

ICTS-RRI Math Circle, Saturday 10th June, 2023

Infinite Series

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The Harmonic Series $\{\sum_{n=1}^{\infty} 1/n, n = 1, 2, 3, \dots\}$ is important because it is on the border between convergent series and divergent ones. It only just diverges. In fact, the infinite sum $\{\zeta(s) = \sum_{n=1}^{\infty} 1/n^s, n = 1, 2, 3, \dots\}$, the famous Riemann zeta function has a pole at $s = 1$.

Last time we looked at some physical realizations of the Harmonic series. This time we will probe a little deeper mathematically, going over standard ways to distinguish between convergent and divergent series.

Starting with the Arithmetic and Geometric progressions, we will look at partial sums, the notion of a limit and comparison tests.

Infinite series have caused a lot of headache for mathematicians over the centuries. In ancient Greece, Zeno formulated a series of paradoxes showing that motion was impossible and that the fleet Achilles could not overtake a tortoise! We will discuss these topics and go over the history of infinite series.

Tea break: 11:15

Last time there were several questions raised about the meaning of a divergent sum. We will explore some of these apparent paradoxes in the coming session. How do we give meaning to a divergent sum? Are these definitions useful in any sense? What are the pitfalls? Do they lead to contradictions? With this newfound wisdom, we will return to the Harmonic series to ask if it can be given meaning.