participants. The statement of the problem below should be tempered by the following additional data

1. While Sharmila and Prakash are adept mathematicians, they are only human, like the rest of us. They may not instantly see all the consequences of every piece of data they receive. (Though they will, given enough time.)
2. They don't necessarily reveal everything they know each time they speak.

If you want to use the puzzle in the form below, please add these caveats and also mention that m, n are greater than 1 and that the Sum S is less than 100. This will reduce the amount of labour needed.

If you prefer an alternative formulation, here it is:

https://en.wikipedia.org/wiki/Sum_and_Product_Puzzle

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Tea break: 4:00

This session was conducted by Kshitij Gajjar to make a beginning on graph theory. With five children and four ribbons, they were asked to form connections that included all the five. When they realised it was impossible, they were asked to prove it was impossible.